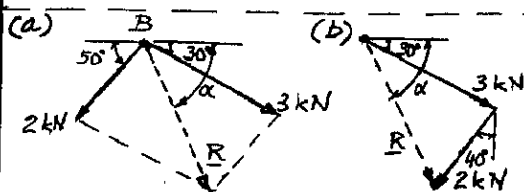


2.1

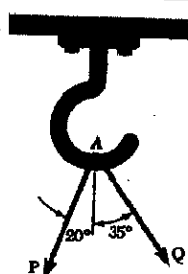


GIVEN: FORCES SHOWN  
DETERMINE GRAPHICALLY  
THEIR RESULTANT,  
USING  
(a) THE PARALLELOGRAM LAW  
(b) THE TRIANGLE RULE

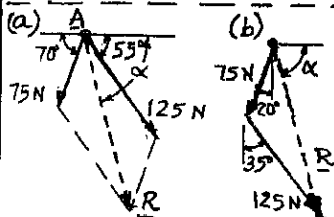


WE MEASURE:  $R = 3.30 \text{ kN}$ ,  $\alpha = 66.6^\circ$   
 $R = 3.30 \text{ kN} \angle 66.6^\circ$

2.2

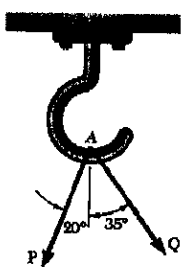


GIVEN:  
 $P = 75 \text{ N}$ ,  $Q = 125 \text{ N}$   
DETERMINE GRAPHICALLY  
THE RESULTANT OF P  
AND Q, USING  
(a) THE PARALLELOGRAM  
LAW  
(b) THE TRIANGLE RULE

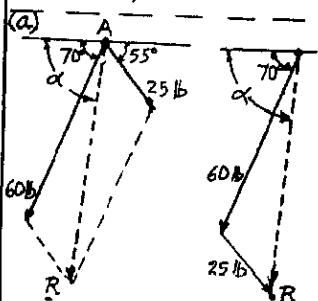


WE MEASURE:  
 $R = 179 \text{ N}$   
 $\alpha = 75.1^\circ$   
 $R = 179 \text{ N} \angle 75.1^\circ$

2.3

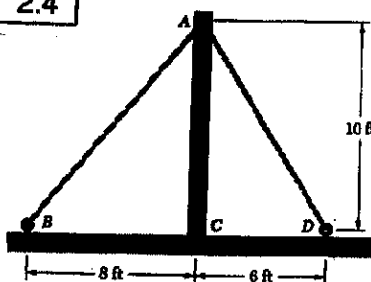


GIVEN:  
 $P = 60 \text{ lb}$ ,  $Q = 25 \text{ lb}$   
DETERMINE GRAPHICALLY  
THE RESULTANT OF P  
AND Q, USING  
(a) THE PARALLELOGRAM  
LAW  
(b) THE TRIANGLE RULE

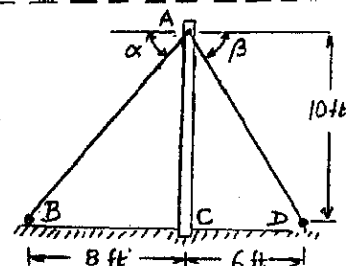


WE MEASURE:  
 $R = 77.1 \text{ lb}$   
 $\alpha = 85.4^\circ$   
 $R = 77.1 \text{ lb} \angle 85.4^\circ$

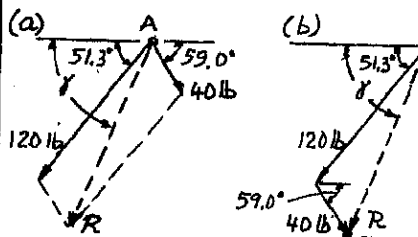
2.4



GIVEN:  
 $T_{AB} = 120 \text{ lb}$   
 $T_{AD} = 40 \text{ lb}$   
DETERMINE  
GRAPHICALLY  
THE RESULTANT  
AT A, USING  
(a) THE PARALLELOGRAM LAW  
(b) THE TRIANGLE RULE

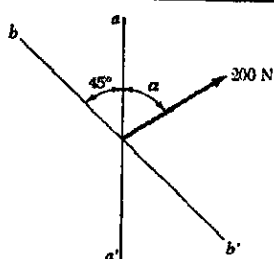


WE MEASURE THE  
ANGLES  $\alpha$  AND  $\beta$ :  
 $\alpha = 51.3^\circ$   
 $\beta = 59.0^\circ$



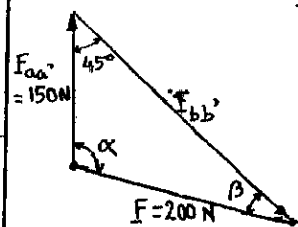
WE MEASURE:  
 $R = 139.1 \text{ lb}$   
 $\gamma = 67.0^\circ$   
 $R = 139.1 \text{ lb} \angle 67.0^\circ$

2.5



GIVEN:  
COMPONENT OF  
200-N FORCE ALONG  
 $a-a'$  MUST BE 150 N.  
DETERMINE BY  
TRIGONOMETRY  
(a) ANGLE  $\alpha$   
(b) COMPONENT  
ALONG  $b-b'$ .

(a) USING TRIANGLE RULE AND LAW OF SINES:



$$\frac{\sin \beta}{150 \text{ N}} = \frac{\sin 45^\circ}{200 \text{ N}}$$

$$\sin \beta = 0.53033$$

$$\beta = 32.03^\circ$$

$$\alpha + \beta + 45^\circ = 180^\circ$$

$$\alpha = 180^\circ - 45^\circ - 32.03^\circ$$

$$\alpha = 102.97^\circ$$

$$\alpha = 103.0^\circ$$

(b) LAW OF SINES:

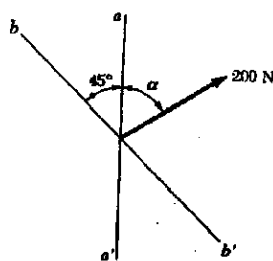
$$\frac{F_{bb'}}{\sin \alpha} = \frac{200 \text{ N}}{\sin 45^\circ}$$

$$F_{bb'} = (200 \text{ N}) \frac{\sin 102.97^\circ}{\sin 45^\circ}$$

$$= 275.63 \text{ N}$$

$$F_{bb'} = 276 \text{ N}$$

2.6



GIVEN:  
COMPONENT OF 200-N  
FORCE ALONG  $b-b'$   
MUST BE 120 N.  
DETERMINE BY  
TRIGONOMETRY  
(a) ANGLE  $\alpha$   
(b) COMPONENT  
ALONG  $a-a'$ .

(a) USING TRIANGLE RULE AND LAW  
OF SINES:

$$\frac{\sin \alpha}{120 \text{ N}} = \frac{\sin 45^\circ}{200 \text{ N}}$$

$$\sin \alpha = 0.42426$$

$$\alpha = 25.1^\circ$$

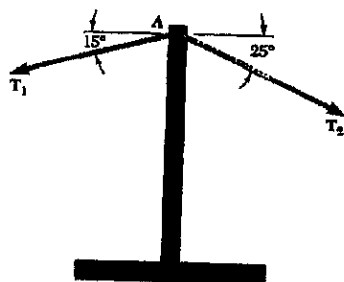
(b)  $\beta = 180^\circ - 45^\circ - 25.1^\circ = 109.9^\circ$

LAW OF SINES:  $\frac{F_{aa'}}{\sin \beta} = \frac{200 \text{ N}}{\sin 45^\circ}$

$$F_{aa'} = (200 \text{ N}) \frac{\sin 109.9^\circ}{\sin 45^\circ}$$

$$F_{aa'} = 266 \text{ N}$$

2.7



GIVEN:  
RESULTANT  $R$  OF  
 $T_1$  AND  $T_2$  MUST  
BE VERTICAL AND  
 $T_1 = 800 \text{ lb}$

FIND:  
(a)  $T_2$   
(b)  $R$

TRIANGLE RULE AND LAW OF SINES:

$$\frac{T_1}{\sin 65^\circ} = \frac{T_2}{\sin 75^\circ} = \frac{R}{\sin 40^\circ}$$

$$\frac{800 \text{ lb}}{\sin 65^\circ} = \frac{T_2}{\sin 75^\circ} = \frac{R}{\sin 40^\circ}$$

(a) SOLVING FOR  $T_2$ :

$$T_2 = (800 \text{ lb}) \frac{\sin 75^\circ}{\sin 65^\circ} = 852.6 \text{ lb}$$

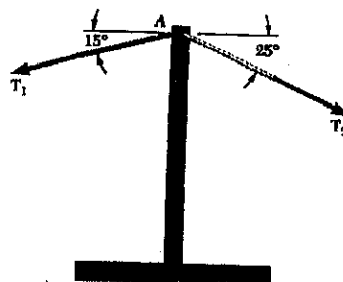
$$T_2 = 853 \text{ lb}$$

(b) SOLVING FOR  $R$ :

$$R = (800 \text{ lb}) \frac{\sin 40^\circ}{\sin 65^\circ} = 567.4 \text{ lb}$$

$$R = 567 \text{ lb}$$

2.8

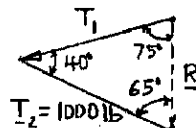


GIVEN:  
RESULTANT  $R$   
OF  $T_1$  AND  $T_2$  MUST  
BE VERTICAL AND  
 $T_2 = 1000 \text{ lb}$

FIND:

(a)  $T_1$   
(b)  $R$

TRIANGLE RULE AND LAW  
OF SINES:



$$\frac{T_1}{\sin 65^\circ} = \frac{1000 \text{ lb}}{\sin 75^\circ} = \frac{R}{\sin 40^\circ}$$

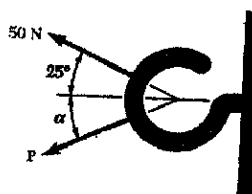
(a) SOLVING FOR  $T_1$ :

$$T_1 = (1000 \text{ lb}) \frac{\sin 65^\circ}{\sin 75^\circ} = 938.28 \text{ lb}, T_1 = 938 \text{ lb}$$

(b) SOLVING FOR  $R$ :

$$R = (1000 \text{ lb}) \frac{\sin 40^\circ}{\sin 75^\circ} = 665.46 \text{ lb}, R = 665 \text{ lb}$$

2.9

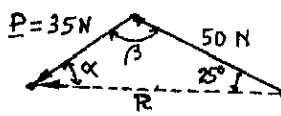


GIVEN:  
RESULTANT  $R$  OF THE  
TWO FORCES MUST BE  
HORIZONTAL AND  
 $P = 35 \text{ N}$

FIND:

(a) ANGLE  $\alpha$   
(b)  $R$

TRIANGLE RULE:



(a) LAW OF SINES:

$$\frac{\sin \alpha}{50 \text{ N}} = \frac{\sin 25^\circ}{35 \text{ N}}$$

$$\sin \alpha = \frac{50 \text{ N} \sin 25^\circ}{35 \text{ N}}$$

$$\sin \alpha = 0.60374, \alpha = 37.14^\circ$$

$$\alpha \approx 37.1^\circ$$

$$(b) \beta = 180^\circ - 25^\circ - 37.14^\circ = 117.86^\circ$$

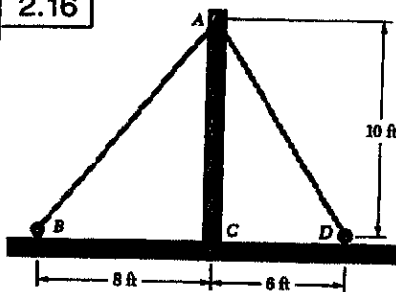
LAW OF SINES:

$$\frac{R}{\sin \beta} = \frac{35 \text{ N}}{\sin 25^\circ}$$

$$R = (35 \text{ N}) \frac{\sin 117.86^\circ}{\sin 25^\circ} = 73.218 \text{ N}$$

$$R = 73.2 \text{ N}$$

2.16



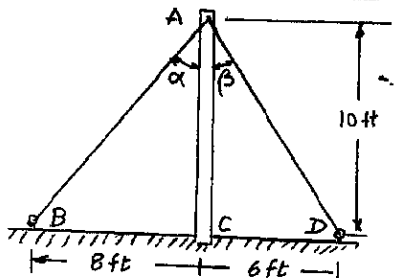
GIVEN:

$$T_{AB} = 120 \text{ lb}$$

$$T_{AD} = 40 \text{ lb}$$

FIND:

RESULTANT  $R$   
OF THE FORCES  
EXERTED AT A  
BY AB AND AD



$$\tan \alpha = \frac{10}{8}$$

$$\alpha = 38.66^\circ$$

$$\tan \beta = \frac{6}{10}$$

$$\beta = 30.96^\circ$$

FROM FORCE TRIANGLE:

LAW OF COSINES:

$$R^2 = (120)^2 + (40)^2 - 2(120)(40) \cos 110.38^\circ$$

$$= 14,400 + 1600 - 9600(-0.3420)$$

$$R^2 = 19,343 \quad R = 139.08 \text{ lb}$$

LAW OF SINES:

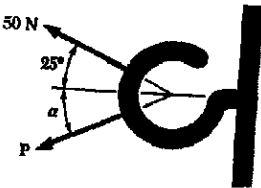
$$\frac{\sin \gamma}{40 \text{ lb}} = \frac{\sin 110.38^\circ}{139.08 \text{ lb}}$$

$$\sin \gamma = 0.26960 \quad \gamma = 15.64^\circ$$

$$\phi = (90^\circ - \alpha) + \gamma = 51.34^\circ + 15.64^\circ = 66.98^\circ$$

$$R = 139.11 \text{ lb} \angle 67.0^\circ$$

2.17

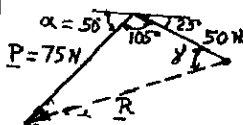


GIVEN:

$$P = 75 \text{ N}, \alpha = 50^\circ$$

FIND:

RESULTANT  $R$   
OF THE TWO FORCES  
SHOWN.



FROM FORCE TRIANGLE:

LAW OF COSINES:

$$R^2 = (75)^2 + (50)^2 - 2(75)(50) \cos 105^\circ$$

$$= 5625 + 2500 - 7500(-0.2598)$$

$$R^2 = 10,066 \quad R = 100.33 \text{ N}$$

LAW OF SINES:

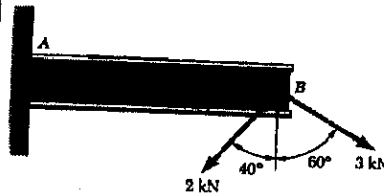
$$\frac{\sin \gamma}{75 \text{ N}} = \frac{\sin 105^\circ}{100.33 \text{ N}}$$

$$\sin \gamma = 0.72206 \quad \gamma = 46.22^\circ$$

$$R \angle = \gamma - 25^\circ = 46.22^\circ - 25^\circ = 21.22^\circ$$

$$R = 100.3 \text{ N} \angle 21.2^\circ$$

2.18

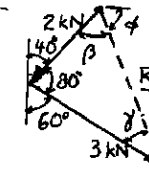


GIVEN:

FORCES SHOWN

FIND:

THEIR RESULTANT



FROM FORCE TRIANGLE:

LAW OF COSINES:

$$R^2 = (2)^2 + (3)^2 - 2(2)(3) \cos 80^\circ$$

$$R^2 = 10.916 \quad R = 3.304 \text{ kN}$$

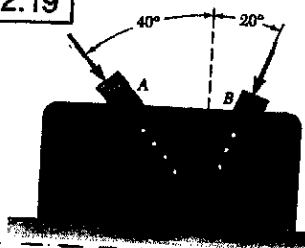
LAW OF SINES:

$$\frac{\sin \gamma}{2 \text{ kN}} = \frac{\sin 80^\circ}{3.304 \text{ kN}} \quad \gamma = 36.59^\circ$$

$$\beta = 180^\circ - (80^\circ + 36.59^\circ) = 63.41^\circ \quad \phi = 180^\circ - (\beta + 50^\circ) = 66.59^\circ$$

$$R = 3.30 \text{ kN} \angle 66.6^\circ$$

2.19



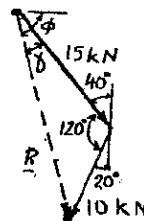
GIVEN:

$$F_A = 15 \text{ kN}$$

$$F_B = 10 \text{ kN}$$

FIND:

RESULTANT OF FORCES  
EXERTED ON BRACKET  
BY MEMBERS A AND B.



FROM FORCE TRIANGLE:

LAW OF COSINES:

$$R^2 = (15)^2 + (10)^2 - 2(15)(10) \cos 120^\circ$$

$$R^2 = 475 \quad R = 21.794 \text{ kN}$$

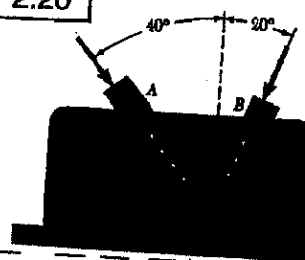
LAW OF SINES:

$$\frac{\sin \gamma}{10 \text{ kN}} = \frac{\sin 120^\circ}{21.794 \text{ kN}} \quad \gamma = 23.41^\circ$$

$$\phi = 50^\circ + \gamma = 50^\circ + 23.41^\circ = 73.41^\circ$$

$$R = 21.8 \text{ kN} \angle 73.4^\circ$$

2.20



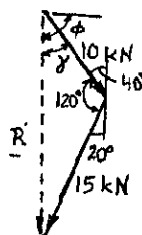
GIVEN:

$$F_A = 10 \text{ kN}$$

$$F_B = 15 \text{ kN}$$

FIND:

RESULTANT OF FORCES  
EXERTED ON BRACKET  
BY MEMBERS A AND B.



FROM FORCE TRIANGLE:

LAW OF COSINES:

$$R^2 = (10)^2 + (15)^2 - 2(10)(15) \cos 120^\circ$$

$$R^2 = 475 \quad R = 21.794 \text{ kN}$$

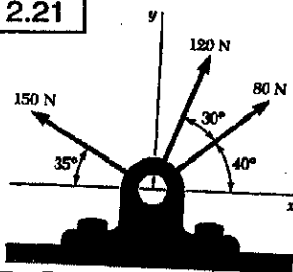
LAW OF SINES:

$$\frac{\sin \gamma}{15 \text{ kN}} = \frac{\sin 120^\circ}{21.794 \text{ kN}} \quad \gamma = 36.59^\circ$$

$$\phi = 50^\circ + \gamma = 50^\circ + 36.59^\circ = 86.59^\circ$$

$$R = 21.8 \text{ kN} \angle 86.6^\circ$$

2.21



**GIVEN:**  
MAGNITUDES AND DIRECTIONS OF FORCES

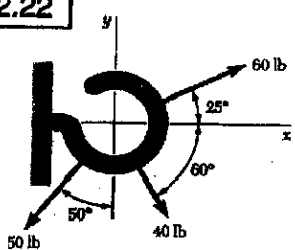
**FIND:**  
X AND Y COMPONENTS OF THE FORCES.

**80-N FORCE:**  $F_x = +(80\text{ N}) \cos 40^\circ$ ,  $F_x = +61.3\text{ N}$   
 $F_y = +(80\text{ N}) \sin 40^\circ$ ,  $F_y = +51.4\text{ N}$

**120-N FORCE:**  $F_x = +(120\text{ N}) \cos 70^\circ$ ,  $F_x = +41.0\text{ N}$   
 $F_y = +(120\text{ N}) \sin 70^\circ$ ,  $F_y = +112.8\text{ N}$

**150-N FORCE:**  $F_x = -(150\text{ N}) \cos 35^\circ$ ,  $F_x = -122.9\text{ N}$   
 $F_y = +(150\text{ N}) \sin 35^\circ$ ,  $F_y = +86.0\text{ N}$

2.22



**GIVEN:**  
MAGNITUDES AND DIRECTIONS OF FORCES

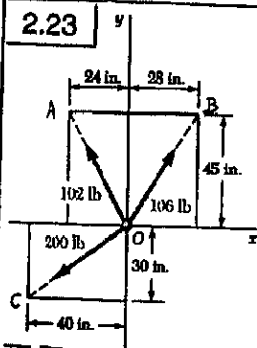
**FIND:**  
X AND Y COMPONENTS OF THE FORCES.

**40-lb FORCE:**  $F_x = +(40\text{ lb}) \cos 60^\circ = +20.0\text{ lb}$ ,  $F_x = +20.0\text{ lb}$   
 $F_y = -(40\text{ lb}) \sin 60^\circ = -34.64\text{ lb}$ ,  $F_y = -34.6\text{ lb}$

**50-lb FORCE:**  $F_x = -(50\text{ lb}) \sin 50^\circ = -38.30\text{ lb}$ ,  $F_x = -38.3\text{ lb}$   
 $F_y = -(50\text{ lb}) \cos 50^\circ = -32.14\text{ lb}$ ,  $F_y = -32.1\text{ lb}$

**60-lb FORCE:**  $F_x = +(60\text{ lb}) \cos 25^\circ = +54.38\text{ lb}$ ,  $F_x = +54.4\text{ lb}$   
 $F_y = +(60\text{ lb}) \sin 25^\circ = +25.36\text{ lb}$ ,  $F_y = +25.4\text{ lb}$

2.23



**GIVEN:**  
FORCES AND DIMENSIONS SHOWN.  
**FIND:**  
X AND Y COMPONENTS OF FORCES.

WE COMPUTE THE FOLLOWING DISTANCES:

$$OA = \sqrt{(24)^2 + (45)^2} = 51\text{ in.}$$

$$OB = \sqrt{(28)^2 + (45)^2} = 53\text{ in.}$$

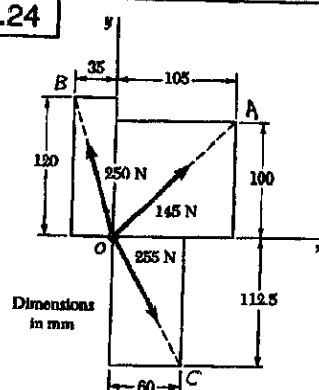
$$OC = \sqrt{(40)^2 + (30)^2} = 50\text{ in.}$$

**102-lb FORCE:**  $F_x = -(102\text{ lb}) \frac{24}{51}$ ,  $F_x = -48.0\text{ lb}$   
 $F_y = +(102\text{ lb}) \frac{45}{51}$ ,  $F_y = +90.0\text{ lb}$

**106-lb FORCE:**  $F_x = +(106\text{ lb}) \frac{28}{53}$ ,  $F_x = +56.0\text{ lb}$   
 $F_y = +(106\text{ lb}) \frac{45}{53}$ ,  $F_y = +90.0\text{ lb}$

**200-lb FORCE:**  $F_x = -(200\text{ lb}) \frac{40}{50}$ ,  $F_x = -160.0\text{ lb}$   
 $F_y = -(200\text{ lb}) \frac{30}{50}$ ,  $F_y = -120.0\text{ lb}$

2.24



**GIVEN:**  
FORCES AND DIMENSIONS SHOWN.

**FIND:**  
X AND Y COMPONENTS OF FORCES

**145-N FORCE:**  $OA = \sqrt{(105)^2 + (100)^2} = 145\text{ mm}$

$$F_x = +(145\text{ N}) \frac{105\text{ mm}}{145\text{ mm}}, \quad F_x = +105.0\text{ N}$$

$$F_y = +(145\text{ N}) \frac{100\text{ mm}}{145\text{ mm}}, \quad F_y = +100.0\text{ N}$$

**250-N FORCE:**  $OB = \sqrt{(35)^2 + (120)^2} = 125\text{ mm}$

$$F_x = -(250\text{ N}) \frac{35\text{ mm}}{125\text{ mm}}, \quad F_x = -70.0\text{ N}$$

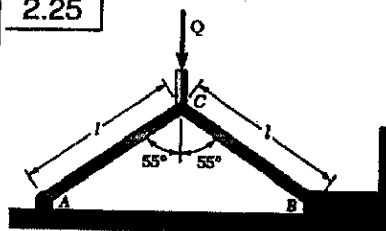
$$F_y = +(250\text{ N}) \frac{120\text{ mm}}{125\text{ mm}}, \quad F_y = +240\text{ N}$$

**255-N FORCE:**  $OC = \sqrt{(60)^2 + (112.5)^2} = 127.5\text{ mm}$

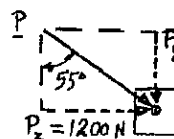
$$F_x = +(255\text{ N}) \frac{60\text{ mm}}{127.5\text{ mm}}, \quad F_x = +120.0\text{ N}$$

$$F_y = -(255\text{ N}) \frac{112.5\text{ mm}}{127.5\text{ mm}}, \quad F_y = -225\text{ N}$$

2.25



**GIVEN:**  
(1) CB EXERTS FORCE P ON B ALONG CB.  
(2) HORIZONTAL COMPONENT OF P IS  $P_x = 1200\text{ N}$ .  
**FIND:**  
(a) MAGNITUDE P  
(b) VERT. COMP.  $P_y$

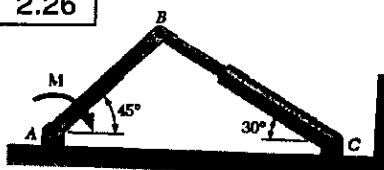


(a)  $P_x = P \sin 55^\circ$ ,  $P = \frac{P_x}{\sin 55^\circ} = \frac{1200\text{ N}}{\sin 55^\circ} = 1464.9\text{ N}$   
 $P = 1465\text{ N}$

(b)  $P_x = P_y \tan 55^\circ$ ,  $P_y = \frac{P_x}{\tan 55^\circ} = \frac{1200\text{ N}}{\tan 55^\circ} = 840.2\text{ N}$   
 $P_y = 840\text{ N} \downarrow$



2.26



GIVEN:

- (1) FORCE  $P$  EXERTED BY BC ON AB IS DIRECTED ALONG BC.  
 (2) COMPONENT OF  $P \perp AB$  IS 600 N

FIND: (a)  $P$ (b) COMP. OF  $P$  ALONG AB

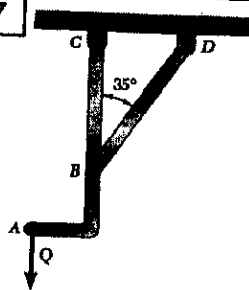
$$(a) P = \frac{P_x}{\cos 15^\circ} = \frac{600 \text{ N}}{\cos 15^\circ}$$

$$P = 621 \text{ N}$$

$$(b) P_y = P_x \tan 15^\circ = (600 \text{ N}) \tan 15^\circ$$

$$P_y = 160.8 \text{ N}$$

2.27



GIVEN:

- (1) FORCE  $P$  EXERTED BY BD ON ABC IS DIRECTED ALONG BD.  
 (2) HORIZ. COMPONENT OF  $P$  IS  $P_x = 300 \text{ lb}$ .

FIND:

(a) MAGNITUDE  $P$ (b) VERT. COMP.  $P_y$ 

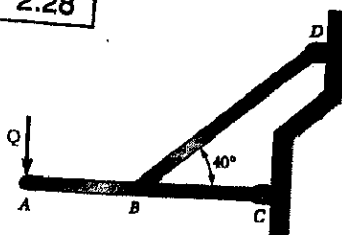
$$(a) P = \frac{P_x}{\sin 35^\circ} = \frac{300 \text{ lb}}{\sin 35^\circ}$$

$$P = 523 \text{ lb}$$

$$(b) P_y = \frac{P_x}{\tan 35^\circ} = \frac{300 \text{ lb}}{\tan 35^\circ}$$

$$P_y = 428 \text{ lb}$$

2.28



GIVEN:

- (1) FORCE  $P$  EXERTED BY BD ON ABC IS DIRECTED ALONG BD.  
 (2) VERT. COMPONENT OF  $P$  IS  $P_y = 240 \text{ lb}$ .

FIND:

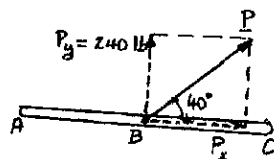
(a) MAGNITUDE  $P$ (b) HORIZ. COMP.  $P_x$ 

$$(a) P = \frac{P_y}{\sin 40^\circ} = \frac{240 \text{ lb}}{\sin 40^\circ}$$

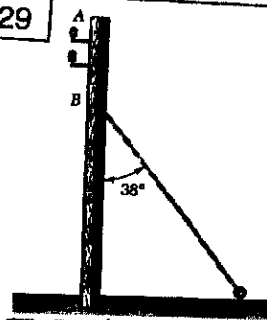
$$P = 373 \text{ lb}$$

$$(b) P_x = \frac{P_y}{\tan 40^\circ} = \frac{240 \text{ lb}}{\tan 40^\circ}$$

$$P_x = 286 \text{ lb}$$



2.29



GIVEN:

- (1) FORCE  $P$  EXERTED BY BD ON POLE IS DIRECTED ALONG BD.  
 (2) COMPONENT OF  $P \perp$  TO AC IS 120 N.

FIND:

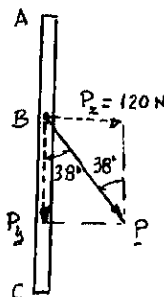
(a) MAGNITUDE  $P$ (b) COMPONENT OF  $P$  ALONG AC.

$$(a) P = \frac{P_x}{\sin 38^\circ} = \frac{120 \text{ N}}{\sin 38^\circ} = 194.91 \text{ N}$$

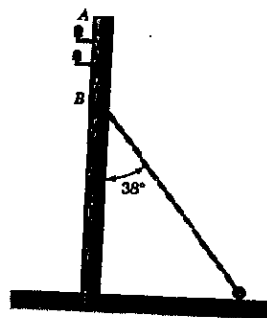
$$P = 194.9 \text{ N}$$

$$(b) P_y = \frac{P_x}{\tan 38^\circ} = \frac{120 \text{ N}}{\tan 38^\circ} = 153.59 \text{ N}$$

$$P_y = 153.6 \text{ N}$$



2.30



GIVEN:

- (1) FORCE  $P$  EXERTED BY BD ON POLE IS DIRECTED ALONG BD.  
 (2) COMPONENT OF  $P$  ALONG AC IS 180 N.

FIND:

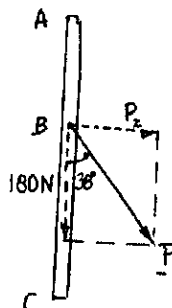
(a) MAGNITUDE  $P$ (b) COMPONENT OF  $P \perp$  TO AC.

$$(a) P = \frac{180 \text{ N}}{\cos 38^\circ} = 228.4 \text{ N}$$

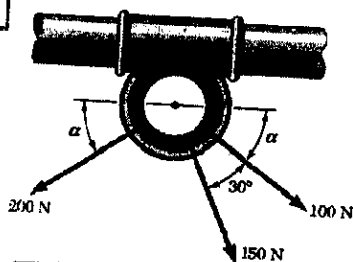
$$P = 228 \text{ N}$$

$$(b) P_x = (180 \text{ N}) \tan 30^\circ = 140.63 \text{ N}$$

$$P_x = 140.6 \text{ N}$$



2.35



GIVEN:

$$\alpha = 35^\circ$$

FIND:

RESULTANT  
OF THE THREE  
FORCES SHOWN.

100-N FORCE:

$$F_x = +(100 \text{ N}) \cos 35^\circ = +81.92 \text{ N}, \quad F_y = -(100 \text{ N}) \sin 35^\circ = -57.36 \text{ N}$$

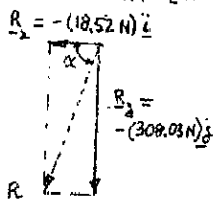
150-N FORCE:

$$F_x = +(150 \text{ N}) \cos 65^\circ = +63.39 \text{ N}, \quad F_y = -(150 \text{ N}) \sin 65^\circ = -135.45 \text{ N}$$

200-N FORCE:

$$F_x = -(200 \text{ N}) \cos 35^\circ = -163.03 \text{ N}, \quad F_y = -(200 \text{ N}) \sin 35^\circ = -114.72 \text{ N}$$

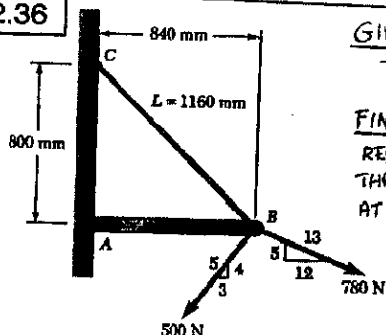
FORCE	x COMP. (N)	y COMP. (N)
100 N	+81.92	-57.36
150 N	+63.39	-135.45
200 N	-163.03	-114.72
	$R_x = -18.52$	$R_y = -308.03$



$$\tan \alpha = \frac{308.03 \text{ N}}{18.52 \text{ N}} \quad \alpha = 86.56^\circ$$

$$R = \frac{308.03 \text{ N}}{\sin 86.56^\circ} = 308.6 \text{ N} \quad R = 309 \text{ N} \angle 86.6^\circ$$

2.36



GIVEN:

$$T_{BC} = 725 \text{ N}$$

FIND:

RESULTANT OF THE  
THREE FORCES EXERTED  
AT POINT B OF BEAM AB.

FORCE EXERTED BY CABLE BC:

$$F_x = -(725 \text{ N}) \frac{840 \text{ mm}}{1160 \text{ mm}} = -525 \text{ N}, \quad F_y = +(725 \text{ N}) \frac{800 \text{ mm}}{1160 \text{ mm}} = +500 \text{ N}$$

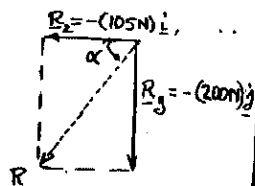
500-N FORCE:

$$F_x = -(500 \text{ N}) \frac{5}{13} = -300 \text{ N}, \quad F_y = -(500 \text{ N}) \frac{12}{13} = -400 \text{ N}$$

780-N FORCE:

$$F_x = +(780 \text{ N}) \frac{12}{13} = +720 \text{ N}, \quad F_y = -(780 \text{ N}) \frac{5}{13} = -300 \text{ N}$$

FORCE	x COMP. (N)	y COMP. (N)
$T_{BC} = 725 \text{ N}$	-525	+500
500 N	-300	-400
780 N	+720	-300
	$R_x = -105$	$R_y = -200$

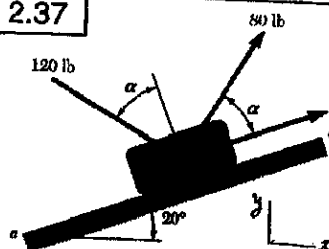


$$\tan \alpha = \frac{200 \text{ N}}{105 \text{ N}} \quad \alpha = 62.30^\circ$$

$$R = \frac{200 \text{ N}}{\sin 62.30^\circ} = 225.9 \text{ N}$$

$$R = 226 \text{ N} \angle 62.3^\circ$$

2.37



GIVEN:

$$\alpha = 40^\circ$$

FIND:

RESULTANT OF THE  
THREE FORCES SHOWN

60-lb FORCE:

$$F_x = +(60 \text{ lb}) \cos 20^\circ = +56.38 \text{ lb}, \quad F_y = +(60 \text{ lb}) \sin 20^\circ = +20.52 \text{ lb}$$

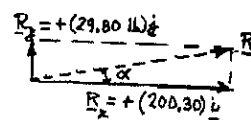
80-lb FORCE:

$$F_x = +(80 \text{ lb}) \cos 60^\circ = +40.00 \text{ lb}, \quad F_y = +(80 \text{ lb}) \sin 60^\circ = +69.28 \text{ lb}$$

120-lb FORCE:

$$F_x = +(120 \text{ lb}) \cos 30^\circ = +103.92 \text{ lb}, \quad F_y = -(120 \text{ lb}) \sin 30^\circ = -60.00 \text{ lb}$$

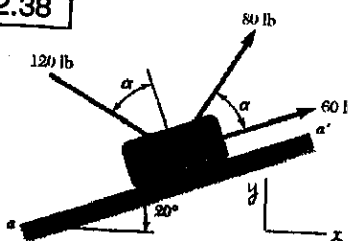
FORCE	x COMP. (lb)	y COMP. (lb)
60 lb	+56.38	+20.52
80 lb	+40.00	+69.28
120 lb	+103.92	-60.00
	$R_x = +200.30$	$R_y = +29.80$



$$\tan \alpha = \frac{29.80 \text{ lb}}{200.30 \text{ lb}} \quad \alpha = 8.462^\circ$$

$$R = \frac{29.80 \text{ lb}}{\sin 8.462^\circ} = 202.51 \text{ lb} \quad R = 203 \text{ lb} \angle 8.46^\circ$$

2.38



GIVEN:

$$\alpha = 75^\circ$$

FIND:

RESULTANT OF THE  
THREE FORCES SHOWN.

60-lb FORCE:

$$F_x = +(60 \text{ lb}) \cos 20^\circ = +56.38 \text{ lb}, \quad F_y = +(60 \text{ lb}) \sin 20^\circ = +20.52 \text{ lb}$$

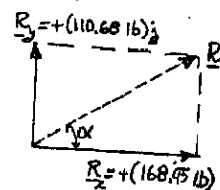
80-lb FORCE:

$$F_x = +(80 \text{ lb}) \cos 95^\circ = -6.97 \text{ lb}, \quad F_y = +(80 \text{ lb}) \sin 95^\circ = +79.70 \text{ lb}$$

120-lb FORCE:

$$F_x = +(120 \text{ lb}) \cos 5^\circ = +119.54 \text{ lb}, \quad F_y = +(120 \text{ lb}) \sin 5^\circ = +10.46 \text{ lb}$$

FORCE	x COMP. (lb)	y COMP. (lb)
60 lb	+56.38	+20.52
80 lb	-6.97	+79.70
120 lb	+119.54	+10.46
	$R_x = +168.95$	$R_y = +110.68$

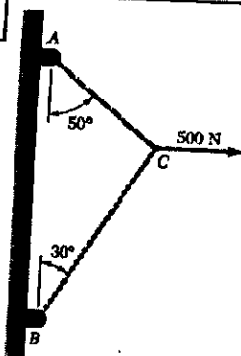


$$\tan \alpha = \frac{110.68 \text{ lb}}{168.95 \text{ lb}} \quad \alpha = 33.23^\circ$$

$$R = \frac{110.68 \text{ lb}}{\sin 33.23^\circ} = 201.98 \text{ lb}$$

$$R = 202 \text{ lb} \angle 33.2^\circ$$

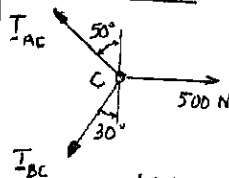
2.43



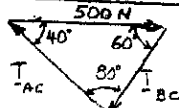
GIVEN:  
CABLES AC AND BC  
ARE LOADED AS SHOWN

FIND:  
(a) TENSION IN AC.  
(b) TENSION IN BC.

F.B. DIAGRAM



FORCE TRIANGLE



LAW OF SINES:  $\frac{T_{AC}}{\sin 60^\circ} = \frac{T_{BC}}{\sin 40^\circ} = \frac{500 \text{ N}}{\sin 80^\circ}$

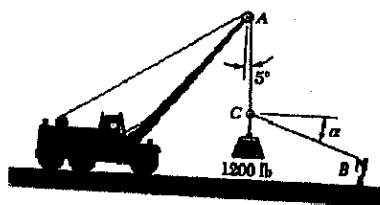
(a)  $T_{AC} = \frac{500 \text{ N}}{\sin 80^\circ} \sin 60^\circ = 439.7 \text{ N}$

$T_{AC} = 440 \text{ N}$

(b)  $T_{BC} = \frac{500 \text{ N}}{\sin 80^\circ} \sin 40^\circ = 326.4 \text{ N}$

$T_{BC} = 326 \text{ N}$

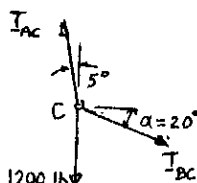
2.45



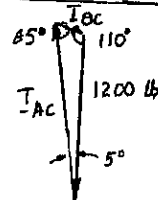
GIVEN:  
 $\alpha = 20^\circ$

FIND:  
TENSION IN  
(a) AC  
(b) BC

F.B. DIAGRAM



FORCE TRIANGLE



LAW OF SINES:  $\frac{T_{AC}}{\sin 110^\circ} = \frac{T_{BC}}{\sin 5^\circ} = \frac{1200 \text{ lb}}{\sin 65^\circ}$

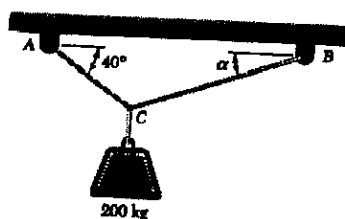
(a)  $T_{AC} = \frac{1200 \text{ lb}}{\sin 65^\circ} \sin 110^\circ = 1244.2 \text{ lb}$

$T_{AC} = 1244 \text{ lb}$

(b)  $T_{BC} = \frac{1200 \text{ lb}}{\sin 65^\circ} \sin 5^\circ = 115.40 \text{ lb}$

$T_{BC} = 115.4 \text{ lb}$

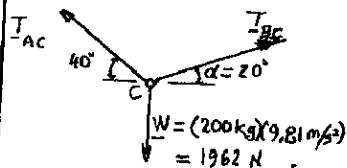
2.44



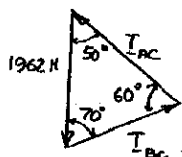
GIVEN  
(1) CABLES AC  
AND BC ARE  
LOADED AS SHOWN  
(2)  $\alpha = 20^\circ$

FIND:  
TENSION IN  
(a) AC  
(b) BC

F.B. DIAGRAM



FORCE TRIANGLE



LAW OF SINES:  $\frac{T_{AC}}{\sin 70^\circ} = \frac{T_{BC}}{\sin 50^\circ} = \frac{1962 \text{ N}}{\sin 60^\circ}$

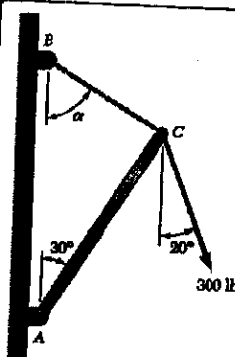
(a)  $T_{AC} = \frac{1962 \text{ N}}{\sin 60^\circ} \sin 70^\circ = 2128.9 \text{ N}$

$T_{AC} = 2,13 \text{ kN}$

(b)  $T_{BC} = \frac{1962 \text{ N}}{\sin 60^\circ} \sin 50^\circ = 1735.49 \text{ N}$

$T_{BC} = 1,735 \text{ kN}$

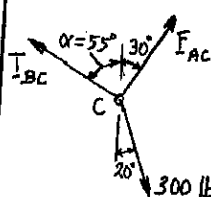
2.46



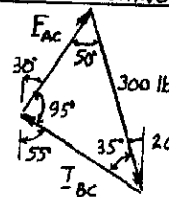
GIVEN:  
(1)  $\alpha = 55^\circ$   
(2) BOOM AC EXERTS  
ON PIN C A FORCE  
ALONG AC.

FIND:  
(a)  $F_{AC}$   
(b)  $T_{BC}$

F.B. DIAGRAM



FORCE TRIANGLE



LAW OF SINES:  $\frac{F_{AC}}{\sin 35^\circ} = \frac{T_{BC}}{\sin 50^\circ} = \frac{300 \text{ lb}}{\sin 95^\circ}$

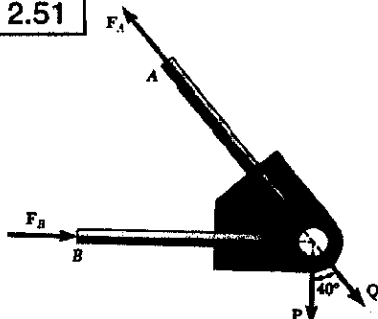
(a)  $F_{AC} = \frac{300 \text{ lb}}{\sin 95^\circ} \sin 35^\circ = 172.73 \text{ lb}$

$F_{AC} = 172.7 \text{ lb}$

(b)  $T_{BC} = \frac{300 \text{ lb}}{\sin 95^\circ} \sin 50^\circ = 230.7 \text{ lb}$

$T_{BC} = 231 \text{ lb}$

2.51

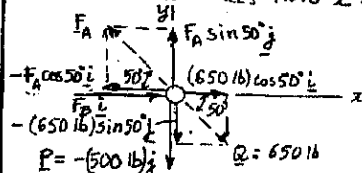


**GIVEN:**  
 (1) CONNECTION IN EQUILIBRIUM UNDER FOUR FORCES  
 (2)  $P = 500 \text{ lb}$   
 $Q = 650 \text{ lb}$

**FIND:**  
 $F_A$  AND  $F_B$

**FREE-BODY DIAGRAM**

RESOLVING THE FORCES INTO  $x$  AND  $y$  COMPONENTS:



$$R = F_A + F_B + P + Q = 0$$

$$-F_A \cos 50^\circ \hat{i} + F_A \sin 50^\circ \hat{j} + F_B \cos 50^\circ \hat{i} - 500 \hat{j} + 650 \cos 50^\circ \hat{i} - 650 \sin 50^\circ \hat{j} = 0$$

$$\text{EQUATING TO ZERO THE COEFF. OF } \hat{i} \text{ AND } \hat{j}:$$

$$(1) F_A \sin 50^\circ - 500 - 650 \sin 50^\circ = 0$$

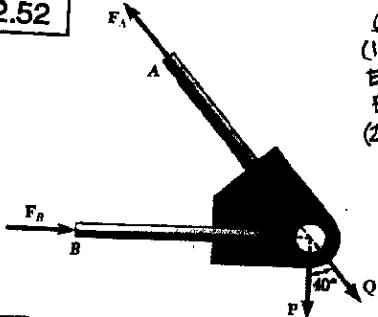
$$F_A = 1303 \text{ lb}$$

$$(2) -F_A \cos 50^\circ + F_B + 650 \cos 50^\circ = 0$$

$$F_B = (1303 \text{ lb}) \cos 50^\circ - (650 \text{ lb}) \cos 50^\circ$$

$$F_B = 420 \text{ lb}$$

2.52

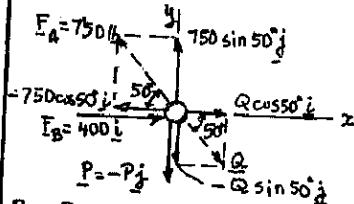


**GIVEN:**  
 (1) CONNECTION IN EQUILIBRIUM UNDER FOUR FORCES  
 (2)  $F_A = 750 \text{ lb}$   
 $F_B = 400 \text{ lb}$

**FIND:**  
 $P$  AND  $Q$

**FREE-BODY DIAGRAM:**

RESOLVING THE FORCES INTO  $x$  AND  $y$  COMPONENTS



$$R = P + Q + F_A + F_B = 0$$

$$-P \hat{j} + Q \cos 50^\circ \hat{i} - Q \sin 50^\circ \hat{j} - 750 \cos 50^\circ \hat{i} + 750 \sin 50^\circ \hat{j} + 400 \hat{i} = 0$$

$$\text{EQUATING TO ZERO THE COEFF. OF } \hat{i} \text{ AND } \hat{j}:$$

$$(1) Q \cos 50^\circ - 750 \cos 50^\circ + 400 = 0$$

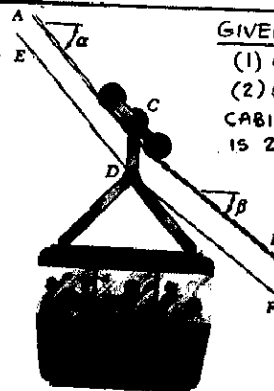
$$Q = 127.7 \text{ lb}$$

$$(2) -P - Q \sin 50^\circ + 750 \sin 50^\circ = 0$$

$$P = -(127.7 \text{ lb}) \sin 50^\circ + (750 \text{ lb}) \sin 50^\circ$$

$$P = 477 \text{ lb}$$

2.53



**GIVEN:**

(1)  $\alpha = 45^\circ$ ,  $\beta = 40^\circ$   
 (2) COMBINED WEIGHT OF CABIN AND PASSENGERS IS  $22.5 \text{ kN}$ .

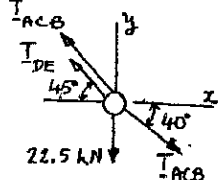
(3)  $T_{DF} \approx 0$

**FIND:**

(a)  $T_{ACB}$

(b)  $T_{DE}$

**FREE-BODY DIAGRAM (CABIN CONSIDERED AS PARTICLE)**



$$\Sigma F_x = 0:$$

$$T_{ACB} \cos 45^\circ - T_{DE} \cos 45^\circ - T_{DE} \cos 45^\circ = 0$$

$$0.05894 T_{ACB} - 0.7071 T_{DE} = 0 \quad (1)$$

$$\Sigma F_y = 0:$$

$$-T_{ACB} \sin 40^\circ + T_{DE} \sin 45^\circ + T_{DE} \sin 45^\circ - 22.5 = 0$$

$$0.06432 T_{ACB} + 0.7071 T_{DE} = 22.5 \quad (2)$$

$$(a) \text{ ADD (1) AND (2): } 0.12326 T_{ACB} = 22.5 \quad T_{ACB} = 182.5 \text{ kN}$$

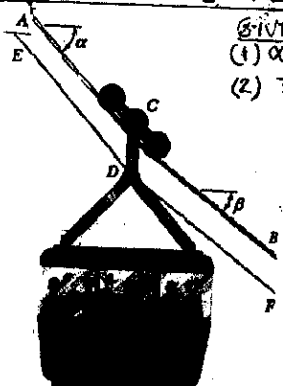
$$T_{ACB} = 182.5 \text{ kN}$$

$$(b) \text{ FROM (1): } T_{DE} = \frac{0.05894}{0.7071} (182.5)$$

$$T_{DE} = 15.22 \text{ kN}$$

**NOTE:** IN PROBS. 2.53 AND 2.54 THE CABIN IS CONSIDERED AS A PARTICLE. IF CONSIDERED AS A RIGID BODY (CHAP. 4) IT WOULD BE FOUND THAT ITS CENTER OF GRAVITY SHOULD BE LOCATED TO THE LEFT OF  $Q$  FOR  $QD$  TO BE VERTICAL.

2.54



**GIVEN:**

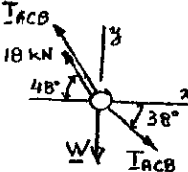
(1)  $\alpha = 48^\circ$ ,  $\beta = 38^\circ$

(2)  $T_{BE} = 18 \text{ kN}$ ,  $T_{DF} \approx 0$

**FIND:**

(a) COMBINED WEIGHT OF CABIN, PASSENGERS, AND SUPPORT SYSTEM  
 (b)  $T_{ACB}$

**FREE-BODY DIAGRAM (CABIN CONSIDERED AS PARTICLE)**



$$(b) \Sigma F_x = 0:$$

$$T_{ACB} \cos 38^\circ - T_{BE} \cos 48^\circ - (18 \text{ kN}) \cos 48^\circ = 0$$

$$0.1189 T_{ACB} - 12.044 \text{ kN} = 0$$

$$(b) T_{ACB} = 101.3 \text{ kN}$$

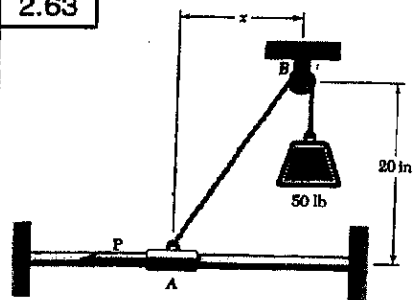
$$(a) \Sigma F_y = 0: T_{ACB} \sin 48^\circ - T_{BE} \sin 38^\circ + (18 \text{ kN}) \sin 48^\circ - W = 0$$

$$W = (101.3 \text{ kN})(\sin 48^\circ - \sin 38^\circ) + (18 \text{ kN}) \sin 48^\circ$$

$$= 26.24 \text{ kN}$$

$$(a) W = 26.3 \text{ kN}$$

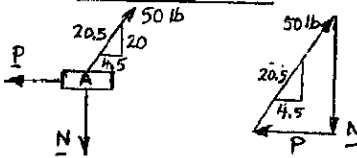
2.63



GIVEN:  
SYSTEM SHOWN  
IS IN EQUILIBRIUM

FIND  
P WHEN  
(a)  $x = 4.5$  in.  
(b)  $x = 15$  in.

(a) FREE BODY: COLLAR A

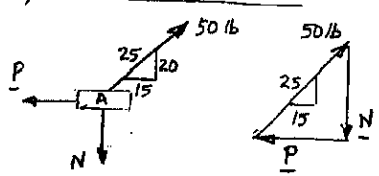


FORCE TRIANGLE

$$\frac{P}{4.5} = \frac{50}{20.5}$$

$$P = 10.98 \text{ lb}$$

(b) FREE BODY: COLLAR A

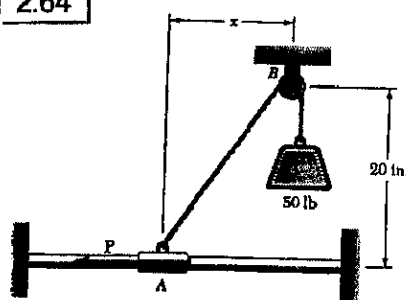


FORCE TRIANGLE

$$\frac{P}{15} = \frac{50}{25}$$

$$P = 30.0 \text{ lb}$$

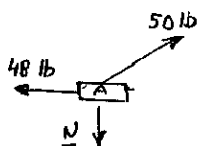
2.64



GIVEN:  
SYSTEM SHOWN  
IS IN EQUILIBRIUM  
WITH  $P = 48$  lb.

FIND:  $x$ 

FREE BODY: COLLAR A



FORCE TRIANGLE



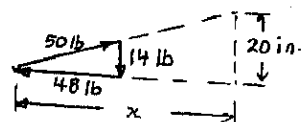
$$N^2 = (50)^2 - (48)^2 = 196$$

$$N = 14.00 \text{ lb}$$

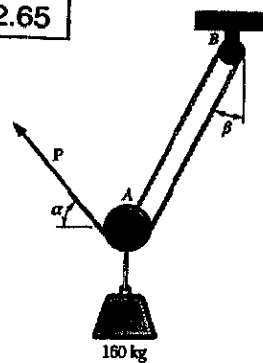
SIMILAR TRIANGLES:

$$\frac{x}{20 \text{ in.}} = \frac{48 \text{ lb}}{14 \text{ lb}}$$

$$x = 68.6 \text{ in.}$$



2.65



GIVEN:

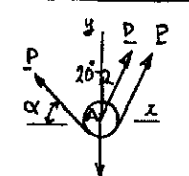
$$\beta = 20^\circ$$

ALSO: T IS THE SAME IN  
ALL PORTIONS OF THE ROPE

FIND:

MAGNITUDE AND  
DIRECTION OF  $\underline{P}$

FREE BODY: PULLEY A



$$\Sigma F_x = 0: 2P \sin 20^\circ - P \cos \alpha = 0$$

$$\cos \alpha = 2 \sin 20^\circ \quad \alpha = \pm 46.84^\circ$$

$$\text{FOR } \alpha = +46.84^\circ$$

$$\Sigma F_y = 0: 2P \cos 20^\circ + P \sin 46.84^\circ - 1569.6 \text{ N} = 0$$

$$P = \frac{1569.6 \text{ N}}{2.609} = 601.6 \text{ N}$$

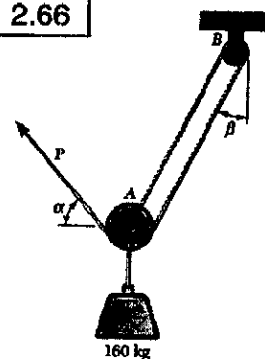
$$\underline{P} = 602 \text{ N } \nearrow 46.8^\circ$$

$$W = (160 \text{ kg})(9.81 \text{ m/s}^2) = 1569.6 \text{ N}$$

$$\text{FOR } \alpha = -46.84^\circ: \Sigma F_y = 0: 2P \cos 20^\circ + P \sin(-46.84^\circ) - 1569.6 \text{ N} = 0$$

$$P = \frac{1569.6 \text{ N}}{1.1499} = 1364.9 \text{ N} \quad \underline{P} = 1365 \text{ N } \searrow 46.8^\circ$$

2.66



GIVEN:

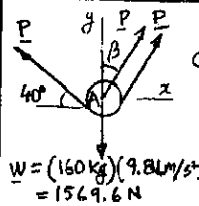
$$\alpha = 40^\circ$$

ALSO: T IS THE SAME IN  
ALL PORTIONS OF THE ROPE.

FIND:

(a) ANGLE  $\beta$ (b) MAGNITUDE OF  $\underline{P}$ 

FREE BODY: PULLEY A



$$\Sigma F_x = 0: 2P \sin \beta - P \cos 40^\circ = 0$$

$$(a) \sin \beta = \frac{1}{2} \cos 40^\circ \quad \beta = 22.52^\circ$$

$$\beta = 22.5^\circ$$

$$(b) \Sigma F_y = 0:$$

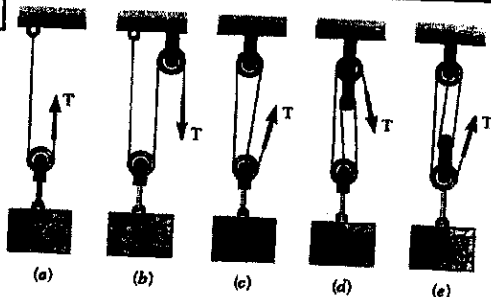
$$P \sin 40^\circ + 2P \cos 22.52^\circ - 1569.6 \text{ N} = 0$$

$$P = \frac{1569.6 \text{ N}}{2.4903} = 630.3 \text{ N}$$

$$\underline{P} = 630 \text{ N}$$

$$W = (160 \text{ kg})(9.81 \text{ m/s}^2) = 1569.6 \text{ N}$$

2.67



GIVEN: 600-lb CRATE SUPPORTED BY ONE OF THE ROPE-AND-PULLEY ARRANGEMENTS SHOWN.

FIND: TENSION IN THE ROPE FOR EACH ARRANGEMENT.

FREE-BODY: PULLEY

(a)  $\sum F_y = 0:$   
 $2T - 600 \text{ lb} = 0$

$T = 300 \text{ lb}$

(b)  $\sum F_y = 0:$   
 $2T - 600 \text{ lb} = 0$

$T = 300 \text{ lb}$

(c)  $\sum F_y = 0:$   
 $3T - 600 \text{ lb} = 0$

$T = 200 \text{ lb}$

(d)  $\sum F_y = 0:$   
 $3T - 600 \text{ lb} = 0$

$T = 200 \text{ lb}$

(e)  $\sum F_y = 0:$   
 $4T - 600 \text{ lb} = 0$

$T = 150 \text{ lb}$

2.68

GIVEN: ASSUME THAT IN PARTS b AND d OF PROB. 2.67 THE FREE END OF THE ROPE IS ATTACHED TO THE CRATE.

FIND: TENSION IN ROPE.

FREE-BODY: PULLEY AND CRATE

(b)  $\sum F_y = 0:$   
 $3T - 600 \text{ lb} = 0$

$T = 200 \text{ lb}$

(d)  $\sum F_y = 0:$   
 $4T - 600 \text{ lb} = 0$

$T = 150 \text{ lb}$

2.69

PULLEY C CAN ROLL ON CABLE ACB.

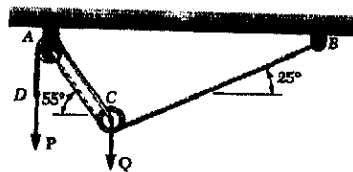
GIVEN:

$P = 750 \text{ N}$

FIND:

(a)  $T_{ACB}$

(b)  $Q$



NOTE: (1) THE TENSION IS THE SAME IN BOTH PORTIONS OF CABLE ACB.

(2) THE TENSION IN CABLE DAC IS EQUAL TO P.

FREE BODY: PULLEY C

(a)  $\sum F_x = 0:$   
 $T_{ACB} \cos 25^\circ - T_{ACB} \cos 55^\circ - (750 \text{ N}) \cos 55^\circ = 0$   
 $T_{ACB} (\cos 25^\circ - \cos 55^\circ) = 750 \cos 55^\circ$

$T_{ACB} = (750 \text{ N}) \frac{0.5736}{0.3327}$   $T_{ACB} = 1293 \text{ N}$

(b)  $\sum F_y = 0: (T_{ACB} + T_{DAC}) \sin 55^\circ + T_{ACB} \sin 25^\circ - Q = 0$   
 $Q = (1293 \text{ N} + 750 \text{ N}) \sin 55^\circ + (1293 \text{ N}) \sin 25^\circ = 2220.0 \text{ N}$   
 $Q = 2220 \text{ N}$

2.70

PULLEY C CAN ROLL ON CABLE ACB.

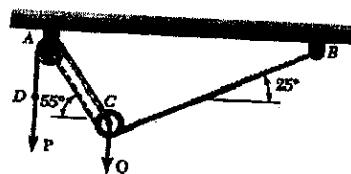
GIVEN:

$Q = 1800 \text{ N}$

FIND:

(a)  $T_{ACB}$

(b)  $P$



NOTE: (1) THE TENSION IS THE SAME IN BOTH PORTIONS OF CABLE ACB.

(2) THE TENSION IN CABLE DAC IS EQUAL TO P.

FREE BODY: PULLEY C

$\sum F_x = 0:$   
 $T_{ACB} \cos 25^\circ - T_{ACB} \cos 55^\circ - P \cos 55^\circ = 0$   
 $P = T_{ACB} \frac{\cos 25^\circ - \cos 55^\circ}{\cos 55^\circ}$   
 $P = 0.5801 T_{ACB}$  (1)

$\sum F_y = 0: (T_{ACB} + P) \sin 55^\circ + T_{ACB} \sin 25^\circ - 1800 \text{ N} = 0$  (2)

(a) SUBSTITUTE FOR P FROM (1) INTO (2):  
 $(1.5801 \sin 55^\circ + \sin 25^\circ) T_{ACB} = 1800 \text{ N}$

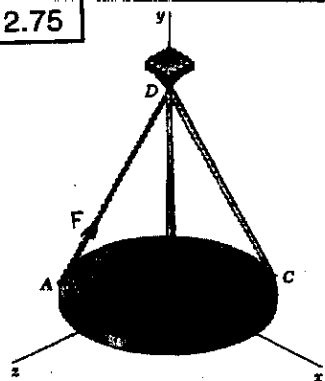
$T_{ACB} = 1048.4 \text{ N}$

$T_{ACB} = 1048 \text{ N}$

(b) CARRY INTO (1):  
 $P = 0.5801 (1048.4 \text{ N})$

$P = 608 \text{ N}$

2.75



GIVEN:

- (1) WIRES FORM  $30^\circ$  ANGLES WITH VERTICAL  
(2) FORCE EXERTED BY AD ON PLATE HAS COMPONENT  $F_x = 110.3 \text{ N}$ .

FIND:

- (a) TENSION IN AD  
(b) ANGLES  $\theta_x, \theta_y, \theta_z$  THAT FORCE EXERTED AT A FORMS WITH THE COORDINATE AXES.

$$(a) F_x = F \sin 30^\circ \sin 50^\circ = 110.3 \text{ N (GIVEN)}$$

$$F = \frac{110.3 \text{ N}}{\sin 30^\circ \sin 50^\circ} = 287.97 \text{ N} \quad F = 288 \text{ N} \quad \blacktriangleleft$$

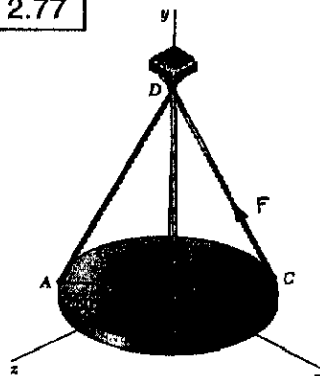
$$(b) \cos \theta_x = \frac{F_x}{F} = \frac{110.3 \text{ N}}{287.97 \text{ N}} = 0.3830 \quad \theta_x = 67.5^\circ \quad \blacktriangleleft$$

$$F_y = F \cos 30^\circ, \quad \cos \theta_y = \frac{F_y}{F} = \cos 30^\circ. \text{ Thus: } \theta_y = 30.0^\circ \quad \blacktriangleleft$$

$$F_z = -F \sin 30^\circ \cos 50^\circ = -(287.97 \text{ N}) \sin 30^\circ \cos 50^\circ = -92.552 \text{ N}$$

$$\cos \theta_z = \frac{F_z}{F} = \frac{-92.552 \text{ N}}{287.97 \text{ N}} = -0.3214 \quad \theta_z = 108.7^\circ \quad \blacktriangleleft$$

2.77



GIVEN:

- (1) WIRES FORM  $30^\circ$  ANGLES WITH VERTICAL  
(2) TENSION IN CD IS  $60 \text{ lb}$ .

FIND:

- (a) COMPONENTS OF FORCE EXERTED AT C.  
(b) ANGLES  $\theta_x, \theta_y, \theta_z$  THAT FORCE FORMS WITH THE COORDINATE AXES.

$$(a) F_x = -(60 \text{ lb}) \sin 30^\circ \cos 60^\circ \quad F_x = -15.00 \text{ lb} \quad \blacktriangleleft$$

$$F_y = (60 \text{ lb}) \cos 30^\circ = 51.96 \text{ lb} \quad F_y = +52.0 \text{ lb} \quad \blacktriangleleft$$

$$F_z = (60 \text{ lb}) \sin 30^\circ \sin 60^\circ = 25.98 \text{ lb} \quad F_z = +26.0 \text{ lb} \quad \blacktriangleleft$$

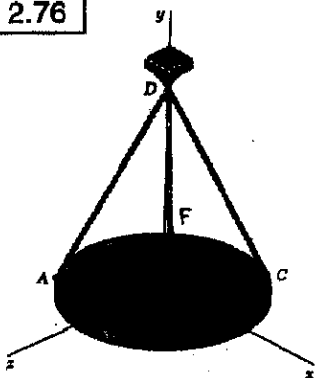
$$(b) \cos \theta_x = \frac{F_x}{F} = \frac{-15.00 \text{ lb}}{60 \text{ lb}} = -0.2500, \quad \theta_x = 104.5^\circ \quad \blacktriangleleft$$

$$\cos \theta_y = \frac{F_y}{F} = \frac{+51.96 \text{ lb}}{60 \text{ lb}} = 0.8660, \quad \theta_y = 30.0^\circ \quad \blacktriangleleft$$

$$\cos \theta_z = \frac{F_z}{F} = \frac{+25.98 \text{ lb}}{60 \text{ lb}} = 0.4330 \quad \theta_z = 64.3^\circ \quad \blacktriangleleft$$

NOTE: VALUE OBTAINED FOR  $\theta_y$  CHECKS WITH GIVEN DATA.

2.76



GIVEN:

- (1) WIRES FORM  $30^\circ$  ANGLES WITH VERTICAL  
(2) FORCE EXERTED BY BD ON PLATE HAS COMPONENT  $F_x = -32.14 \text{ N}$ .

FIND:

- (a) TENSION IN BD  
(b) ANGLES  $\theta_x, \theta_y, \theta_z$  THAT FORCE EXERTED AT B FORMS WITH THE COORDINATE AXES.

$$(a) F_x = -F \sin 30^\circ \cos 40^\circ = -32.14 \text{ N (GIVEN)}$$

$$F = \frac{32.14 \text{ N}}{\sin 30^\circ \cos 40^\circ} = 100.0 \text{ N} \quad F = 100.0 \text{ N} \quad \blacktriangleleft$$

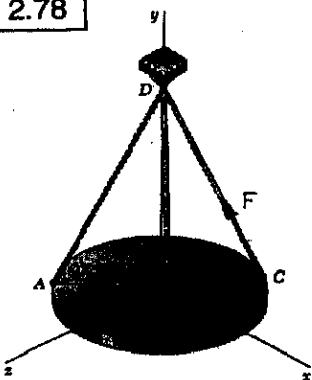
$$(b) F_z = -F \sin 30^\circ \cos 40^\circ = -(100.0 \text{ N}) \sin 30^\circ \cos 40^\circ = -38.30 \text{ N}$$

$$\cos \theta_x = \frac{F_x}{F} = \frac{-38.30 \text{ N}}{100.0 \text{ N}} = -0.3830 \quad \theta_x = 112.5^\circ \quad \blacktriangleleft$$

$$F_y = F \cos 30^\circ, \quad \cos \theta_y = \frac{F_y}{F} = \cos 30^\circ, \text{ Thus: } \theta_y = 30.0^\circ \quad \blacktriangleleft$$

$$\cos \theta_z = \frac{F_z}{F} = \frac{-32.14 \text{ N}}{100.0 \text{ N}} = -0.3214 \quad \theta_z = 108.7^\circ \quad \blacktriangleleft$$

2.78



GIVEN:

- (1) WIRES FORM  $30^\circ$  ANGLES WITH VERTICAL.  
(2) FORCE EXERTED BY CD ON PLATE HAS COMPONENT  $F_x = -20.0 \text{ lb}$ .

FIND:

- (a) TENSION IN CD.  
(b) ANGLES  $\theta_x, \theta_y, \theta_z$  THAT FORCE EXERTED AT C FORMS WITH THE COORDINATE AXES.

$$(a) F_x = -F \sin 30^\circ \cos 60^\circ = -20.0 \text{ lb (GIVEN)}$$

$$F = \frac{20.0 \text{ lb}}{\sin 30^\circ \cos 60^\circ} = 80.0 \text{ lb} \quad F = 80.0 \text{ lb} \quad \blacktriangleleft$$

$$(b) \cos \theta_x = \frac{F_x}{F} = \frac{-20.0 \text{ lb}}{80.0 \text{ lb}} = -0.2500 \quad \theta_x = 104.5^\circ \quad \blacktriangleleft$$

$$F_y = F \cos 30^\circ, \quad \cos \theta_y = \frac{F_y}{F} = \cos 30^\circ, \text{ Thus: } \theta_y = 30.0^\circ \quad \blacktriangleleft$$

$$F_z = F \sin 30^\circ \sin 60^\circ = (80.0 \text{ lb}) \sin 30^\circ \sin 60^\circ = 34.641 \text{ lb}$$

$$\cos \theta_z = \frac{F_z}{F} = \frac{34.641 \text{ lb}}{80.0 \text{ lb}} = 0.4330 \quad \theta_z = 64.3^\circ \quad \blacktriangleleft$$

**2.79** GIVEN:  $\mathbf{F} = (260\text{N})\mathbf{i} - (320\text{N})\mathbf{j} + (800\text{N})\mathbf{k}$   
 FIND: MAGNITUDE AND DIRECTION OF  $\mathbf{F}$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{(260)^2 + (320)^2 + (800)^2}, F = 900\text{N}$$

$$\cos \theta_x = \frac{F_x}{F} = \frac{260\text{N}}{900\text{N}} = 0.2889 \quad \theta_x = 73.2^\circ$$

$$\cos \theta_y = \frac{F_y}{F} = \frac{-320\text{N}}{900\text{N}} = -0.3556 \quad \theta_y = 110.8^\circ$$

$$\cos \theta_z = \frac{F_z}{F} = \frac{800\text{N}}{900\text{N}} = 0.8889 \quad \theta_z = 27.3^\circ$$

**2.80** GIVEN:  $\mathbf{F} = (320\text{N})\mathbf{i} + (400\text{N})\mathbf{j} - (250\text{N})\mathbf{k}$   
 FIND: MAGNITUDE AND DIRECTION OF  $\mathbf{F}$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{(320)^2 + (400)^2 + (250)^2}, F = 570\text{N}$$

$$\cos \theta_x = \frac{F_x}{F} = \frac{320\text{N}}{570\text{N}} = 0.5614 \quad \theta_x = 55.8^\circ$$

$$\cos \theta_y = \frac{F_y}{F} = \frac{400\text{N}}{570\text{N}} = 0.7018 \quad \theta_y = 45.4^\circ$$

$$\cos \theta_z = \frac{F_z}{F} = \frac{-250\text{N}}{570\text{N}} = -0.4386 \quad \theta_z = 116.0^\circ$$

**2.81** GIVEN: FORCE WITH  $\theta_x = 69.3^\circ$ ,  $\theta_z = 57.9^\circ$   
 AND  $F_y = -174.0\text{ lb}$ .  
 FIND: (a)  $\theta_y$ , (b)  $F_x$ ,  $F_z$ , AND  $F$ .

(a) TO DETERMINE  $\theta_y$  WE USE THE RELATION  
 $\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1 \quad \cos^2 \theta_y = 1 - \cos^2 \theta_x - \cos^2 \theta_z$   
 SINCE  $F_y < 0$ , WE MUST HAVE  $\cos \theta_y < 0$ . THUS:  
 $\cos \theta_y = -\sqrt{1 - \cos^2 69.3^\circ - \cos^2 57.9^\circ} = -0.7699, \theta_y = 140.3^\circ$

(b)  $F = \frac{F_y}{\cos \theta_y} = \frac{-174.0\text{ lb}}{-0.7699} = 226.0\text{ lb} \quad F = 226\text{ lb}$   
 $F_x = F \cos \theta_x = (226.0\text{ lb}) \cos 69.3^\circ \quad F_x = 79.9\text{ lb}$   
 $F_z = F \cos \theta_z = (226.0\text{ lb}) \cos 57.9^\circ \quad F_z = 120.1\text{ lb}$

**2.82** GIVEN: FORCE WITH  $\theta_x = 70.9^\circ$ ,  $\theta_y = 144.9^\circ$   
 AND  $F_z = -52.0\text{ lb}$   
 FIND: (a)  $\theta_z$ , (b)  $F_x$ ,  $F_y$ , AND  $F$ .

(a) TO DETERMINE  $\theta_z$  WE USE THE RELATION  
 $\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1, \cos^2 \theta_z = 1 - \cos^2 \theta_x - \cos^2 \theta_y$   
 SINCE  $F_z < 0$ , WE MUST HAVE  $\cos \theta_z < 0$ . THUS:  
 $\cos \theta_z = -\sqrt{1 - \cos^2 70.9^\circ - \cos^2 144.9^\circ} = -0.4728, \theta_z = 118.2^\circ$

(b)  $F = \frac{F_z}{\cos \theta_z} = \frac{-52.0\text{ lb}}{-0.4728} = 110.0\text{ lb} \quad F = 110\text{ lb}$   
 $F_x = F \cos \theta_x = (110.0\text{ lb}) \cos 70.9^\circ \quad F_x = 36.0\text{ lb}$   
 $F_y = F \cos \theta_y = (110.0\text{ lb}) \cos 144.9^\circ \quad F_y = -90.0\text{ lb}$

**2.83** GIVEN:  $F = 230\text{N}$ ,  $\theta_x = 32.5^\circ$ ,  $F_y = -60\text{N}$ ,  $F_z > 0$   
 FIND: (a)  $F_x$  AND  $F_z$ , (b)  $\theta_y$  AND  $\theta_z$

(a)  $F_x = F \cos \theta_x = (230\text{N}) \cos 32.5^\circ \quad F_x = 194.0\text{N}$   
 $F^2 = F_x^2 + F_y^2 + F_z^2 \quad (230\text{N})^2 = (194.0\text{N})^2 + (-60\text{N})^2 + F_z^2$   
 $F_z = +\sqrt{(230)^2 - (194)^2 - (60)^2} \quad F_z = +108.0\text{N}$

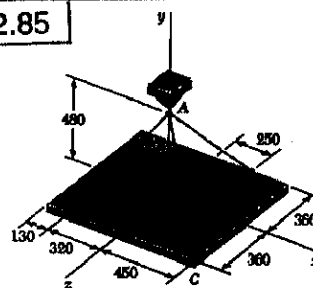
(b)  $\cos \theta_y = F_y/F = -60/230 = -0.2609 \quad \theta_y = 105.1^\circ$   
 $\cos \theta_z = F_z/F = 108/230 = +0.4696 \quad \theta_z = 62.0^\circ$

**2.84** GIVEN:  $F = 210\text{N}$ ,  $F_x = 80\text{N}$ ,  $\theta_z = 151.2^\circ$ ,  $F_y < 0$   
 FIND: (a)  $F_y$  AND  $F_z$ , (b)  $\theta_x$  AND  $\theta_y$ .

(a)  $F_z = F \cos \theta_z = (210\text{N}) \cos 151.2^\circ \quad F_z = -184.0\text{N}$   
 $F^2 = F_x^2 + F_y^2 + F_z^2 \quad (210\text{N})^2 = (80\text{N})^2 + F_y^2 + (-184.0\text{N})^2$   
 $F_y = -\sqrt{(210)^2 - (80)^2 - (184.0)^2} \quad F_y = -62.0\text{N}$

(b)  $\cos \theta_x = F_x/F = 80/210 = +0.3810 \quad \theta_x = 67.6^\circ$   
 $\cos \theta_y = F_y/F = -62.0/210 = - \quad \theta_y = 107.2^\circ$

**2.85** GIVEN: TENSION IN CABLE AB IS 408 N.  
 FIND: COMPONENTS OF FORCE EXERTED ON PLATE AT B.



Dimensions in mm

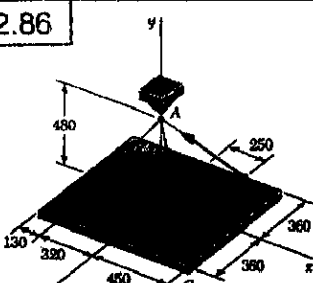
$$\mathbf{BA} = 320\mathbf{i} + 480\mathbf{j} - 360\mathbf{k} \quad BA = \sqrt{(320)^2 + (480)^2 + (360)^2} = 680$$

$$\mathbf{F} = F \frac{\mathbf{BA}}{BA} = F \frac{\mathbf{BA}}{680\text{ mm}} = \frac{408\text{ N}}{680\text{ mm}} [(320\text{ mm})\mathbf{i} + (480\text{ mm})\mathbf{j} - (360\text{ mm})\mathbf{k}]$$

$$\mathbf{F} = (192\text{ N})\mathbf{i} + (288\text{ N})\mathbf{j} - (216\text{ N})\mathbf{k}$$

$$F_x = +192\text{ N}, F_y = +288\text{ N}, F_z = -216\text{ N}$$

**2.86** GIVEN: TENSION IN CABLE AD IS 429 N.  
 FIND: COMPONENTS OF FORCE EXERTED ON PLATE AT D.



Dimensions in mm

$$\mathbf{DA} = -250\mathbf{i} + 480\mathbf{j} + 360\mathbf{k} \quad DA = \sqrt{(250)^2 + (480)^2 + (360)^2} = 650$$

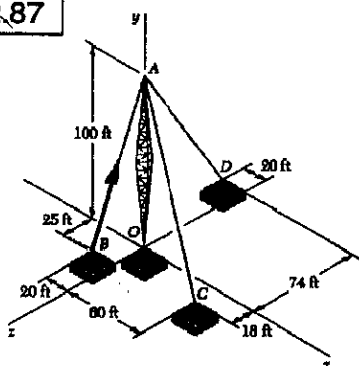
$$\mathbf{F} = F \frac{\mathbf{DA}}{DA} = F \frac{\mathbf{DA}}{650\text{ mm}} = \frac{429\text{ N}}{650\text{ mm}} [(-250\text{ mm})\mathbf{i} + (480\text{ mm})\mathbf{j} + (360\text{ mm})\mathbf{k}]$$

$$\mathbf{F} = (-165\text{ N})\mathbf{i} + (316.8\text{ N})\mathbf{j} + (237.6\text{ N})\mathbf{k}$$

$$F_x = -165\text{ N}, F_y = +317\text{ N}, F_z = +238\text{ N}$$



2.87



GIVEN:  
TENSION IN WIRE  
AB IS 525 lb.

FIND:  
COMPONENTS OF  
FORCE EXERTED  
ON BOLT B BY  
WIRE AB.

$$\vec{BA} = (20\text{ ft})\underline{i} + (100\text{ ft})\underline{j} - (25\text{ ft})\underline{k}$$

$$BA = \sqrt{(20)^2 + (100)^2 + (25)^2}$$

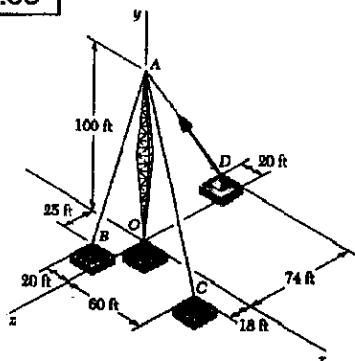
$$BA = 105\text{ ft}$$

$$\underline{F} = F \frac{\vec{BA}}{BA} = F \frac{\vec{BA}}{105\text{ ft}} = \frac{525\text{ lb}}{105\text{ ft}} [(20\text{ ft})\underline{i} + (100\text{ ft})\underline{j} - (25\text{ ft})\underline{k}]$$

$$\underline{F} = (100\text{ lb})\underline{i} + (500\text{ lb})\underline{j} - (125\text{ lb})\underline{k}$$

$$F_x = +100\text{ lb}, F_y = +500\text{ lb}, F_z = -125\text{ lb}$$

2.88



GIVEN:  
TENSION IN WIRE  
AD IS 315 lb.

FIND:  
COMPONENTS OF  
FORCE EXERTED  
ON BOLT D BY  
WIRE AD.

$$\vec{DA} = (20\text{ ft})\underline{i} + (100\text{ ft})\underline{j} + (74\text{ ft})\underline{k}$$

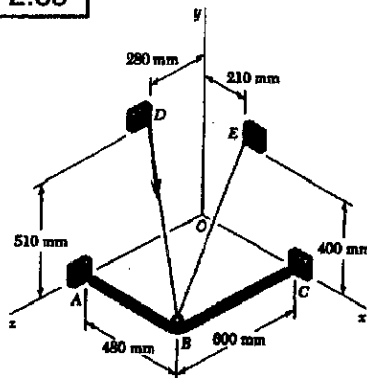
$$DA = \sqrt{(20)^2 + (100)^2 + (74)^2} = 126\text{ ft}$$

$$\underline{F} = F \frac{\vec{DA}}{DA} = F \frac{\vec{DA}}{126\text{ ft}} = \frac{315\text{ lb}}{126\text{ ft}} [(20\text{ ft})\underline{i} + (100\text{ ft})\underline{j} + (74\text{ ft})\underline{k}]$$

$$\underline{F} = (50\text{ lb})\underline{i} + (250\text{ lb})\underline{j} + (185\text{ lb})\underline{k}$$

$$F_x = +50\text{ lb}, F_y = +250\text{ lb}, F_z = +185\text{ lb}$$

2.89



GIVEN:  
TENSION IN  
CABLE DBE  
IS 385 N.

FIND:  
COMPONENTS OF  
FORCE EXERTED  
BY CABLE ON D.

$$\vec{DB} = (480\text{ mm})\underline{i} - (510\text{ mm})\underline{j} + (320\text{ mm})\underline{k}$$

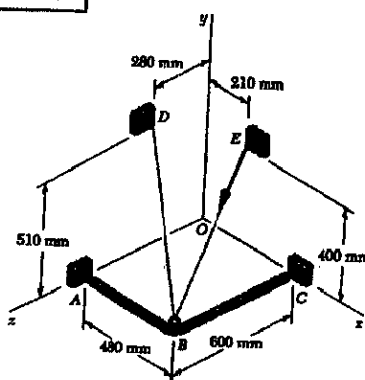
$$DB = \sqrt{(480)^2 + (510)^2 + (320)^2} = 770\text{ mm}$$

$$\underline{F} = F \frac{\vec{DB}}{DB} = F \frac{\vec{DB}}{770\text{ mm}} = \frac{385\text{ N}}{770\text{ mm}} [(480\text{ mm})\underline{i} - (510\text{ mm})\underline{j} + (320\text{ mm})\underline{k}]$$

$$\underline{F} = (240\text{ N})\underline{i} - (255\text{ N})\underline{j} + (160\text{ N})\underline{k}$$

$$F_x = +240\text{ N}, F_y = -255\text{ N}, F_z = +160\text{ N}$$

2.90



GIVEN:  
TENSION IN  
CABLE DBE  
IS 385 N.

FIND:  
COMPONENTS OF  
FORCE EXERTED  
BY CABLE ON E.

$$\vec{EB} = (270\text{ mm})\underline{i} - (400\text{ mm})\underline{j} + (600\text{ mm})\underline{k}$$

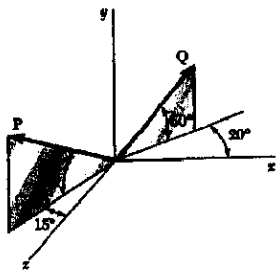
$$EB = \sqrt{(270)^2 + (400)^2 + (600)^2} = 770\text{ mm}$$

$$\underline{F} = F \frac{\vec{EB}}{EB} = F \frac{\vec{EB}}{770\text{ mm}} = \frac{385\text{ N}}{770\text{ mm}} [(270\text{ mm})\underline{i} - (400\text{ mm})\underline{j} + (600\text{ mm})\underline{k}]$$

$$\underline{F} = (135\text{ N})\underline{i} - (200\text{ N})\underline{j} + (300\text{ N})\underline{k}$$

$$F_x = +135\text{ N}, F_y = -200\text{ N}, F_z = +300\text{ N}$$

2.91



GIVEN:

$$P = 300 \text{ N}$$

$$Q = 400 \text{ N}$$

FIND:

MAGNITUDE AND  
DIRECTION OF  
RESULTANT  
OF  $\underline{P}$  AND  $\underline{Q}$ .

FORCE  $\underline{P}$ :  $P_x = -(300 \text{ N}) \cos 30^\circ \sin 15^\circ = -67.24 \text{ N}$   
 $P_y = +(300 \text{ N}) \sin 30^\circ = +150.00 \text{ N}$   
 $P_z = +(300 \text{ N}) \cos 30^\circ \cos 15^\circ = +250.95 \text{ N}$   
 $\underline{P} = (-67.24 \text{ N})\underline{i} + (150.00 \text{ N})\underline{j} + (250.95 \text{ N})\underline{k}$

FORCE  $\underline{Q}$ :  $Q_x = +(400 \text{ N}) \cos 50^\circ \cos 20^\circ = +241.61 \text{ N}$   
 $Q_y = +(400 \text{ N}) \sin 50^\circ = +306.42 \text{ N}$   
 $Q_z = -(400 \text{ N}) \cos 50^\circ \sin 20^\circ = -87.94 \text{ N}$   
 $\underline{Q} = (+241.61 \text{ N})\underline{i} + (306.42 \text{ N})\underline{j} - (87.94 \text{ N})\underline{k}$

RESULTANT:

$$\underline{R} = \underline{P} + \underline{Q} = (174.37 \text{ N})\underline{i} + (456.42 \text{ N})\underline{j} + (163.01 \text{ N})\underline{k}$$

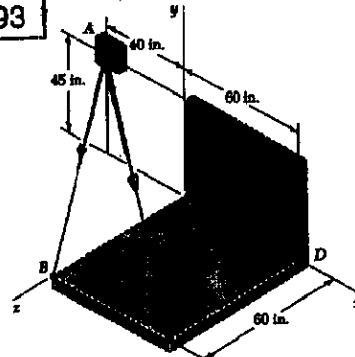
$$R = \sqrt{(174.37)^2 + (456.42)^2 + (163.01)^2} = 515.07 \text{ N}, R = 515 \text{ N}$$

$$\cos \theta_x = R_x/R = (174.37 \text{ N})/(515.07 \text{ N}) = 0.3385, \theta_x = 70.2^\circ$$

$$\cos \theta_y = R_y/R = (456.42 \text{ N})/(515.07 \text{ N}) = 0.8861, \theta_y = 27.6^\circ$$

$$\cos \theta_z = R_z/R = (163.01 \text{ N})/(515.07 \text{ N}) = 0.3165, \theta_z = 71.5^\circ$$

2.93



GIVEN:

$$T_{AB} = 425 \text{ lb}$$

$$T_{AC} = 510 \text{ lb}$$

FIND:

MAGNITUDE AND  
DIRECTION OF  
RESULTANT OF  
FORCES AT A.

$$\underline{AB} = (40 \text{ in.})\underline{i} - (45 \text{ in.})\underline{j} + (60 \text{ in.})\underline{k} \quad AB = \sqrt{(40)^2 + (45)^2 + (60)^2} = 85 \text{ in.}$$

$$\underline{AC} = (100 \text{ in.})\underline{i} - (45 \text{ in.})\underline{j} + (60 \text{ in.})\underline{k} \quad AC = \sqrt{(100)^2 + (45)^2 + (60)^2} = 125 \text{ in.}$$

$$\underline{F}_{AB} = F_{AB} \frac{\underline{AB}}{AB} = F_{AB} \frac{\underline{AB}}{85 \text{ in.}} = \frac{425 \text{ lb}}{85 \text{ in.}} [(40 \text{ in.})\underline{i} - (45 \text{ in.})\underline{j} + (60 \text{ in.})\underline{k}]$$

$$\underline{F}_{AB} = (200 \text{ lb})\underline{i} - (225 \text{ lb})\underline{j} + (300 \text{ lb})\underline{k}$$

$$\underline{F}_{AC} = F_{AC} \frac{\underline{AC}}{AC} = \frac{510 \text{ lb}}{125 \text{ in.}} [(100 \text{ in.})\underline{i} - (45 \text{ in.})\underline{j} + (60 \text{ in.})\underline{k}]$$

$$\underline{F}_{AC} = (408 \text{ lb})\underline{i} - (183.6 \text{ lb})\underline{j} + (244.8 \text{ lb})\underline{k}$$

$$\underline{R} = \underline{F}_{AB} + \underline{F}_{AC} = (608 \text{ lb})\underline{i} - (408.6 \text{ lb})\underline{j} + (544.8 \text{ lb})\underline{k}, R = 912.92 \text{ lb}$$

$$R = 913 \text{ lb}$$

$$\cos \theta_x = R_x/R = 608/912.92 = 0.6660$$

$$\theta_x = 48.2^\circ$$

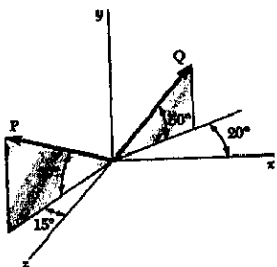
$$\cos \theta_y = R_y/R = -408.6/912.92 = -0.4476$$

$$\theta_y = 116.6^\circ$$

$$\cos \theta_z = R_z/R = 544.8/912.92 = 0.5968$$

$$\theta_z = 53.4^\circ$$

2.92



GIVEN:

$$P = 400 \text{ N}$$

$$Q = 300 \text{ N}$$

FIND:

MAGNITUDE AND  
DIRECTION OF  
RESULTANT  
OF  $\underline{P}$  AND  $\underline{Q}$ .

FORCE  $\underline{P}$ :  $P_x = -(400 \text{ N}) \cos 30^\circ \sin 15^\circ = -89.66 \text{ N}$   
 $P_y = +(400 \text{ N}) \sin 30^\circ = +200.00 \text{ N}$   
 $P_z = +(400 \text{ N}) \cos 30^\circ \cos 15^\circ = +334.61 \text{ N}$   
 $\underline{P} = (-89.66 \text{ N})\underline{i} + (200.00 \text{ N})\underline{j} + (334.61 \text{ N})\underline{k}$

FORCE  $\underline{Q}$ :  $Q_x = +(300 \text{ N}) \cos 50^\circ \cos 20^\circ = +181.21 \text{ N}$   
 $Q_y = +(300 \text{ N}) \sin 50^\circ = +229.81 \text{ N}$   
 $Q_z = -(300 \text{ N}) \cos 50^\circ \sin 20^\circ = -65.45 \text{ N}$   
 $\underline{Q} = (181.21 \text{ N})\underline{i} + (229.81 \text{ N})\underline{j} - (65.45 \text{ N})\underline{k}$

RESULTANT:

$$\underline{R} = \underline{P} + \underline{Q} = (91.55 \text{ N})\underline{i} + (429.81 \text{ N})\underline{j} + (268.66 \text{ N})\underline{k}$$

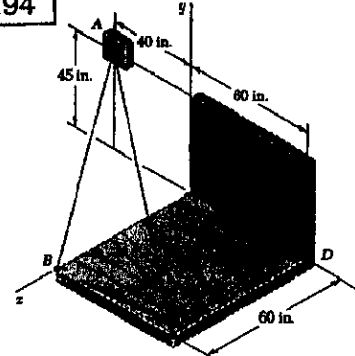
$$R = \sqrt{(91.55)^2 + (429.81)^2 + (268.66)^2} = 515.07 \text{ N}, R = 515 \text{ N}$$

$$\cos \theta_x = R_x/R = (91.55 \text{ N})/(515.07 \text{ N}) = 0.1777, \theta_x = 79.8^\circ$$

$$\cos \theta_y = R_y/R = (429.81 \text{ N})/(515.07 \text{ N}) = 0.8345, \theta_y = 33.4^\circ$$

$$\cos \theta_z = R_z/R = (268.66 \text{ N})/(515.07 \text{ N}) = 0.5216, \theta_z = 58.6^\circ$$

2.94



GIVEN:

$$T_{AB} = 510 \text{ lb}$$

$$T_{AC} = 425 \text{ lb}$$

FIND:

MAGNITUDE AND  
DIRECTION OF  
RESULTANT OF  
FORCES AT A.

$$\underline{AB} = (40 \text{ in.})\underline{i} - (45 \text{ in.})\underline{j} + (60 \text{ in.})\underline{k} \quad AB = \sqrt{(40)^2 + (45)^2 + (60)^2} = 85 \text{ in.}$$

$$\underline{AC} = (100 \text{ in.})\underline{i} - (45 \text{ in.})\underline{j} + (60 \text{ in.})\underline{k} \quad AC = \sqrt{(100)^2 + (45)^2 + (60)^2} = 125 \text{ in.}$$

$$\underline{F}_{AB} = F_{AB} \frac{\underline{AB}}{AB} = \frac{510 \text{ lb}}{85 \text{ in.}} [(40 \text{ in.})\underline{i} - (45 \text{ in.})\underline{j} + (60 \text{ in.})\underline{k}]$$

$$\underline{F}_{AB} = (240 \text{ lb})\underline{i} - (270 \text{ lb})\underline{j} + (360 \text{ lb})\underline{k}$$

$$\underline{F}_{AC} = F_{AC} \frac{\underline{AC}}{AC} = \frac{425 \text{ lb}}{125 \text{ in.}} [(100 \text{ in.})\underline{i} - (45 \text{ in.})\underline{j} + (60 \text{ in.})\underline{k}]$$

$$\underline{F}_{AC} = (340 \text{ lb})\underline{i} - (153 \text{ lb})\underline{j} + (204 \text{ lb})\underline{k}$$

$$\underline{R} = \underline{F}_{AB} + \underline{F}_{AC} = (580 \text{ lb})\underline{i} - (423 \text{ lb})\underline{j} + (564 \text{ lb})\underline{k}, R = 912.92 \text{ lb}$$

$$R = 913 \text{ lb}$$

$$\cos \theta_x = R_x/R = 580/912.92 = 0.6353$$

$$\theta_x = 50.6^\circ$$

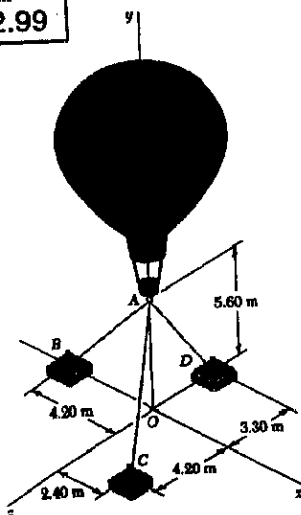
$$\cos \theta_y = R_y/R = -423/912.92 = -0.4633$$

$$\theta_y = 117.6^\circ$$

$$\cos \theta_z = R_z/R = 564/912.92 = 0.6178$$

$$\theta_z = 51.8^\circ$$

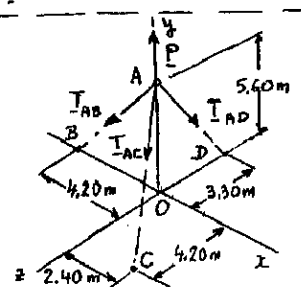
2.99



GIVEN:  
 $T_{AB} = 259 \text{ N}$

FIND:  
VERTICAL FORCE  $P$   
EXERTED AT A BY THE  
BALLOON.

FIG. P 2.99, P 2.100,  
P 2.101, AND P 2.102



FREE BODY: A

FORCES APPLIED AT A  
ARE  $T_{AB}$ ,  $T_{AC}$ ,  $T_{AD}$ ,  
AND  $P$ , WHERE  $P = P_j$ .  
TO EXPRESS THE OTHER  
FORCES IN TERMS OF THE  
UNIT VECTORS, WE WRITE

$$\begin{aligned}\vec{AB} &= -(4.20\text{ m})\hat{i} - (5.60\text{ m})\hat{j} & AB &= 7.00 \text{ m} \\ \vec{AC} &= (2.40\text{ m})\hat{i} - (5.60\text{ m})\hat{j} + (4.20\text{ m})\hat{k}, & AC &= 7.40 \text{ m} \\ \vec{AD} &= -(5.60\text{ m})\hat{j} - (3.30\text{ m})\hat{k}, & AD &= 6.50 \text{ m}\end{aligned}$$

$$\begin{aligned}T_{AB} &= T_{AB} \frac{\vec{AB}}{AB} = T_{AB} \frac{-(4.20\hat{i} - 5.60\hat{j})}{7.00} = (-0.6\hat{i} - 0.8\hat{j}) T_{AB} \\ T_{AC} &= T_{AC} \frac{\vec{AC}}{AC} = T_{AC} \frac{(2.40\hat{i} - 5.60\hat{j} + 4.20\hat{k})}{7.40} = \left(\frac{24}{74}\hat{i} - \frac{56}{74}\hat{j} + \frac{42}{74}\hat{k}\right) T_{AC} \\ T_{AD} &= T_{AD} \frac{\vec{AD}}{AD} = T_{AD} \frac{-(5.60\hat{j} - 3.30\hat{k})}{6.50} = \left(-\frac{56}{65}\hat{j} - \frac{33}{65}\hat{k}\right) T_{AD}\end{aligned}$$

EQUILIBRIUM CONDITION:

$$\Sigma \vec{F} = 0: T_{AB} + T_{AC} + T_{AD} + P\hat{j} = 0$$

SUBSTITUTING THE EXPRESSIONS OBTAINED FOR  $T_{AB}$ ,  
 $T_{AC}$ , AND  $T_{AD}$  AND FACTORING  $\hat{i}$ ,  $\hat{j}$ , AND  $\hat{k}$ :

$$\begin{aligned}(-0.6 T_{AB} + \frac{24}{74} T_{AC})\hat{i} \\ + (-0.8 T_{AB} - \frac{56}{74} T_{AC} - \frac{56}{65} T_{AD} + P)\hat{j} \\ + (\frac{42}{74} T_{AC} - \frac{33}{65} T_{AD})\hat{k} = 0\end{aligned}$$

SETTING TO ZERO THE COEFFICIENTS OF  $\hat{i}$ ,  $\hat{j}$ ,  $\hat{k}$ :

$$(1) \quad -0.6 T_{AB} + \frac{24}{74} T_{AC} = 0 \quad (1)$$

$$(2) \quad -0.8 T_{AB} - \frac{56}{74} T_{AC} - \frac{56}{65} T_{AD} + P = 0 \quad (2)$$

$$(3) \quad \frac{42}{74} T_{AC} - \frac{33}{65} T_{AD} = 0 \quad (3)$$

CONTINUED

2.99 CONTINUED

MAKING  $T_{AB} = 259 \text{ N}$  IN EQ. (1) AND SOLVING FOR  $T_{AC}$ :

$$T_{AC} = \frac{74}{24} (0.6)(259 \text{ N}) \quad T_{AC} = 479.15 \text{ N}$$

CARRYING INTO EQ. (3) AND SOLVING FOR  $T_{AD}$ :

$$T_{AD} = \frac{65}{33} \frac{42}{74} (479.15 \text{ N}) \quad T_{AD} = 535.66 \text{ N}$$

SUBSTITUTING FOR  $T_{AB}$ ,  $T_{AC}$ ,  $T_{AD}$  INTO (2) AND SOLVING  
FOR  $P$ :

$$P = 0.8(259 \text{ N}) + \frac{56}{74}(479.15 \text{ N}) + \frac{56}{65}(535.66 \text{ N}) = 1031.3 \text{ N}$$

$$P = 1031 \text{ N} \uparrow$$

2.100

GIVEN:  $T_{AC} = 444 \text{ N}$

(SEE FIGURE  
ON LEFT)

FIND: VERTICAL FORCE  $P$  EXERTED  
AT A BY THE BALLOON

SEE LEFT-HAND COLUMN FOR DERIVATION OF Eqs. (1), (2), (3).  
MAKING  $T_{AC} = 444 \text{ N}$  IN Eqs. (1) AND (3) AND SOLVING  
FOR  $T_{AB}$  AND  $T_{AD}$ :

$$\begin{aligned}T_{AB} &= \frac{24}{0.6(74)} (444 \text{ N}) & T_{AD} &= \frac{65}{33} \frac{42}{74} (444 \text{ N}) \\ T_{AB} &= 240 \text{ N} & T_{AD} &= 496.36 \text{ N}\end{aligned}$$

SUBSTITUTING FOR  $T_{AB}$ ,  $T_{AC}$ ,  $T_{AD}$  INTO (2) AND SOLVING  
FOR  $P$ :

$$P = 0.8(240 \text{ N}) + \frac{56}{74}(444 \text{ N}) + \frac{56}{65}(496.36 \text{ N}) = 955.6 \text{ N}$$

$$P = 956 \text{ N} \uparrow$$

2.101

(SEE FIGURE ON UPPER LEFT)

GIVEN:  $T_{AD} = 481 \text{ N}$

FIND: VERTICAL FORCE  $P$  EXERTED AT A BY THE BALLOON

SEE LEFT-HAND COLUMN FOR DERIVATION OF Eqs. (1), (2), (3).

MAKING  $T_{AD} = 481 \text{ N}$  IN EQ. (3) AND SOLVING FOR  $T_{AC}$ :

$$T_{AC} = \frac{74}{42} \frac{33}{65} (481 \text{ N}) \quad T_{AC} = 430.26 \text{ N}$$

CARRYING INTO EQ. (1) AND SOLVING FOR  $T_{AB}$ :

$$T_{AB} = \frac{24}{0.6(74)} (430.26 \text{ N}) \quad T_{AB} = 232.57 \text{ N}$$

SUBSTITUTING FOR  $T_{AB}$ ,  $T_{AC}$ ,  $T_{AD}$  INTO (2) AND SOLVING  
FOR  $P$ :

$$P = 0.8(232.57 \text{ N}) + \frac{56}{74}(430.26 \text{ N}) + \frac{56}{65}(481 \text{ N}) = 926.06 \text{ N}$$

$$P = 926 \text{ N} \uparrow$$

2.102

(SEE FIGURE ON UPPER LEFT)

GIVEN: BALLOON EXERTS FORCE  $P = 800 \text{ N}$  AT A.

FIND: TENSION IN EACH CABLE

SEE LEFT-HAND COLUMN FOR DERIVATION OF Eqs. (1), (2), (3).

$$\text{FROM EQ. (1): } T_{AB} = \frac{24}{0.6(74)} T_{AC} \quad T_{AB} = 0.54054 T_{AC}$$

$$\text{FROM EQ. (3): } T_{AD} = \frac{65}{33} \frac{42}{74} T_{AC} \quad T_{AD} = 1.1179 T_{AC}$$

SUBSTITUTE FOR  $T_{AB}$  AND  $T_{AD}$  INTO EQ. (2):

$$-0.8(0.54054 T_{AC}) - \frac{56}{74} T_{AC} - \frac{56}{65} (1.1179 T_{AC}) + P = 0$$

$$2.1523 T_{AC} = P \quad T_{AC} = \frac{800 \text{ N}}{2.1523} \quad T_{AC} = 371.69 \text{ N}$$

SUBSTITUTE INTO EXPRESSIONS FOR  $T_{AB}$  AND  $T_{AD}$ :

$$T_{AB} = 0.54054(371.69 \text{ N}) = 200.91 \text{ N}$$

$$T_{AD} = 1.1179(371.69 \text{ N}) = 415.51 \text{ N}$$

$$T_{AB} = 201 \text{ N}, T_{AC} = 372 \text{ N}, T_{AD} = 416 \text{ N}$$

## 2.111 CONTINUED

WE REPEAT THE LAST EGS:

$$-160 \text{ lb} + \frac{60}{118} T_{AC} - \frac{10}{126} T_{AD} = 0 \quad (1)$$

$$-800 \text{ lb} - \frac{100}{118} T_{AC} - \frac{100}{126} T_{AD} + P = 0 \quad (2)$$

$$200 \text{ lb} + \frac{18}{118} T_{AC} - \frac{74}{126} T_{AD} = 0 \quad (3)$$

MULTIPLY EQ.(1) BY -3, EQ.(3) BY 10, AND ADD:

$$2400 \text{ lb} - \frac{180}{118} T_{AC} = 0 \quad T_{AC} = 459.529 \text{ lb}$$

$$\text{SUBSTITUTE INTO (1) AND SOLVE FOR } T_{AD}: \quad T_{AD} = 458.118 \text{ lb}$$

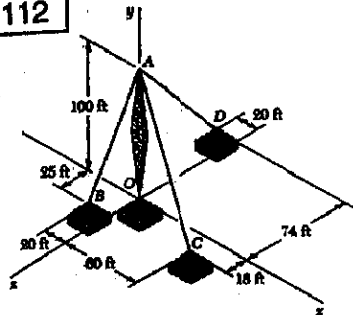
$$T_{AC} = \frac{118}{60} (160 + \frac{10}{126} \times 459.529) \quad T_{AC} = 458.118 \text{ lb}$$

$$\text{SUBSTITUTE FOR THE TENSIONS IN (2) AND SOLVE FOR } P:$$

$$P = 800 \text{ lb} + \frac{100}{118} (458.118 \text{ lb}) + \frac{100}{126} (459.529 \text{ lb}) = 1552.94 \text{ lb}$$

$$\text{WEIGHT OF PLATE} = P = 1553 \text{ lb}$$

## 2.112

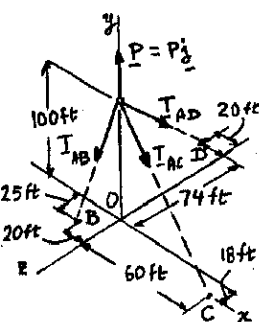


GIVEN:

$$T_{AC} = 590 \text{ lb}$$

FIND:

VERTICAL FORCE  
P EXERTED BY  
TOWER ON PIN A.



FREE BODY: A

$$\Sigma F = 0:$$

$$T_{AB} + T_{AC} + T_{AD} + P_j = 0$$

$$\vec{AB} = -20\hat{i} - 100\hat{j} + 25\hat{k} \quad AB = 105 \text{ ft}$$

$$\vec{AC} = 60\hat{i} - 100\hat{j} + 18\hat{k} \quad AC = 118 \text{ ft}$$

$$\vec{AD} = -20\hat{i} - 100\hat{j} - 74\hat{k} \quad AD = 126 \text{ ft}$$

WE WRITE

$$T_{AB} = T_{AB} \frac{\vec{AB}}{AB} = T_{AB} \frac{(-20\hat{i} - 100\hat{j} + 25\hat{k})}{105}$$

$$T_{AC} = T_{AC} \frac{\vec{AC}}{AC} = T_{AC} \frac{(60\hat{i} - 100\hat{j} + 18\hat{k})}{118}$$

$$T_{AD} = T_{AD} \frac{\vec{AD}}{AD} = T_{AD} \frac{(-20\hat{i} - 100\hat{j} - 74\hat{k})}{126}$$

SUBSTITUTING INTO THE EQ.  $\Sigma F = 0$  AND FACTORING  $\hat{i}, \hat{j}, \hat{k}$ :

$$\begin{aligned} & \left( -\frac{4}{21} T_{AB} + \frac{60}{118} T_{AC} - \frac{20}{126} T_{AD} \right) \hat{i} \\ & + \left( -\frac{20}{21} T_{AB} - \frac{100}{118} T_{AC} - \frac{100}{126} T_{AD} + P \right) \hat{j} \\ & + \left( \frac{5}{21} T_{AB} + \frac{18}{118} T_{AC} - \frac{74}{126} T_{AD} \right) \hat{k} = 0 \end{aligned}$$

SETTING THE COEFF. OF  $\hat{i}, \hat{j}, \hat{k}$  EQUAL TO ZERO:

$$\textcircled{1} -\frac{4}{21} T_{AB} + \frac{60}{118} T_{AC} - \frac{20}{126} T_{AD} = 0 \quad (1)$$

$$\textcircled{2} -\frac{20}{21} T_{AB} - \frac{100}{118} T_{AC} - \frac{100}{126} T_{AD} + P = 0 \quad (2)$$

$$\textcircled{3} \frac{5}{21} T_{AB} + \frac{18}{118} T_{AC} - \frac{74}{126} T_{AD} = 0 \quad (3)$$

CONTINUED

## 2.112 CONTINUED

MAKING  $T_{AC} = 590 \text{ lb}$  IN EQS. (1), (2), AND (3):

$$-\frac{4}{21} T_{AB} - \frac{20}{126} T_{AD} + 300 \text{ lb} = 0 \quad (1')$$

$$-\frac{20}{21} T_{AB} - \frac{100}{126} T_{AD} - 500 \text{ lb} + P = 0 \quad (2')$$

$$\frac{5}{21} T_{AB} - \frac{74}{126} T_{AD} + 90 \text{ lb} = 0 \quad (3')$$

MULTIPLY EQ.(1') BY 5, EQ.(3') BY 4, AND ADD:

$$-\frac{20}{21} T_{AB} + 1860 \text{ lb} = 0 \quad T_{AB} = 591.818 \text{ lb}$$

$$\text{SUBSTITUTE INTO (1') AND SOLVE FOR } T_{AD}: \quad T_{AD} = 1081.82 \text{ lb}$$

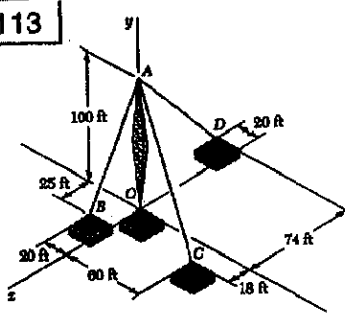
$$T_{AB} = \frac{21}{5} (300 \text{ lb} - \frac{20}{126} \times 591.818 \text{ lb}) \quad T_{AB} = 1081.82 \text{ lb}$$

$$\text{SUBSTITUTE FOR THE TENSIONS IN (2') AND SOLVE FOR } P:$$

$$P = 500 \text{ lb} + \frac{20}{21} (1081.82 \text{ lb}) + \frac{100}{126} (591.818 \text{ lb}) = 2000.00 \text{ lb}$$

$$\text{WEIGHT OF PLATE} = P = 2000 \text{ lb}$$

## 2.113



GIVEN:

TOWER EXERTS ON A  
AN UPWARD VERTICAL  
FORCE P OF 1800 lb.

FIND:

TENSION IN EACH  
WIRE.

SEE COLUMN ON THE LEFT FOR DERIVATION OF EQS. (1), (2), AND (3). MAKING  $P = 1800 \text{ lb}$  IN EQ. (2), WE HAVE

$$-\frac{4}{21} T_{AB} + \frac{60}{118} T_{AC} - \frac{20}{126} T_{AD} = 0 \quad (1)$$

$$-\frac{20}{21} T_{AB} - \frac{100}{118} T_{AC} - \frac{100}{126} T_{AD} + 1800 \text{ lb} = 0 \quad (2)$$

$$\frac{5}{21} T_{AB} + \frac{18}{118} T_{AC} - \frac{74}{126} T_{AD} = 0 \quad (3)$$

MULTIPLY (1) BY -74, (3) BY 20, AND ADD:

$$\frac{396}{21} T_{AB} - \frac{4080}{118} T_{AC} = 0 \quad T_{AC} = 0.545378 T_{AB} \quad (4)$$

SUBSTITUTE INTO (1):

$$\left[ -\frac{4}{21} + \frac{60}{118} (0.545378) \right] T_{AB} - \frac{20}{126} T_{AD} = 0$$

$$0.0868347 T_{AB} - \frac{20}{126} T_{AD} = 0 \quad T_{AD} = 0.547059 T_{AB} \quad (5)$$

SUBSTITUTE FOR  $T_{AC}$  AND  $T_{AD}$  INTO (2) AND SOLVE FOR  $T_{AB}$ :

$$-\frac{20}{21} T_{AB} - \frac{100}{118} (0.545378 T_{AB}) - \frac{100}{126} (0.547059 T_{AB}) + 1800 \text{ lb} = 0$$

$$1.84814 T_{AB} = 1800 \text{ lb} \quad T_{AB} = 973.636 \text{ lb} \quad (6)$$

$$T_{AB} = 974 \text{ lb}$$

SUBSTITUTING FROM (6) INTO (4):

$$T_{AC} = 0.545378 (973.636 \text{ lb}) = 531.000 \text{ lb}$$

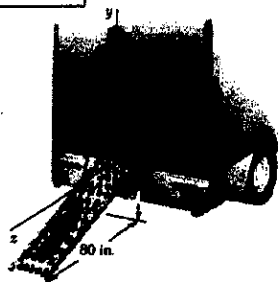
$$T_{AC} = 531 \text{ lb}$$

SUBSTITUTING FROM (6) INTO (5):

$$T_{AD} = 0.547059 (973.636 \text{ lb}) = 532.637 \text{ lb}$$

$$T_{AD} = 533 \text{ lb}$$

2.119



GIVEN:

(1) 200-lb COUNTERWEIGHT IS IN EQUILIBRIUM UNDER FORCES EXERTED BY ROPES AND FORCE PERPENDICULAR TO CHUTE.

(2) COORDINATES OF A, B, C ARE

A (0, -20 in., 40 in.)

B (-40 in., 50 in., 0)

C (45 in., 40 in., 0)

FIND:

TENSION IN EACH ROPE.

FREE BODY: COUNTERWEIGHT

$$\Sigma \mathbf{F} = \mathbf{0}:$$

$$T_{AB} + T_{AC} + \mathbf{W} + \mathbf{N} = \mathbf{0}$$

WHERE

$$\mathbf{W} = -(200 \text{ lb}) \hat{j}$$

$$\mathbf{N} = \left( \frac{2}{\sqrt{5}} \hat{j} + \frac{1}{\sqrt{5}} \hat{k} \right) N$$

WE NOTE THAT

$$\mathbf{AB} = -(40 \text{ in.}) \hat{i} + (70 \text{ in.}) \hat{j} - (40 \text{ in.}) \hat{k}$$

$$AB = 90 \text{ in.}$$

$$\mathbf{AC} = (45 \text{ in.}) \hat{i} + (60 \text{ in.}) \hat{j} - (40 \text{ in.}) \hat{k}$$

$$AC = 85 \text{ in.}$$

$$\text{THUS: } T_{AB} = T_{AB} \frac{\mathbf{AB}}{AB} = \left( -\frac{4}{9} \hat{i} + \frac{7}{9} \hat{j} - \frac{4}{9} \hat{k} \right) T_{AB}$$

$$T_{AC} = T_{AC} \frac{\mathbf{AC}}{AC} = \left( \frac{9}{17} \hat{i} + \frac{12}{17} \hat{j} - \frac{8}{17} \hat{k} \right) T_{AC}$$

SUBSTITUTE FOR  $T_{AB}$ ,  $T_{AC}$ ,  $\mathbf{N}$ , AND  $\mathbf{W}$  INTO  $\Sigma \mathbf{F} = \mathbf{0}$  AND FACTOR  $\hat{i}$ ,  $\hat{j}$ ,  $\hat{k}$ :

$$\left( -\frac{4}{9} T_{AB} + \frac{9}{17} T_{AC} \right) \hat{i} + \left( \frac{7}{9} T_{AB} + \frac{12}{17} T_{AC} + \frac{2}{\sqrt{5}} N - 200 \text{ lb} \right) \hat{j} + \left( -\frac{4}{9} T_{AB} - \frac{8}{17} T_{AC} + \frac{1}{\sqrt{5}} N \right) \hat{k} = \mathbf{0}$$

EQUATING TO ZERO THE COEFFICIENTS OF  $\hat{i}$ ,  $\hat{j}$ ,  $\hat{k}$ :

$$\textcircled{1} -\frac{4}{9} T_{AB} + \frac{9}{17} T_{AC} = 0 \quad (1)$$

$$\textcircled{2} \frac{7}{9} T_{AB} + \frac{12}{17} T_{AC} + \frac{2}{\sqrt{5}} N - 200 \text{ lb} = 0 \quad (2)$$

$$\textcircled{3} -\frac{4}{9} T_{AB} - \frac{8}{17} T_{AC} + \frac{1}{\sqrt{5}} N = 0 \quad (3)$$

MULTIPLY (3) BY -2 AND ADD (2):

$$\frac{15}{9} T_{AB} + \frac{28}{17} T_{AC} - 200 \text{ lb} = 0 \quad (4)$$

MULTIPLY (1) BY 15, (4) BY 4, AND ADD:

$$\frac{247}{17} T_{AC} - 800 \text{ lb} = 0 \quad T_{AC} = 55.061 \text{ lb} \quad (5)$$

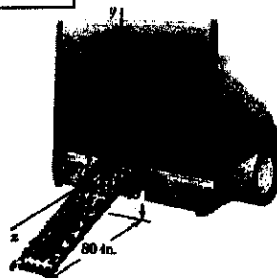
SUBSTITUTE FROM (5) INTO (1) AND SOLVE FOR  $T_{AB}$ :

$$T_{AB} = \frac{9}{4} \cdot \frac{2}{17} (55.061 \text{ lb}) = 65.587 \text{ lb}$$

THE TENSIONS IN THE ROPES ARE

$$T_{AB} = 65.6 \text{ lb}, \quad T_{AC} = 55.1 \text{ lb}$$

2.120



GIVEN:

(1) 200-lb COUNTERWEIGHT IS IN EQUILIBRIUM UNDER FORCES EXERTED BY THE TWO WORKERS SHOWN, BY A THIRD WORKER WHO PUSHES WITH  $\mathbf{P} = -(40 \text{ lb}) \hat{i}$ , AND A FORCE PERPENDICULAR TO THE CHUTE.

(2) COORDINATES OF A, B, C ARE

A (0, -20 in., 40 in.)

B (-40 in., 50 in., 0)

C (45 in., 40 in., 0)

FIND: TENSION IN ROPES AB AND AC.

FREE BODY: COUNTERWEIGHT

$$\Sigma \mathbf{F} = \mathbf{0}:$$

$$T_{AB} + T_{AC} + \mathbf{W} + \mathbf{P} + \mathbf{N} = \mathbf{0}$$

WHERE

$$\mathbf{W} = -(200 \text{ lb}) \hat{j}$$

$$\mathbf{P} = -(40 \text{ lb}) \hat{i}$$

$$\mathbf{N} = \left( \frac{2}{\sqrt{5}} \hat{j} + \frac{1}{\sqrt{5}} \hat{k} \right) N$$

WE NOTE THAT

$$\mathbf{AB} = -(40 \text{ in.}) \hat{i} + (70 \text{ in.}) \hat{j} - (40 \text{ in.}) \hat{k}$$

$$AB = 90 \text{ in.}$$

$$\mathbf{AC} = (45 \text{ in.}) \hat{i} + (60 \text{ in.}) \hat{j} - (40 \text{ in.}) \hat{k}$$

$$AC = 85 \text{ in.}$$

$$\text{THUS: } T_{AB} = T_{AB} \frac{\mathbf{AB}}{AB} = \left( -\frac{4}{9} \hat{i} + \frac{7}{9} \hat{j} - \frac{4}{9} \hat{k} \right) T_{AB}$$

$$T_{AC} = T_{AC} \frac{\mathbf{AC}}{AC} = \left( \frac{9}{17} \hat{i} + \frac{12}{17} \hat{j} - \frac{8}{17} \hat{k} \right) T_{AC}$$

SUBSTITUTE FOR  $T_{AB}$ ,  $T_{AC}$ ,  $\mathbf{N}$ ,  $\mathbf{P}$ , AND  $\mathbf{W}$  INTO  $\Sigma \mathbf{F} = \mathbf{0}$  AND FACTOR  $\hat{i}$ ,  $\hat{j}$ ,  $\hat{k}$ :

$$\left( -\frac{4}{9} T_{AB} + \frac{9}{17} T_{AC} - 40 \text{ lb} \right) \hat{i} + \left( \frac{7}{9} T_{AB} + \frac{12}{17} T_{AC} + \frac{2}{\sqrt{5}} N - 200 \text{ lb} \right) \hat{j} + \left( -\frac{4}{9} T_{AB} - \frac{8}{17} T_{AC} + \frac{1}{\sqrt{5}} N \right) \hat{k} = \mathbf{0}$$

EQUATING TO ZERO THE COEFFICIENTS OF  $\hat{i}$ ,  $\hat{j}$ ,  $\hat{k}$ :

$$\textcircled{1} -\frac{4}{9} T_{AB} + \frac{9}{17} T_{AC} - 40 \text{ lb} = 0 \quad (1)$$

$$\textcircled{2} \frac{7}{9} T_{AB} + \frac{12}{17} T_{AC} + \frac{2}{\sqrt{5}} N - 200 \text{ lb} = 0 \quad (2)$$

$$\textcircled{3} -\frac{4}{9} T_{AB} - \frac{8}{17} T_{AC} + \frac{1}{\sqrt{5}} N = 0 \quad (3)$$

MULTIPLY (3) BY -2 AND ADD (2):

$$\frac{15}{9} T_{AB} + \frac{28}{17} T_{AC} - 200 \text{ lb} = 0 \quad (4)$$

MULTIPLY (1) BY 15, (4) BY 4, AND ADD:

$$\frac{247}{17} T_{AC} - 1400 \text{ lb} = 0 \quad T_{AC} = 96.3563 \text{ lb} \quad (5)$$

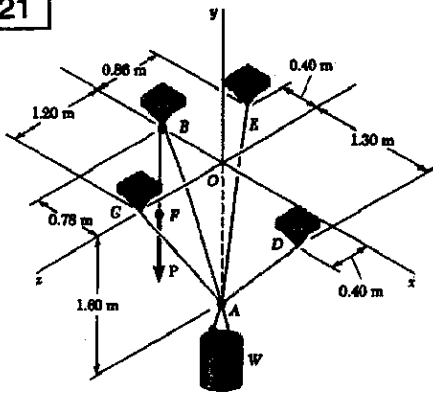
SUBSTITUTE FROM (5) INTO (1) AND SOLVE FOR  $T_{AB}$ :

$$T_{AB} = \frac{9}{4} \left[ \frac{9}{17} (96.3563 \text{ lb}) - 40 \text{ lb} \right] = 24.777 \text{ lb}$$

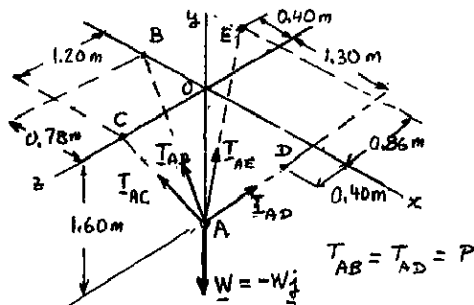
THE TENSIONS IN THE ROPES ARE

$$T_{AB} = 24.8 \text{ lb}, \quad T_{AC} = 96.4 \text{ lb}$$

## 2.121



GIVEN: CONTAINER OF WEIGHT  $W = 1000 \text{ N}$  IS SUSPENDED FROM RING A. CABLES AC AND AE ARE ATTACHED TO RING. CABLE FBAD PASSES THROUGH RING AND OVER PULLEY B. FIND: MAGNITUDE OF FORCE  $P$ .



FREE BODY: RING A

$$\sum \mathbf{F} = 0: T_{AB} + T_{AC} + T_{AD} + T_{AE} - W\mathbf{j} = 0$$

WE HAVE

$$\begin{aligned} \vec{AB} &= -(0.78\text{ m})\mathbf{i} + (1.60\text{ m})\mathbf{j} & AB &= 1.78 \text{ m} \\ \vec{AC} &= (1.60\text{ m})\mathbf{j} + (1.20\text{ m})\mathbf{k} & AC &= 2.00 \text{ m} \\ \vec{AD} &= (1.30\text{ m})\mathbf{i} + (1.60\text{ m})\mathbf{j} + (0.40\text{ m})\mathbf{k} & AD &= 2.10 \text{ m} \\ \vec{AE} &= -(0.40\text{ m})\mathbf{i} + (1.60\text{ m})\mathbf{j} - (0.86\text{ m})\mathbf{k} & AE &= 1.86 \text{ m} \end{aligned}$$

$$T_{AB} = P \frac{\vec{AB}}{AB} = P \frac{-(0.78)\mathbf{i} + (1.6)\mathbf{j}}{1.78} = \left( -\frac{0.78}{1.78}P + \frac{1.6}{1.78}P \right) \mathbf{j}$$

$$T_{AC} = P \frac{\vec{AC}}{AC} = P \frac{(1.6)\mathbf{j} + (1.2)\mathbf{k}}{2.0} = (0.8\mathbf{j} + 0.6\mathbf{k})P$$

$$T_{AD} = P \frac{\vec{AD}}{AD} = P \frac{(1.3)\mathbf{i} + (1.6)\mathbf{j} + (0.4)\mathbf{k}}{2.1} = \left( \frac{1.3}{2.1}\mathbf{i} + \frac{1.6}{2.1}\mathbf{j} + \frac{0.4}{2.1}\mathbf{k} \right)P$$

$$T_{AE} = P \frac{\vec{AE}}{AE} = P \frac{-(0.4)\mathbf{i} + (1.6)\mathbf{j} - (0.86)\mathbf{k}}{1.86} = \left( -\frac{0.4}{1.86}\mathbf{i} + \frac{1.6}{1.86}\mathbf{j} - \frac{0.86}{1.86}\mathbf{k} \right)P$$

SUBSTITUTING FOR THE TENSIONS IN  $\sum \mathbf{F} = 0$  AND FACTORING  $\mathbf{i}, \mathbf{j}, \mathbf{k}$ :

$$\begin{aligned} &\left( -\frac{0.78}{1.78}P + \frac{1.3}{2.1}P - \frac{0.4}{1.86}T_{AE} \right)\mathbf{i} \\ &+ \left( \frac{1.6}{1.78}P + 0.8T_{AC} + \frac{1.6}{2.1}P + \frac{1.6}{1.86}T_{AE} - W \right)\mathbf{j} \\ &+ \left( 0.6T_{AC} + \frac{0.4}{2.1}P - \frac{0.86}{1.86}T_{AE} \right)\mathbf{k} = 0 \end{aligned}$$

EQUATING TO ZERO THE COEFFICIENTS OF  $\mathbf{i}, \mathbf{j}, \mathbf{k}$ , WE OBTAIN AFTER REDUCTIONS:

CONTINUED

## 2.121 CONTINUED

$$\textcircled{1} 0.180845P - 0.215054T_{AE} \quad (1)$$

$$\textcircled{4} 1.66078P + 0.8T_{AC} + 0.860215T_{AE} - W = 0 \quad (2)$$

$$\textcircled{5} 0.190476P + 0.6T_{AC} - 0.462366T_{AE} = 0 \quad (3)$$

$$\text{SOLVING (1) FOR } T_{AE}: T_{AE} = 0.840931P$$

CARRYING INTO Eqs. (2) AND (3):

$$2.38416P + 0.8T_{AC} - W = 0 \quad (4)$$

$$-0.198342P + 0.6T_{AC} = 0 \quad (5)$$

MULTIPLY (4) BY 3, (5) BY -4, AND ADD:

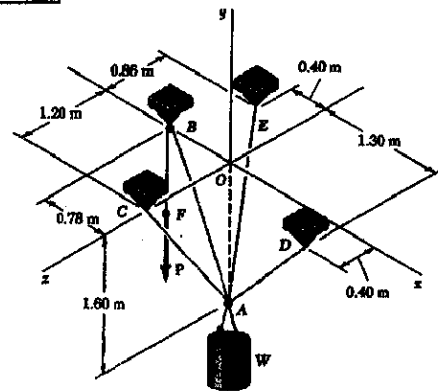
$$7.94525P - 3W = 0$$

MAKING  $W = 1000 \text{ N}$ :

$$7.94525P - 3000 \text{ N} = 0 \quad P = 377.556 \text{ N}$$

$$P = 378 \text{ N}$$

## 2.122



GIVEN:

(1) CONTAINER IS SUSPENDED FROM RING A.

CABLES AC AND AE ARE ATTACHED TO RING.

CABLE FBAD PASSES THROUGH RING AND OVER PULLEY B.

(2)  $T_{AC} = 150 \text{ N}$ .

FIND:

(a) MAGNITUDE OF FORCE  $P$

(b) WEIGHT  $W$  OF CONTAINER

SEE SOLUTION OF PROB. 2.121 LEADING TO Eqs. (4) AND (5):

$$2.38416P + 0.8T_{AC} - W = 0 \quad (4)$$

$$-0.198342P + 0.6T_{AC} = 0 \quad (5)$$

(a) MAKE  $T_{AC} = 150 \text{ N}$  IN EQ. (5):

$$-0.198342P + 0.6(150 \text{ N}) = 0$$

$$P = 453.762 \text{ N}$$

$$P = 454 \text{ N}$$

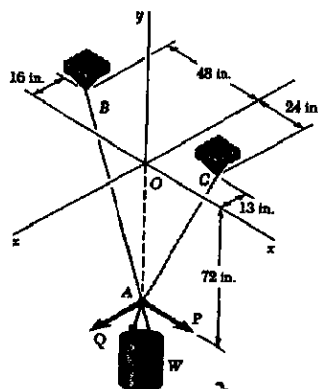
(b) CARRY THE VALUES OF  $T_{AC}$  AND  $P$  INTO EQ. (4):

$$2.38416(453.762 \text{ N}) + 0.8(150 \text{ N}) - W = 0$$

$$W = 1201.84 \text{ N}$$

$$W = 1202 \text{ N}$$

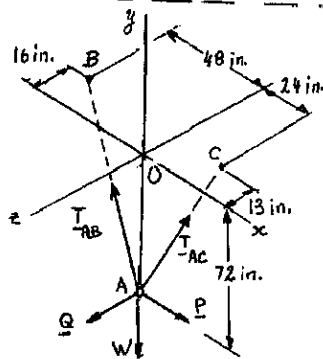
2.123



GIVEN: CONTAINER OF WEIGHT  $W = 270 \text{ lb}$  IS SUSPENDED FROM RING A.

CABLE BAC PASSES THROUGH RING A.

FIND:  $P$  AND  $Q$  FOR EQUILIBRIUM POSITION SHOWN.



FREE BODY: RING A

$$\sum \underline{F} = 0:$$

$$T_{AB} + T_{AC} + P + Q + W = 0$$

WHERE  $P = P \underline{i}$

$$Q = Q \underline{k}$$

$$W = -W \underline{j}$$

$$T_{AB} = T \underline{\lambda}_{AB}$$

$$T_{AC} = T \underline{\lambda}_{AC}$$

(SAME TENSION  $T$  IN BOTH PORTIONS OF CABLE)

WE HAVE

$$\underline{AB} = -(48 \text{ in.}) \underline{i} + (72 \text{ in.}) \underline{j} - (16 \text{ in.}) \underline{k} \quad AB = 88 \text{ in.}$$

$$\underline{AC} = (24 \text{ in.}) \underline{i} + (72 \text{ in.}) \underline{j} - (13 \text{ in.}) \underline{k} \quad AC = 77 \text{ in.}$$

$$T_{AB} = T \underline{\lambda}_{AB} = T \frac{\underline{AB}}{AB} = \left( -\frac{6}{11} \underline{i} + \frac{9}{11} \underline{j} - \frac{2}{11} \underline{k} \right) T$$

$$T_{AC} = T \underline{\lambda}_{AC} = T \frac{\underline{AC}}{AC} = \left( \frac{24}{77} \underline{i} + \frac{72}{77} \underline{j} - \frac{13}{77} \underline{k} \right) T$$

SUBSTITUTING FOR  $T_{AB}$ ,  $T_{AC}$ ,  $P$ ,  $Q$ , AND  $W$  INTO  $\sum \underline{F} = 0$  AND FACTORING  $\underline{i}$ ,  $\underline{j}$ ,  $\underline{k}$ :

$$\left( -\frac{6}{11} T + \frac{24}{77} T + P \right) \underline{i} + \left( \frac{9}{11} T + \frac{72}{77} T - W \right) \underline{j} + \left( -\frac{2}{11} T - \frac{13}{77} T + Q \right) \underline{k} = 0$$

SETTING THE COEFFICIENTS OF  $\underline{i}$ ,  $\underline{j}$ ,  $\underline{k}$  EQUAL TO ZERO AND REDUCING:

$$\textcircled{1} \quad -\frac{10}{77} T + P = 0 \quad (1)$$

$$\textcircled{2} \quad \frac{135}{77} T - W = 0 \quad (2)$$

$$\textcircled{3} \quad -\frac{27}{77} T + Q = 0 \quad (3)$$

MAKING  $W = 270 \text{ lb}$  IN EQ. (2) AND SOLVING FOR  $T$ :

$$T = \frac{77}{135} (270 \text{ lb}) = 154.0 \text{ lb}$$

SUBSTITUTING FOR  $T$  IN EQS. (1) AND (3), WE OBTAIN

$$P = 36.0 \text{ lb}, \quad Q = 54.0 \text{ lb}$$

2.124

(SEE FIGURE ON THE LEFT)

GIVEN: (1)  $Q = 36 \text{ lb}$ .

(2) CABLE BAC PASSES THROUGH RING A.

FIND:  $W$  AND  $P$ .

SEE SOLUTION AT LEFT FOR DERIVATION OF EQS. (1), (2), (3). MAKING  $Q = 36 \text{ lb}$  IN EQ. (3):

$$-\frac{27}{77} T + 36 \text{ lb} = 0 \quad T = \frac{77}{27} (36 \text{ lb}) \quad T = 102.667 \text{ lb}$$

SUBSTITUTING FOR  $T$  IN EQS. (1) AND (2):

$$-\frac{10}{77} (102.667 \text{ lb}) + P = 0$$

$$P = 24.0 \text{ lb}$$

$$\frac{135}{77} (102.667 \text{ lb}) - W = 0$$

$$W = 180.0 \text{ lb}$$

2.125

GIVEN:

(1) COLLARS A AND B CONNECTED BY WIRE OF LENGTH  $525 \text{ mm}$

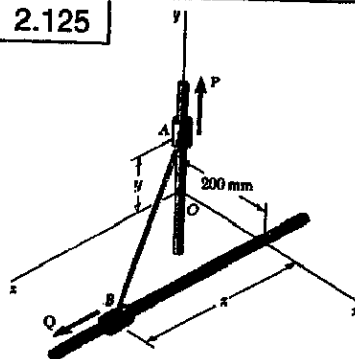
(2)  $P = (341 \text{ N}) \underline{i}$

(3)  $y = 155 \text{ mm}$

FIND:

(a)  $T_{AB}$

(b)  $Q$  FOR EQUILIBRIUM



$$(AB)^2 = x^2 + y^2 + z^2: (525 \text{ mm})^2 = (200 \text{ mm})^2 + (155 \text{ mm})^2 + z^2$$

$$z = 460 \text{ mm}$$

$$\underline{AB} = (200 \text{ mm}) \underline{i} - (155 \text{ mm}) \underline{j} + (460 \text{ mm}) \underline{k} \quad AB = 525 \text{ mm}$$

$$\underline{\lambda}_{AB} = \frac{\underline{AB}}{AB} = \frac{200}{525} \underline{i} - \frac{155}{525} \underline{j} + \frac{460}{525} \underline{k}$$

(a) FREE BODY: COLLAR A

$$\sum \underline{F} = 0;$$

$$N_x \underline{i} + P \underline{j} + N_z \underline{k} + T_{AB} \underline{\lambda}_{AB} = 0$$

SUBSTITUTING FOR  $\underline{\lambda}_{AB}$  AND SETTING THE COEFF. OF  $\underline{j}$  EQUAL TO ZERO:

$$P + \left( -\frac{155}{525} T_{AB} \right) = 0$$

MAKING  $P = 341 \text{ N}$  AND SOLVING FOR  $T_{AB}$ :

$$T_{AB} = \frac{525}{155} (341 \text{ N})$$

$$T_{AB} = 1155 \text{ N}$$

(b) FREE BODY: COLLAR B

$$\sum \underline{F} = 0$$

$$N'_x \underline{i} + N'_y \underline{j} + Q \underline{k} - T_{AB} \underline{\lambda}_{AB} = 0$$

SUBSTITUTING FOR  $\underline{\lambda}_{AB}$  AND SETTING THE COEFF. OF  $\underline{k}$  EQUAL TO ZERO:

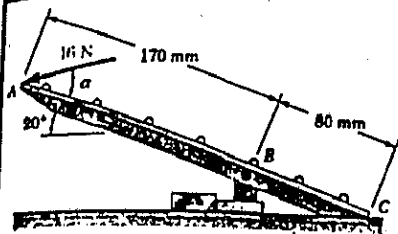
$$Q - \left( \frac{460}{525} T_{AB} \right) = 0$$

MAKING  $T_{AB} = 1155 \text{ N}$  AND SOLVING FOR  $Q$ :

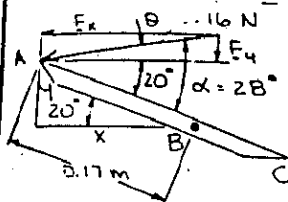
$$Q = \frac{460}{525} (1155 \text{ N})$$

$$Q = 1012 \text{ N}$$

3.1



GIVEN:  $\alpha = 28^\circ$   
 FIND: MOMENT OF FORCE ABOUT B (RESOLVE FORCE INTO HORIZONTAL AND VERTICAL COMPONENTS)



FIRST NOTE THAT  $\theta = 28^\circ - 20^\circ = 8^\circ$

THEN

$$F_x = (16 \text{ N}) \cos 8^\circ = 15.8443 \text{ N}$$

$$F_y = (16 \text{ N}) \sin 8^\circ = 2.2268 \text{ N}$$

$$\text{AND } x = (0.17 \text{ m}) \cos 20^\circ = 0.159748 \text{ m}$$

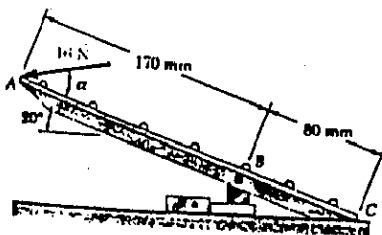
$$y = (0.17 \text{ m}) \sin 20^\circ = 0.058143 \text{ m}$$

NOTING THAT THE DIRECTION OF THE MOMENT OF EACH FORCE COMPONENT ABOUT B IS COUNTERCLOCKWISE, HAVE

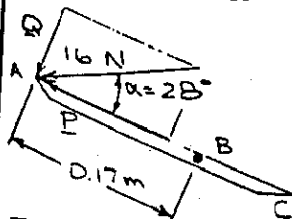
$$\begin{aligned} M_B &= xF_y + yF_x \\ &= (0.159748 \text{ m})(2.2268 \text{ N}) + (0.058143 \text{ m})(15.8443 \text{ N}) \\ &= 1.277 \text{ N} \end{aligned}$$

$$\text{OR } \underline{M_B = 1.277 \text{ N} \cdot \text{m}}$$

3.2



GIVEN:  $\alpha = 28^\circ$   
 FIND: MOMENT OF FORCE ABOUT B (RESOLVE FORCE INTO COMPONENTS PARALLEL AND PERPENDICULAR TO ABC)



FIRST RESOLVE THE 16-N FORCE INTO COMPONENTS P AND Q, WHERE  $Q = (16 \text{ N}) \sin 28^\circ = 7.5115 \text{ N}$

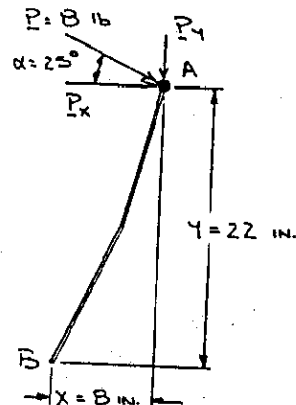
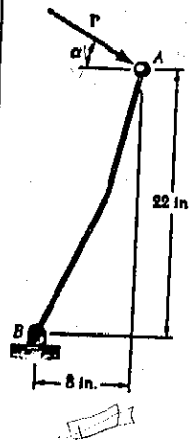
$$\text{THEN } M_B = r_{AB} Q$$

$$\begin{aligned} &= (0.17 \text{ m})(7.5115 \text{ N}) \\ &= 1.277 \text{ N} \cdot \text{m} \end{aligned}$$

$$\text{OR } \underline{M_B = 1.277 \text{ N} \cdot \text{m}}$$

3.3

GIVEN:  $P = 8 \text{ lb}$ ,  $\alpha = 25^\circ$   
 FIND: MOMENT OF FORCE ABOUT B



FIRST NOTE --

$$P_x = (8 \text{ lb}) \cos 25^\circ = 7.2505 \text{ lb}$$

$$P_y = (8 \text{ lb}) \sin 25^\circ = 3.3809 \text{ lb}$$

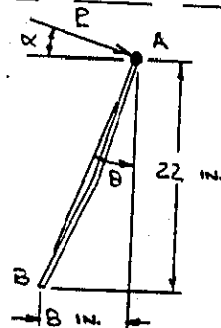
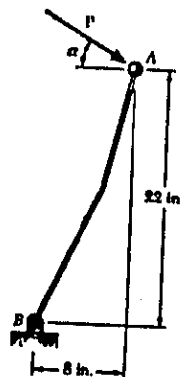
NOTING THAT THE DIRECTION OF THE MOMENT OF EACH FORCE COMPONENT ABOUT B IS CLOCKWISE, HAVE

$$\begin{aligned} M_B &= -xP_y - yP_x \\ &= -(8 \text{ in.})(3.3809 \text{ lb}) - (22 \text{ in.})(7.2505 \text{ lb}) \\ &= -186.6 \text{ lb} \cdot \text{in.} \end{aligned}$$

$$\text{OR } \underline{M_B = 186.6 \text{ lb} \cdot \text{in.}}$$

3.4

GIVEN:  $M_B = 210 \text{ lb} \cdot \text{in.}$   
 FIND:  $(P)_{\min}$



FOR P TO BE MINIMUM, IT MUST BE PERPENDICULAR TO THE LINE JOINING POINTS A AND B. THUS,

$$\begin{aligned} \alpha &= \theta \\ &= \tan^{-1} \frac{8}{22} \\ &= 19.98^\circ \end{aligned}$$

$$\text{AND } M_B = d P_{\min}$$

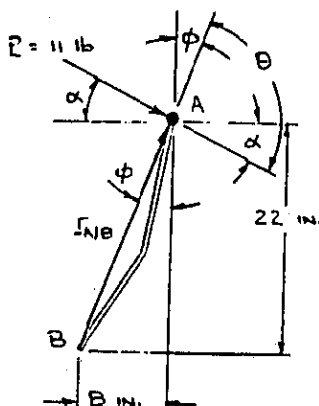
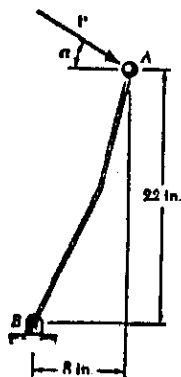
$$\text{WHERE } d = r_{AB} = \sqrt{(8 \text{ in.})^2 + (22 \text{ in.})^2} = 23.409 \text{ in.}$$

$$\begin{aligned} \text{THEN } P_{\min} &= \frac{210 \text{ lb} \cdot \text{in.}}{23.409 \text{ in.}} \\ &= 8.97 \text{ lb} \end{aligned}$$

$$\underline{P_{\min} = 8.97 \text{ lb} \quad \angle 19.98^\circ}$$



3.5

GIVEN:  $P = 11 \text{ lb}$ ,  $M_B = 250 \text{ lb}\cdot\text{in.}$ FIND:  $\alpha$ 

BY DEFINITION...  $M_B = r_{A/B} P \sin \theta$   
 WHERE  $\theta = \alpha + (90^\circ - \phi)$   
 AND  $\phi = \tan^{-1} \frac{8}{22} = 19.9831^\circ$

ALSO --  $r_{A/B} = \sqrt{(8 \text{ in.})^2 + (22 \text{ in.})^2} = 23.409 \text{ in.}$   
 THEN...  $250 \text{ lb}\cdot\text{in.} = (23.409 \text{ in.})(11 \text{ lb})$   
 $\times \sin(\alpha + 90^\circ - 19.9831^\circ)$

OR  $\sin(\alpha + 70.0169^\circ) = 0.97588$

OR  $\alpha + 70.0169^\circ = 76.1391^\circ$

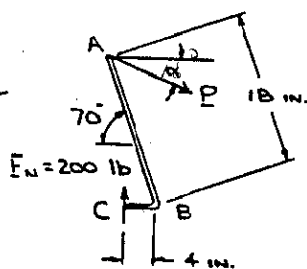
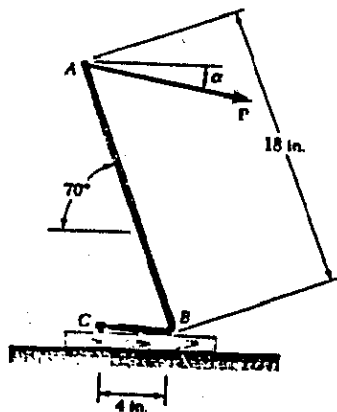
AND  $\alpha + 70.0169^\circ = 103.861^\circ$

$\alpha = 6.12^\circ, 33.8^\circ$

3.6

GIVEN:  $F_N = 200 \text{ lb}$  ↑  
FIND:

- (a) MOMENT  $M_B$  OF  $F_N$  ABOUT B  
 (b)  $P$  GIVEN  $M_B$   
 AND  $\alpha = 10^\circ$   
 (c)  $P_{\min}$  GIVEN  $M_B$

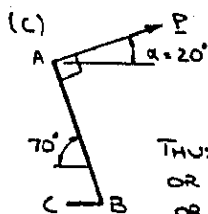


(a) HAVE  $M_B = r_{A/B} F_N$   
 $= (4 \text{ in.})(200 \text{ lb})$   
 $= 800 \text{ lb}\cdot\text{in.}$   
 OR  $M_B = 800 \text{ lb}\cdot\text{in.}$

(b) BY DEFINITION  $M_B = r_{A/B} P \sin \theta$   
 WHERE  $\theta = 10^\circ + (180^\circ - 70^\circ) = 120^\circ$   
 THEN  $800 \text{ lb}\cdot\text{in.} = (18 \text{ in.})$   
 $\times P \sin 120^\circ$   
 OR  $P = 51.3 \text{ lb}$

(CONTINUED)

3.6 CONTINUED



FOR  $P$  TO BE MINIMUM,  
 IT MUST BE PERPENDICULAR  
 TO THE LINE JOINING POINTS  
 A AND B. THUS,  $P$  MUST  
 BE DIRECTED AS SHOWN.

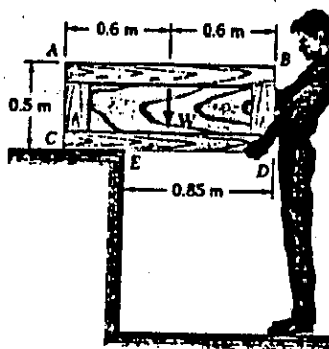
THUS --  $M_B = d P_{\min}$   $d = r_{A/B}$   
 OR  $800 \text{ lb}\cdot\text{in.} = (18 \text{ in.}) P_{\min}$   
 OR  $P_{\min} = 44.4 \text{ lb}$   
 $P_{\min} = 44.4 \text{ lb}$   $\angle 20^\circ$

3.7

GIVEN: MASS  $m$  OF  
CRATE = 80 kg

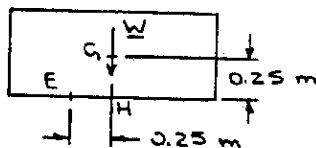
FIND:

- (a) MOMENT  $M_E$   
 OF WEIGHT  $W$   
 ABOUT E  
 (b)  $(F_B)_{\min}$  GIVEN  
 $-M_E$

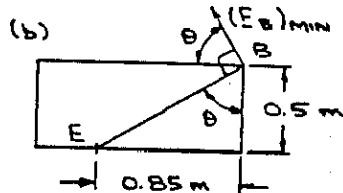


FIRST NOTE --

$W = mg$   
 $= (80 \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2})$   
 $= 784.8 \text{ N}$



(a) HAVE  $M_E = r_{G/E} W$   
 $= (0.25 \text{ m})(784.8 \text{ N})$   
 $= 196.2 \text{ N}\cdot\text{m}$   
 OR  $M_E = 196.2 \text{ N}\cdot\text{m}$



B AND E. THEN, WITH  $F_B$  DIRECTED AS  
 SHOWN, HAVE

$(-M_E) = r_{B/E} (F_B)_{\min}$

WHERE  $r_{B/E} = \sqrt{(0.85 \text{ m})^2 + (0.5 \text{ m})^2}$   
 $= 0.98615 \text{ m}$

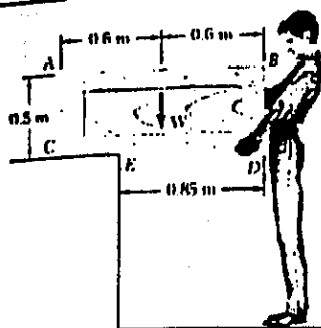
THEN  $196.2 \text{ N}\cdot\text{m} = (0.98615 \text{ m})(F_B)_{\min}$   
 OR  $(F_B)_{\min} = 199.0 \text{ N}$

ALSO --  $\tan \theta = \frac{0.85 \text{ m}}{0.5 \text{ m}}$

OR  $\theta = 59.5^\circ$

$(F_B)_{\min} = 199.0 \text{ N}$   $\angle 59.5^\circ$

3.8



GIVEN: MASS  $m$  OF CRATE = 80 kg

FIND:

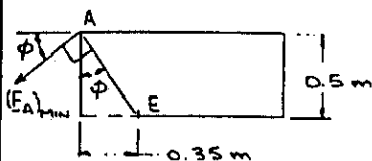
- (a) MOMENT  $M_E$  OF WEIGHT  $W$  ABOUT  $E$   
 (b)  $(F_A)_{\min}$  GIVEN  $-M_E$   
 (c)  $(F_{\text{VERTICAL}})_{\min}$  GIVEN  $-M_E$

FIRST NOTE...

$$W = mg = (80 \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2}) = 784.8 \text{ N}$$

(a) HAVE  $M_E = r_{W/E} W = (0.25 \text{ m})(784.8 \text{ N}) = 196.2 \text{ N}\cdot\text{m}$   
 OR  $M_E = 196.2 \text{ N}\cdot\text{m}$

(b)



FOR  $F_A$  TO BE MINIMUM, IT MUST BE PERPENDICULAR TO THE LINE JOINING POINTS A AND E. THEN,

WITH  $F_A$  DIRECTED AS SHOWN, HAVE  $(-M_E) = r_{A/E} (F_A)_{\min}$

WHERE  $r_{A/E} = \sqrt{(0.35 \text{ m})^2 + (0.5 \text{ m})^2} = 0.61033 \text{ m}$   
 THEN  $196.2 \text{ N}\cdot\text{m} = (0.61033 \text{ m})(F_A)_{\min}$   
 OR  $(F_A)_{\min} = 321 \text{ N}$

ALSO...  $\tan \phi = \frac{0.35 \text{ m}}{0.5 \text{ m}}$  OR  $\phi = 35.0^\circ$

$(F_A)_{\min} = 321 \text{ N} \nearrow 35.0^\circ$

(c) FOR  $F_{\text{VERTICAL}}$  TO BE MINIMUM, THE PERPENDICULAR DISTANCE FROM ITS LINE OF ACTION TO POINT E MUST BE MAXIMUM. THUS, APPLY  $(F_{\text{VERTICAL}})_{\min}$  AT POINT D, AND THEN

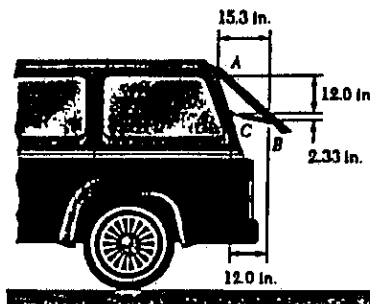
$(-M_E) = r_{D/E} (F_{\text{VERTICAL}})_{\min}$

$196.2 \text{ N}\cdot\text{m} = (0.85 \text{ m})(F_{\text{VERTICAL}})_{\min}$

OR  $(F_{\text{VERTICAL}})_{\min} = 231 \text{ N}$  AT POINT D

3.9

GIVEN:  $F_{CB} = 125 \text{ lb}$   
 FIND: MOMENT OF  $F_{CB}$  ABOUT A



FIRST NOTE...  $d_{CB} = \sqrt{(12 \text{ in.})^2 + (2.33 \text{ in.})^2} = 12.2241 \text{ in.}$

THEN  $\cos \theta = \frac{12.0}{12.2241}$

$\sin \theta = \frac{2.33}{12.2241}$

AND  $F_{CB} = F_{CB} \cos \theta \mathbf{i} - F_{CB} \sin \theta \mathbf{j}$   
 $= \frac{125 \text{ lb}}{12.2241} (12.0 \mathbf{i} - 2.33 \mathbf{j})$

NOW...  $M_A = r_{B/A} \times F_{CB}$

WHERE  $r_{B/A} = (15.3 \text{ in.}) \mathbf{i} - (14.33 \text{ in.}) \mathbf{j}$

THEN...  $M_A = [(15.3 \text{ in.}) \mathbf{i} - (14.33 \text{ in.}) \mathbf{j}]$

$\times \frac{125 \text{ lb}}{12.2241} (12.0 \mathbf{i} - 2.33 \mathbf{j})$

$= -(364.54 \text{ lb}\cdot\text{in.}) \mathbf{k} + (1758.41 \text{ lb}\cdot\text{in.}) \mathbf{k}$

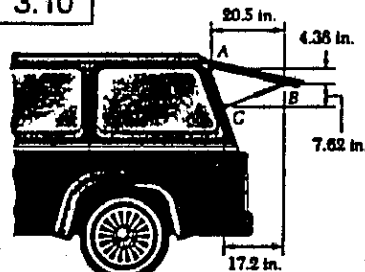
$= (1393.87 \text{ lb}\cdot\text{in.}) \mathbf{k}$

$= (116.2 \text{ lb}\cdot\text{ft}) \mathbf{k}$

$M_A = 116.2 \text{ lb}\cdot\text{ft}$

3.10

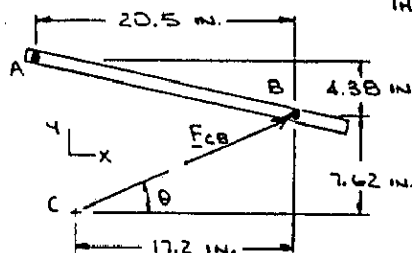
GIVEN:  $F_{CB} = 125 \text{ lb}$   
 FIND: MOMENT OF  $F_{CB}$  ABOUT A



FIRST NOTE...  $d_{CB} = \sqrt{(17.2 \text{ in.})^2 + (7.62 \text{ in.})^2} = 18.8123 \text{ in.}$

THEN  $\cos \theta = \frac{17.2}{18.8123}$

$\sin \theta = \frac{7.62}{18.8123}$



(CONTINUED)

### 3.10 CONTINUED

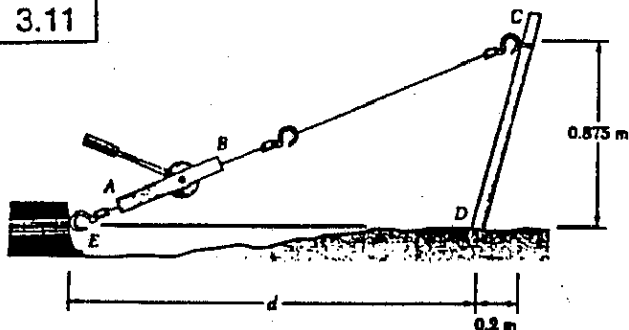
$$\text{AND } F_{CB} = F_{CB} \cos \theta \mathbf{i} + F_{CB} \sin \theta \mathbf{j} \\ = \frac{125 \text{ lb}}{18.8123} (17.2 \mathbf{i} + 7.62 \mathbf{j})$$

$$\text{NOW.. } \underline{M_A} = \underline{r_{B/A}} \times \underline{F_{CB}}$$

$$\text{WHERE } \underline{r_{B/A}} = (20.5 \text{ in.}) \mathbf{i} - (4.38 \text{ in.}) \mathbf{j}$$

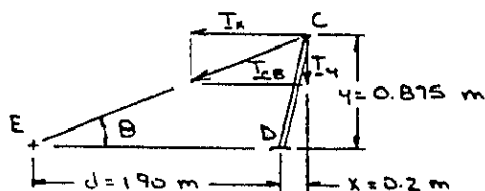
$$\text{THEN.. } \underline{M_A} = [(20.5 \text{ in.}) \mathbf{i} - (4.38 \text{ in.}) \mathbf{j}] \\ \times \frac{125 \text{ lb}}{18.8123} (17.2 \mathbf{i} + 7.62 \mathbf{j}) \\ = (1037.95 \text{ lb} \cdot \text{in.}) \mathbf{k} + (500.58 \text{ lb} \cdot \text{in.}) \mathbf{k} \\ = (1538.53 \text{ lb} \cdot \text{in.}) \mathbf{k} \\ = (128.2 \text{ lb} \cdot \text{ft}) \mathbf{k} \\ \underline{M_A} = 128.2 \text{ lb} \cdot \text{ft} \quad \blacktriangleleft$$

### 3.11



GIVEN:  $T_{CB} = 1040 \text{ N}$ ,  $d = 1.90 \text{ m}$ .

FIND: Moment of  $T_{CB}$  about D; RESOLVE  $T_{CB}$  INTO HORIZONTAL AND VERTICAL COMPONENTS APPLIED AT  
(a) POINT C  
(b) POINT E



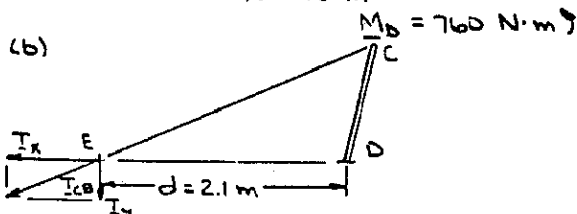
$$\text{FIRST NOTE.. } d_{CE} = \sqrt{(2.1 \text{ m})^2 + (0.875 \text{ m})^2} \\ = 2.275 \text{ m}$$

$$\text{THEN } \cos \theta = \frac{2.1}{2.275} = \frac{12}{13} \quad \sin \theta = \frac{0.875}{2.275} = \frac{5}{13}$$

$$\text{AND } T_x = T_{CB} \cos \theta = (1040 \text{ N}) \left( \frac{12}{13} \right) = 960 \text{ N}$$

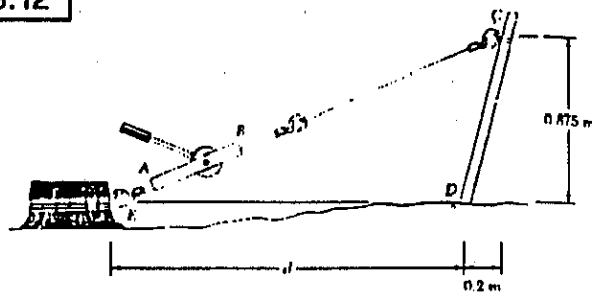
$$T_y = T_{CB} \sin \theta = (1040 \text{ N}) \left( \frac{5}{13} \right) = 400 \text{ N}$$

$$\text{(a) BY OBSERVATION.. } M_D = -x T_y + y T_x \\ \text{OR } M_D = (0.2 \text{ m})(400 \text{ N}) + (0.875 \text{ m})(960 \text{ N}) \\ = 760 \text{ N} \cdot \text{m} \quad \blacktriangleleft$$



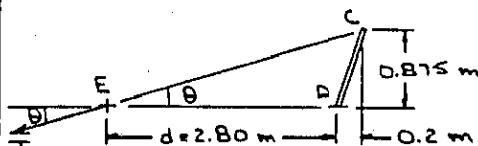
$$\text{BY OBSERVATION.. } M_D = d T_y \\ = (1.90 \text{ m})(400 \text{ N}) \\ = 760 \text{ N} \cdot \text{m} \\ \underline{M_D} = 760 \text{ N} \cdot \text{m} \quad \blacktriangleleft$$

### 3.12



GIVEN: Moment of  $T_{BA}$  about D =  $960 \text{ N} \cdot \text{m}$   
 $d = 2.80 \text{ m}$

FIND:  $T_{BA}$



$$\text{FIRST NOTE.. } d_{CE} = \sqrt{(3.0 \text{ m})^2 + (0.875 \text{ m})^2} \\ = 3.125 \text{ m}$$

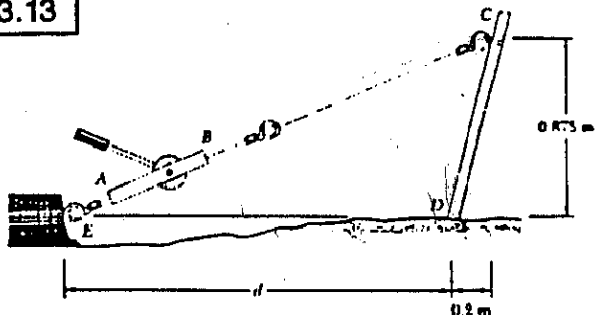
$$\text{THEN } \sin \theta = \frac{0.875}{3.125} = \frac{7}{25}$$

WITH  $T_{BA}$  APPLIED AT POINT E, HAVE

$$M_D = d (T_{BA} \sin \theta)$$

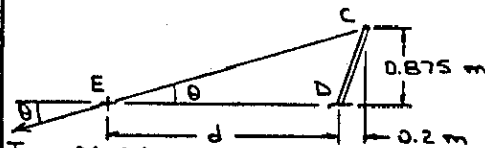
$$\text{OR } 960 \text{ N} \cdot \text{m} = (2.80 \text{ m}) (T_{BA} \cdot \frac{7}{25}) \\ \text{OR } T_{BA} = 1224 \text{ N} \quad \blacktriangleleft$$

### 3.13



GIVEN: Moment of  $T_{BA}$  about D =  $960 \text{ N} \cdot \text{m}$   
 $(T_{BA})_{\text{max}} = 2400 \text{ N}$

FIND:  $d_{\text{min}}$



$$T_{BA} = 2400 \text{ N}$$

WITH  $T_{BA}$  APPLIED AT POINT E, HAVE

$$M_D = d (T_{BA} \sin \theta)$$

$$\text{WHERE } \sin \theta = \frac{0.875}{\sqrt{(d+0.2)^2 + (0.875)^2}}$$

$$\text{THEN.. } 960 \text{ N} \cdot \text{m} = (d \text{ m}) (2400 \text{ N}) \left( \frac{0.875}{\sqrt{(d+0.2)^2 + (0.875)^2}} \right)$$

$$\text{OR } \sqrt{(d+0.2)^2 + (0.875)^2} = 2.1875 d$$

SQUARING BOTH SIDES OF THE EQUATION..

$$d^2 + 0.4d + 0.04 + 0.7656 = 4.7852 d^2$$

$$\text{OR } 3.7852 d^2 - 0.4d - 0.8056 = 0$$

(CONTINUED)

### 3.13 CONTINUED

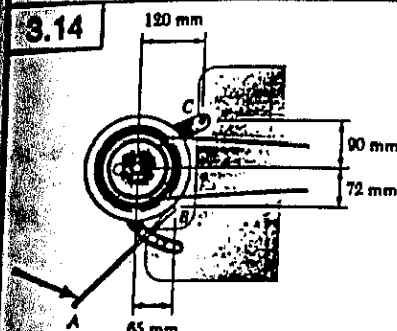
THEN  $d = \frac{0.4 \pm \sqrt{(-0.4)^2 - 4(3.7852)(-0.8056)}}{2(3.7852)}$

REJECTING THE NEGATIVE ROOT

$d = 0.517 \text{ m}$

$d = 517 \text{ mm}$

### 3.14



GIVEN:  $F_B = 485 \text{ N}$ ,  
LINE OF  
ACTION OF  $F_B$   
PASSES  
THROUGH O  
FIND: MOMENT OF  
 $F_B$  ABOUT C

FIRST NOTE...

$d_{OB} = \sqrt{(65 \text{ mm})^2 + (72 \text{ mm})^2}$

$= 97 \text{ mm}$

THEN  $\cos \theta = \frac{65}{97}$   
 $\sin \theta = \frac{72}{97}$

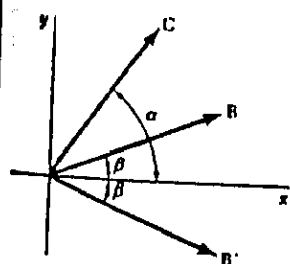
AND  $F_x = F_B \cos \theta = (485 \text{ N})\left(\frac{65}{97}\right) = 325 \text{ N}$   
 $F_y = F_B \sin \theta = (485 \text{ N})\left(\frac{72}{97}\right) = 360 \text{ N}$

BY OBSERVATION...  $M_C = -x F_y - y F_x$   
WHERE  $x = 0.12 \text{ m} - 0.065 \text{ m} = 0.055 \text{ m}$   
 $y = 0.072 \text{ m} + 0.09 \text{ m} = 0.162 \text{ m}$

THEN  $M_C = -(0.055 \text{ m})(360 \text{ N}) - (0.162 \text{ m})(325 \text{ N})$   
 $= -72.45 \text{ N}\cdot\text{m}$

$M_C = 72.5 \text{ N}\cdot\text{m}$

### 3.15



GIVEN: VECTORS  $B$ ,  $B'$

AND  $C$   
PROVE:  $\sin \alpha \cos \beta$   
 $= \frac{1}{2} \sin(\alpha + \beta)$   
 $+ \frac{1}{2} \sin(\alpha - \beta)$

FIRST NOTE...  $B = B(\cos \beta \mathbf{i} + \sin \beta \mathbf{j})$

$B' = B(\cos \beta \mathbf{i} - \sin \beta \mathbf{j})$

$C = C(\cos \alpha \mathbf{i} + \sin \alpha \mathbf{j})$

BY DEFINITION...  $|B \times C| = BC \sin(\alpha - \beta)$  (1)

$|B' \times C| = BC \sin(\alpha + \beta)$  (2)

(CONTINUED)

### 3.15 CONTINUED

NOW  $B \times C = B(\cos \beta \mathbf{i} + \sin \beta \mathbf{j}) \times C(\cos \alpha \mathbf{i} + \sin \alpha \mathbf{j})$   
 $= BC(\cos \beta \sin \alpha - \sin \beta \cos \alpha) \mathbf{k}$  (3)  
AND  $B' \times C = B(\cos \beta \mathbf{i} - \sin \beta \mathbf{j}) \times C(\cos \alpha \mathbf{i} + \sin \alpha \mathbf{j})$   
 $= BC(\cos \beta \sin \alpha + \sin \beta \cos \alpha) \mathbf{k}$  (4)

EQUATING THE RIGHT-HAND SIDES OF EQS. (1) AND (2) TO THE MAGNITUDES OF THE RIGHT-HAND SIDES OF EQS. (3) AND (4), RESPECTIVELY, YIELDS...

$BC \sin(\alpha - \beta) = BC(\cos \beta \sin \alpha - \sin \beta \cos \alpha)$  (5)

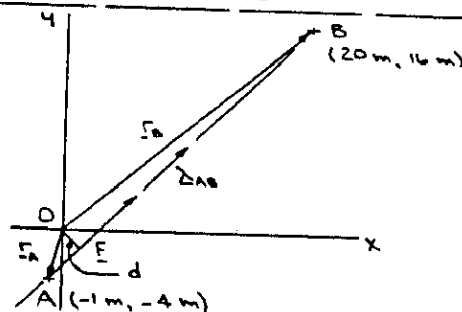
$BC \sin(\alpha + \beta) = BC(\cos \beta \sin \alpha + \sin \beta \cos \alpha)$  (6)

(5) + (6)  $\Rightarrow \sin(\alpha - \beta) + \sin(\alpha + \beta) = 2 \cos \beta \sin \alpha$   
OR  $\sin \alpha \cos \beta = \frac{1}{2} \sin(\alpha + \beta) + \frac{1}{2} \sin(\alpha - \beta)$

### 3.16

GIVEN: POINTS (20 m, 16 m) AND (-1 m, -4 m)

FIND: PERPENDICULAR DISTANCE  $d$  FROM THE ORIGIN TO THE LINE DRAWN THROUGH THE POINTS



FIRST NOTE...  $d_{AB} = \sqrt{[20 \text{ m} - (-1 \text{ m})]^2 + [16 \text{ m} - (-4 \text{ m})]^2}$   
 $= 29 \text{ m}$

NOW ASSUME THAT A FORCE  $F$ , OF MAGNITUDE  $F$ , ACTS AT POINT A AND IS DIRECTED FROM A TO B. THEN  $F = F \lambda_{AB}$  ( $F$  IN N)

WHERE  $\lambda_{AB} = \frac{r_B - r_A}{d_{AB}}$   
 $= \frac{1}{29} (21 \mathbf{i} + 20 \mathbf{j})$

BY DEFINITION...  $M_O = |r_A \times F| = dF$   
WHERE  $r_A = (-1 \text{ m}) \mathbf{i} - (4 \text{ m}) \mathbf{j}$

THEN  $M_O = [(-1 \text{ m}) \mathbf{i} - (4 \text{ m}) \mathbf{j}] \times \frac{F}{29} (21 \mathbf{i} + 20 \mathbf{j}) (\text{N})$   
 $= \frac{F}{29} [-(20) \mathbf{k} + (84) \mathbf{k}] \text{ N}\cdot\text{m}$   
 $= \left(\frac{64}{29} F \text{ N}\cdot\text{m}\right) \mathbf{k}$

FINALLY...  $\left(\frac{64}{29} F\right) \text{ N}\cdot\text{m} = d(F \text{ N})$

OR  $d = \frac{64}{29} \text{ m}$   
 $d = 2.21 \text{ m}$

3.17

GIVEN: VECTORS  $\underline{A}$  AND  $\underline{B}$ FIND: UNIT VECTOR  $\underline{\lambda}$  NORMAL TO THE PLANE DEFINED BY  $\underline{A}$  AND  $\underline{B}$  WHEN

$$\begin{aligned} \text{(a)} \quad & \underline{A} = \underline{i} + 2\underline{j} - 5\underline{k} \\ & \underline{B} = 4\underline{i} - 7\underline{j} - 5\underline{k} \\ \text{(b)} \quad & \underline{A} = 3\underline{i} - 3\underline{j} + 2\underline{k} \\ & \underline{B} = -2\underline{i} + 6\underline{j} - 4\underline{k} \end{aligned}$$

BY DEFINITION, THE VECTOR  $\underline{A} \times \underline{B}$  IS NORMAL TO THE PLANE DEFINED BY  $\underline{A}$  AND  $\underline{B}$  THUS,

$$\underline{\lambda} = \frac{\underline{A} \times \underline{B}}{|\underline{A} \times \underline{B}|}$$

$$\begin{aligned} \text{(a) HAVE} \quad & \underline{A} \times \underline{B} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & 2 & -5 \\ 4 & -7 & -5 \end{vmatrix} \\ & = (-10 - 35)\underline{j} + (-20 + 5)\underline{j} \\ & \quad + (-7 - 8)\underline{k} \\ & = -45\underline{j} - 15\underline{j} - 15\underline{k} \end{aligned}$$

$$\text{THEN } |\underline{A} \times \underline{B}| = 15 \sqrt{(-3)^2 + (-1)^2 + (-1)^2}$$

$$= 15\sqrt{11}$$

$$\therefore \underline{\lambda} = \frac{1}{\sqrt{11}}(-3\underline{j} - \underline{j} - \underline{k})$$

$$\begin{aligned} \text{(b) HAVE} \quad & \underline{A} \times \underline{B} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 3 & -3 & 2 \\ -2 & 6 & -4 \end{vmatrix} \\ & = (12 - 12)\underline{j} + (-4 + 12)\underline{j} \\ & \quad + (18 - 6)\underline{k} \\ & = 8\underline{j} + 12\underline{k} \end{aligned}$$

$$\text{THEN } |\underline{A} \times \underline{B}| = 4 \sqrt{(2)^2 + (3)^2}$$

$$= 4\sqrt{13}$$

$$\therefore \underline{\lambda} = \frac{1}{\sqrt{13}}(2\underline{j} + 3\underline{k})$$

3.18

GIVEN: ADJACENT SIDES  $\underline{P}$  AND  $\underline{Q}$  OF A PARALLELOGRAM

FIND: AREA OF PARALLELOGRAM WHEN

$$\begin{aligned} \text{(a)} \quad & \underline{P} = -7\underline{i} + 3\underline{j} - 3\underline{k} \\ & \underline{Q} = 2\underline{i} + 2\underline{j} + 5\underline{k} \\ \text{(b)} \quad & \underline{P} = 6\underline{i} - 5\underline{j} - 2\underline{k} \\ & \underline{Q} = -2\underline{j} + 5\underline{j} - \underline{k} \end{aligned}$$

HAVE... AREA  $A = |\underline{P} \times \underline{Q}|$ 

$$\begin{aligned} \text{(a)} \quad & \underline{P} \times \underline{Q} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -7 & 3 & -3 \\ 2 & 2 & 5 \end{vmatrix} \\ & = (15 + 6)\underline{j} + (-6 + 35)\underline{j} + (-14 - 6)\underline{k} \\ & = 21\underline{j} + 29\underline{j} - 20\underline{k} \end{aligned}$$

$$\text{THEN } A = \sqrt{(20)^2 + (29)^2 + (-20)^2} \quad A = 41.0$$

$$\begin{aligned} \text{(b)} \quad & \underline{P} \times \underline{Q} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 6 & -5 & -2 \\ -2 & 5 & -1 \end{vmatrix} \\ & = (5 + 10)\underline{j} + (4 + 6)\underline{j} + (30 - 10)\underline{k} \\ & = 15\underline{j} + 10\underline{j} + 20\underline{k} \end{aligned}$$

$$\text{THEN } A = \sqrt{(15)^2 + (10)^2 + (20)^2} \quad A = 26.9$$

3.19

GIVEN: FORCE  $\underline{F} = 6\underline{i} + 4\underline{j} - \underline{k}$  ACTING AT POINT AFIND: MOMENT OF  $\underline{F}$  ABOUT ORIGIN O WHEN

$$\begin{aligned} \text{(a)} \quad & \underline{r}_A = -2\underline{i} + 6\underline{j} + 3\underline{k} \\ \text{(b)} \quad & \underline{r}_A = 5\underline{i} - 3\underline{j} + 7\underline{k} \\ \text{(c)} \quad & \underline{r}_A = -9\underline{i} - 6\underline{j} + 1.5\underline{k} \end{aligned}$$

BY DEFINITION  $\underline{M}_O = \underline{r}_A \times \underline{F}$ 

$$\begin{aligned} \text{(a) HAVE} \quad & \underline{M}_O = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -2 & 6 & 3 \\ 6 & 4 & -1 \end{vmatrix} \\ & = (-6 - 12)\underline{j} + (18 - 2)\underline{j} + (-8 - 36)\underline{k} \\ & = -18\underline{j} + 16\underline{j} - 44\underline{k} \end{aligned}$$

$$\begin{aligned} \text{(b) HAVE} \quad & \underline{M}_O = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 5 & -3 & 7 \\ 6 & 4 & -1 \end{vmatrix} \\ & = (3 - 28)\underline{j} + (42 + 5)\underline{j} + (20 + 18)\underline{k} \\ & = -25\underline{j} + 47\underline{j} + 38\underline{k} \end{aligned}$$

$$\begin{aligned} \text{(c) HAVE} \quad & \underline{M}_O = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -9 & -6 & 1.5 \\ 6 & 4 & -1 \end{vmatrix} \\ & = (6 - 6)\underline{j} + (9 - 9)\underline{j} + (-36 + 36)\underline{k} \\ & = 0 \end{aligned}$$

NOTE: THE ANSWER TO PART C IS AS EXPECTED SINCE  $\underline{r}_A$  AND  $\underline{F}$  ARE PROPORTIONAL (THUS, THEIR LINES OF ACTION ARE PARALLEL).

3.20

GIVEN: FORCE  $\underline{F} = 2\underline{i} - 7\underline{j} - 3\underline{k}$  ACTING AT POINT AFIND: MOMENT OF  $\underline{F}$  ABOUT ORIGIN O WHEN

$$\begin{aligned} \text{(a)} \quad & \underline{r}_A = 4\underline{i} - 3\underline{j} - 5\underline{k} \\ \text{(b)} \quad & \underline{r}_A = -8\underline{i} - 2\underline{j} + \underline{k} \\ \text{(c)} \quad & \underline{r}_A = \underline{i} - 3.5\underline{j} - 1.5\underline{k} \end{aligned}$$

BY DEFINITION  $\underline{M}_O = \underline{r}_A \times \underline{F}$ 

$$\begin{aligned} \text{(a) HAVE} \quad & \underline{M}_O = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 4 & -3 & -5 \\ 2 & -7 & -3 \end{vmatrix} \\ & = (9 - 35)\underline{j} + (-10 + 12)\underline{j} + (-28 + 6)\underline{k} \\ & = -26\underline{j} + 2\underline{j} - 22\underline{k} \end{aligned}$$

$$\begin{aligned} \text{(b) HAVE} \quad & \underline{M}_O = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -8 & -2 & 1 \\ 2 & -7 & -3 \end{vmatrix} \\ & = (6 + 7)\underline{j} + (2 - 24)\underline{j} + (56 + 4)\underline{k} \\ & = 13\underline{j} - 22\underline{j} + 60\underline{k} \end{aligned}$$

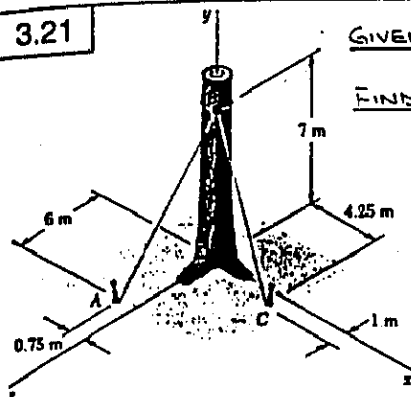
$$\begin{aligned} \text{(c) HAVE} \quad & \underline{M}_O = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & -3.5 & -1.5 \\ 2 & -7 & -3 \end{vmatrix} \\ & = (10.5 - 10.5)\underline{j} + (-3 + 3)\underline{j} + (-7 + 7)\underline{k} \\ & = 0 \end{aligned}$$

(CONTINUED)

## 3.20 CONTINUED

NOTE: THE ANSWER TO PART C IS AS EXPECTED SINCE SA AND E ARE PROPORTIONAL (THUS, THEIR LINES OF ACTION ARE PARALLEL).

3.21



GIVEN:  $T_{BA} = 555 \text{ N}$   
 $T_{EC} = 660 \text{ N}$

FIND: MOMENT OF  
 $(T_{BA} + T_{EC})$   
 ABOUT O

FIRST NOTE --  $d_{\text{eq}} = \sqrt{(-0.75)^2 + (-7)^2 + (6)^2}$   
 $= 9.25 \text{ m}$

$$d_{BC} = \sqrt{(4.25)^2 + (-7)^2 + (1)^2}$$
$$= 8.25 \text{ m}$$

$$\text{Now.. } \underline{T}_{BA} = \frac{T_{BA}}{d_{BA}} \underline{BA} = \frac{555 \text{ N}}{9.25} (-0.75\mathbf{j} - 7\mathbf{j} + 6\mathbf{k})$$

$$= -(45 \text{ N})\mathbf{j} - (420 \text{ N})\mathbf{j} + (360 \text{ N})\mathbf{k}$$

AND  $\underline{T}_{BC} = \frac{T_{AC}}{d_{BC}} \underline{BC} = \frac{660 \text{ N}}{0.25} (4.25\hat{i} - 7\hat{j} + 8\hat{k})$   
 $= (340 \text{ N})\hat{i} - (560 \text{ N})\hat{j} + (800 \text{ N})\hat{k}$

THEN  $R = T_{BA} + T_{BC}$   
 $= -(295 \text{ N})\mathbf{i} - (980 \text{ N})\mathbf{j} + (440 \text{ N})\mathbf{k}$

FINALLY..  $\underline{M}_O = (\underline{F}_{BO} \times \underline{R})$  WHERE  $\underline{F}_{BO} = (7m)\underline{j}$   
 $= (7m)\underline{j} \times [-(295 N)\underline{i}$   
 $\quad \quad \quad -(980 N)\underline{j} + (440 N)\underline{k}]$   
 $= (3080 N \cdot m)\underline{i} - (2065 N \cdot m)\underline{k}$   
 $M_O = (3080 N \cdot m)\underline{i} - (2070 N \cdot m)\underline{k}$

## 3.22 CONTINUED

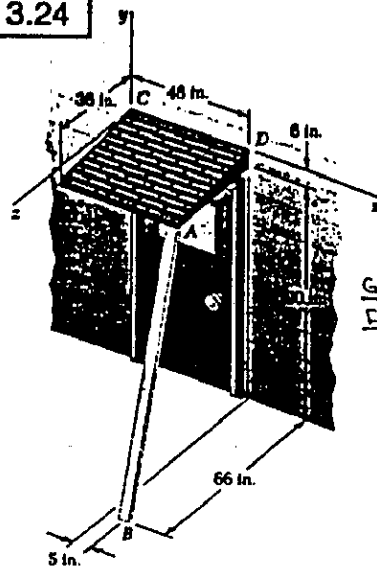
FIRST NOTE ..  $T_{CD} = T_{CE} = W_{AIRC} = mg$   
WHERE  $m = 26 \text{ kg}$ ,  $g = 9.81 \frac{m}{s^2}$

Now..  $d_{CE} = \sqrt{(1.5)^2 + (-6.0)^2 + (-2.0)^2} = 6.5 \text{ m}$

$$\text{THEN } \underline{T}_{CE} = \frac{T_{CE}}{d_{CE}} \vec{CE} = \frac{26.9}{6.5} (1.5\hat{i} - 6\hat{j} - 2\hat{k})$$

$$= 9(6\hat{i} - 24\hat{j} - 8\hat{k}) \cdot (N)$$

3.24



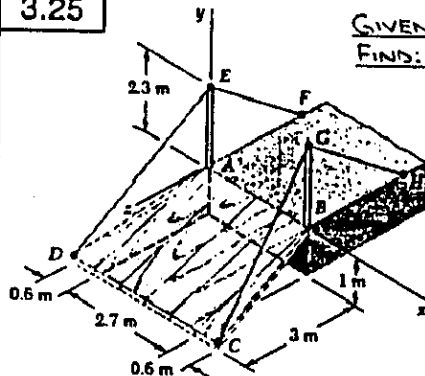
GIVEN:  $T_{BA} = 57 \text{ lb}$   
 FIND: MOMENT  
 ABOUT C OF  $T_{BA}$   
 AT A

FIRST NOTE...  $d_{BA} = \sqrt{(-5)^2 + (90)^2 + (30)^2} = 95 \text{ in.}$   
 THEN  $T_{BA} = \frac{T_{BA}}{d_{BA}} \vec{BA} = \frac{57 \text{ lb}}{95} (-5\mathbf{j} + 90\mathbf{j} - 30\mathbf{k})$   
 $= 3[-(1/10)\mathbf{j} + (18/10)\mathbf{j} - (6/10)\mathbf{k}]$

NOW...  $M_C = [r_{AC} \times T_{BA}]$   
 WHERE  $r_{AC} = (48 \text{ in.})\mathbf{j} - (6 \text{ in.})\mathbf{j} + (36 \text{ in.})\mathbf{k}$

THEN  $M_C = (6)(3) \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -1 & 6 \\ -1 & 18 & -6 \end{vmatrix}$   
 $= 18[(1-108)\mathbf{i} + (-6+48)\mathbf{j} + (144-1)\mathbf{k}]$   
 $= -(1836 \text{ lb}\cdot\text{in.})\mathbf{i} + (736 \text{ lb}\cdot\text{in.})\mathbf{j} + (2541 \text{ lb}\cdot\text{in.})\mathbf{k}$   
 OR  $M_C = -(153.0 \text{ lb}\cdot\text{ft})\mathbf{i} + (61.3 \text{ lb}\cdot\text{ft})\mathbf{j} + (211 \text{ lb}\cdot\text{ft})\mathbf{k}$

3.25



GIVEN:  $T_{DE} = T_{CG} = 810 \text{ N}$   
 FIND: MOMENT ABOUT  
 A OF  
 (a)  $T_{DE}$  AT D  
 (b)  $T_{CG}$  AT C

FIRST NOTE...  $d_{DE} = \sqrt{(0.6)^2 + (3.3)^2 + (-3)^2} = 4.5 \text{ m}$   
 $d_{CG} = \sqrt{(-0.6)^2 + (3.3)^2 + (-3)^2} = 4.5 \text{ m}$

THEN  $T_{DE} = \frac{T_{DE}}{d_{DE}} \vec{DE} = \frac{810 \text{ N}}{4.5} (0.6\mathbf{j} + 3.3\mathbf{j} - 3\mathbf{k})$   
 $= 54[(12 \text{ N})\mathbf{j} + (11 \text{ N})\mathbf{j} - (10 \text{ N})\mathbf{k}]$

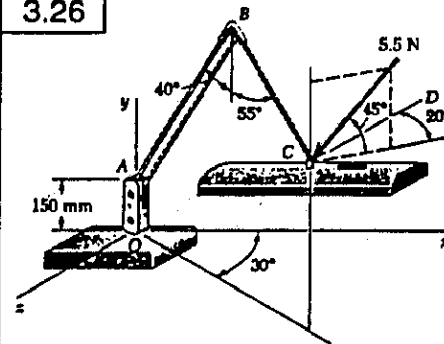
SIMILARLY,  $T_{CG} = 54[-(2 \text{ N})\mathbf{j} + (11 \text{ N})\mathbf{j} - (10 \text{ N})\mathbf{k}]$

(a) NOW...  $M_A = [r_{EA} \times T_{DE}]$  WHERE  $r_{EA} = (2.3 \text{ m})\mathbf{j}$   
 $= 2.3\mathbf{j} \times 54[-(2)\mathbf{j} + (11)\mathbf{j} - (10)\mathbf{k}]$   
 OR  $M_A = -(1242 \text{ N}\cdot\text{m})\mathbf{i} - (248 \text{ N}\cdot\text{m})\mathbf{k}$   
 (CONTINUED)

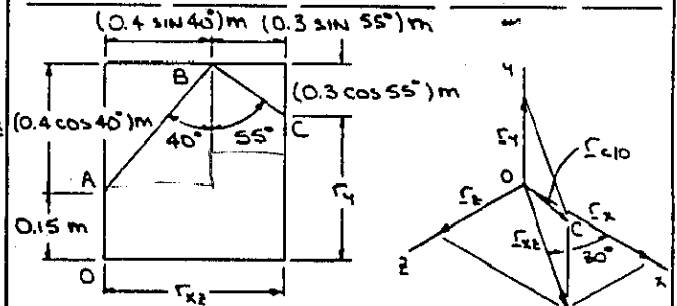
3.25 CONTINUED

(b) NOW...  $M_A = [r_{EA} \times T_{CG}]$   
 WHERE  $r_{EA} = (2.7 \text{ m})\mathbf{j} + (2.3 \text{ m})\mathbf{j}$   
 THEN...  $M_A = 54 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.7 & 2.3 & 0 \\ -2 & 11 & -10 \end{vmatrix}$   
 $= 54[-23\mathbf{j} + 27\mathbf{j} + (29.7 + 4.6)\mathbf{k}]$   
 OR  $M_A = -(1242 \text{ N}\cdot\text{m})\mathbf{i} + (1458 \text{ N}\cdot\text{m})\mathbf{j} + (1852 \text{ N}\cdot\text{m})\mathbf{k}$

3.26



GIVEN:  $AB = 0.4 \text{ m}$   
 $BC = 0.3 \text{ m}$   
 FIND: MOMENT  
 OF FORCE  
 ABOUT O

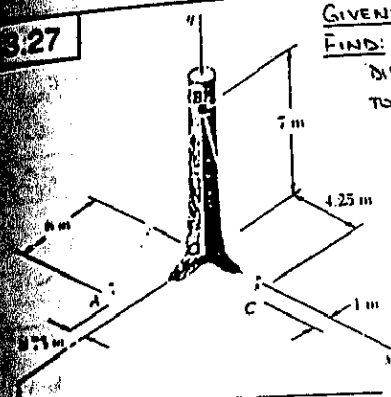


HAVE...  $r_{AO} = [(0.4 \sin 40^\circ + 0.3 \sin 55^\circ) \cos 30^\circ]\mathbf{j}$   
 $+ [0.15 + 0.4 \cos 40^\circ - 0.3 \cos 55^\circ]\mathbf{j}$   
 $+ [(0.4 \sin 40^\circ + 0.3 \sin 55^\circ) \sin 30^\circ]\mathbf{k}$   
 $= (0.43549 \text{ m})\mathbf{j} + (0.28434 \text{ m})\mathbf{j}$   
 $+ (0.25143 \text{ m})\mathbf{k}$

ALSO...  $F = 5.5(-\cos 45^\circ \sin 20^\circ \mathbf{i}$   
 $- \sin 45^\circ \mathbf{j}$   
 $+ \cos 45^\circ \cos 20^\circ \mathbf{k})$   
 $= \frac{5.5 \text{ N}}{\sqrt{2}}(-\sin 20^\circ \mathbf{i} - \mathbf{j}$   
 $+ \cos 20^\circ \mathbf{k})$

NOW...  $M_O = r_{AO} \times F$   
 $= \frac{5.5}{\sqrt{2}} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.43549 & 0.28434 & 0.25143 \\ -\sin 20^\circ & -1 & \cos 20^\circ \end{vmatrix}$   
 $= \frac{5.5}{\sqrt{2}} [(0.28434 \cos 20^\circ + 0.25143)\mathbf{j}$   
 $+ (-0.25143 \sin 20^\circ - 0.43549 \cos 20^\circ)\mathbf{j}$   
 $+ (-0.43549 + 0.28434 \sin 20^\circ)\mathbf{k}]$   
 OR  $M_O = (2.02 \text{ N}\cdot\text{m})\mathbf{i} - (1.926 \text{ N}\cdot\text{m})\mathbf{j} - (1.315 \text{ N}\cdot\text{m})\mathbf{k}$

3.27



GIVEN:  $T_{BA}, T_{AB} = 555 \text{ N}$   
 FIND: PERPENDICULAR  
 DISTANCE FROM O  
 TO CABLE AB

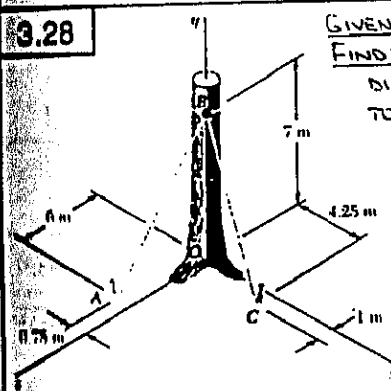
FROM THE SOLUTION TO PROBLEM 3.21

$$\begin{aligned} \mathbf{T}_{BA} &= -(45 \text{ N})\mathbf{i} - (420 \text{ N})\mathbf{j} + (360 \text{ N})\mathbf{k} \\ \text{Now } \mathbf{M}_O &= \mathbf{r}_{O/A} \times \mathbf{T}_{BA} \quad \mathbf{r}_{O/A} = (7 \text{ m})\mathbf{j} \\ &= 7\mathbf{j} \times (-45\mathbf{i} - 420\mathbf{j} + 360\mathbf{k}) \\ &= 7[(360 \text{ N}\cdot\text{m})\mathbf{i} + (45 \text{ N}\cdot\text{m})\mathbf{k}] \\ \text{Then } \mathbf{M}_O &= 7\sqrt{(360)^2 + (45)^2} \\ &= 2539.6 \text{ N}\cdot\text{m} \end{aligned}$$



$$\begin{aligned} \text{Also } \mathbf{M}_O &= dT_{BA} \\ \text{OR } 2539.6 \text{ N}\cdot\text{m} &= d \cdot 555 \text{ N} \\ \text{OR } d &= 4.58 \text{ m} \end{aligned}$$

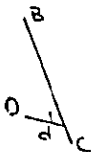
3.28



GIVEN:  $T_{BC}, T_{CB} = 660 \text{ N}$   
 FIND: PERPENDICULAR  
 DISTANCE FROM O  
 TO CABLE BC

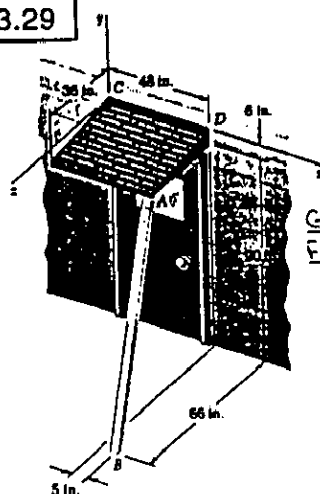
FROM THE SOLUTION TO PROBLEM 3.21

$$\begin{aligned} \mathbf{T}_{BC} &= (340 \text{ N})\mathbf{i} - (560 \text{ N})\mathbf{j} + (80 \text{ N})\mathbf{k} \\ \text{Now } \mathbf{M}_O &= \mathbf{r}_{O/C} \times \mathbf{T}_{BC} \quad \mathbf{r}_{O/C} = (7 \text{ m})\mathbf{j} \\ &= 7\mathbf{j} \times (340\mathbf{i} - 560\mathbf{j} + 80\mathbf{k}) \\ &= 7[(80 \text{ N}\cdot\text{m})\mathbf{i} - (340 \text{ N}\cdot\text{m})\mathbf{k}] \\ \text{Then } \mathbf{M}_O &= 7\sqrt{(80)^2 + (-340)^2} \\ &= 2445.0 \text{ N}\cdot\text{m} \end{aligned}$$



$$\begin{aligned} \text{Also } \mathbf{M}_O &= dT_{BC} \\ \text{OR } 2445.0 \text{ N}\cdot\text{m} &= d \cdot 660 \text{ N} \\ \text{OR } d &= 3.70 \text{ m} \end{aligned}$$

3.29



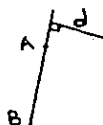
GIVEN:  $T_{BA}, T_{AB} = 57 \text{ lb}$   
 FIND: PERPENDICULAR  
 DISTANCE FROM B TO  
 A LINE THROUGH A AND  
 C

FROM THE SOLUTION TO PROBLEM 3.24

$$\begin{aligned} \mathbf{T}_{BA} &= 3[-(1 \text{ lb})\mathbf{i} + (18 \text{ lb})\mathbf{j} - (6 \text{ lb})\mathbf{k}] \\ \text{Now } \mathbf{M}_B &= \mathbf{r}_{A/B} \times \mathbf{T}_{BA} \\ \text{WHERE } \mathbf{r}_{A/B} &= -(6 \text{ in.})\mathbf{j} + (36 \text{ in.})\mathbf{k} \\ &= 6[-(1 \text{ in.})\mathbf{j} + (6 \text{ in.})\mathbf{k}] \end{aligned}$$

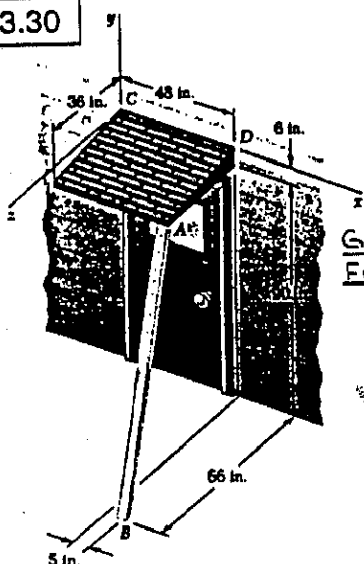
$$\begin{aligned} \text{Then } \mathbf{M}_B &= (3)(6) \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -1 & 6 \\ -1 & 18 & -6 \end{vmatrix} \\ &= 18[(6 - 108)\mathbf{i} - 6\mathbf{j} - \mathbf{k}] \\ &= 18[(-102 \text{ lb}\cdot\text{in.})\mathbf{i} - (6 \text{ lb}\cdot\text{in.})\mathbf{j} \\ &\quad - (1 \text{ lb}\cdot\text{in.})\mathbf{k}] \end{aligned}$$

$$\begin{aligned} \text{AND } M_B &= 18\sqrt{(-102)^2 + (-6)^2 + (-1)^2} \\ &= 1839.26 \text{ lb}\cdot\text{in.} \end{aligned}$$



$$\begin{aligned} \text{Also } \mathbf{M}_B &= dT_{BA} \\ \text{OR } 1839.26 \text{ lb}\cdot\text{in.} &= d \cdot 57 \text{ lb} \\ \text{OR } d &= 32.3 \text{ in.} \end{aligned}$$

3.30



GIVEN:  $M_C, T_{CA} = 57 \text{ lb}$   
 FIND: PERPENDICULAR  
 DISTANCE FROM C  
 TO A LINE  
 THROUGH A AND B

FROM THE SOLUTION TO PROBLEM 3.24

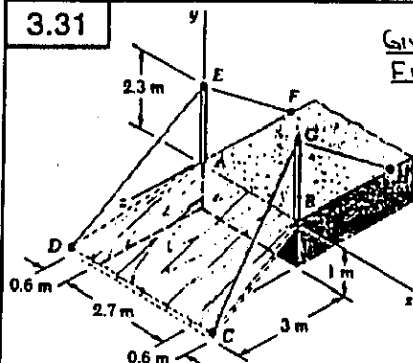
$$\begin{aligned} \mathbf{M}_C &= -(1836 \text{ lb}\cdot\text{in.})\mathbf{i} + (756 \text{ lb}\cdot\text{in.})\mathbf{j} + (2574 \text{ lb}\cdot\text{in.})\mathbf{k} \\ \text{Then } M_C &= \sqrt{(-1836)^2 + (756)^2 + (2574)^2} \\ &\quad (\text{CONTINUED}) \end{aligned}$$



### 3.30 CONTINUED

OR  $M_C = 3250.8 \text{ lb} \cdot \text{in.}$   
 ALSO..  $M_C = d T_{BA}$   
 OR  $3250.8 \text{ lb} \cdot \text{in.} = d \cdot 57 \text{ lb}$   
 OR  $d = 57.0 \text{ in.}$

### 3.31



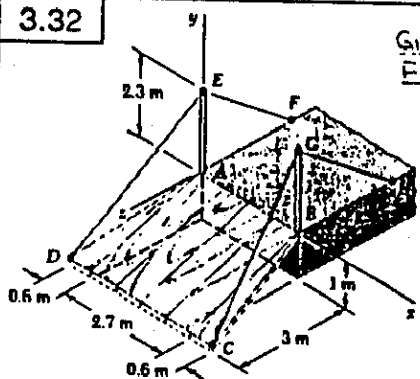
GIVEN:  $M_A, T_{DE} = 810 \text{ N}$   
 FIND: PERPENDICULAR  
 DISTANCE FROM  
 A TO CABLE  
 DE

FROM THE SOLUTION TO PROBLEM 3.25(a)  
 $M_A = -(1242 \text{ N} \cdot \text{m})\mathbf{i} - (248 \text{ N} \cdot \text{m})\mathbf{j}$   
 THEN  $M_A = \sqrt{(-1242)^2 + (-248)^2}$   
 $= 1266.52 \text{ N} \cdot \text{m}$



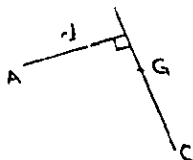
ALSO..  $M_A = d T_{DE}$   
 OR  $1266.52 \text{ N} \cdot \text{m} = d \cdot 810 \text{ N}$   
 OR  $d = 1.564 \text{ m}$

### 3.32



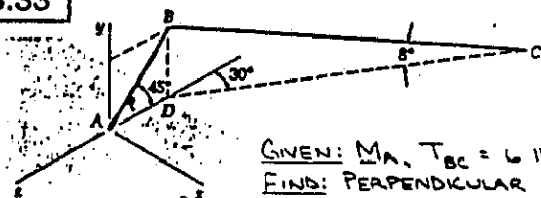
GIVEN:  $M_A, T_{CG} = 810 \text{ N}$   
 FIND: PERPENDICULAR  
 DISTANCE FROM  
 A TO A LINE  
 THROUGH C  
 AND G

FROM THE SOLUTION TO PROBLEM 3.25(b)  
 $M_A = -(1242 \text{ N} \cdot \text{m})\mathbf{i} + (1458 \text{ N} \cdot \text{m})\mathbf{j} + (1852 \text{ N} \cdot \text{m})\mathbf{k}$   
 THEN  $M_A = \sqrt{(-1242)^2 + (1458)^2 + (1852)^2}$   
 $= 2664.3 \text{ N} \cdot \text{m}$



ALSO..  $M_A = d T_{CG}$   
 OR  $2664.3 \text{ N} \cdot \text{m} = d \cdot 810 \text{ N}$   
 OR  $d = 3.29 \text{ m}$

### 3.33



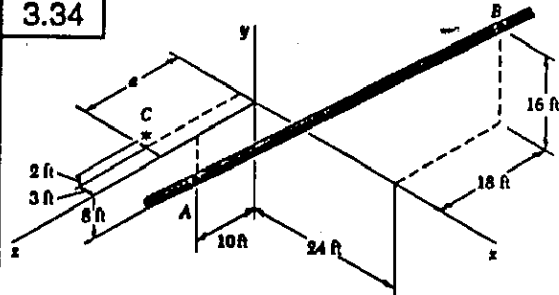
GIVEN:  $M_A, T_{BC} = 6 \text{ lb}$   
 FIND: PERPENDICULAR  
 DISTANCE FROM A TO  
 A LINE THROUGH C  
 AND G

FROM THE SOLUTION TO PROBLEM 3.23  
 $M_A = -(25.4 \text{ lb} \cdot \text{ft})\mathbf{i} - (12.60 \text{ lb} \cdot \text{ft})\mathbf{j} - (12.60 \text{ lb} \cdot \text{ft})\mathbf{k}$   
 THEN  $M_A = \sqrt{(-25.4)^2 + (-12.60)^2 + (-12.60)^2}$   
 $= 31.027 \text{ lb} \cdot \text{ft}$



ALSO..  $M_A = d T_{BC}$   
 OR  $31.027 \text{ lb} \cdot \text{ft} = d \cdot 6 \text{ lb}$   
 OR  $d = 5.17 \text{ ft}$

### 3.34



GIVEN: SECTION OF PIPELINE  
 FIND: Q SO THAT PERPENDICULAR  
 DISTANCE d FROM C TO A LINE  
 THROUGH A AND B IS A MINIMUM

FIRST NOTE..  $d_{AB} = \sqrt{(24-0)^2 + (16-(-8))^2 + (-18-10)^2}$   
 $= 44 \text{ ft}$

NOW ASSUME THAT A FORCE  $\mathbf{F}$ , OF  
 MAGNITUDE F, ACTS AT POINT A AND IS  
 DIRECTED FROM A TO B. THEN

$\mathbf{F} = F \Delta_{AB} \quad (F \text{ in lb})$   
 $= \frac{F}{44} (24\mathbf{i} + 24\mathbf{j} - 28\mathbf{k})$   
 $= \frac{F}{11} (6\mathbf{i} + 6\mathbf{j} - 7\mathbf{k})$

BY DEFINITION..  $M_C = |\Sigma \mathbf{r}_{AC} \times \mathbf{F}| = d F$   
 WHERE  $\Sigma \mathbf{r}_{AC} = (3\mathbf{i})\mathbf{j} - (10\mathbf{j})\mathbf{i} + [(10-Q)\mathbf{k}]\mathbf{j}$

THEN  $M_C = \frac{F}{11} \begin{vmatrix} 1 & 1 & 1 \\ 3 & -10 & (10-Q) \\ 0 & 0 & -7 \end{vmatrix}$   
 $= \frac{F}{11} \{ [70 - 6(10-Q)]\mathbf{j} + [6(10-Q) + 21]\mathbf{i} + [18 + 60]\mathbf{k} \}$   
 $= \frac{F}{11} \{ [(10+6Q)16\mathbf{k}] + [(81-6Q)16\mathbf{k}] + [7816\mathbf{k}]\mathbf{j} \}$

THEN..  $(\frac{F}{11})^2 [(10+6Q)^2 + (81-6Q)^2 + (78)^2] = (d F)^2$   
 OR  $d^2 = \frac{1}{121} [(10+6Q)^2 + (81-6Q)^2 + (78)^2] \quad (\text{ft})^2$   
 FINALLY..  $\frac{d(d^2)}{dQ} = \frac{1}{121} [2(6)(10+6Q) + 2(-6)(81-6Q)] = 0$

(CONTINUED)

### 3.34 CONTINUED

OR  $(10+6a) - (81-6a) = 0$   
SO THAT FOR  $d_{\min}$   $a = 5.92 \text{ ft}$

### 3.35

GIVEN:  $\underline{P} = 4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$

$\underline{Q} = -\mathbf{j} + 4\mathbf{j} - 5\mathbf{k}$

$\underline{S} = \mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$

FIND:  $\underline{P} \cdot \underline{Q}$ ,  $\underline{P} \cdot \underline{S}$ ,  $\underline{Q} \cdot \underline{S}$

HAVE..  $\underline{P} \cdot \underline{Q} = (4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}) \cdot (-\mathbf{j} + 4\mathbf{j} - 5\mathbf{k})$   
 $= (4)(-1) + (3)(4) + (-2)(-5)$   
OR  $\underline{P} \cdot \underline{Q} = 18$

$\underline{P} \cdot \underline{S} = (4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}) \cdot (\mathbf{i} + 4\mathbf{j} + 3\mathbf{k})$   
 $= (4)(1) + (3)(4) + (-2)(3)$   
OR  $\underline{P} \cdot \underline{S} = 10$

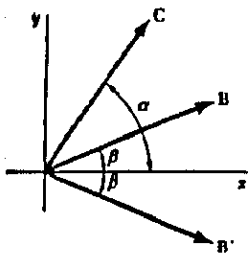
$\underline{Q} \cdot \underline{S} = (-\mathbf{j} + 4\mathbf{j} - 5\mathbf{k}) \cdot (\mathbf{i} + 4\mathbf{j} + 3\mathbf{k})$   
 $= (-1)(1) + (4)(4) + (-5)(3)$   
OR  $\underline{Q} \cdot \underline{S} = 0$

THUS,  $\underline{Q}$  AND  $\underline{S}$  ARE PERPENDICULAR

### 3.36

GIVEN:  $\underline{B}$ ,  $\underline{B}'$ , AND  $\underline{C}$

PROVE:  $\cos \alpha \cos \beta$   
 $= \frac{1}{2} \cos(\alpha + \beta)$   
 $+ \frac{1}{2} \cos(\alpha - \beta)$



FIRST NOTE..  $\underline{B} = B(\cos \beta \mathbf{i} + \sin \beta \mathbf{j})$   
 $\underline{B}' = B(\cos \beta \mathbf{i} - \sin \beta \mathbf{j})$   
 $\underline{C} = C(\cos \alpha \mathbf{i} + \sin \alpha \mathbf{j})$

BY DEFINITION..  $\underline{B} \cdot \underline{C} = BC \cos(\alpha - \beta)$  (1)  
 $\underline{B}' \cdot \underline{C} = BC \cos(\alpha + \beta)$  (2)

NOW  $\underline{B} \cdot \underline{C} = B(\cos \beta \mathbf{i} + \sin \beta \mathbf{j}) \cdot C(\cos \alpha \mathbf{i} + \sin \alpha \mathbf{j})$   
 $= BC(\cos \beta \cos \alpha + \sin \beta \sin \alpha)$  (3)

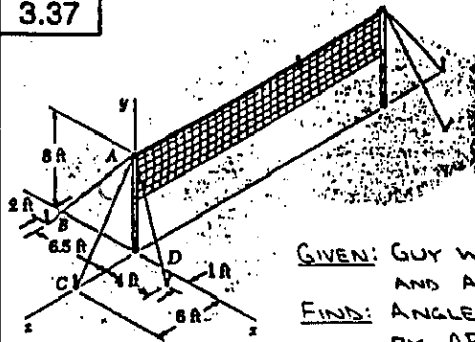
AND  $\underline{B}' \cdot \underline{C} = B(\cos \beta \mathbf{i} - \sin \beta \mathbf{j}) \cdot C(\cos \alpha \mathbf{i} + \sin \alpha \mathbf{j})$   
 $= BC(\cos \beta \cos \alpha - \sin \beta \sin \alpha)$  (4)

EQUATING THE RIGHT-HAND SIDES OF EQS. (1) AND (2) TO THE RIGHT-HAND SIDES OF EQS. (3) AND (4), RESPECTIVELY, YIELDS  
 $BC \cos(\alpha - \beta) = BC(\cos \beta \cos \alpha + \sin \beta \sin \alpha)$  (5)  
 $BC \cos(\alpha + \beta) = BC(\cos \beta \cos \alpha - \sin \beta \sin \alpha)$  (6)

(5) + (6)  $\Rightarrow \cos(\alpha - \beta) + \cos(\alpha + \beta) = 2 \cos \beta \cos \alpha$

OR  $\cos \alpha \cos \beta = \frac{1}{2} \cos(\alpha + \beta) + \frac{1}{2} \cos(\alpha - \beta)$

### 3.37



GIVEN: GUY WIRES AB AND AC

FIND: ANGLE  $\theta$  FORMED BY AB AND AC

FIRST NOTE..  $AB = \sqrt{(-6.5)^2 + (-8)^2 + (2)^2}$   
 $= 10.5 \text{ ft}$

$AC = \sqrt{(0)^2 + (-8)^2 + (6)^2}$   
 $= 10 \text{ ft}$

AND  $\underline{AB} = (-6.5 \text{ ft})\mathbf{i} - (8 \text{ ft})\mathbf{j} + (2 \text{ ft})\mathbf{k}$

$\underline{AC} = -(8 \text{ ft})\mathbf{j} - (6 \text{ ft})\mathbf{k}$

BY DEFINITION..  $\underline{AB} \cdot \underline{AC} = (AB)(AC) \cos \theta$

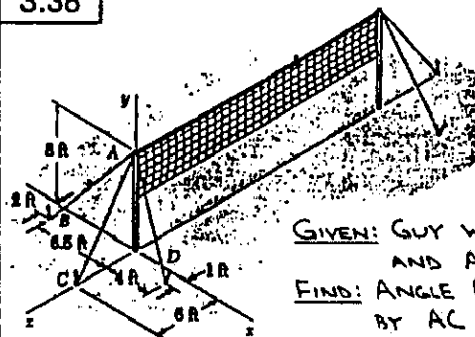
OR  $(-6.5\mathbf{i} - 8\mathbf{j} + 2\mathbf{k}) \cdot (-8\mathbf{j} - 6\mathbf{k}) = (10.5)(10) \cos \theta$

$(-6.5)(0) + (-8)(-8) + (2)(-6) = 105 \cos \theta$

OR  $\cos \theta = 0.723 \text{ BI}$

OR  $\theta = 43.6^\circ$

### 3.38



GIVEN: GUY WIRES AC AND AD

FIND: ANGLE  $\theta$  FORMED BY AC AND AD

FIRST NOTE..  $AC = \sqrt{(0)^2 + (-8)^2 + (6)^2}$   
 $= 10 \text{ ft}$

$AD = \sqrt{(4)^2 + (-8)^2 + (1)^2}$   
 $= 9 \text{ ft}$

AND  $\underline{AC} = -(8 \text{ ft})\mathbf{j} - (6 \text{ ft})\mathbf{k}$

$\underline{AD} = (4 \text{ ft})\mathbf{i} - (8 \text{ ft})\mathbf{j} + (1 \text{ ft})\mathbf{k}$

BY DEFINITION..  $\underline{AC} \cdot \underline{AD} = (AC)(AD) \cos \theta$

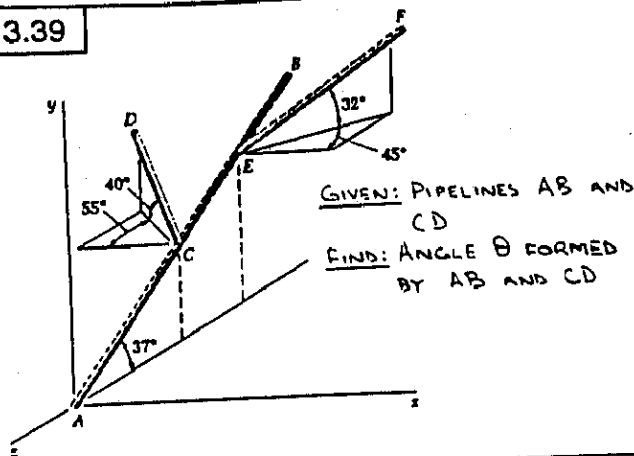
OR  $(-8\mathbf{j} - 6\mathbf{k}) \cdot (4\mathbf{i} - 8\mathbf{j} + \mathbf{k}) = (10)(9) \cos \theta$

$(0)(4) + (-8)(-8) + (-6)(1) = 90 \cos \theta$

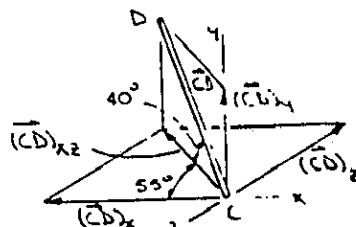
OR  $\cos \theta = 0.777 \text{ BI}$

OR  $\theta = 38.9^\circ$

3.39

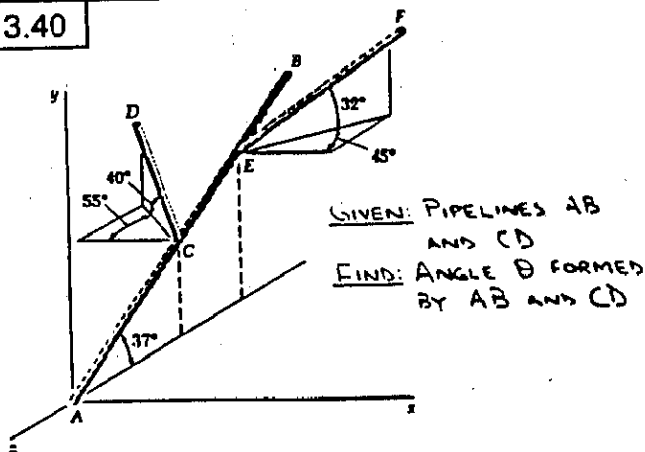


FIRST NOTE.  $\vec{AB} = AB(\sin 37^\circ \hat{j} - \cos 37^\circ \hat{k})$   
 $\vec{CD} = CD(-\cos 40^\circ \cos 55^\circ \hat{i} + \sin 40^\circ \hat{j} - \cos 40^\circ \sin 55^\circ \hat{k})$

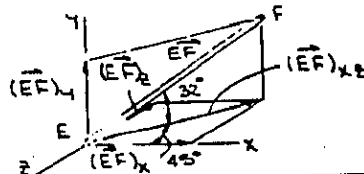


NOW.  $\vec{AB} \cdot \vec{CD} = (AB)(CD) \cos \theta$   
 OR  $AB(\sin 37^\circ \hat{j} - \cos 37^\circ \hat{k}) \cdot CD(-\cos 40^\circ \cos 55^\circ \hat{i} + \sin 40^\circ \hat{j} - \cos 40^\circ \sin 55^\circ \hat{k})$   
 $= (AB)(CD) \cos \theta$   
 OR  $\cos \theta = (\sin 37^\circ \sin 40^\circ) + (-\cos 37^\circ)(-\cos 40^\circ \sin 55^\circ)$   
 $= 0.88799$   
 OR  $\theta = 27.4^\circ$

3.40



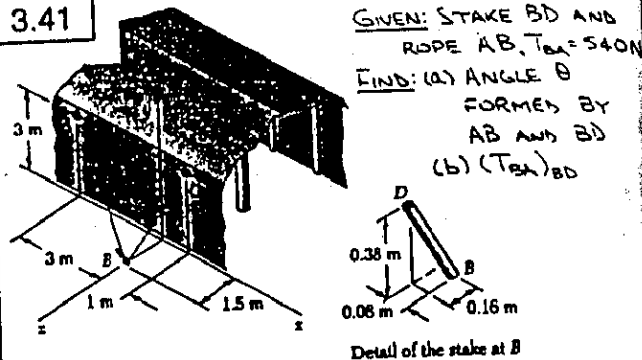
FIRST NOTE.  $\vec{AB} = AB(\sin 37^\circ \hat{j} - \cos 37^\circ \hat{k})$   
 $\vec{EF} = EF(\cos 32^\circ \cos 45^\circ \hat{i} + \sin 32^\circ \hat{j} - \cos 32^\circ \sin 45^\circ \hat{k})$



3.40 CONTINUED

NOW  $\vec{AB} \cdot \vec{EF} = (AB)(EF) \cos \theta$   
 OR  $AB(\sin 37^\circ \hat{j} - \cos 37^\circ \hat{k}) \cdot EF(\cos 32^\circ \cos 45^\circ \hat{i} + \sin 32^\circ \hat{j} - \cos 32^\circ \sin 45^\circ \hat{k})$   
 $= (AB)(EF) \cos \theta$   
 OR  $\cos \theta = (\sin 37^\circ)(\sin 32^\circ) + (-\cos 37^\circ)(-\cos 32^\circ \sin 45^\circ)$   
 $= 0.79782$   
 OR  $\theta = 37.1^\circ$

3.41



FIRST NOTE.  $BA = \sqrt{(-1)^2 + (3)^2 + (-1.5)^2} = 4.5$   
 $BD = \sqrt{(0.08)^2 + (0.38)^2 + (0.16)^2}$   
 $= 0.42$

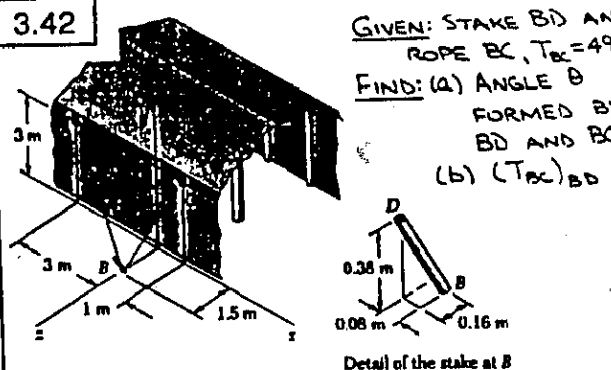
THEN  $T_{AB} = \frac{T_{AB}}{4.5}(-3\hat{i} + 3\hat{j} - 1.5\hat{k})$   
 $= \frac{T_{AB}}{3}(-2\hat{i} + 2\hat{j} - \hat{k})$   
 $\Delta_{BD} = \frac{BD}{0.42}(-0.08\hat{i} + 0.38\hat{j} + 0.16\hat{k})$   
 $= \frac{1}{21}(-4\hat{i} + 19\hat{j} + 8\hat{k})$

(a) HAVE  $T_{AB} \cdot \Delta_{BD} = T_{AB} \cos \theta$

OR  $\frac{T_{AB}}{3}(-2\hat{i} + 2\hat{j} - \hat{k}) \cdot \frac{1}{21}(-4\hat{i} + 19\hat{j} + 8\hat{k}) = T_{AB} \cos \theta$   
 OR  $\cos \theta = \frac{1}{63}[-(2)(4) + (2)(19) + (-1)(8)]$   
 $= 0.60317$   
 OR  $\theta = 52.9^\circ$

(b) HAVE  $(T_{AB})_{BD} = T_{AB} \cdot \Delta_{BD}$   
 $= T_{AB} \cos \theta$   
 $= (540 \text{ N})(0.60317)$   
 OR  $(T_{AB})_{BD} = 326 \text{ N}$

3.42



FIRST NOTE.  $BC = \sqrt{(1)^2 + (3)^2 + (-1.5)^2} = 3.5$   
 (CONTINUED)

# 3.42 CONTINUED

$$BD = \sqrt{(-0.08)^2 + (0.38)^2 + (0.16)^2}$$

$$= 0.42 \text{ m}$$

THEN  $T_{BC} = \frac{T_{BC}}{3.5} (\hat{i} + 3\hat{j} - 4\hat{k})$

$$= \frac{T_{BC}}{7} (2\hat{i} + 6\hat{j} - 3\hat{k})$$

$$\Delta BD = \frac{BD}{BD} = \frac{1}{0.42} (-0.08\hat{i} + 0.38\hat{j} + 0.16\hat{k})$$

$$= \frac{1}{21} (-4\hat{i} + 19\hat{j} + 8\hat{k})$$

(a) HAVE  $T_{BC} \cdot \Delta BD = T_{BC} \cos \theta$

OR  $\frac{T_{BC}}{7} (2\hat{i} + 6\hat{j} - 3\hat{k}) \cdot \frac{1}{21} (-4\hat{i} + 19\hat{j} + 8\hat{k}) = T_{BC} \cos \theta$

OR  $\cos \theta = \frac{1}{147} [(2)(-4) + (6)(19) + (-3)(8)]$

$$= 0.55782$$

OR  $\theta = 56.1^\circ$

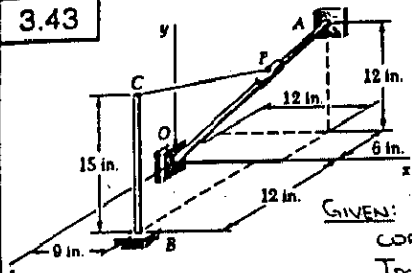
(b) HAVE  $(T_{BC})_{BD} = T_{BC} \cdot \Delta BD$

$$= T_{BC} \cos \theta$$

$$= (490 \text{ N})(0.55782)$$

OR  $(T_{BC})_{BD} = 273 \text{ N}$

3.43



GIVEN: ROD OA AND  
CORD PC,  $OP = 6 \text{ in.}$   
 $T_{PC} = 3 \text{ lb}$

FIND: (a) ANGLE  $\theta$  FORMED  
BY OA AND PC

(b)  $(T_{PC})_{OA}$

FIRST NOTE..  $OA = \sqrt{(12)^2 + (12)^2 + (-6)^2} = 18 \text{ in.}$

THEN..  $\Delta OA = \frac{OA}{OA} = \frac{1}{18} (12\hat{i} + 12\hat{j} - 6\hat{k})$

$$= \frac{1}{3} (2\hat{i} + 2\hat{j} - \hat{k})$$

NOW  $OP = 6 \text{ in.} \Rightarrow OP = \frac{1}{3} (OA)$

$\therefore$  THE COORDINATES OF POINT P ARE  
(4 in., 4 in., -2 in.)

SO THAT  $\vec{PC} = (5\hat{i} + 11\hat{j} + 14\hat{k}) - (4\hat{i} + 4\hat{j} - 2\hat{k}) = \hat{i} + 7\hat{j} + 16\hat{k}$

AND  $PC = \sqrt{(5)^2 + (11)^2 + (14)^2} = \sqrt{342} \text{ in.}$

(a) HAVE..  $\vec{PC} \cdot \Delta OA = (PC) \cos \theta$

OR  $(5\hat{i} + 11\hat{j} + 14\hat{k}) \cdot \frac{1}{3} (2\hat{i} + 2\hat{j} - \hat{k}) = \sqrt{342} \cos \theta$

OR  $\cos \theta = \frac{1}{3\sqrt{342}} [(5)(2) + (11)(2) + (14)(-1)]$

$$= 0.32444$$

OR  $\theta = 71.1^\circ$

(b) HAVE..  $(T_{PC})_{OA} = T_{PC} \cdot \Delta OA$

$$= (T_{PC} \Delta PC) \cdot \Delta OA$$

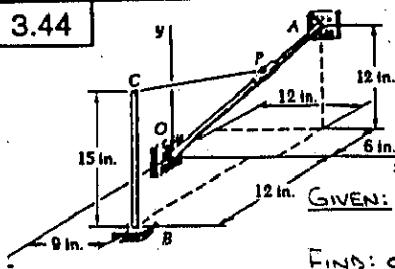
$$= T_{PC} \frac{PC}{PC} \cdot \Delta OA$$

$$= T_{PC} \cos \theta$$

$$= (3 \text{ lb})(0.32444)$$

OR  $(T_{PC})_{OA} = 0.973 \text{ lb}$

3.44



GIVEN: ROD OA AND  
POINT P

FIND:  $\Delta OP$  SO THAT OA  
AND PC ARE  
PERPENDICULAR

FIRST NOTE..  $OA = \sqrt{(12)^2 + (12)^2 + (-6)^2} = 18 \text{ in.}$

THEN..  $\Delta OA = \frac{OA}{OA} = \frac{1}{18} (12\hat{i} + 12\hat{j} - 6\hat{k})$

$$= \frac{1}{3} (2\hat{i} + 2\hat{j} - \hat{k})$$

LET THE COORDINATES OF POINT P BE

(x in., y in., z in.). THEN

$$\vec{PC} = [(9-x)\hat{i} + (15-y)\hat{j} + (12-z)\hat{k}]$$

ALSO,  $\vec{OP} = \Delta OP \Delta OA$

$$= \frac{\Delta OP}{3} (2\hat{i} + 2\hat{j} - \hat{k})$$

AND  $\vec{OP} = (x\hat{i} + y\hat{j} + z\hat{k})$

$$\therefore x = \frac{2}{3} \Delta OP \quad y = \frac{2}{3} \Delta OP \quad z = -\frac{1}{3} \Delta OP$$

THE REQUIREMENT THAT OA AND PC BE  
PERPENDICULAR IMPLIES THAT

$$\Delta OA \cdot \vec{PC} = 0$$

OR  $\frac{1}{3} (2\hat{i} + 2\hat{j} - \hat{k}) \cdot [(9-x)\hat{i} + (15-y)\hat{j} + (12-z)\hat{k}] = 0$

OR  $(2)(9 - \frac{2}{3} \Delta OP) + (2)(15 - \frac{2}{3} \Delta OP) + (-1)[12 - (-\frac{1}{3} \Delta OP)] = 0$

OR  $\Delta OP = 12 \text{ in.}$

3.45

GIVEN: VECTORS  $\vec{P}$ ,  $\vec{Q}$ , AND  $\vec{S}$

FIND: VOLUME OF THE PARALLELOGRAM  
DEFINED BY  $\vec{P}$ ,  $\vec{Q}$ , AND  $\vec{S}$  WHEN

(a)  $\vec{P} = 4\hat{i} - 3\hat{j} + 2\hat{k}$

$$\vec{Q} = -2\hat{i} - 5\hat{j} + \hat{k}$$

$$\vec{S} = 7\hat{i} + \hat{j} - \hat{k}$$

(b)  $\vec{P} = 5\hat{i} - \hat{j} + 6\hat{k}$

$$\vec{Q} = 2\hat{i} + 3\hat{j} + \hat{k}$$

$$\vec{S} = -3\hat{i} - 2\hat{j} + 4\hat{k}$$

AS EXPLAINED IN SEC. 3.10, THE VOLUME  $V$   
OF THE PARALLELOGRAM IS GIVEN BY

$$V = |\vec{P} \cdot (\vec{Q} \times \vec{S})|$$

(a) HAVE

$$\vec{P} \cdot (\vec{Q} \times \vec{S}) = \begin{vmatrix} 4 & -3 & 2 \\ -2 & -5 & 1 \\ 7 & 1 & -1 \end{vmatrix}$$

$$= 20 - 21 - 4 + 70 + 6 - 4$$

$$= 67$$

$$\therefore V = 67$$

(b) HAVE

$$\vec{P} \cdot (\vec{Q} \times \vec{S}) = \begin{vmatrix} 5 & -1 & 6 \\ 2 & 3 & 1 \\ -3 & -2 & 4 \end{vmatrix}$$

$$= 60 + 3 - 24 + 54 + 8 + 10$$

$$= 111$$

$$\therefore V = 111$$

3.46

GIVEN:  $P = 3\mathbf{i} - \mathbf{j} + \mathbf{k}$  $Q = 4\mathbf{i} + Q_4\mathbf{j} - 2\mathbf{k}$  $S = 2\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ FIND:  $Q_4$  SO THAT  $P$ ,  $Q$ , AND  $S$  ARE COPLANARIF  $P$ ,  $Q$ , AND  $S$  ARE COPLANAR, THEN  $P$  MUST BE PERPENDICULAR TO  $(Q \times S)$ .

$$\therefore P \cdot (Q \times S) = 0$$

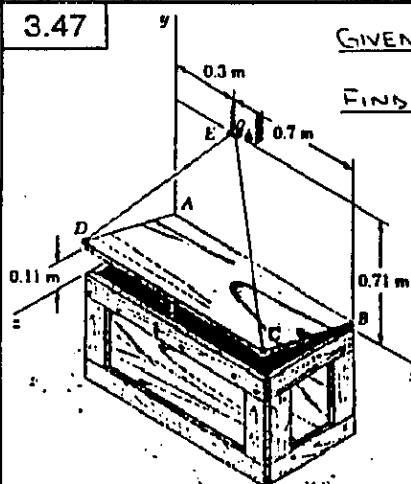
(OR, THE VOLUME OF THE PARALLELOGRAM DEFINED BY  $P$ ,  $Q$ , AND  $S$  IS ZERO). THEN

$$\begin{vmatrix} 3 & -1 & 1 \\ 4 & Q_4 & -2 \\ 2 & -2 & 2 \end{vmatrix} = 0$$

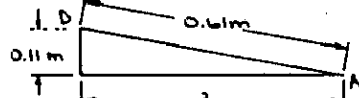
$$\text{OR } 6Q_4 + 4 - 8 - 2Q_4 + 8 - 12 = 0$$

$$\text{OR } Q_4 = 2$$

3.47

GIVEN:  $0.61 = 1.00\text{-m LID}$  $T_{DE} = 66\text{ N}$ FIND:  $M_x, M_y, M_z$  OF IDE AT D

FIRST NOTE..  
 $z = \sqrt{(0.61)^2 - (0.11)^2}$   
 $= 0.60\text{ m}$



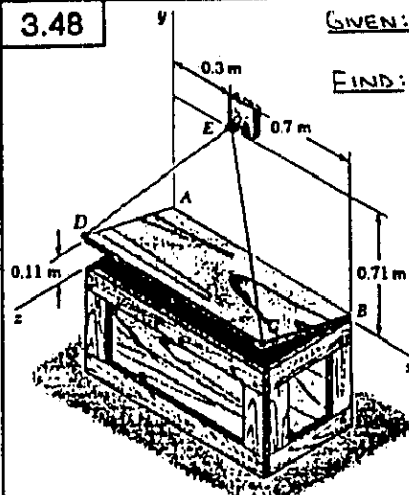
THEN  $d_{DE} = \sqrt{(0.3)^2 + (0.6)^2 + (-0.6)^2} = 0.9\text{ m}$   
 AND  $T_{DE} = \frac{66\text{ N}}{0.9} (0.3\mathbf{i} + 0.6\mathbf{j} - 0.6\mathbf{k})$   
 $= 22[(1\text{ N})\mathbf{i} + (2\text{ N})\mathbf{j} - (2\text{ N})\mathbf{k}]$

NOW..  $M_A = \sum \mathbf{r}_{D/A} \times T_{DE}$   
 WHERE  $\sum \mathbf{r}_{D/A} = (0.11\text{ m})\mathbf{j} + (0.60\text{ m})\mathbf{k}$

THEN..  $M_A = 22 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.11 & 0.60 \\ 1 & 2 & -2 \end{vmatrix}$   
 $= 22[(-0.22 - 1.20)\mathbf{j} + 0.60\mathbf{j} - 0.11\mathbf{k}]$   
 $= -(31.24\text{ N}\cdot\text{m})\mathbf{j} + (13.20\text{ N}\cdot\text{m})\mathbf{j}$   
 $- (2.42\text{ N}\cdot\text{m})\mathbf{k}$

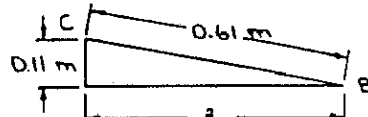
$\therefore M_x = -31.2\text{ N}\cdot\text{m}, M_y = 13.20\text{ N}\cdot\text{m}, M_z = -2.42\text{ N}\cdot\text{m}$

3.48

GIVEN:  $0.61 = 1.00\text{-m LID}$  $T_{CE} = 66\text{ N}$ FIND:  $M_x, M_y, M_z$  OF ICE AT C

FIRST NOTE..

$$z = \sqrt{(0.61)^2 - (0.11)^2} = 0.60\text{ m}$$



THEN  $d_{CE} = \sqrt{(-0.7)^2 + (0.6)^2 + (-0.6)^2} = 1.1\text{ m}$   
 AND  $T_{CE} = \frac{66\text{ N}}{1.1} (-0.7\mathbf{i} + 0.6\mathbf{j} - 0.6\mathbf{k})$   
 $= 6[(-7\text{ N})\mathbf{i} + (6\text{ N})\mathbf{j} - (6\text{ N})\mathbf{k}]$

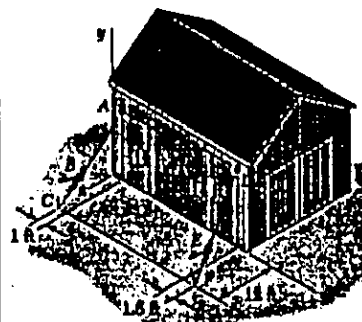
NOW..  $M_A = \sum \mathbf{r}_{E/A} \times T_{CE}$   
 WHERE  $\sum \mathbf{r}_{E/A} = (0.3\text{ m})\mathbf{i} + (0.71\text{ m})\mathbf{j}$

THEN..  $M_A = 6 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.3 & 0.71 & 0 \\ -7 & 6 & -6 \end{vmatrix}$   
 $= 6[-4.26\mathbf{j} + 1.8\mathbf{j} + (1.8 + 4.97)\mathbf{k}]$   
 $= -(25.56\text{ N}\cdot\text{m})\mathbf{j} + (10.80\text{ N}\cdot\text{m})\mathbf{j}$   
 $+ (40.62\text{ N}\cdot\text{m})\mathbf{k}$

$\therefore M_x = -25.6\text{ N}\cdot\text{m}, M_y = 10.80\text{ N}\cdot\text{m}, M_z = 40.6\text{ N}\cdot\text{m}$

3.49 and 3.50

GIVEN:  $T_{AB}, M_x$  OF  
 $T_{AB}$  (AT A) AND  
 $T_{DE}$  (AT D)  
 $= 4728\text{ lb}\cdot\text{ft}$

FIND:  $T_{DE}$ 

FIRST NOTE..  $d_{AC} = \sqrt{(-1)^2 + (-12)^2 + (12)^2} = 17\text{ ft}$   
 $d_{DE} = \sqrt{(1.5)^2 + (-14)^2 + (12)^2} = 18.5\text{ ft}$

THEN..  $T_{AB} = \frac{T_{AB}}{17} (-\mathbf{j} - 12\mathbf{j} + 12\mathbf{k})$  (1b)

$T_{DE} = \frac{T_{DE}}{18.5} (1.5\mathbf{i} - 14\mathbf{j} + 12\mathbf{k})$  (1b)

NOW..  $M_x = \sum (\mathbf{r}_{A/G} \times T_{AB}) + \sum (\mathbf{r}_{D/H} \times T_{DE})$

(CONTINUED)

### 3.49 and 3.50 CONTINUED

WHERE  $\Sigma F_{AG} = (12 \text{ ft})\mathbf{j}$   
 $\Sigma F_{DH} = (14 \text{ ft})\mathbf{j}$

THEN..  $M_x = \mathbf{i} \cdot [12\mathbf{j} \times \frac{T_{AB}}{17}(-\mathbf{i} - 12\mathbf{j} + 12\mathbf{k}) + 14\mathbf{j} \times \frac{T_{DE}}{18.5}(1.5\mathbf{i} - 14\mathbf{j} + 12\mathbf{k})]$   
 $= \frac{144}{17} T_{AB} + \frac{168}{18.5} T_{DE} \quad (16 \cdot \text{ft}) \quad (1)$

3.49 SUBSTITUTING INTO EQ. (1) WITH  
 $T_{AB} = 255 \text{ lb} \quad M_x = 4728 \text{ lb} \cdot \text{ft}$

HAVE  $4728 = \frac{144}{17} (255) + \frac{168}{18.5} T_{DE}$   
 OR  $T_{DE} = 283 \text{ lb}$

3.50 SUBSTITUTING INTO EQ. (1) WITH  
 $T_{AB} = 306 \text{ lb} \quad M_x = 4728 \text{ lb} \cdot \text{ft}$

HAVE  $4728 = \frac{144}{17} (306) + \frac{168}{18.5} T_{DE}$   
 OR  $T_{DE} = 235 \text{ lb}$

3.51

GIVEN:  $M_y = 120 \text{ N} \cdot \text{m}$ ,  $M_z = -460 \text{ N} \cdot \text{m}$   
 OF  $T_{BA}$  AT B

FIND:  $a$

FIRST NOTE..

$\mathbf{BA} = (2.2 \text{ m})\mathbf{i} - (3.2 \text{ m})\mathbf{j} - (a \text{ m})\mathbf{k}$

NOW..  $\mathbf{M}_0 = \Sigma \mathbf{r}_{A/O} \times T_{BA}$

WHERE

$\Sigma \mathbf{r}_{A/O} = (2.2 \text{ m})\mathbf{i} + (1.6 \text{ m})\mathbf{j}$

$T_{BA} = \frac{T_{BA}}{d_{BA}} (2.2\mathbf{i} - 3.2\mathbf{j} - a\mathbf{k}) \quad (\text{N})$

THEN..  $\mathbf{M}_0 = \frac{T_{BA}}{d_{BA}} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.2 & 1.6 & 0 \\ 2.2 & -3.2 & -a \end{vmatrix}$   
 $= \frac{T_{BA}}{d_{BA}} \{-1.6a\mathbf{i} + 2.2a\mathbf{j} + [(2.2)(-3.2) - (1.6)(2.2)]\mathbf{k}\}$

THUS..  $M_y = 2.2 \frac{T_{BA}}{d_{BA}} a \quad (\text{N} \cdot \text{m})$

$M_z = -10.56 \frac{T_{BA}}{d_{BA}} \quad (\text{N} \cdot \text{m})$

THEN.. FORMING THE RATIO  $\frac{M_y}{M_z}$  ..

$\frac{120 \text{ N} \cdot \text{m}}{-460 \text{ N} \cdot \text{m}} = \frac{2.2 \frac{T_{BA}}{d_{BA}} a \quad (\text{N} \cdot \text{m})}{-10.56 \frac{T_{BA}}{d_{BA}} \quad (\text{N} \cdot \text{m})}$

OR  $a = 1.252 \text{ m}$

3.52

GIVEN:  $T_{DA} = 195 \text{ N}$ ,  
 $M_y = 132 \text{ N} \cdot \text{m}$  OF  
 $T_{BA}$  AT B

FIND:  $a$

FIRST NOTE..

$d_{BA} = \sqrt{(2.2)^2 + (-3.2)^2 + (-a)^2}$   
 $= \sqrt{15.08 + a^2} \text{ m}$

AND

$T_{BA} = \frac{195 \text{ N}}{d_{BA}} (2.2\mathbf{i} - 3.2\mathbf{j} - a\mathbf{k})$

NOW  $M_y = \mathbf{j} \cdot (\Sigma \mathbf{r}_{A/O} \times T_{BA})$

WHERE  $\Sigma \mathbf{r}_{A/O} = (2.2 \text{ m})\mathbf{i} + (1.6 \text{ m})\mathbf{j}$

THEN..  $M_y = \frac{195}{d_{BA}} \begin{vmatrix} 0 & 1 & 0 \\ 2.2 & 1.6 & 0 \\ 2.2 & -3.2 & -a \end{vmatrix}$   
 $= \frac{195}{d_{BA}} (2.2a) \quad (\text{N} \cdot \text{m})$

SUBSTITUTING FOR  $M_y$  AND  $d_{BA}$  ..

$132 \text{ N} \cdot \text{m} = \frac{195}{\sqrt{15.08 + a^2}} (2.2a)$

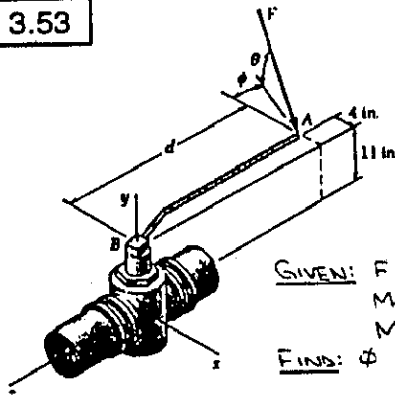
OR  $0.30769 \sqrt{15.08 + a^2} = a$

SQUARING BOTH SIDES OF THE EQUATION..

$0.094675(15.08 + a^2) = a^2$

OR  $a = 1.256 \text{ m}$

3.53



GIVEN:  $F = 70 \text{ lb}$ ,  $\theta = 25^\circ$ ,  
 $M_x = -61 \text{ lb} \cdot \text{ft}$ ,  
 $M_z = -43 \text{ lb} \cdot \text{ft}$

FIND:  $\phi$  AND  $d$

HAVE..  $\mathbf{M}_0 = \Sigma \mathbf{r}_{A/O} \times \mathbf{F}$

WHERE  $\Sigma \mathbf{r}_{A/O} = -(4 \text{ in.})\mathbf{j} + (11 \text{ in.})\mathbf{j} - (d \text{ in.})\mathbf{k}$

AND  $\mathbf{F} = (70 \text{ lb})(\cos \theta \cos \phi \mathbf{i} - \sin \theta \mathbf{j} + \cos \theta \sin \phi \mathbf{k})$

THEN..  $\mathbf{M}_0 = 70 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 11 & -d \\ \cos \theta \cos \phi & -\sin \theta & \cos \theta \sin \phi \end{vmatrix}$   
 $= 70[(11 \cos \theta \sin \phi - d \sin \theta)\mathbf{i} + (-d \cos \theta \cos \phi + 4 \cos \theta \sin \phi)\mathbf{j} + (4 \sin \theta - 11 \cos \theta \cos \phi)\mathbf{k}] \quad (\text{lb} \cdot \text{in.})$

NOW CONSIDER THE Z AND X COMPONENTS OF  $\mathbf{M}_0$ . HAVE..

$M_z: -43 \text{ lb} \cdot \text{ft} = \frac{12 \text{ in.}}{1 \text{ ft}} = 70(4 \sin 25^\circ - 11 \cos 25^\circ \cos \phi) \quad (\text{lb} \cdot \text{in.})$

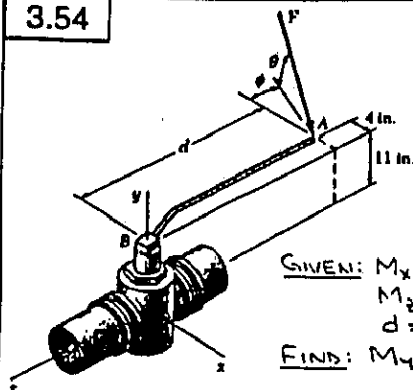
OR  $\cos \phi = 0.90897$

OR  $\phi = 24.637^\circ$

$M_x: -61 \text{ lb} \cdot \text{ft} = \frac{12 \text{ in.}}{1 \text{ ft}} = 70(11 \cos 25^\circ \sin 24.637^\circ - d \sin 25^\circ) \quad (\text{lb} \cdot \text{in.})$

OR  $d = 34.6 \text{ in.}$

3.54



GIVEN:  $M_x = -77 \text{ lb}\cdot\text{ft}$ ,  
 $M_z = -81 \text{ lb}\cdot\text{ft}$ ,  
 $d = 27 \text{ in.}$

FIND:  $M_y$

HAVE...  $\underline{M}_O = \Sigma \underline{r}_{AO} \times \underline{F}$

WHERE  $\underline{r}_{AO} = -(4 \text{ in.})\underline{i} + (11 \text{ in.})\underline{j} - (27 \text{ in.})\underline{k}$

AND  $\underline{F} = F(\cos\theta \cos\phi \underline{i} - \sin\theta \underline{j} + \cos\theta \sin\phi \underline{k})$

$$\text{THEN... } \underline{M}_O = F \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -4 & 11 & -27 \\ \cos\theta \cos\phi & -\sin\theta & \cos\theta \sin\phi \end{vmatrix}$$

$$= F[(11 \cos\theta \sin\phi - 27 \sin\theta)\underline{i} + (-27 \cos\theta \cos\phi + 4 \cos\theta \sin\phi)\underline{j} + (4 \sin\theta - 11 \cos\theta \cos\phi)](16 \text{ in})$$

SO THAT  $M_x = F(11 \cos\theta \sin\phi - 27 \sin\theta)$  (1)

$M_y = F(-27 \cos\theta \cos\phi + 4 \cos\theta \sin\phi)$  (2)

$M_z = F(4 \sin\theta - 11 \cos\theta \cos\phi)$  (3)

WHERE  $M_x$ ,  $M_y$ , AND  $M_z$  ARE IN  $\text{lb}\cdot\text{in.}$  NOW...

EQ. (1)  $\Rightarrow \cos\theta \sin\phi = \frac{1}{11} \left( \frac{M_x}{F} + 27 \sin\theta \right)$  (4)

EQ. (3)  $\Rightarrow \cos\theta \cos\phi = \frac{1}{11} \left( 4 \sin\theta - \frac{M_z}{F} \right)$  (5)

SUBSTITUTING EQS. (4) AND (5) INTO EQ. (2) YIELDS

$$M_y = F \left\{ -27 \left[ \frac{1}{11} \left( 4 \sin\theta - \frac{M_z}{F} \right) \right] + 4 \left[ \frac{1}{11} \left( \frac{M_x}{F} + 27 \sin\theta \right) \right] \right\}$$

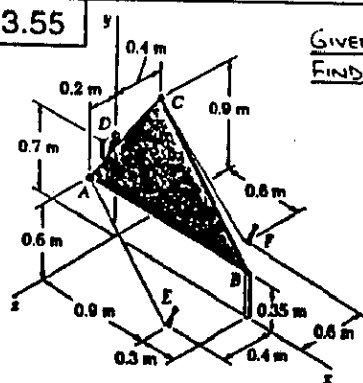
$$= \frac{1}{11} (27 M_z + 4 M_x)$$

NOTING THAT THE RATIOS  $\frac{27}{11}$  AND  $\frac{4}{11}$  ARE THE RATIOS OF LENGTHS, HAVE...

$$M_y = \frac{27}{11} (-81 \text{ lb}\cdot\text{ft}) + \frac{4}{11} (-77 \text{ lb}\cdot\text{ft})$$

$$\text{OR } M_y = -227 \text{ lb}\cdot\text{ft}$$

3.55



GIVEN:  $T_{AE} = 55 \text{ N}$

FIND: MOMENT OF  $\underline{T}_{AE}$   
 AT A ABOUT LINE  
 JOINING D AND B

FIRST NOTE...  $d_{AE} = \sqrt{(0.9)^2 + (-0.6)^2 + (0.2)^2} = 1.1 \text{ m}$

$$\text{THEN... } \underline{T}_{AE} = \frac{55 \text{ N}}{1.1} (0.9 \underline{i} - 0.6 \underline{j} + 0.2 \underline{k})$$

$$= 5[(9 \text{ N})\underline{i} - (6 \text{ N})\underline{j} + (2 \text{ N})\underline{k}]$$

(CONTINUED)

3.55 CONTINUED

$$\text{ALSO... } DB = \sqrt{(1.2)^2 + (-0.35)^2 + (0)^2} = 1.25 \text{ m}$$

$$\text{THEN } \underline{\lambda}_{DB} = \frac{\underline{DB}}{DB} = \frac{1}{1.25} (1.2 \underline{i} - 0.35 \underline{j})$$

$$= \frac{1}{25} (24 \underline{i} - 7 \underline{j})$$

NOW...  $M_{DB} = \underline{\lambda}_{DB} \cdot (\Sigma \underline{r}_{AD} \times \underline{T}_{AE})$

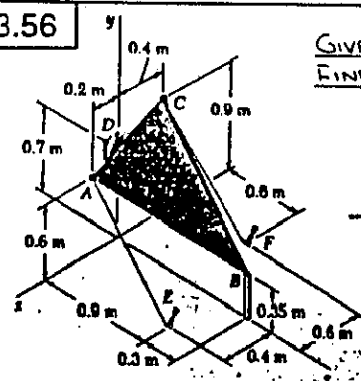
WHERE  $\Sigma \underline{r}_{AD} = -(0.1 \text{ m})\underline{j} - (0.2 \text{ m})\underline{k}$

$$\text{THEN... } M_{DB} = \frac{1}{25} (5) \begin{vmatrix} 24 & -7 & 0 \\ 0 & -0.1 & 0.2 \\ 9 & -6 & 2 \end{vmatrix}$$

$$= \frac{1}{25} (-4.8 - 12.6 + 28.8)$$

$$\text{OR } M_{DB} = 2.28 \text{ N}\cdot\text{m}$$

3.56



GIVEN:  $T_{CF} = 33 \text{ N}$

FIND: MOMENT OF  $\underline{T}_{CF}$   
 AT C ABOUT LINE  
 JOINING D AND B

FIRST NOTE...  $d_{CF} = \sqrt{(0.6)^2 + (-0.9)^2 + (-0.2)^2} = 1.1 \text{ m}$

$$\text{THEN... } \underline{T}_{CF} = \frac{33 \text{ N}}{1.1} (0.6 \underline{i} - 0.9 \underline{j} - 0.2 \underline{k})$$

$$= 3[(6 \text{ N})\underline{i} - (9 \text{ N})\underline{j} - (2 \text{ N})\underline{k}]$$

ALSO...  $DB = \sqrt{(1.2)^2 + (-0.35)^2 + (0)^2} = 1.25 \text{ m}$

$$\text{THEN } \underline{\lambda}_{DB} = \frac{\underline{DB}}{DB} = \frac{1}{1.25} (1.2 \underline{i} - 0.35 \underline{j})$$

$$= \frac{1}{25} (24 \underline{i} - 7 \underline{j})$$

NOW...  $M_{DB} = \underline{\lambda}_{DB} \cdot (\Sigma \underline{r}_{CD} \times \underline{T}_{CF})$

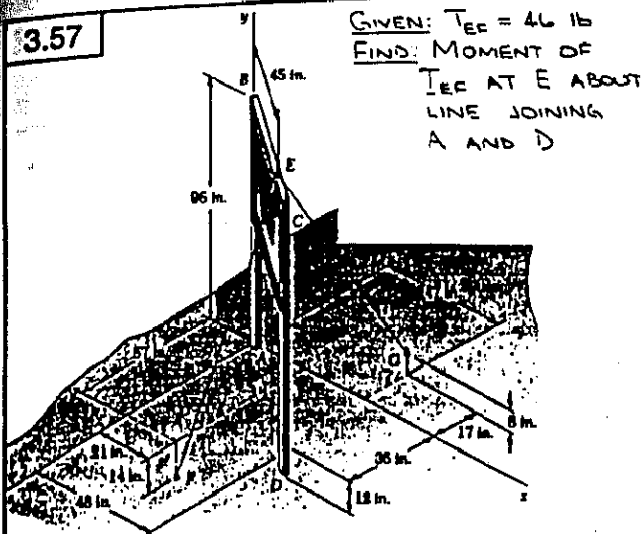
WHERE  $\Sigma \underline{r}_{CD} = (0.2 \text{ m})\underline{j} - (0.4 \text{ m})\underline{k}$

$$\text{THEN... } M_{DB} = \frac{1}{25} (3) \begin{vmatrix} 24 & -7 & 0 \\ 0 & 0.2 & -0.4 \\ 6 & -9 & -2 \end{vmatrix}$$

$$= \frac{3}{25} (-9.6 + 16.8 - 86.4)$$

$$\text{OR } M_{DB} = -950 \text{ N}\cdot\text{m}$$

3.57



GIVEN:  $T_{EG} = 46 \text{ lb}$   
 FIND: MOMENT OF  
 $T_{EG}$  AT E ABOUT  
 LINE JOINING  
 A AND D

FIRST NOTE THAT  $BC = \sqrt{(48)^2 + (36)^2} = 60 \text{ in.}$   
 AND THAT  $\frac{BE}{BC} = \frac{45}{60} = \frac{3}{4}$  THE COORDINATES

OF POINT E ARE THEN  $(\frac{3}{4} \times 48, \frac{3}{4} \times 36)$   
 OR  $(36 \text{ in.}, 96 \text{ in.}, 27 \text{ in.})$ . THEN..

$$d_{ED} = \sqrt{(-15)^2 + (-110)^2 + (30)^2} = 115 \text{ in.}$$

$$\text{THEN.. } T_{EG} = \frac{46 \text{ lb}}{115} (-15\mathbf{j} - 110\mathbf{j} + 30\mathbf{k})$$

$$= 2 [(-3 \text{ lb})\mathbf{j} - (22 \text{ lb})\mathbf{j} + (6 \text{ lb})\mathbf{k}]$$

$$\text{ALSO.. } AD = \sqrt{(48)^2 + (-12)^2 + (36)^2} = 12\sqrt{26} \text{ in.}$$

$$\text{THEN } \lambda_{AD} = \frac{AD}{AD} = \frac{1}{12\sqrt{26}} (48\mathbf{j} - 12\mathbf{j} + 36\mathbf{k})$$

$$= \frac{1}{\sqrt{26}} (4\mathbf{j} - \mathbf{j} + 3\mathbf{k})$$

$$\text{NOW.. } M_{AD} = \lambda_{AD} \cdot (\sum E/A \times T_{EG})$$

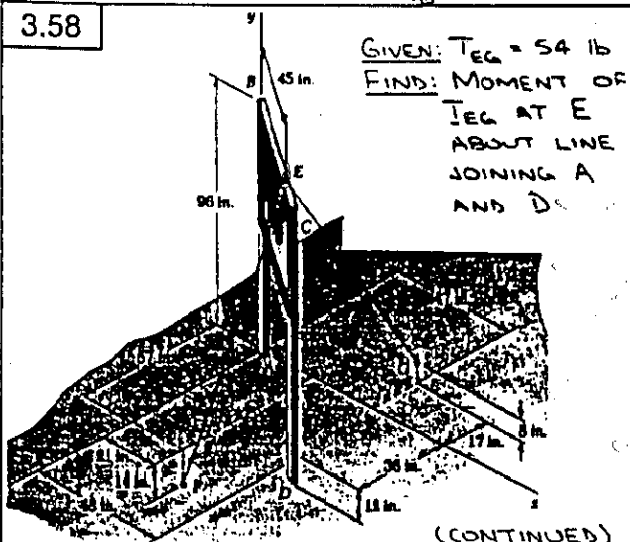
$$\text{WHERE.. } \sum E/A = (30 \text{ in.})\mathbf{j} + (96 \text{ in.})\mathbf{j} + (27 \text{ in.})\mathbf{k}$$

$$\text{THEN.. } M_{AD} = \frac{1}{\sqrt{26}} (2) \begin{vmatrix} 4 & -1 & 3 \\ 36 & 96 & 27 \\ -3 & -22 & 6 \end{vmatrix}$$

$$= \frac{2}{\sqrt{26}} (2304 + 81 - 232 - 864 - 216 + 2576)$$

$$\text{OR } M_{AD} = 1359 \text{ lb}\cdot\text{in.}$$

3.58



GIVEN:  $T_{EG} = 54 \text{ lb}$   
 FIND: MOMENT OF  
 $T_{EG}$  AT E ABOUT  
 LINE JOINING  
 A AND D

(CONTINUED)

3.58 CONTINUED

FIRST NOTE THAT  $BC = \sqrt{(48)^2 + (36)^2} = 60 \text{ in.}$   
 AND THAT  $\frac{BE}{BC} = \frac{45}{60} = \frac{3}{4}$ . THE COORDINATES OF

POINT E ARE THEN  $(\frac{3}{4} \times 48, \frac{3}{4} \times 36)$  OR  
 $(36 \text{ in.}, 96 \text{ in.}, 27 \text{ in.})$ . THEN..

$$d_E = \sqrt{(11)^2 + (-88)^2 + (-44)^2} = 99 \text{ in.}$$

$$\text{THEN.. } T_{EG} = \frac{54 \text{ lb}}{99} (11\mathbf{j} - 88\mathbf{j} - 44\mathbf{k})$$

$$= 6 [(1 \text{ lb})\mathbf{j} - (8 \text{ lb})\mathbf{j} - (4 \text{ lb})\mathbf{k}]$$

$$\text{ALSO.. } AD = \sqrt{(48)^2 + (-12)^2 + (36)^2} = 12\sqrt{26} \text{ in.}$$

$$\text{THEN } \lambda_{AD} = \frac{AD}{AD} = \frac{1}{12\sqrt{26}} (48\mathbf{j} - 12\mathbf{j} + 36\mathbf{k})$$

$$= \frac{1}{\sqrt{26}} (4\mathbf{j} - \mathbf{j} + 3\mathbf{k})$$

$$\text{NOW.. } M_{AD} = \lambda_{AD} \cdot (\sum E/A \times T_{EG})$$

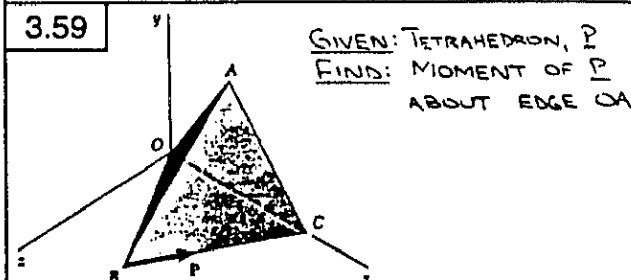
$$\text{WHERE } \sum E/A = (36 \text{ in.})\mathbf{j} + (96 \text{ in.})\mathbf{j} + (27 \text{ in.})\mathbf{k}$$

$$\text{THEN } M_{AD} = \frac{1}{\sqrt{26}} (6) \begin{vmatrix} 4 & -1 & 3 \\ 36 & 96 & 27 \\ 1 & -8 & -4 \end{vmatrix}$$

$$= \frac{6}{\sqrt{26}} (-1536 - 27 - 864 - 288 - 144 - 864)$$

$$\text{OR } M_{AD} = -2350 \text{ lb}\cdot\text{in.}$$

3.59



GIVEN: TETRAHEDRON, P  
 FIND: MOMENT OF P  
 ABOUT EDGE OA

FIRST CONSIDER TRIANGLE OBC. WITH THE  
 LENGTH OF THE SIDES OF THE  
 TRIANGLE EQUAL TO  $a$ , HAVE..

$$BC = a \cos 60^\circ \mathbf{j} - a \sin 60^\circ \mathbf{k}$$

$$\text{THEN } \lambda_{BC} = \frac{BC}{BC} = \frac{1}{2} (\mathbf{j} - \sqrt{3} \mathbf{k})$$

$$\text{AND } \mathbf{P} = P \lambda_{BC} = \frac{P}{2} (\mathbf{j} - \sqrt{3} \mathbf{k})$$

TO DETERMINE  $\lambda_{OA}$ , FIRST OBSERVE THAT  
 $\angle AOC = 60^\circ$ . THE PROJECTION OF  $OA$  ON THE  
 X AXIS IS THEN

$$(OA)_x = a \cos 60^\circ = \frac{a}{2}$$

ALSO, THE PROJECTION OF  $OA$  ONTO THE  
 XZ PLANE BISECTS  $\angle BOC$ , WHERE  $\angle BOC$   
 $= 60^\circ$ . THEN, FROM THE SKETCH..

$$(OA)_x = \frac{a}{2} \quad (OA)_z = (OA)_x \tan 30^\circ = \frac{a}{2\sqrt{3}}$$

$$\text{NOW.. } (OA)^2 = (OA)_x^2 + (OA)_y^2 + (OA)_z^2$$

$$\text{SUBSTITUTING... } a^2 = \left(\frac{a}{2}\right)^2 + (OA)_y^2 + \left(\frac{a}{2\sqrt{3}}\right)^2$$

$$\text{OR } (OA)_y = a \sqrt{\frac{2}{3}}$$

$$\text{THEN.. } \vec{OA} = \frac{a}{2} \mathbf{i} + a \sqrt{\frac{2}{3}} \mathbf{j} + \frac{a}{2\sqrt{3}} \mathbf{k}$$

$$\text{SO THAT } \lambda_{OA} = \frac{1}{2} \mathbf{i} + \sqrt{\frac{2}{3}} \mathbf{j} + \frac{1}{2\sqrt{3}} \mathbf{k}$$

(CONTINUED)



### 3.59 CONTINUED

FINALLY..  $M_{OA} = \Delta_{OA} \cdot (\Sigma C_{10} \cdot P)$

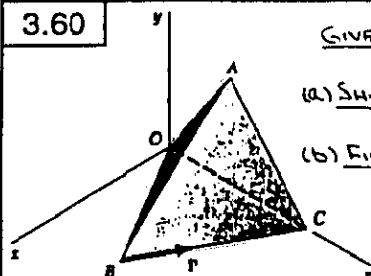
WHERE  $\Sigma C_{10} = a_i$

$$\text{THEN.. } M_{OA} = a \left( \frac{P}{2} \right) \begin{vmatrix} \frac{1}{2} & \sqrt{\frac{2}{3}} & \frac{1}{2\sqrt{3}} \\ 1 & 0 & 0 \\ \frac{1}{2} & 0 & -\sqrt{3} \end{vmatrix}$$

$$= \frac{1}{2} a P \left( \sqrt{\frac{2}{3}} \cdot \sqrt{3} \right)$$

$$\text{OR } M_{OA} = \frac{aP}{\sqrt{2}}$$

### 3.60

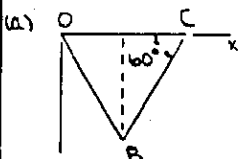


GIVEN: TETRAHEDRON, P,

$M_{OA}$  OF P

(a) SHOW:  $\vec{OA}$  AND  $\vec{BC}$  ARE PERPENDICULAR

(b) FIND: PERPENDICULAR DISTANCE BETWEEN  $\vec{OA}$  AND  $\vec{BC}$



FIRST CONSIDER TRIANGLE OBC. WITH THE LENGTH OF THE SIDES EQUAL TO  $a$ , HAVE-

$$\vec{BC} = a \cos 60^\circ \vec{i} - a \sin 60^\circ \vec{k}$$

$$= \frac{a}{2} (\vec{i} - \sqrt{3} \vec{k})$$

TO DETERMINE  $\vec{OA}$ , FIRST OBSERVE THAT  $\angle AOX = 60^\circ$ . THE PROJECTION OF  $\vec{OA}$  ON THE X AXIS IS THEN

$$(\vec{OA})_x = a \cos 60^\circ = \frac{a}{2}$$

ALSO, THE PROJECTION OF  $\vec{OA}$  ONTO THE XZ PLANE BISECTS  $\angle BOC$ , WHERE  $\angle BOC = 60^\circ$ . THEN, FROM THE SKETCH..

$$(\vec{OA})_z = (\vec{OA})_x \tan 30^\circ = \frac{a}{2\sqrt{3}}$$

$$\text{THEN.. } \vec{OA} = \frac{a}{2} \vec{i} + (\vec{OA})_y \vec{j} + \frac{a}{2\sqrt{3}} \vec{k}$$

IF  $\vec{BC}$  AND  $\vec{OA}$  ARE PERPENDICULAR,

$$\vec{BC} \cdot \vec{OA} = 0$$

$$\text{THUS, } \vec{BC} \cdot \vec{OA} = \frac{a}{2} (\vec{i} - \sqrt{3} \vec{k}) \cdot \left[ \frac{a}{2} \vec{i} + (\vec{OA})_y \vec{j} + \frac{a}{2\sqrt{3}} \vec{k} \right]$$

$$= \frac{a}{2} \left[ (1) \left( \frac{a}{2} \right) + (0) (\vec{OA})_y + (-\sqrt{3}) \left( \frac{a}{2\sqrt{3}} \right) \right]$$

$$\therefore \vec{BC} \cdot \vec{OA} = 0 \Rightarrow \vec{BC} \text{ AND } \vec{OA} \text{ ARE PERPENDICULAR}$$

(b) SINCE  $\vec{OA}$  IS PERPENDICULAR TO  $\vec{BC}$ , AND THUS TO P, IT FOLLOWS THAT

$$M_{OA} = dP$$

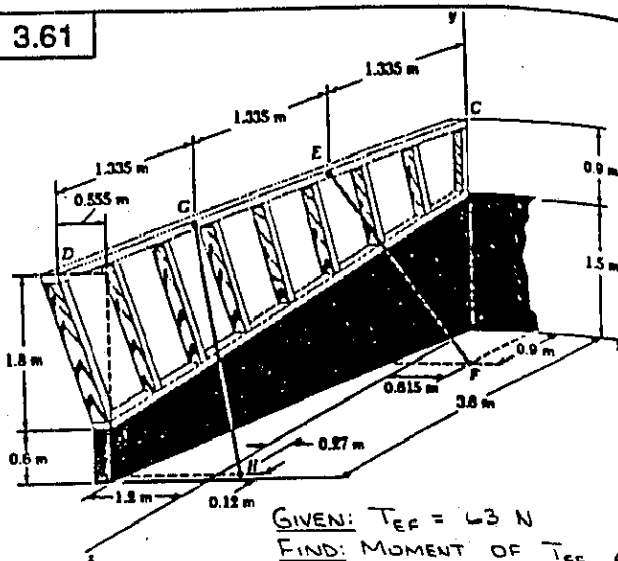
WHERE  $d$  IS THE PERPENDICULAR DISTANCE BETWEEN  $\vec{OA}$  AND  $\vec{BC}$  AND FROM THE SOLUTION TO PROBLEM 3.59

$$M_{OA} = \frac{1}{\sqrt{2}} aP$$

$$\text{THEN.. } \frac{1}{\sqrt{2}} aP = dP$$

$$\text{OR } d = \frac{a}{\sqrt{2}}$$

### 3.61



GIVEN:  $T_{EF} = 63 \text{ N}$

FIND: MOMENT OF  $T_{EF}$  AT E ABOUT SILL AB

FIRST NOTE THAT

$$CE = \frac{1}{3} CD$$

THEN...

$$d_{EC} = \left\{ \left[ \frac{1}{3} (0.555 + 1.2) + 0.615 \right]^2 + (-2.4)^2 + [0.9 - (\frac{1}{3} \times 3.6)]^2 \right\}^{\frac{1}{2}}$$

$$= \sqrt{(1.2)^2 + (-2.4)^2 + (-0.3)^2} = 2.7 \text{ m}$$

$$\text{AND } T_{EF} = \frac{63 \text{ N}}{2.7} (1.2 \vec{i} - 2.4 \vec{j} - 0.3 \vec{k})$$

$$= 7 [(4 \text{ N}) \vec{i} - (8 \text{ N}) \vec{j} - (1 \text{ N}) \vec{k}]$$

$$\text{ALSO.. } AB = \sqrt{(1.2)^2 + (0.9)^2 + (-3.6)^2} = 3.9 \text{ m}$$

$$\text{THEN } \Delta_{AB} = \frac{1}{3.9} (1.2 \vec{i} + 0.9 \vec{j} - 3.6 \vec{k})$$

$$= \frac{1}{13} (4 \vec{i} + 3 \vec{j} - 12 \vec{k})$$

$$\text{NOW.. } M_{AB} = \Delta_{AB} \cdot (\Sigma F_{10} + T_{EF})$$

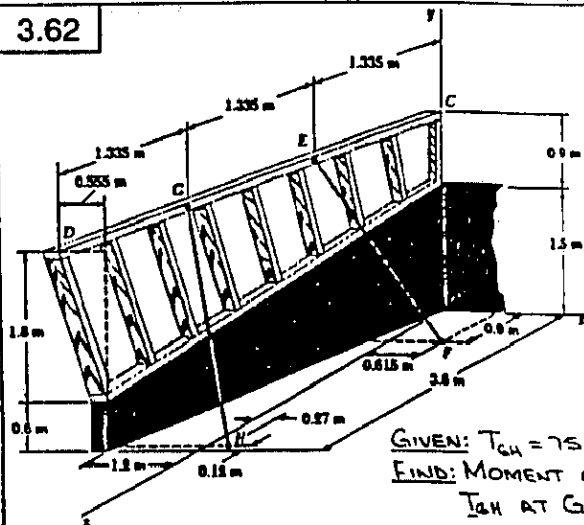
$$\text{WHERE } \Sigma F_{10} = (0.615 \text{ m}) \vec{i} - (1.5 \text{ m}) \vec{j} + (0.9 \text{ m}) \vec{k}$$

$$\text{THEN.. } M_{AB} = \frac{1}{13} (7) \begin{vmatrix} 4 & 3 & -12 \\ 0.615 & -1.5 & 0.9 \\ 4 & -8 & -1 \end{vmatrix}$$

$$= \frac{7}{13} (6 + 10.8 + 59.04 - 72 + 1.845 + 28.8)$$

$$\text{OR } M_{AB} = 18.57 \text{ N}\cdot\text{m}$$

### 3.62



GIVEN:  $T_{GH} = 75 \text{ N}$

FIND: MOMENT OF  $T_{GH}$  AT G ABOUT SILL AB

(CONTINUED)

### 3.62 CONTINUED

FIRST NOTE THAT  $CG = \frac{2}{3} CD$

THEN..  $d_{GH} = \left\{ \left[ \frac{2}{3}(0.555 + 1.2) + 0.27 \right]^2 + (-2.4)^2 \right. \\ \left. + \left[ (3.6 - 0.12) - \left( \frac{2}{3} + 3.6 \right) \right]^2 \right\}^{\frac{1}{2}}$

AND  $I_{GH} = \frac{75 N}{3} (1.44 \hat{i} - 2.4 \hat{j} + 1.08 \hat{k}) \\ = 3[(12 N)\hat{i} - (20 N)\hat{j} + (9 N)\hat{k}]$

ALSO..  $AB = \sqrt{(1.2)^2 + (0.9)^2 + (-3.6)^2} = 3.9 \text{ m}$

THEN..  $\Delta_{AB} = \frac{1}{3.9} (1.2 \hat{i} + 0.9 \hat{j} - 3.6 \hat{k}) \\ = \frac{1}{13} (4 \hat{i} + 3 \hat{j} - 12 \hat{k})$

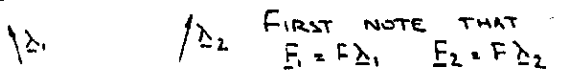
NOW..  $M_{AB} = \Delta_{AB} \cdot (\Sigma H/A \times I_{GH})$   
WHERE  $\Sigma H/A = (1.47 \text{ m})\hat{i} - (0.6 \text{ m})\hat{j} - (0.12 \text{ m})\hat{k}$

THEN..  $M_{AB} = \frac{1}{13} (3) \begin{vmatrix} 4 & 3 & -12 \\ 1.47 & -0.6 & -0.12 \\ 12 & -20 & 9 \end{vmatrix} \\ = \frac{3}{13} (-216 - 4.32 + 552.8 - 86.4 - 39.69 - 9.6) \\ \text{OR } M_{AB} = 44.1 \text{ N}\cdot\text{m}$

### 3.63

GIVEN: FORCES  $F_1$  AND  $F_2$ ,  $F_1 = F_2 = F$

SHOW:  $M_{\lambda_1}$  OF  $F_2 = M_{\lambda_2}$  OF  $F_1$



FIRST NOTE THAT  $F_1 = F \lambda_1$   $F_2 = F \lambda_2$

NOW, BY DEFINITION..

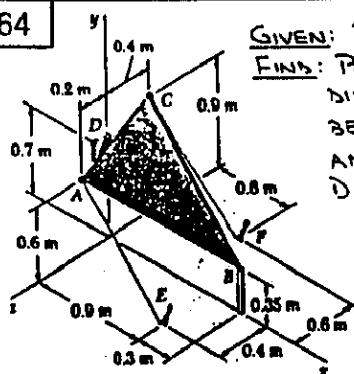
$M_{\lambda_1} = \lambda_1 \cdot (\Sigma \times F_2) \\ = \lambda_1 \cdot (\Sigma \times \lambda_2) F$

AND  $M_{\lambda_2} = \lambda_2 \cdot (\Sigma \times F_1) \\ = \lambda_2 \cdot (\Sigma \times \lambda_1) F$

USING EQ. (3.39)  $\lambda_2 \cdot (\Sigma \times \lambda_1) = \lambda_1 \cdot (\Sigma \times \lambda_2)$   
SO THAT  $M_{\lambda_2} = \lambda_1 \cdot (\Sigma \times \lambda_2) F$

$\therefore M_{\lambda_1} = M_{\lambda_2}$

### \* 3.64



GIVEN:  $I_{AE}$ ,  $\Delta_{DB}$ ,  $M_{DB}$

FIND: PERPENDICULAR DISTANCE  $d$  BETWEEN CABLE AE AND LINE JOINING D AND B

FROM THE SOLUTION TO PROBLEM 3.55..  $I_{AE} = 55 \text{ N}$ ,  $I_{AE} = 5[(9 \text{ N})\hat{i} - (6 \text{ N})\hat{j} + (2 \text{ N})\hat{k}]$

$M_{DB} = 2.28 \text{ N}\cdot\text{m}$   $\Delta_{DB} = \frac{1}{25} (24 \hat{i} - 7 \hat{j})$

BASED ON THE DISCUSSION OF SEC. 3.11, IT (CONTINUED)

### 3.64 CONTINUED

FOLLOWS THAT ONLY THE PERPENDICULAR COMPONENT OF  $I_{AE}$  WILL CONTRIBUTE TO THE MOMENT OF  $I_{AE}$  ABOUT LINE DB. NOW

$(I_{AE})_{\text{PARALLEL}} = I_{AE} \cdot \Delta_{DB} \\ = 5(9 \hat{i} - 6 \hat{j} + 2 \hat{k}) \cdot \frac{1}{25} (24 \hat{i} - 7 \hat{j}) \\ = \frac{1}{5} [(9)(24) + (-6)(-7)] \\ = 51.6 \text{ N}$

ALSO..  $I_{AE} = (I_{AE})_{\text{PARALLEL}} + (I_{AE})_{\text{PERP.}}$

SO THAT  $(I_{AE})_{\text{PERP.}} = \sqrt{(55)^2 - (51.6)^2} \\ = 19.0379 \text{ N}$

SINCE  $\Delta_{DB}$  AND  $(I_{AE})_{\text{PERP.}}$  ARE PERPENDICULAR, IT FOLLOWS THAT

$M_{DB} = d(I_{AE})_{\text{PERP.}}$   
OR  $2.28 \text{ N}\cdot\text{m} = d \cdot 19.0379 \text{ N}$   
OR  $d = 0.1198 \text{ m}$

### ALTERNATIVE SOLUTION

LET THE PERPENDICULAR LINE, DRAWN FROM LINE DB TO THE LINE OF ACTION OF  $I_{AE}$ , BE REPRESENTED BY

$d = x \hat{i} + y \hat{j} + z \hat{k}$   $x, y, z$  IN  $\text{m}$

NOW..  $d \perp I_{AE} \Rightarrow d \cdot I_{AE} = 0$   
OR  $(x \hat{i} + y \hat{j} + z \hat{k}) \cdot 5(9 \hat{i} - 6 \hat{j} + 2 \hat{k}) = 0$   
OR  $9x - 6y + 2z = 0$  (1)

AND  $d \perp \Delta_{DB} \Rightarrow d \cdot \Delta_{DB} = 0$   
OR  $(x \hat{i} + y \hat{j} + z \hat{k}) \cdot \frac{1}{25} (24 \hat{i} - 7 \hat{j}) = 0$   
OR  $24x - 7y = 0 \Rightarrow y = \frac{24}{7}x$  (2)

SUBSTITUTING EQ. (2) INTO EQ. (1)..  $9x - 6(\frac{24}{7}x) + 2z = 0 \Rightarrow z = \frac{81}{14}x$  (3)

NOW..  $M_{DB} = \Delta_{DB} \cdot (d \times I_{AE})$   
 $= \frac{1}{25} (5) \begin{vmatrix} 24 & -7 & 0 \\ x & y & z \\ 9 & -6 & 2 \end{vmatrix}$   
 $= \frac{1}{25} (48y - 63z + 14x + 144z)$   
 $= \frac{1}{25} (48y + 14x + 81z)$

SUBSTITUTING FOR  $M_{DB}$  AND USING EQS (2)

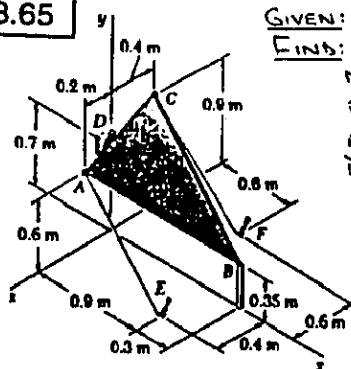
AND (3) YIELDS..  $2.28 = \frac{1}{25} [48(\frac{24}{7}x) + 14x + 81(\frac{81}{14}x)]$   
OR  $x = 0.017614 \text{ m}$

AND THEN (2)  $\Rightarrow y = 0.060391 \text{ m}$

(3)  $\Rightarrow z = 0.101909 \text{ m}$

FINALLY,  $d = \sqrt{x^2 + y^2 + z^2} \\ = \sqrt{(0.017614)^2 + (0.060391)^2 + (0.101909)^2}^{\frac{1}{2}}$   
OR  $d = 0.1198 \text{ m}$

\* 3.65



GIVEN:  $T_{CF}$ ,  $\Delta_{DB}$ ,  $M_{DB}$   
FIND: PERPENDICULAR  
DISTANCE  $d$   
BETWEEN CABLE  $CF$   
AND LINE JOINING  
 $D$  AND  $B$

FROM THE SOLUTION TO PROBLEM 3.56..

$$T_{CF} = 33 \text{ N} \quad T_{CF} = 3[(16 \text{ N})_i - (9 \text{ N})_j - (2 \text{ N})_k]$$

$$|M_{DB}| = 9.50 \text{ N}\cdot\text{m} \quad \Delta_{DB} = \frac{1}{25}(24i - 7j)$$

BASED ON THE DISCUSSION OF SEC. 3.11, IT  
FOLLOWS THAT ONLY THE PERPENDICULAR  
COMPONENT OF  $T_{CF}$  WILL CONTRIBUTE TO  
THE MOMENT OF  $T_{CF}$  ABOUT LINE  $DB$ . NOW..

$$(T_{CF})_{\text{PARALLEL}} = T_{CF} \cdot \Delta_{DB}$$

$$= 3(16i - 9j - 2k) \cdot \frac{1}{25}(24i - 7j)$$

$$= \frac{3}{25}[(16)(24) + (-9)(-7)]$$

$$= 24.84 \text{ N}$$

ALSO..  $T_{CF} = (T_{CF})_{\text{PARALLEL}} + (T_{CF})_{\text{PERP.}}$   
SO THAT  $(T_{CF})_{\text{PERP.}} = \sqrt{(33)^2 - (24.84)^2}$   
 $= 21.725 \text{ N}$

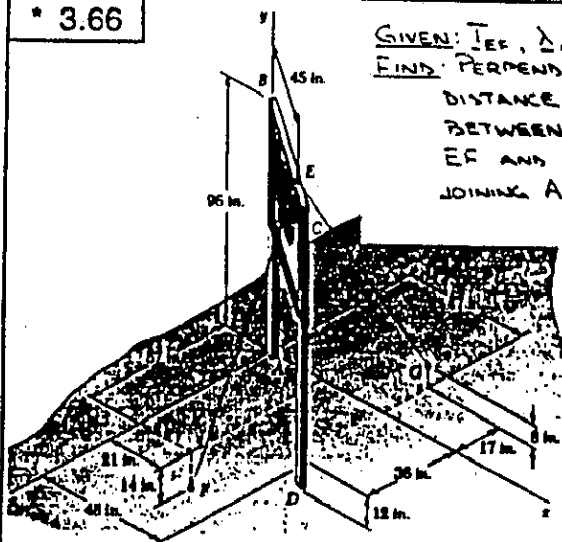
SINCE  $\Delta_{DB}$  AND  $(T_{CF})_{\text{PERP.}}$  ARE PERPENDICULAR,  
IT FOLLOWS THAT

$$|M_{DB}| = d(T_{CF})_{\text{PERP.}}$$

OR  $9.50 \text{ N}\cdot\text{m} = d \cdot 21.725 \text{ N}$   
OR  $d = 0.437 \text{ m}$

FOR A SECOND METHOD OF SOLUTION, SEE  
THE SOLUTION TO PROBLEM 3.64.

\* 3.66



GIVEN:  $T_{EF}$ ,  $\Delta_{AD}$ ,  $M_{AD}$   
FIND: PERPENDICULAR  
DISTANCE  $d$   
BETWEEN CABLE  
 $EF$  AND LINE  
JOINING  $A$  AND  $D$

FROM THE SOLUTION TO PROBLEM 3.57..

$$T_{EF} = 46 \text{ lb} \quad T_{EF} = 2[-(3 \text{ lb})_i - (22 \text{ lb})_j + (6 \text{ lb})_k]$$

(CONTINUED)

3.66 CONTINUED

$$M_{AD} = 1359 \text{ lb}\cdot\text{in.} \quad \Delta_{AD} = \frac{1}{\sqrt{26}}(4i - j + 3k)$$

BASED ON THE DISCUSSION OF SEC. 3.11, IT  
FOLLOWS THAT ONLY THE PERPENDICULAR  
COMPONENT OF  $T_{EF}$  WILL CONTRIBUTE TO  
THE MOMENT OF  $T_{EF}$  ABOUT LINE  $AD$ . NOW

$$(T_{EF})_{\text{PARALLEL}} = T_{EF} \cdot \Delta_{AD}$$

$$= 2(-3i - 22j + 6k) \cdot \frac{1}{\sqrt{26}}(4i - j + 3k)$$

$$= \frac{2}{\sqrt{26}}[(-3)(4) + (-22)(-1) + (6)(3)]$$

$$= 10.9825 \text{ lb}$$

ALSO..  $T_{EF} = (T_{EF})_{\text{PARALLEL}} + (T_{EF})_{\text{PERP.}}$   
SO THAT  $(T_{EF})_{\text{PERP.}} = \sqrt{(46)^2 - (10.9825)^2}$   
 $= 44.670 \text{ lb}$

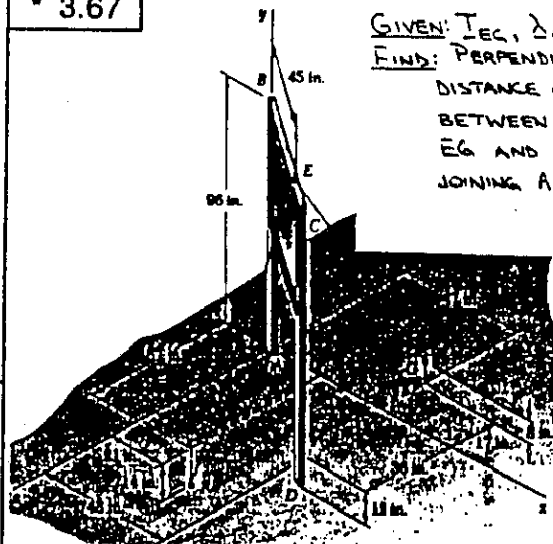
SINCE  $\Delta_{AD}$  AND  $(T_{EF})_{\text{PERP.}}$  ARE PERPENDICULAR  
IT FOLLOWS THAT

$$M_{AD} = d(T_{EF})_{\text{PERP.}}$$

OR  $1359 \text{ lb}\cdot\text{in.} = d \cdot 44.670 \text{ lb}$   
OR  $d = 30.4 \text{ in.}$

FOR A SECOND METHOD OF SOLUTION, SEE  
THE SOLUTION TO PROBLEM 3.64.

\* 3.67



GIVEN:  $T_{EG}$ ,  $\Delta_{AD}$ ,  $M_{AD}$   
FIND: PERPENDICULAR  
DISTANCE  $d$   
BETWEEN CABLE  
 $EG$  AND LINE  
JOINING  $A$  AND  $D$

FROM THE SOLUTION TO PROBLEM 3.58..

$$T_{EG} = 54 \text{ lb} \quad T_{EG} = 6[(1 \text{ lb})_i - (8 \text{ lb})_j - (4 \text{ lb})_k]$$

$$|M_{AD}| = 2350 \text{ lb}\cdot\text{in.} \quad \Delta_{AD} = \frac{1}{\sqrt{26}}(4i - j + 3k)$$

BASED ON THE DISCUSSION OF SEC. 3.11, IT  
FOLLOWS THAT ONLY THE PERPENDICULAR  
COMPONENT OF  $T_{EG}$  WILL CONTRIBUTE TO  
THE MOMENT OF  $T_{EG}$  ABOUT LINE  $AD$ . NOW

$$(T_{EG})_{\text{PARALLEL}} = T_{EG} \cdot \Delta_{AD}$$

$$= 6(1i - 8j - 4k) \cdot \frac{1}{\sqrt{26}}(4i - j + 3k)$$

$$= \frac{6}{\sqrt{26}}[(1)(4) + (-8)(-1) + (-4)(3)]$$

$$= 0$$

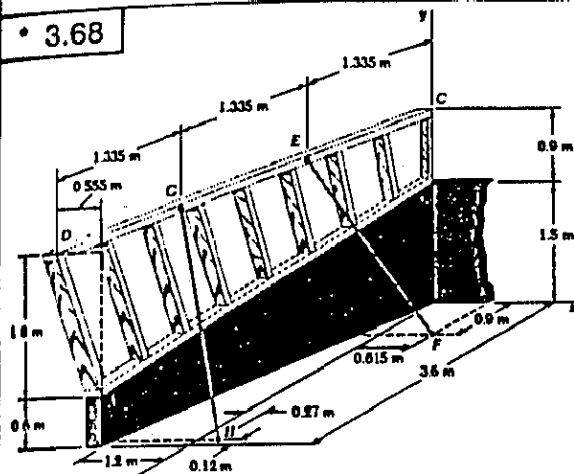
THUS,  $(T_{EG})_{\text{PERP.}} = T_{EG} = 54 \text{ lb}$   
(CONTINUED)

### 3.67 CONTINUED

SINCE  $\Delta_{AD}$  AND  $(T_{EG})_{\text{PERP}}$  ARE PERPENDICULAR,  
IT FOLLOWS THAT  
 $|M_{AB}| = d(T_{EG})_{\text{PERP}}$   
OR  $2350 \text{ lb}\cdot\text{in.} = d \times 54 \text{ lb}$   
OR  $d = 43.5 \text{ in.}$

FOR A SECOND METHOD OF SOLUTION, SEE  
THE SOLUTION TO PROBLEM 3.64.

### 3.68



GIVEN:  $T_{EF}$ ,  $\Delta_{AB}$ ,  $M_{AB}$   
FIND: PERPENDICULAR  
DISTANCE  $d$  BETWEEN  
CABLE EF AND SILL AB

FROM THE SOLUTION TO PROBLEM 3.61--

$$T_{EF} = 63 \text{ N} \quad T_{EF} = 7[(4 \text{ N})\mathbf{i} - (8 \text{ N})\mathbf{j} - (1 \text{ N})\mathbf{k}]$$

$$M_{AB} = 18.57 \text{ N}\cdot\text{m} \quad \Delta_{AB} = \frac{1}{13}(4\mathbf{i} + 3\mathbf{j} - 12\mathbf{k})$$

BASED ON THE DISCUSSION OF SEC. 3.11, IT  
FOLLOWS THAT ONLY THE PERPENDICULAR  
COMPONENT OF  $T_{EF}$  WILL CONTRIBUTE TO  
THE MOMENT OF  $T_{EF}$  ABOUT SILL AB. NOW..

$$(T_{EF})_{\text{PARALLEL}} = T_{EF} \cdot \Delta_{AB}$$

$$= 7(4\mathbf{i} - 8\mathbf{j} - \mathbf{k}) \cdot \frac{1}{13}(4\mathbf{i} + 3\mathbf{j} - 12\mathbf{k})$$

$$= \frac{7}{13}\{4(4) + (-8)(3) + (-1)(-12)\}$$

$$= 2.1538 \text{ N}$$

ALSO..  $T_{EF} = (T_{EF})_{\text{PARALLEL}} + (T_{EF})_{\text{PERP}}$   
SO THAT  $(T_{EF})_{\text{PERP}} = \sqrt{(63)^2 - (2.1538)^2}$   
 $= 62.963 \text{ N}$

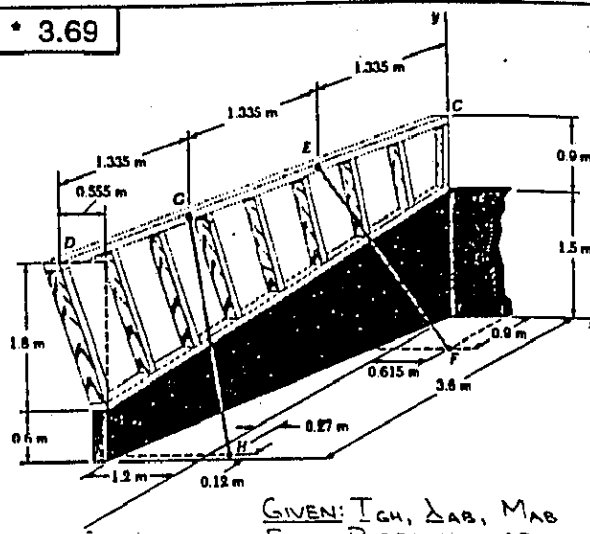
SINCE  $\Delta_{AB}$  AND  $(T_{EF})_{\text{PERP}}$  ARE PERPENDICULAR,  
IT FOLLOWS THAT

$$M_{AB} = d(T_{EF})_{\text{PERP}}$$

OR  $18.57 \text{ N}\cdot\text{m} = d \cdot 62.963 \text{ N}$   
OR  $d = 0.295 \text{ m}$

FOR A SECOND METHOD OF SOLUTION, SEE  
THE SOLUTION TO PROBLEM 3.64.

### 3.69



GIVEN:  $T_{GH}$ ,  $\Delta_{AB}$ ,  $M_{AB}$   
FIND: PERPENDICULAR  
DISTANCE  $d$  BETWEEN  
CABLE GH AND SILL AB

FROM THE SOLUTION TO PROBLEM 3.62..

$$T_{GH} = 75 \text{ N} \quad T_{GH} = 3[(12 \text{ N})\mathbf{i} - (20 \text{ N})\mathbf{j} + (9 \text{ N})\mathbf{k}]$$

$$M_{AB} = 44.1 \text{ N}\cdot\text{m} \quad \Delta_{AB} = \frac{1}{13}(4\mathbf{i} + 3\mathbf{j} - 12\mathbf{k})$$

BASED ON THE DISCUSSION OF SEC. 3.11, IT  
FOLLOWS THAT ONLY THE PERPENDICULAR  
COMPONENT OF  $T_{GH}$  WILL CONTRIBUTE TO  
THE MOMENT OF  $T_{GH}$  ABOUT SILL AB. NOW..

$$(T_{GH})_{\text{PARALLEL}} = T_{GH} \cdot \Delta_{AB}$$

$$= 3(12\mathbf{i} - 20\mathbf{j} + 9\mathbf{k}) \cdot \frac{1}{13}(4\mathbf{i} + 3\mathbf{j} - 12\mathbf{k})$$

$$= \frac{3}{13}\{12(4) + (-20)(3) + (9)(-12)\}$$

$$= -27.692 \text{ N}$$

$$\text{ALSO.. } T_{GH} = (T_{GH})_{\text{PARALLEL}} + (T_{GH})_{\text{PERP}}$$

$$\text{SO THAT.. } (T_{GH})_{\text{PERP}} = \sqrt{(75)^2 - (-27.692)^2}$$

$$= 69.700 \text{ N}$$

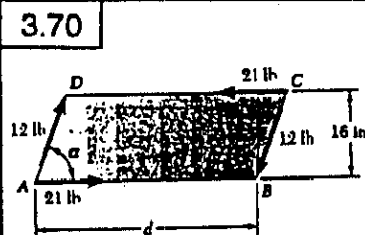
SINCE  $\Delta_{AB}$  AND  $(T_{GH})_{\text{PERP}}$  ARE PERPENDICULAR,  
IT FOLLOWS THAT

$$M_{AB} = d(T_{GH})_{\text{PERP}}$$

OR  $44.1 \text{ N}\cdot\text{m} = d \cdot 69.700 \text{ N}$   
OR  $d = 0.633 \text{ m}$

FOR A SECOND METHOD OF SOLUTION, SEE  
THE SOLUTION TO PROBLEM 3.64.

### 3.70

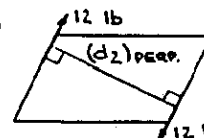


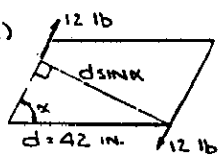
GIVEN:  $F_1 = 21 \text{ lb}$   
 $F_2 = 12 \text{ lb}$

FIND: (a)  $M_1$   
(b)  $(d_2)_{\text{PERP}}$ , GIVEN  
 $M_1 + M_2 = 0$   
(c)  $x$ , GIVEN  
 $M_1 + M_2 = 72 \text{ lb}\cdot\text{in.}$   
 $d = 42 \text{ in.}$

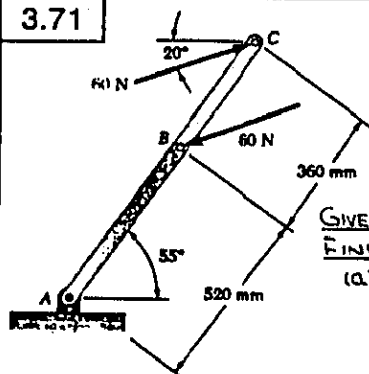
(a) HAVE  $M_1 = d_1 F_1$  WHERE  $d_1 = 16 \text{ in.}$   
 $= (16 \text{ in.})(21 \text{ lb})$   
OR  $M_1 = 336 \text{ lb}\cdot\text{in.}$   
(CONTINUED)

### 3.70 CONTINUED

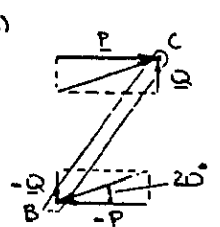
(b)  HAVE...  $M_1 = M_2 = 0$   
OR  $336 \text{ lb}\cdot\text{in.} - (d_2)_{\text{PERP}}(12 \text{ lb}) = 0$   
OR  $(d_2)_{\text{PERP}} = 28 \text{ in.}$

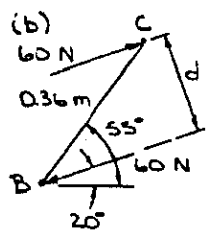
(c)  HAVE...  $M_{\text{TOTAL}} = M_1 + M_2$   
OR  $-72 \text{ lb}\cdot\text{in.} = 336 \text{ lb}\cdot\text{in.}$   
 $-[(42 \text{ in.} \times \sin \alpha)(12 \text{ lb})]$   
OR  $\sin \alpha = 0.80952$   
OR  $\alpha = 54.0^\circ$

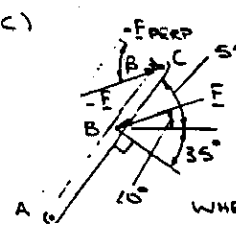
### 3.71



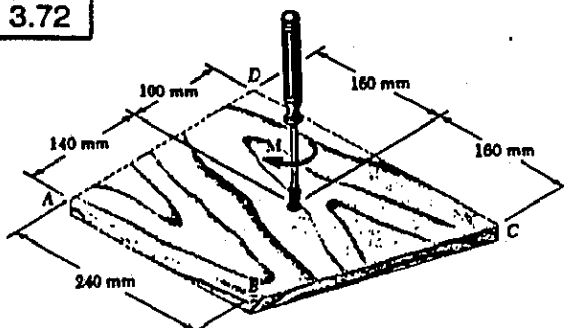
GIVEN: 60-N FORCES  
FIND: MOMENT OF COUPLE  
(a) BY RESOLVING FORCES INTO HORIZONTAL AND VERTICAL COMPONENTS  
(b) BY USING  $d_{\text{PERP}}$   
(c) BY SUMMING MOMENTS ABOUT A

(a)  EACH 60-N FORCE IS FIRST RESOLVED INTO HORIZONTAL (P) AND VERTICAL (Q) COMPONENTS, WHERE  
 $P = (60 \text{ N}) \cos 20^\circ$   
 $Q = (60 \text{ N}) \sin 20^\circ$   
SINCE P AND -P AND Q AND -Q ARE BOTH COUPLES, THE TOTAL MOMENT IS GIVEN BY..  
 $M = -[(0.36 \text{ m})(\sin 55^\circ)][(60 \text{ N})(\cos 20^\circ)]$   
 $+ [(0.36 \text{ m})(\cos 55^\circ)][(60 \text{ N})(\sin 20^\circ)]$   
 $= -(0.36)(60) \sin(55^\circ - 20^\circ) \text{ N}\cdot\text{m}$   
OR  $M = 12.39 \text{ N}\cdot\text{m}$

(b)  HAVE...  $M = -dF$   
WHERE  $d = (0.36 \text{ m}) \sin(55^\circ - 20^\circ)$   
THEN..  
 $M = -(0.36 \text{ m} \times \sin 35^\circ)(60 \text{ N})$   
OR  $M = 12.39 \text{ N}\cdot\text{m}$

(c)  SINCE ONLY THE PERPENDICULAR COMPONENTS OF THE FORCES WILL CONTRIBUTE TO THE MOMENT ABOUT A, HAVE..  
 $M_A = r_{B/A} F_{\text{PERP}} - r_{C/A} F_{\text{PERP}}$   
WHERE  $F_{\text{PERP}} = F \cos \beta$   
 $= (60 \text{ N}) \cos(35^\circ + 20^\circ)$   
THEN..  $M_A = (0.52 - 0.88) \text{ m} \cdot (60 \text{ N}) \cos 55^\circ$   
OR  $M = M_A = 12.39 \text{ N}\cdot\text{m}$

### 3.72

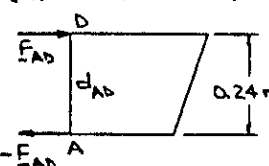


GIVEN:  $M = 18 \text{ N}\cdot\text{m}$

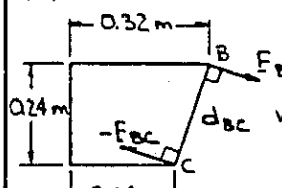
FIND: TWO SMALLEST FORCES EQUIVALENT TO M AND APPLIED AT  
(a) CORNERS A AND D  
(b) CORNERS B AND C  
(c) ANYWHERE ON THE BLOCK

FIRST NOTE THAT IF THE TWO FORCES ARE TO BE EQUIVALENT TO M, THEY MUST FORM A COUPLE. FURTHER, THE FORCES WILL BE MINIMUM WHEN THEY ARE PERPENDICULAR TO THE LINE JOINING THEIR POINTS OF APPLICATION. THUS, FOR EACH PART OF THE PROBLEM --  $M = dF$

(a) FORCES AT A AND D

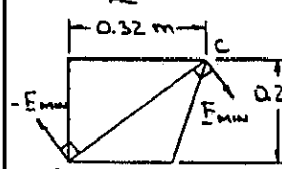
 HAVE...  $M = d_{AD} F_{AD}$   
OR  $18 \text{ N}\cdot\text{m} = 0.24 \text{ m} \cdot F_{AD}$   
OR  $F_{AD} = 75 \text{ N}$

(b) FORCES AT B AND C

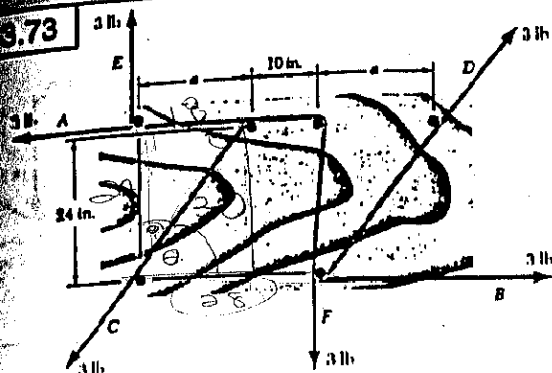
 HAVE...  $M = d_{BC} F_{BC}$   
WHERE  $d_{BC} = \sqrt{(0.32 - 0.24)^2 + (0.24)^2}$   
 $= 0.25298 \text{ m}$   
THEN..  $18 \text{ N}\cdot\text{m} = 0.25298 \text{ m} \cdot F_{BC}$   
OR  $F_{BC} = 71.2 \text{ N}$

(c)  $F_{\text{MIN}}$

FOR  $F_{\text{MIN}}$ , WANT d TO BE MAXIMUM. THUS,  $d = d_{AC}$

 HAVE...  $M = d_{AC} F_{\text{MIN}}$   
WHERE  $d_{AC} = \sqrt{(0.32)^2 + (0.24)^2}$   
 $= 0.4 \text{ m}$   
THEN..  $18 \text{ N}\cdot\text{m} = 0.4 \text{ m} \cdot F_{\text{MIN}}$   
OR  $F_{\text{MIN}} = 45 \text{ N}$

3.73



GIVEN:  $d_{peg} = 2$  in.,  $F = 3$  lb,  $\alpha = 18$  in

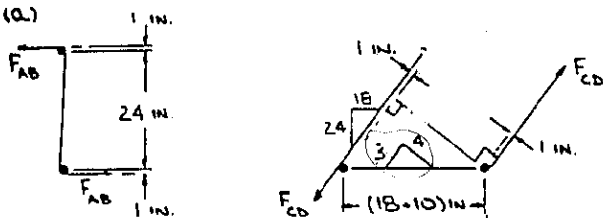
FIND:  $M$  FOR

(a) WIRES AB AND CD

(b) WIRES AB, CD, AND EF

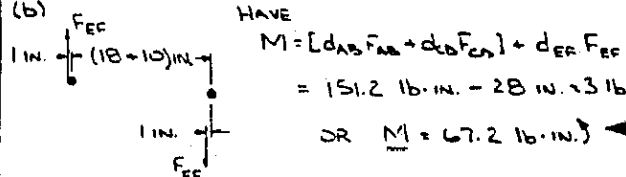
IN GENERAL,  $M = \sum dF$ , WHERE  $d$  IS THE PERPENDICULAR DISTANCE BETWEEN THE LINES OF ACTION OF THE TWO FORCES ACTING ON A GIVEN WIRE.

(a)



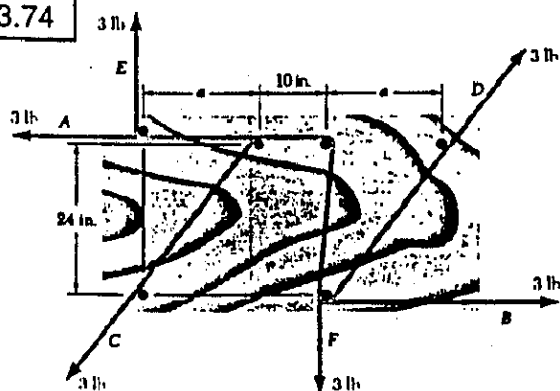
HAVE...  $M = d_{AB}F_{AB} + d_{CD}F_{CD}$   
 $= (2 + 24) \text{ in.} \times 3 \text{ lb} + (2 + 18) \text{ in.} \times 3 \text{ lb}$   
 OR  $M = 151.2 \text{ lb} \cdot \text{in.}$

(b)



HAVE  
 $M = [d_{AB}F_{AB} + d_{CD}F_{CD}] + d_{EF}F_{EF}$   
 $= 151.2 \text{ lb} \cdot \text{in.} - 28 \text{ in.} \times 3 \text{ lb}$   
 OR  $M = 67.2 \text{ lb} \cdot \text{in.}$

3.74



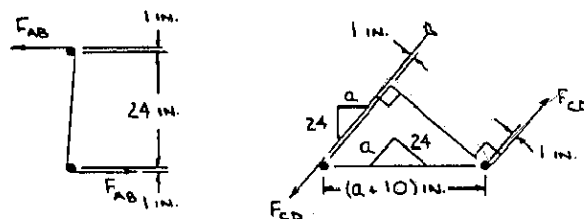
GIVEN:  $d_{peg} = 2$  in.,  $F_{AB} = F_{CD} = 3$  lb,  
 $M = 159.6 \text{ lb} \cdot \text{in.}$

FIND:  $\alpha_{min}$

HAVE...  $M = d_{AB}F_{AB} + d_{CD}F_{CD}$   
 (CONTINUED)

3.74 CONTINUED

WHERE  $d_{AB}$  AND  $d_{CD}$  ARE THE PERPENDICULAR DISTANCES BETWEEN THE LINES OF ACTION OF THE FORCES ACTING ON WIRES AB AND CD, RESPECTIVELY.



THEN...  $159.6 \text{ lb} \cdot \text{in.} = (2 + 24) \text{ in.} \times 3 \text{ lb}$   
 $+ [2 + \frac{24}{\sqrt{24^2 + \alpha^2}} (\alpha + 10)] \text{ in.} \times 3 \text{ lb}$

OR  $25.2 = \frac{24(\alpha + 10)}{\sqrt{576 + \alpha^2}}$

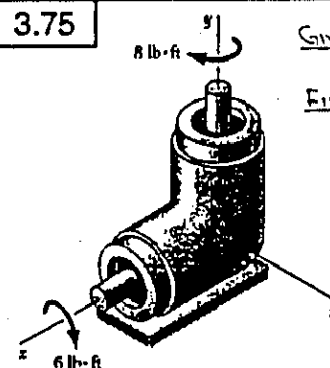
OR  $(25.2)^2 (576 + \alpha^2) = (576)(\alpha + 10)^2$   
 OR  $59.04\alpha^2 - 11520\alpha + 308183 = 0$

OR  $\alpha = \frac{11520 \pm \sqrt{(-11520)^2 - 4(59.04)(308183)}}{2(59.04)}$

SOLVING YIELDS...  $\alpha = 32.0$  in.,  $\alpha = 16.1$  in.

TAKING THE SMALLER ROOT...  $\alpha = 16.1$  in.

3.75



GIVEN:  $M_1 = 8 \text{ lb} \cdot \text{ft}$   
 $M_2 = 6 \text{ lb} \cdot \text{ft}$

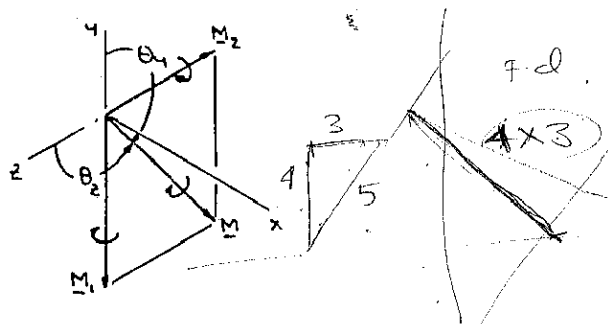
FIND:  $|M_1 + M_2|$ ,  $\theta_x$ ,  $\theta_y$ ,  $\theta_z$

HAVE...  $M = M_1 + M_2$   
 $= -(8 \text{ lb} \cdot \text{ft})\hat{j} - (6 \text{ lb} \cdot \text{ft})\hat{k}$

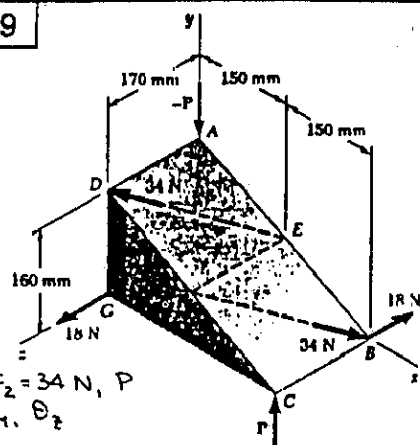
THEN...  $M = \sqrt{(0)^2 + (-8)^2 + (-6)^2}$   
 OR  $M = 10 \text{ lb} \cdot \text{ft}$

AND

$\cos \theta_x = 0$   $\cos \theta_y = -\frac{8}{10}$   $\cos \theta_z = -\frac{6}{10}$   
 OR  $\theta_x = 90^\circ$   $\theta_y = 143.1^\circ$   $\theta_z = 126.9^\circ$



# 3.76 and 3.79



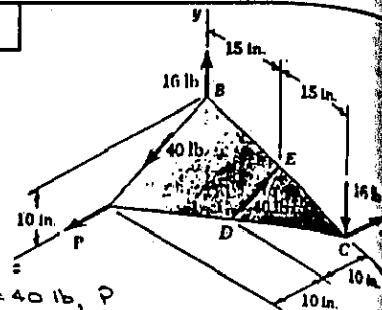
GIVEN:  $F_1 = 18 \text{ N}$ ,  $F_2 = 34 \text{ N}$ ,  $P$   
 FIND:  $M$ ,  $\theta_x$ ,  $\theta_y$ ,  $\theta_z$

HAVE  $\underline{M} = \underline{M}_1 + \underline{M}_2 + \underline{M}_3$   
 WHERE  $\underline{M}_1 = \underline{r}_{C/G} \times \underline{F}_1 = (0.3 \text{ m})\underline{i} \times (18 \text{ N})\underline{k}$   
 $= (5.4 \text{ N}\cdot\text{m})\underline{j}$   
 $\underline{M}_2 = \underline{r}_{E/B} \times \underline{F}_2$   
 WHERE  $\underline{r}_{E/B} = (0.17 \text{ m})\underline{k}$   
 AND  $d_{FB} = \sqrt{(0.15)^2 + (-0.08)^2 + (-0.17)^2}$   
 $= 0.172 \text{ m}$   
 THEN  $\underline{F}_2 = \frac{34 \text{ N}}{0.172} (0.15\underline{j} - 0.08\underline{j} - 0.17\underline{k})$   
 $= \sqrt{2} [(15 \text{ N})\underline{j} - (8 \text{ N})\underline{j} - (17 \text{ N})\underline{k}]$   
 SO THAT  $\underline{M}_2 = 0.172 \times \sqrt{2} (15\underline{j} - 8\underline{j} - 17\underline{k})$   
 $= \sqrt{2} [(1.36 \text{ N}\cdot\text{m})\underline{j} + (2.55 \text{ N}\cdot\text{m})\underline{j}]$   
 $\underline{M}_3 = \underline{r}_C \times \underline{P} = [(0.3 \text{ m})\underline{i} + (0.17 \text{ m})\underline{k}] \times P\underline{j}$   
 $= P(-0.17\underline{j} + 0.3\underline{k}) \text{ (N}\cdot\text{m)}$

3.76  $P=0 \therefore \underline{M} = \underline{M}_1 + \underline{M}_2$   
 OR  $\underline{M} = (5.4\underline{j}) + \sqrt{2} (1.36\underline{j} + 2.55\underline{j})$   
 $= (1.92333 \text{ N}\cdot\text{m})\underline{j} + (9.0062 \text{ N}\cdot\text{m})\underline{j}$   
 THEN  $M = \sqrt{(1.92333)^2 + (9.0062)^2 + (0)^2} = 9.2093 \text{ N}\cdot\text{m}$   
 OR  $M = 9.21 \text{ N}\cdot\text{m}$   
 AND  $\Delta_{\text{axis}} = \frac{\underline{M}}{M} = 0.20885\underline{j} + 0.97795\underline{j}$   
 THEN  $\cos \theta_x = 0.20885$   $\cos \theta_y = 0.97795$   $\cos \theta_z = 0$   
 SO THAT  $\theta_x = 77.9^\circ$   $\theta_y = 12.05^\circ$   $\theta_z = 90^\circ$

3.79  $P=20 \text{ N} \therefore \underline{M} = \underline{M}_1 + \underline{M}_2 + \underline{M}_3$   
 OR  $\underline{M} = (1.92333\underline{j} + 9.0062\underline{j}) + 20(-0.17\underline{j} + 0.3\underline{k})$   
 $= -(1.47667 \text{ N}\cdot\text{m})\underline{j} + (9.0062 \text{ N}\cdot\text{m})\underline{j} + (6 \text{ N}\cdot\text{m})\underline{k}$   
 THEN  $M = \sqrt{(-1.47667)^2 + (9.0062)^2 + (6)^2} = 10.9221 \text{ N}\cdot\text{m}$   
 OR  $M = 10.92 \text{ N}\cdot\text{m}$   
 AND  $\Delta_{\text{axis}} = \frac{\underline{M}}{M} = -0.135200\underline{j} + 0.82459\underline{j} + 0.54934\underline{k}$   
 THEN  $\cos \theta_x = -0.135200$   $\cos \theta_y = 0.82459$   $\cos \theta_z = 0.54934$   
 SO THAT  $\theta_x = 97.8^\circ$   $\theta_y = 34.5^\circ$   $\theta_z = 56.7^\circ$

# 3.77 and 3.78



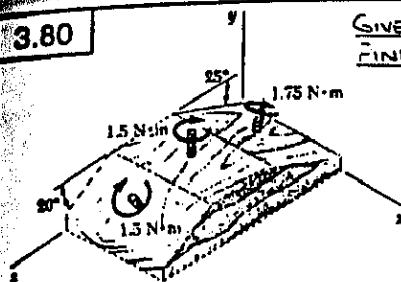
GIVEN:  $F_1 = 16 \text{ lb}$ ,  $F_2 = 40 \text{ lb}$ ,  $P$   
 FIND:  $M$ ,  $\theta_x$ ,  $\theta_y$ ,  $\theta_z$

HAVE  $\underline{M} = \underline{M}_1 + \underline{M}_2 + \underline{M}_3$   
 WHERE  $\underline{M}_1 = \underline{r}_C \times \underline{F}_1 = (30 \text{ in.})\underline{i} \times (-16 \text{ lb})\underline{j}$   
 $= -(480 \text{ lb}\cdot\text{in.})\underline{k}$   
 $\underline{M}_2 = \underline{r}_{E/B} \times \underline{F}_2$   
 WHERE  $\underline{r}_{E/B} = (15 \text{ in.})\underline{j} - (5 \text{ in.})\underline{j}$   
 AND  $d_{DE} = \sqrt{(0)^2 + (5)^2 + (-10)^2} = 5\sqrt{5} \text{ in.}$   
 THEN  $\underline{F}_2 = \frac{40 \text{ lb}}{5\sqrt{5}} (5\underline{j} - 10\underline{k})$   
 $= 8\sqrt{5} [(1 \text{ lb})\underline{j} - (2 \text{ lb})\underline{k}]$   
 SO THAT  $\underline{M}_2 = 8\sqrt{5} \begin{vmatrix} \underline{j} & \underline{j} & \underline{k} \\ 1 & -5 & 0 \\ 0 & 1 & -2 \end{vmatrix}$   
 $= 8\sqrt{5} [(10 \text{ lb}\cdot\text{in.})\underline{j} + (30 \text{ lb}\cdot\text{in.})\underline{j} + (15 \text{ lb}\cdot\text{in.})\underline{k}]$   
 $\underline{M}_3 = \underline{r}_C \times \underline{P} = (30 \text{ in.})\underline{i} \times (-P)\underline{k}$   
 $= (30P)\underline{j} \text{ (lb}\cdot\text{in.)}$

3.77  $P=0 \therefore \underline{M} = \underline{M}_1 + \underline{M}_2$   
 OR  $\underline{M} = -(480)\underline{k} + 8\sqrt{5} (10\underline{j} + 30\underline{j} + 15\underline{k})$   
 $= (178.885 \text{ lb}\cdot\text{in.})\underline{j} + (536.66 \text{ lb}\cdot\text{in.})\underline{j}$   
 $- (211.67 \text{ lb}\cdot\text{in.})\underline{k}$   
 THEN  $M = \sqrt{(178.885)^2 + (536.66)^2 + (-211.67)^2}$   
 $= 603.99 \text{ lb}\cdot\text{in.}$   
 OR  $M = 604 \text{ lb}\cdot\text{in.}$   
 AND  $\Delta_{\text{axis}} = \frac{\underline{M}}{M} = 0.29617\underline{j} + 0.88852\underline{j} - 0.35045\underline{k}$   
 THEN  $\cos \theta_x = 0.29617$   $\cos \theta_y = 0.88852$   $\cos \theta_z = -0.35045$   
 SO THAT  $\theta_x = 72.8^\circ$   $\theta_y = 27.3^\circ$   $\theta_z = 110.5^\circ$

3.78  $P=20 \text{ lb} \therefore \underline{M} = \underline{M}_1 + \underline{M}_2 + \underline{M}_3$   
 OR  $\underline{M} = -(480)\underline{k} + 8\sqrt{5} (10\underline{j} + 30\underline{j} + 15\underline{k})$   
 $+ (30 \times 20)\underline{j}$   
 $= (178.885 \text{ lb}\cdot\text{in.})\underline{j} + (1136.66 \text{ lb}\cdot\text{in.})\underline{j}$   
 $- (211.67 \text{ lb}\cdot\text{in.})\underline{k}$   
 THEN  $M = \sqrt{(178.885)^2 + (1136.66)^2 + (-211.67)^2}$   
 $= 1169.96 \text{ lb}\cdot\text{in.}$   
 OR  $M = 1170 \text{ lb}\cdot\text{in.}$   
 AND  $\Delta_{\text{axis}} = \frac{\underline{M}}{M} = 0.152898\underline{j} + 0.97154\underline{j} - 0.180921\underline{k}$   
 THEN  $\cos \theta_x = 0.152898$   $\cos \theta_y = 0.97154$   $\cos \theta_z = -0.180921$   
 SO THAT  $\theta_x = 81.2^\circ$   $\theta_y = 13.7^\circ$   $\theta_z = 100.4^\circ$

3.80



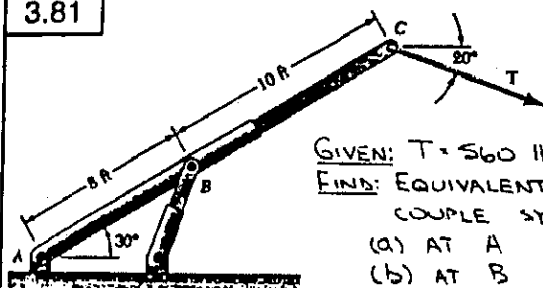
GIVEN:  $M_1, M_2$ , AND  $M_3$   
FIND:  $M, \theta_x, \theta_y, \theta_z$

HAVE..  $M = M_1 + M_2 + M_3$   
OR  $M = 1.5(-\cos 20^\circ \hat{j} - \sin 20^\circ \hat{k})$   
 $- 1.5 \hat{j} + 1.75(-\cos 25^\circ \hat{j} + \sin 25^\circ \hat{k})$   
 $= -(4.4956 \text{ N}\cdot\text{m})\hat{j} + (0.22655 \text{ N}\cdot\text{m})\hat{k}$   
THEN..  $M = \sqrt{(0)^2 + (-4.4956)^2 + (0.22655)^2}$   
 $= 4.5013 \text{ N}\cdot\text{m}$

OR  $M = 4.50 \text{ N}\cdot\text{m}$   
AND  $\Delta_{\text{axis}} = \frac{M}{M} = -0.99873\hat{j} + 0.050330\hat{k}$

THEN..  $\cos \theta_x = 0$   $\cos \theta_y = -0.99873$   $\cos \theta_z = 0.050330$   
SO THAT..  $\theta_x = 90^\circ$   $\theta_y = 177.1^\circ$   $\theta_z = 87.1^\circ$

3.81



GIVEN:  $T = 560 \text{ lb}$   
FIND: EQUIVALENT FORCE-COUPLE SYSTEM

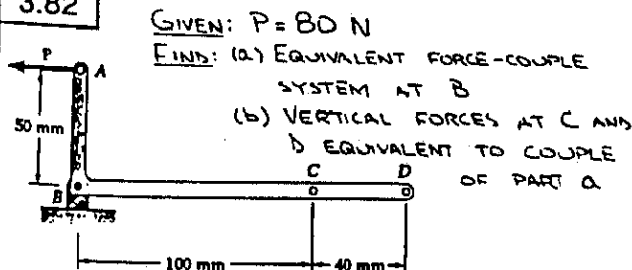
- (a) AT A  
(b) AT B

(a) HAVE..  $E = 560 \text{ lb} \angle 20^\circ$   
AND  $M = M_A$   
 $= -(18 \text{ ft})(560 \text{ lb})\sin 50^\circ$   
 $= -7720 \text{ lb}\cdot\text{ft}$   
.. THE EQUIVALENT FORCE-COUPLE SYSTEM AT A IS  $E = 560 \text{ lb} \angle 20^\circ, M = 7720 \text{ lb}\cdot\text{ft}$

(b) HAVE..  $E = 560 \text{ lb} \angle 20^\circ$   
AND  $M = M_B = -(10 \text{ ft})(560 \text{ lb})\sin 50^\circ$   
 $= -4290 \text{ lb}\cdot\text{ft}$

.. THE EQUIVALENT FORCE-COUPLE SYSTEM AT B IS  $E = 560 \text{ lb} \angle 20^\circ, M = 4290 \text{ lb}\cdot\text{ft}$

3.82



GIVEN:  $P = 80 \text{ N}$

FIND: (a) EQUIVALENT FORCE-COUPLE SYSTEM AT B  
(b) VERTICAL FORCES AT C AND D EQUIVALENT TO COUPLE OF PART (a)

(a) HAVE  $E = 80 \text{ N} \leftarrow$

(CONTINUED)

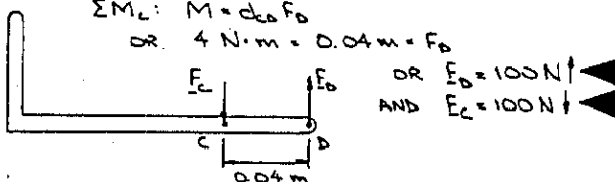
3.82 CONTINUED

AND  $M = M_B = (0.05 \text{ m})(80 \text{ N}) = 4 \text{ N}\cdot\text{m}$   
.. THE EQUIVALENT FORCE-COUPLE SYSTEM AT B IS  $E = 80 \text{ N} \leftarrow, M = 4 \text{ N}\cdot\text{m}$

(b) IF THE TWO VERTICAL FORCES ARE TO BE EQUIVALENT TO  $M$ , THEY MUST BE A COUPLE. FURTHER, THE SENSE OF THE MOMENT OF THIS COUPLE MUST BE COUNTERCLOCKWISE.

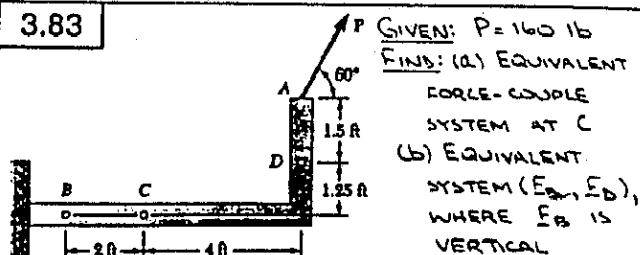
THEN.. WITH  $E_C$  AND  $E_D$  ACTING AS SHOWN, HAVE

$\Sigma M_C: M = d_{CD} F_D$   
OR  $4 \text{ N}\cdot\text{m} = 0.04 \text{ m} \cdot F_D$



OR  $E_D = 100 \text{ N} \uparrow$   
AND  $E_C = 100 \text{ N} \downarrow$

3.83

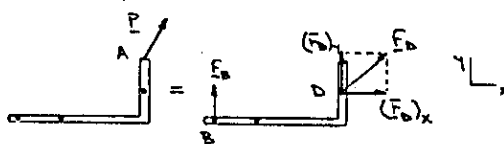


GIVEN:  $P = 160 \text{ lb}$   
FIND: (a) EQUIVALENT FORCE-COUPLE SYSTEM AT C

(b) EQUIVALENT SYSTEM ( $E_B, E_D$ ), WHERE  $E_B$  IS VERTICAL

(a) HAVE..  $E = 160 \text{ lb} \angle 60^\circ$   
AND  $M = M_C = xP_y - yP_x$   
 $y = 2.75 \text{ ft}$  OR  $M = (4 \text{ ft})(160 \text{ lb})\sin 60^\circ$   
 $- (2.75 \text{ ft})(160 \text{ lb})\cos 60^\circ$   
 $= 334.26 \text{ lb}\cdot\text{ft}$   
.. THE EQUIVALENT FORCE-COUPLE SYSTEM AT C IS..  $E = 160 \text{ lb} \angle 60^\circ, M = 334 \text{ lb}\cdot\text{ft}$

(b) REQUIRE



THEN FOR EQUIVALENCE..

$\Sigma F_x: (160 \text{ lb})\cos 60^\circ = (F_D)_x$   
OR  $(F_D)_x = 80 \text{ lb}$

$\Sigma M_A: 0 = (1.5 \text{ ft})(80 \text{ lb}) - (6 \text{ ft})F_D = 0$   
OR  $E_D = 20 \text{ lb} \uparrow$

$\Sigma F_y: (160 \text{ lb})\sin 60^\circ = 20 \text{ lb} + (F_D)_y$   
OR  $(F_D)_y = 118.564 \text{ lb}$

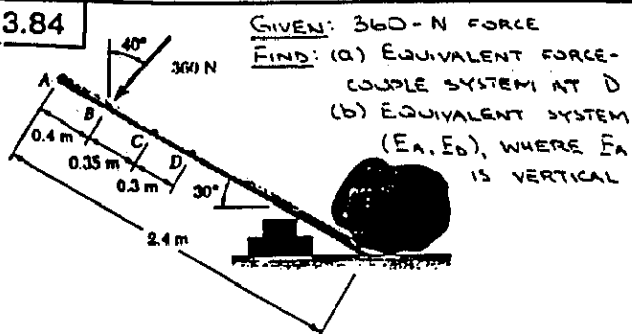
THEN..  $F_D = \sqrt{(80)^2 + (118.564)^2}$   
 $= 143.0 \text{ lb}$

$\tan \theta = \frac{118.564}{80}$   
OR  $\theta = 56.0^\circ$

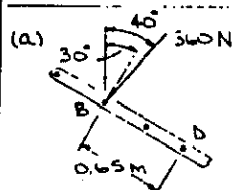
..  $E_D = 143.0 \text{ lb} \angle 56.0^\circ$



3.84



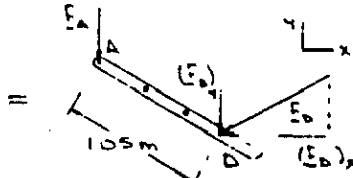
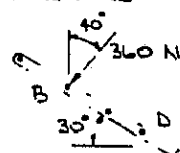
GIVEN: 360-N FORCE  
FIND: (a) EQUIVALENT FORCE-COUPLE SYSTEM AT D  
(b) EQUIVALENT SYSTEM ( $E_A, E_D$ ), WHERE  $E_A$  IS VERTICAL



HAVE  $F = 360 \text{ N} \nearrow 50^\circ$   
AND  $M = M_D$   
 $-(0.65 \text{ m} \times 360 \text{ N}) \cos 10^\circ$   
 $= 230.45 \text{ N}\cdot\text{m}$

$\therefore$  THE EQUIVALENT FORCE-COUPLE SYSTEM AT D IS  
 $F = 360 \text{ N} \nearrow 50^\circ, M = 230 \text{ N}\cdot\text{m}$

(b) REQUIRE



THEN FOR EQUIVALENCE ..

$$\sum M_D: M = (0.65 \cos 30^\circ) F_A$$

$$\text{OR } 230.45 \text{ N}\cdot\text{m} = (1.05 \text{ m}) (\cos 30^\circ) F_A$$

$$\text{OR } F_A = 253.43 \text{ N}$$

THEN..  $F_A = 253 \text{ N}$ 

$$\sum F_x: -(360 \text{ N}) \sin 40^\circ = (F_D)_x$$

$$\text{OR } (F_D)_x = -231.40 \text{ N}$$

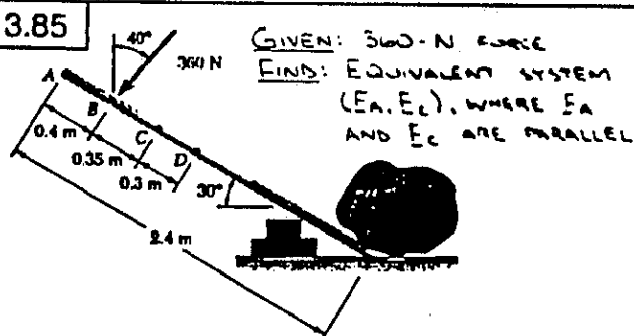
$$\sum F_y: -(360 \text{ N}) \cos 40^\circ = -253.43 \text{ N} - (F_D)_y$$

$$\text{OR } (F_D)_y = -22.35 \text{ N}$$

$$\text{THEN.. } F_D = \sqrt{(-231.40)^2 + (-22.35)^2} = 232 \text{ N}$$

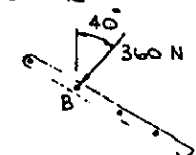
OR  $\theta = 5.52^\circ$  $\therefore F_D = 232 \text{ N} \nearrow 5.52^\circ$ 

3.85



GIVEN: 360-N FORCE  
FIND: EQUIVALENT SYSTEM ( $E_A, E_C$ ), WHERE  $E_A$  AND  $E_C$  ARE PARALLEL

REQUIRE



(CONTINUES)

3.85 CONTINUED

THEN FOR EQUIVALENCE ..

$$\sum F_x: -360 \sin 40^\circ = -F_A \sin \alpha - F_C \sin \alpha \quad (1)$$

$$\sum F_y: -360 \cos 40^\circ = -F_A \cos \alpha - F_C \cos \alpha \quad (2)$$

$$\text{FORMING } \frac{(1)}{(2)} \dots \frac{-360 \sin 40^\circ}{-360 \cos 40^\circ} = \frac{-(F_A + F_C) \sin \alpha}{-(F_A + F_C) \cos \alpha}$$

SIMPLIFYING YIELDS  $\alpha = 40^\circ$ AND THEN  $F_A + F_C = 360 \text{ N} \quad (3)$ 

$$\text{NOW.. } \sum M_D: 0 = (0.4 \text{ m}) F_A \cos 10^\circ - (0.35 \text{ m}) F_C \cos 10^\circ$$

$$\text{OR } F_A = \frac{7}{8} F_C \quad (4)$$

SUBSTITUTING FOR  $F_A$  IN EQ. (3).

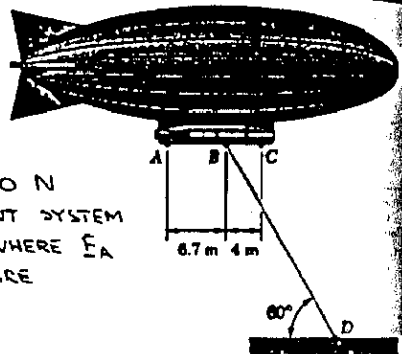
$$\frac{7}{8} F_C + F_C = 360$$

$$\text{OR } F_C = 192 \text{ N}$$

AND THEN  $F_A = 168 \text{ N}$ 

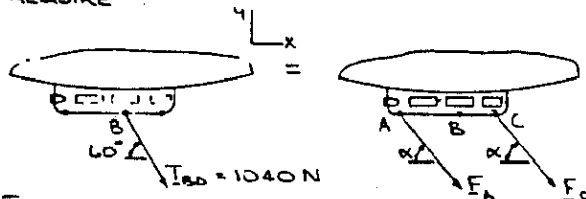
$$\therefore F_A = 168 \text{ N} \nearrow 50^\circ, F_C = 192 \text{ N} \nearrow 50^\circ$$

3.86

GIVEN:  $T_{BD} = 1040 \text{ N}$ 

FIND: EQUIVALENT SYSTEM ( $E_A, E_C$ ), WHERE  $E_A$  AND  $E_C$  ARE PARALLEL

REQUIRE



THEN FOR EQUIVALENCE ..

$$\sum F_x: 1040 \cos 60^\circ = F_A \cos \alpha + F_C \cos \alpha \quad (1)$$

$$\sum F_y: -1040 \sin 60^\circ = -F_A \sin \alpha - F_C \sin \alpha \quad (2)$$

$$\text{FORMING } \frac{(1)}{(2)} \dots \frac{1040 \cos 60^\circ}{-1040 \sin 60^\circ} = \frac{(F_A + F_C) \cos \alpha}{-(F_A + F_C) \sin \alpha}$$

SIMPLIFYING YIELDS  $\alpha = 60^\circ$ AND THEN  $F_A + F_C = 1040 \text{ N} \quad (3)$ 

$$\text{NOW.. } \sum M_B: 0 = (6.7 \text{ m}) F_A \sin 60^\circ - (4 \text{ m}) F_C \sin 60^\circ$$

SUBSTITUTING FOR  $F_C$  FROM EQ. (3)...

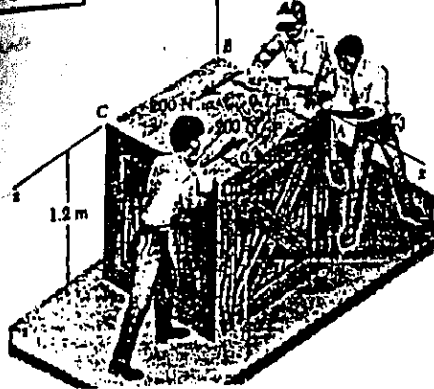
$$6.7 F_A - 4(1040 - F_A) = 0$$

$$\text{OR } F_A = 388.79 \text{ N}$$

AND THEN  $F_C = 651.21 \text{ N}$ 

$$\therefore F_A = 389 \text{ N} \nearrow 60^\circ, F_C = 651 \text{ N} \nearrow 60^\circ$$

3.87



GIVEN:  $1.1 \times 1.2$ -m CRATE

- FIND: (a) EQUIVALENT FORCE-COUPLE SYSTEM AT A IF  $P = 240$  N  
 (b) SINGLE EQUIVALENT FORCE AND POINT OF APPLICATION ON SIDE AB  
 (c)  $P$  IF THREE FORCES ARE EQUIVALENT TO A SINGLE FORCE AT B

- (a) SINCE THE TWO  $200$ -N FORCES FORM A COUPLE, THE THREE FORCES ARE EQUIVALENT TO A FORCE  $E$  AND A COUPLE VECTOR  $M$ , WHERE

$$E = (240 \text{ N})\mathbf{j}$$

$$\text{AND } M = (0.7 - 0.2)\text{m} \cdot 200\text{N} = 100 \text{ N}\cdot\text{m}$$

- $\therefore$  THE EQUIVALENT FORCE-COUPLE SYSTEM AT A IS...

$$E = (240 \text{ N})\mathbf{j}, M = (100 \text{ N}\cdot\text{m})\mathbf{j}$$

- (b) THE SINGLE EQUIVALENT FORCE  $F'$  IS EQUAL TO  $(240 \text{ N})\mathbf{j}$  AND IS APPLIED ALONG AB SO THAT ITS MOMENT ABOUT A IS EQUAL TO  $M$ . THUS

$$M = dF'$$

$$\text{OR } 100 \text{ N}\cdot\text{m} = d(240 \text{ N})$$

$$\text{OR } d = 0.417 \text{ m}$$

$$\therefore F' = (240 \text{ N})\mathbf{j}$$

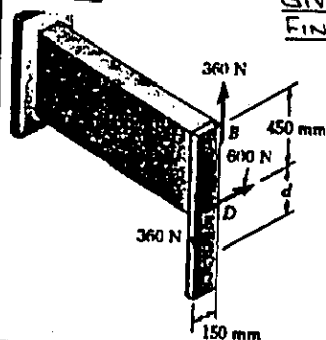
AND IS APPLIED  $0.417$  m FROM A ALONG SIDE AB

- (c) FOR THIS CASE,  $d = 1$  m. THEN..

$$M = dP$$

$$\text{OR } 100 \text{ N}\cdot\text{m} = (1 \text{ m})P \quad \text{OR } P = 100 \text{ N}$$

3.88



GIVEN: FORCE-COUPLE SYSTEM

- FIND: (a) SINGLE EQUIVALENT FORCE  $E$  AT C, DISTANCE  $d$

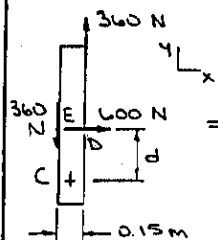
- (b)  $E$  AND  $d$  IF THE DIRECTIONS OF THE TWO  $360$ -N FORCES ARE REVERSED

(CONTINUED)

3.88 CONTINUED

- (a) HAVE  $F = 600$  N

REQUIRE



FOR EQUIVALENCE..

$$\Sigma M_C: (0.15 \text{ m})(360 \text{ N}) - d(600 \text{ N}) = 0$$

$$\text{OR } d = 0.090 \text{ m}$$

$$\therefore E = (600 \text{ N})\mathbf{j}$$

$$\text{AND } d = 90 \text{ mm}$$

BELOW POINTS D AND E

- (b) THE ONLY EFFECT OF REVERSING THE DIRECTIONS OF THE TWO  $360$ -N FORCES WILL BE TO CHANGE THE SENSE OF THE MOMENT OF THE COUPLE. THUS

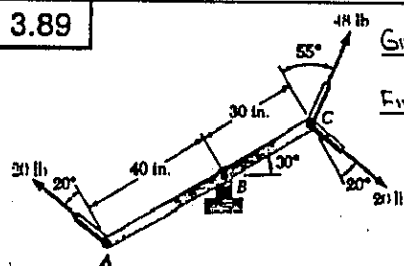
$$E = (600 \text{ N})\mathbf{j}$$

$$\text{AND } \Sigma M_C: -(0.15 \text{ m})(360 \text{ N}) - d(600 \text{ N}) = 0$$

$$\text{OR } d = -0.090 \text{ m}$$

$$\therefore d = 90 \text{ mm ABOVE POINTS D AND E}$$

3.89



GIVEN: FORCE-COUPLE SYSTEM

- FIND: (a) EQUIVALENT FORCE-COUPLE SYSTEM AT B  
 (b) SINGLE EQUIVALENT FORCE, POINT OF APPLICATION

- (a) FIRST NOTE THAT THE TWO  $20$ -lb FORCES FORM A COUPLE. THEN

$$F = 48 \text{ lb}$$

$$\text{WHERE } \theta = 180^\circ - (60^\circ + 55^\circ) = 65^\circ$$

$$\text{AND } M = \Sigma M_B$$

$$= (30 \text{ in.})(48 \text{ lb}) \cos 55^\circ$$

$$- (70 \text{ in.})(20 \text{ lb}) \cos 20^\circ$$

$$= -489.62 \text{ lb}\cdot\text{in.}$$

- $\therefore$  THE EQUIVALENT FORCE-COUPLE SYSTEM AT B IS

$$F = 48 \text{ lb} \angle 65^\circ, M = 490 \text{ lb}\cdot\text{in.}$$

- (b) THE SINGLE EQUIVALENT FORCE  $F'$  IS EQUAL TO  $E$ . FURTHER, SINCE THE SENSE OF  $M$  IS CLOCKWISE,  $F'$  MUST BE APPLIED BETWEEN A AND B. FOR EQUIVALENCE..

$$\Sigma M_B: M = -dF' \cos 55^\circ$$

WHERE  $d$  IS THE DISTANCE FROM B TO THE POINT OF APPLICATION OF  $E'$ . THEN..

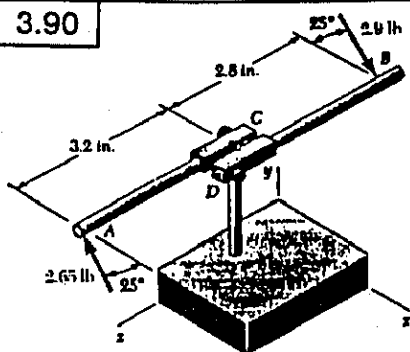
$$-489.62 \text{ lb}\cdot\text{in.} = -d(48 \text{ lb}) \cos 55^\circ$$

$$\text{OR } d = 17.78 \text{ in.}$$

$$\therefore F' = 48 \text{ lb} \angle 65^\circ$$

AND IS APPLIED TO THE LEVER  $17.78$  in. TO THE LEFT OF PIN B

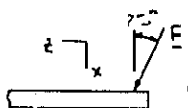
3.90



**GIVEN:** APPLIED FORCES

**FIND:** SINGLE EQUIVALENT FORCE, POINT OF APPLICATION

FIRST TRANSFER THE 2.65-LB FORCE AT A TO B. THE RESULTING FORCE-COUPLE SYSTEM ( $\underline{F}$ ,  $\underline{M}$ ) AT B IS THEN --



$$F = (2.65 - 2.65) \text{ lb} = 0.25 \text{ lb}$$

$$\text{AND } M = M_B = (6 \text{ in.}) (2.65 \text{ lb}) \cos 25^\circ$$

$$\text{OR } M = -(4.4103 \text{ lb} \cdot \text{in.}) \hat{j}$$

THE SINGLE EQUIVALENT FORCE  $\underline{F}'$  IS EQUAL TO  $\underline{F}$ . FURTHER, FOR EQUIVALENCE

$$\sum M_B: M = Q F' \cos 25^\circ$$

WHERE  $Q$  IS THE DISTANCE FROM B TO THE POINT OF APPLICATION OF  $\underline{F}'$ . SINCE  $\underline{M}$  ACTS IN THE  $-\hat{j}$  DIRECTION,  $\underline{F}'$  WOULD HAVE TO BE APPLIED TO THE RIGHT OF B. THEN..

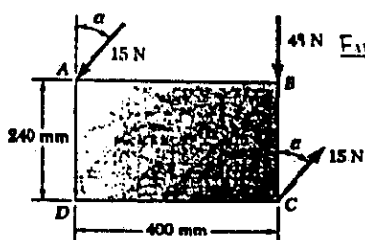
$$-4.4103 \text{ lb} \cdot \text{in.} = -Q (0.25 \text{ lb}) \cos 25^\circ$$

$$\text{OR } Q = 63.6 \text{ in.}$$

$$\therefore \underline{F}' = (0.25 \text{ lb}) (\cos 25^\circ \hat{i} + \sin 25^\circ \hat{j})$$

AND IS APPLIED ON AN EXTENSION OF HANDLE BD AT A DISTANCE OF 63.6 IN. TO THE RIGHT OF B.

3.91



**GIVEN:** FORCE-COUPLE SYSTEM

**FIND:** (a) MAGNITUDE OF SINGLE EQUIVALENT FORCE  $\underline{F}'$  AND ITS LINE OF ACTION FOR  $\alpha = 40^\circ$   
(b)  $\alpha$  IF LINE OF ACTION OF  $\underline{F}'$  INTERSECTS CD 300 mm TO THE RIGHT OF D

(a) THE GIVEN FORCE-COUPLE SYSTEM ( $\underline{F}$ ,  $\underline{M}$ ) AT B IS

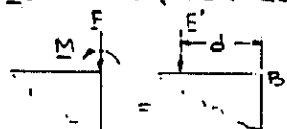
$$\underline{F} = 48 \text{ N} \hat{i}$$

$$\text{AND } M = \sum M_B$$

$$= (0.4 \text{ m}) (15 \text{ N}) \cos 40^\circ + (0.24 \text{ m}) (15 \text{ N}) \sin 40^\circ$$

$$\text{OR } M = 6.9103 \text{ N} \cdot \text{m}$$

THE SINGLE EQUIVALENT FORCE  $\underline{F}'$  IS EQUAL TO  $\underline{F}$ . FURTHER, FOR EQUIVALENCE..



$$\sum M_B: M = d F'$$

$$\text{OR } 6.9103 \text{ N} \cdot \text{m} = d \cdot 48 \text{ N}$$

$$\text{OR } d = 0.14396 \text{ m}$$

$$\therefore \underline{F}' = 48 \text{ N}$$

AND THE LINE OF ACTION OF  $\underline{F}'$  INTERSECTS LINE AB 144 mm TO THE RIGHT OF A.

(CONTINUED)

3.91 CONTINUED

(b) FOLLOWING THE SOLUTION TO PART (a) BUT WITH  $d = 0.1 \text{ m}$  AND  $\alpha$  UNKNOWN, HAVE

$$\sum M_B: (0.4 \text{ m}) (15 \text{ N}) \cos \alpha + (0.24 \text{ m}) (15 \text{ N}) \sin \alpha = (0.1 \text{ m}) (48 \text{ N})$$

$$\text{OR } 5 \cos \alpha + 3 \sin \alpha = 4$$

REARRANGING AND SQUARING..  $25 \cos^2 \alpha = (4 - 3 \sin \alpha)^2$

USING  $\cos^2 \alpha = 1 - \sin^2 \alpha$  AND EXPANDING..

$$25 (1 - \sin^2 \alpha) = 16 - 24 \sin \alpha + 9 \sin^2 \alpha$$

$$\text{OR } 34 \sin^2 \alpha - 24 \sin \alpha - 9 = 0$$

$$\text{THEN } \sin \alpha = \frac{24 \pm \sqrt{(-24)^2 - 4(34)(-9)}}{2(34)}$$

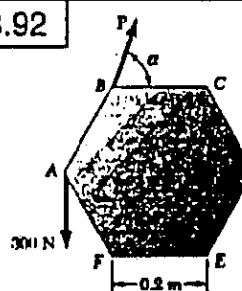
$$\text{OR } \sin \alpha = 0.97686$$

$$\text{OR } \alpha = 77.7^\circ$$

$$\sin \alpha = -0.27098$$

$$\alpha = -15.72^\circ$$

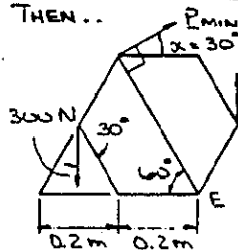
3.92



**GIVEN:** FORCE-COUPLE SYSTEM ( $\underline{P}$ ,  $\underline{M}$ )

**FIND:**  $\underline{P}_{\text{MIN}}$  SO THAT ( $\underline{P}$ ,  $\underline{M}$ ) IS EQUIVALENT TO A SINGLE FORCE AT E

FROM THE STATEMENT OF THE PROBLEM, IT FOLLOWS THAT  $\sum M_E = 0$  FOR THE GIVEN FORCE-COUPLE SYSTEM. FURTHER, FOR  $\underline{P}_{\text{MIN}}$ , MUST REQUIRE THAT  $\underline{P}$  BE PERPENDICULAR TO  $\underline{r}_{BE}$ . THEN..

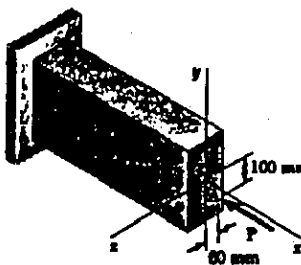


$$\sum M_E: (0.2 \sin 30^\circ + 0.2) \text{ m} \cdot 300 \text{ N} + (0.2 \text{ m}) \sin 30^\circ \cdot 300 \text{ N} - (0.4 \text{ m}) \underline{P}_{\text{MIN}} = 0$$

$$\text{OR } \underline{P}_{\text{MIN}} = 300 \text{ N}$$

$$\therefore \underline{P}_{\text{MIN}} = 300 \text{ N} \angle 30^\circ$$

3.93



**GIVEN:**  $P = 1220 \text{ N}$

**FIND:** EQUIVALENT FORCE-COUPLE SYSTEM ( $\underline{F}$ ,  $\underline{M}$ ) AT G

$$\text{HAVE } \underline{P} = -(1220 \text{ N}) \hat{j}$$

$$\text{NOW.. } \underline{M} = \underline{M}_G$$

$$= \underline{r}_{AG} \times \underline{P}$$

$$= [-(0.1 \text{ m}) \hat{j} - (0.06 \text{ m}) \hat{k}] \times [-(1220 \text{ N}) \hat{j}]$$

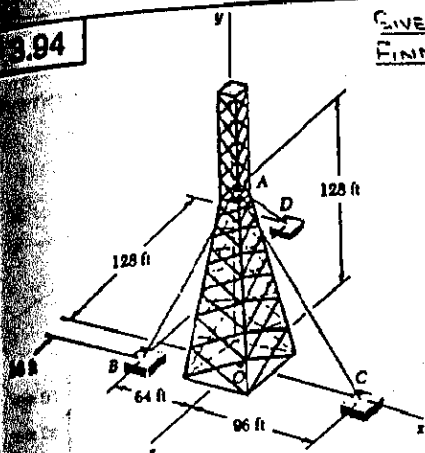
$$= (73.2 \text{ N} \cdot \text{m}) \hat{j} - (122 \text{ N} \cdot \text{m}) \hat{k}$$

$\therefore$  THE EQUIVALENT FORCE-COUPLE SYSTEM AT G IS..

$$\underline{F} = -(1220 \text{ N}) \hat{j}$$

$$\underline{M} = (73.2 \text{ N} \cdot \text{m}) \hat{j} - (122 \text{ N} \cdot \text{m}) \hat{k}$$

3.94

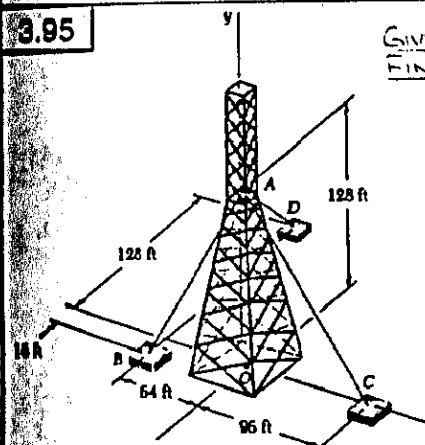


GIVEN:  $T_{AB} = 288 \text{ lb}$   
 FIND: EQUIVALENT  
 FORCE-COUPLE  
 SYSTEM ( $\mathbf{F}$ ,  $\mathbf{M}$ )  
 AT O

HAVE..  $d_{AB} = \sqrt{(-64)^2 + (-128)^2 + (16)^2} = 144 \text{ ft}$   
 THEN  $\mathbf{T}_{AB} = \frac{288 \text{ lb}}{144} (-64\mathbf{j} - 128\mathbf{j} + 16\mathbf{k})$   
 $= (32 \text{ lb})(-4\mathbf{j} - 8\mathbf{j} + \mathbf{k})$

NOW..  $\mathbf{M} = \mathbf{M}_O = \mathbf{r}_{A/O} \times \mathbf{T}_{AB}$   
 $= 128\mathbf{j} \times 32(-4\mathbf{j} - 8\mathbf{j} + \mathbf{k})$   
 $= (4096 \text{ lb}\cdot\text{ft})\mathbf{i} + (16,384 \text{ lb}\cdot\text{ft})\mathbf{k}$   
 THE EQUIVALENT FORCE-COUPLE SYSTEM AT  
 O IS..  $\mathbf{F} = -(128 \text{ lb})\mathbf{j} - (256 \text{ lb})\mathbf{j} + (32 \text{ lb})\mathbf{k}$   
 $\mathbf{M} = (4.10 \text{ kip}\cdot\text{ft})\mathbf{i} + (16.38 \text{ kip}\cdot\text{ft})\mathbf{k}$

3.95



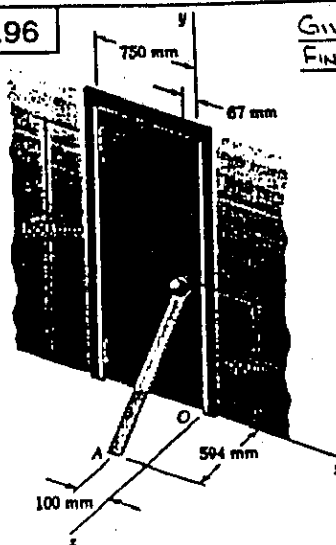
GIVEN:  $T_{AB} = 270 \text{ lb}$   
 FIND: EQUIVALENT  
 FORCE-COUPLE  
 SYSTEM ( $\mathbf{F}$ ,  $\mathbf{M}$ )  
 AT O

HAVE..  $d_{AB} = \sqrt{(-64)^2 + (-128)^2 + (-128)^2} = 192 \text{ ft}$   
 THEN..  $\mathbf{T}_{AB} = \frac{270 \text{ lb}}{192} (-64\mathbf{j} - 128\mathbf{j} - 128\mathbf{k})$   
 $= (90 \text{ lb})(-\mathbf{j} - 2\mathbf{j} - 2\mathbf{k})$

NOW..  $\mathbf{M} = \mathbf{M}_O = \mathbf{r}_{A/O} \times \mathbf{T}_{AB}$   
 $= 128\mathbf{j} \times 90(-\mathbf{j} - 2\mathbf{j} - 2\mathbf{k})$   
 $= -(23,040 \text{ lb}\cdot\text{ft})\mathbf{i} + (11,520 \text{ lb}\cdot\text{ft})\mathbf{k}$

THE EQUIVALENT FORCE-COUPLE SYSTEM AT  
 O IS..  $\mathbf{F} = -(90 \text{ lb})\mathbf{j} - (180 \text{ lb})\mathbf{j} - (180 \text{ lb})\mathbf{k}$   
 $\mathbf{M} = -(23.0 \text{ kip}\cdot\text{ft})\mathbf{i} + (11.52 \text{ kip}\cdot\text{ft})\mathbf{k}$

3.96



GIVEN:  $F_{AB} = 175 \text{ N}$   
 FIND: EQUIVALENT FORCE-  
 COUPLE SYSTEM  
 ( $\mathbf{F}$ ,  $\mathbf{M}$ ) AT C

HAVE  $d_{AB} = \sqrt{(33)^2 + (990)^2 + (-594)^2} = 1155 \text{ mm}$

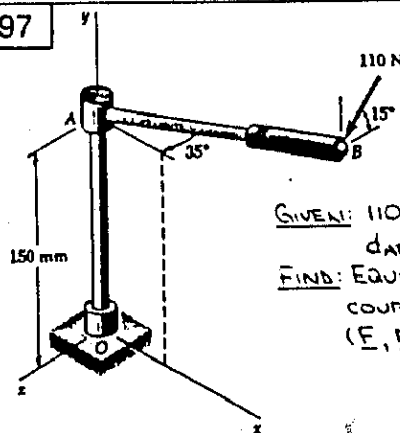
THEN  $\mathbf{F}_{AB} = \frac{175 \text{ N}}{1155} (33\mathbf{i} + 990\mathbf{j} - 594\mathbf{k})$   
 $= (5 \text{ N})(\mathbf{i} + 30\mathbf{j} - 18\mathbf{k})$

NOW..  $\mathbf{M} = \mathbf{M}_C = \mathbf{r}_{B/C} \times \mathbf{F}_{AB}$   
 WHERE  $\mathbf{r}_{B/C} = (0.683 \text{ m})\mathbf{i} - (0.860 \text{ m})\mathbf{j}$

THEN..  $\mathbf{M} = 5 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.683 & -0.860 & 0 \\ 1 & 30 & -18 \end{vmatrix}$   
 $= 5[(-0.860)(-18)]\mathbf{i} + [(-0.683)(-18)]\mathbf{j}$   
 $+ [(0.683)(30) - (-0.860)(1)]\mathbf{k}$   
 $= (77.4 \text{ N}\cdot\text{m})\mathbf{i} + (61.47 \text{ N}\cdot\text{m})\mathbf{j} + (106.75 \text{ N}\cdot\text{m})\mathbf{k}$

THE EQUIVALENT FORCE-COUPLE SYSTEM AT C  
 IS..  $\mathbf{F} = (5 \text{ N})\mathbf{i} + (150 \text{ N})\mathbf{j} - (90 \text{ N})\mathbf{k}$   
 $\mathbf{M} = (77.4 \text{ N}\cdot\text{m})\mathbf{i} + (61.5 \text{ N}\cdot\text{m})\mathbf{j} + (106.8 \text{ N}\cdot\text{m})\mathbf{k}$

3.97



GIVEN: 110-N FORCE  $\mathbf{P}$   
 $d_{AB} = 220 \text{ mm}$   
 FIND: EQUIVALENT FORCE-  
 COUPLE SYSTEM  
 ( $\mathbf{F}$ ,  $\mathbf{M}$ ) AT O

HAVE..  $\mathbf{P} = (110 \text{ N})(-\sin 15^\circ \mathbf{j} + \cos 15^\circ \mathbf{k})$

NOW..  $\mathbf{M} = \mathbf{M}_O = \mathbf{r}_{B/O} \times \mathbf{P}$   
 WHERE  $\mathbf{r}_{B/O} = (0.22 \text{ m})\cos 35^\circ \mathbf{j} + (0.15 \text{ m})\mathbf{j}$   
 $- (0.22 \text{ m})\sin 35^\circ \mathbf{k}$

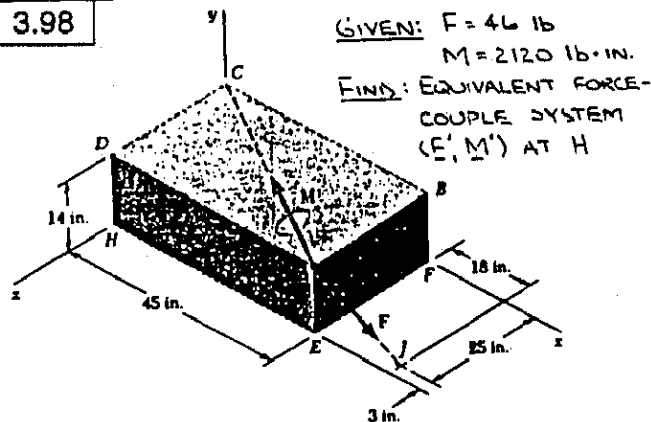
THEN..  $\mathbf{M} = 110 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.22 \cos 35^\circ & 0.15 \\ 0 & -\sin 15^\circ & \cos 15^\circ \end{vmatrix}$

(CONTINUED)

### 3.97 CONTINUED

$$\begin{aligned} \text{OR } \underline{M} &= 110\mathbf{j}[(0.15)(\cos 15^\circ) - (-0.22 \sin 35^\circ)(-\sin 15^\circ)]\mathbf{i} \\ &\quad + [(-0.22 \cos 35^\circ)(\cos 15^\circ)]\mathbf{j} \\ &\quad + [(0.22 \cos 35^\circ)(-\sin 15^\circ)]\mathbf{k} \\ &= (12.345 \text{ N}\cdot\text{m})\mathbf{i} - (19.148 \text{ N}\cdot\text{m})\mathbf{j} - (5.131 \text{ N}\cdot\text{m})\mathbf{k} \\ \therefore \text{THE EQUIVALENT FORCE-COUPLE SYSTEM AT O IS: } \underline{F} &= (110 \text{ N})(-\sin 15^\circ)\mathbf{j} + (\cos 15^\circ)\mathbf{k} \\ &= -(28.5 \text{ N})\mathbf{j} + (106.3 \text{ N})\mathbf{k} \\ \underline{M} &= (12.35 \text{ N}\cdot\text{m})\mathbf{i} - (19.15 \text{ N}\cdot\text{m})\mathbf{j} - (5.13 \text{ N}\cdot\text{m})\mathbf{k} \end{aligned}$$

3.98



GIVEN:  $F = 46 \text{ lb}$   
 $M = 2120 \text{ lb}\cdot\text{in.}$   
 FIND: EQUIVALENT FORCE-COUPLE SYSTEM ( $F'$ ,  $M'$ ) AT H

$$\text{HAVE.. } d_{AH} = \sqrt{(18)^2 + (-14)^2 + (-3)^2} = 23 \text{ in.}$$

$$\text{THEN.. } \underline{F} = \frac{46 \text{ lb}}{23} (18\mathbf{j} - 14\mathbf{j} - 3\mathbf{k}) = (36 \text{ lb})\mathbf{j} - (28 \text{ lb})\mathbf{j} - (6 \text{ lb})\mathbf{k}$$

$$\text{ALSO.. } d_{AC} = \sqrt{(-45)^2 + (0)^2 + (-28)^2} = 53 \text{ in.}$$

$$\text{THEN } \underline{M} = \frac{2120 \text{ lb}\cdot\text{in.}}{53} (-45\mathbf{i} - 28\mathbf{k}) = -(1800 \text{ lb}\cdot\text{in.})\mathbf{i} - (1120 \text{ lb}\cdot\text{in.})\mathbf{k}$$

$$\text{NOW.. } \underline{M}' = \underline{M} + \underline{\Sigma}_{AH} \times \underline{F}$$

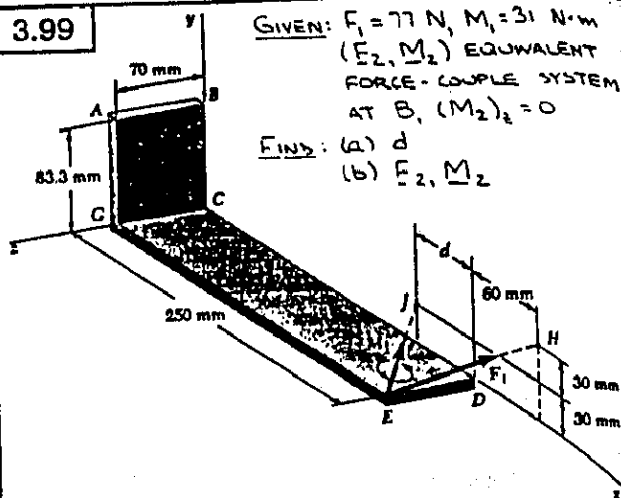
WHERE  $\underline{\Sigma}_{AH} = (45 \text{ in.})\mathbf{i} + (14 \text{ in.})\mathbf{j}$

$$\begin{aligned} \text{THEN.. } \underline{M}' &= (-1800\mathbf{i} - 1120\mathbf{k}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 45 & 14 & 0 \\ 36 & -28 & -6 \end{vmatrix} \\ &= (-1800\mathbf{i} - 1120\mathbf{k}) + \{ [(14)(-6)]\mathbf{i} \\ &\quad + [(45)(-6)]\mathbf{j} \\ &\quad + [(45)(-28) - (14)(36)]\mathbf{k} \} \\ &= (-1800 - 84)\mathbf{i} + (270)\mathbf{j} \\ &\quad + (-1120 - 1764)\mathbf{k} \\ &= -(1884 \text{ lb}\cdot\text{in.})\mathbf{i} + (270 \text{ lb}\cdot\text{in.})\mathbf{j} \\ &\quad - (2884 \text{ lb}\cdot\text{in.})\mathbf{k} \\ &= -(157 \text{ lb}\cdot\text{ft})\mathbf{i} + (22.5 \text{ lb}\cdot\text{ft})\mathbf{j} \\ &\quad - (240 \text{ lb}\cdot\text{ft})\mathbf{k} \end{aligned}$$

$$\therefore \text{THE EQUIVALENT FORCE-COUPLE SYSTEM AT H IS } \underline{F}' = (36 \text{ lb})\mathbf{j} - (28 \text{ lb})\mathbf{j} - (6 \text{ lb})\mathbf{k}$$

$$\underline{M}' = -(157 \text{ lb}\cdot\text{ft})\mathbf{i} + (22.5 \text{ lb}\cdot\text{ft})\mathbf{j} - (240 \text{ lb}\cdot\text{ft})\mathbf{k}$$

3.99



GIVEN:  $F_1 = 77 \text{ N}$ ,  $M_1 = 31 \text{ N}\cdot\text{m}$   
 $(F_2, M_2)$  EQUIVALENT FORCE-COUPLE SYSTEM AT B,  $(M_2)_z = 0$   
 FIND: (a)  $d$   
 (b)  $F_2, M_2$

$$\text{HAVE.. } d_{EH} = \sqrt{(60)^2 + (60)^2 + (-70)^2} = 110 \text{ mm}$$

$$\text{THEN } \underline{F}_1 = \frac{77 \text{ N}}{110} (60\mathbf{i} + 60\mathbf{j} - 70\mathbf{k}) = (42 \text{ N})\mathbf{i} + (42 \text{ N})\mathbf{j} - (49 \text{ N})\mathbf{k}$$

$$\text{ALSO.. } d_{EJ} = \sqrt{(-d)^2 + (30)^2 + (-70)^2} \text{ mm}$$

$$\text{AND } \underline{M}_1 = \frac{31 \text{ N}\cdot\text{m}}{d_{EJ}} [-(d)\mathbf{i} + (30 \text{ mm})\mathbf{j} - (70 \text{ mm})\mathbf{k}]$$

$$\text{(a) HAVE.. } \underline{M}_2 = \underline{M}_1 + \underline{\Sigma}_{HB} \times \underline{F}_1 \quad (1)$$

WHERE  $\underline{\Sigma}_{HB} = (0.31 \text{ m})\mathbf{i} - (0.0233 \text{ m})\mathbf{j}$

$$\begin{aligned} \text{THEN.. } \underline{\Sigma}_{HB} \times \underline{F}_1 &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.31 & -0.0233 & 0 \\ 42 & 42 & -49 \end{vmatrix} \\ &= [(-0.0233)(-49)]\mathbf{i} + [-(0.31)(-49)]\mathbf{j} \\ &\quad + [(0.31)(42) - (-0.0233)(42)]\mathbf{k} \\ &= (1.1417 \text{ N}\cdot\text{m})\mathbf{i} + (15.19 \text{ N}\cdot\text{m})\mathbf{j} \\ &\quad + (13.9986 \text{ N}\cdot\text{m})\mathbf{k} \end{aligned}$$

EQ. (1) CAN THEN BE EXPRESSED AS

$$\begin{aligned} (M_2)_x\mathbf{i} + (M_2)_y\mathbf{j} &= \frac{31 \text{ N}\cdot\text{m}}{d_{EJ}} [-(d)\mathbf{i} + (30 \text{ mm})\mathbf{j} - (70 \text{ mm})\mathbf{k}] \\ &\quad + [(1.1417 \text{ N}\cdot\text{m})\mathbf{i} + (15.19 \text{ N}\cdot\text{m})\mathbf{j} \\ &\quad + (13.9986 \text{ N}\cdot\text{m})\mathbf{k}] \end{aligned}$$

EQUATING THE  $\mathbf{k}$  COEFFICIENTS..

$$0 = \frac{31 \text{ N}\cdot\text{m}}{d_{EJ}} (-70 \text{ mm}) + 13.9986 \text{ N}\cdot\text{m}$$

$$\text{THEN.. } d_{EJ}^2 = \left( \frac{31}{13.9986} \cdot 70 \text{ mm} \right)^2 = [(d)^2 + (30)^2 + (-70)^2] \text{ mm}^2$$

OR  $d = 135.018 \text{ mm}$   $d = 135.0 \text{ mm}$

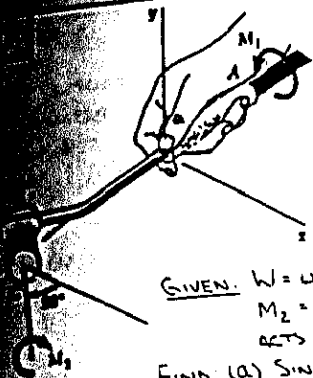
$$\text{(b) FIRST NOTE } d_{EJ} = \sqrt{(-135.018)^2 + (30)^2 + (-70)^2} = 155.016 \text{ mm}$$

USING EQ. (2),  $\underline{M}_2$  IS THEN..

$$\begin{aligned} \underline{M}_2 &= \left( -\frac{31 \times 135.018}{155.016} + 1.1417 \right)\mathbf{i} \\ &\quad + \left( \frac{31 \times 30}{155.016} + 15.19 \right)\mathbf{j} \\ &= -(25.859 \text{ N}\cdot\text{m})\mathbf{i} + (21.189 \text{ N}\cdot\text{m})\mathbf{j} \end{aligned}$$

$$\therefore \underline{F}_2 = (42 \text{ N})\mathbf{i} + (42 \text{ N})\mathbf{j} - (49 \text{ N})\mathbf{k}$$

$$\underline{M}_2 = -(25.9 \text{ N}\cdot\text{m})\mathbf{i} + (21.2 \text{ N}\cdot\text{m})\mathbf{j}$$



GIVEN:  $W = 0.6 \text{ lb}$ ,  $M_1 = 0.68 \text{ lb-in.}$   
 $M_2 = 0.65 \text{ lb-in.}$ ,  $W$   
 ACTS ALONG  $y$  AXIS

FIND: (a) SINGLE EQUIVALENT  
 FORCE  $F$   
 (b) POINT WHERE LINE OF  
 ACTION OF  $F$   
 INTERSECTS  $xz$  PLANE

ASSUME THAT THE GIVEN FORCE  $W$  AND  
 $M_1$  AND  $M_2$  ACT AT THE ORIGIN.

$$W = -W_2 \hat{j}$$

$$M = M_1 \hat{i} + M_2 \hat{k}$$

$$= -(M_2 \cos 25^\circ) \hat{i} + (M_1 - M_2 \sin 25^\circ) \hat{k}$$

THAT SINCE  $W$  AND  $M$  ARE  
 PERPENDICULAR, IT FOLLOWS THAT THEY CAN BE  
 REPLACED WITH A SINGLE EQUIVALENT FORCE.

HAVE  $F = W$  OR  $F = (0.6 \text{ lb}) \hat{j}$

ASSUME THAT THE LINE OF ACTION OF  
 $F$  PASSES THROUGH POINT  $P(x, 0, z)$ . THEN



EQUIVALENCE..

$$M = \Sigma r \times F$$

$$= (0.65 \cos 25^\circ) \hat{i} + (0.68 - 0.65 \sin 25^\circ) \hat{k}$$

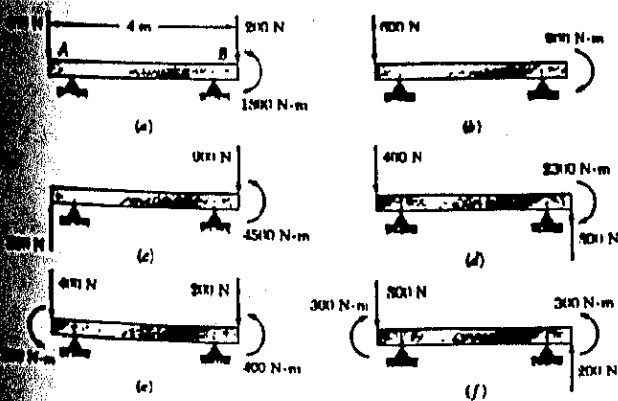
$$= (x \hat{i} + z \hat{k}) \times (-0.6 \hat{j})$$

EQUATING THE  $\hat{i}$  AND  $\hat{k}$  COEFFICIENTS

$$0.65 \cos 25^\circ = 0.6z \text{ OR } z = 0.982 \text{ m}$$

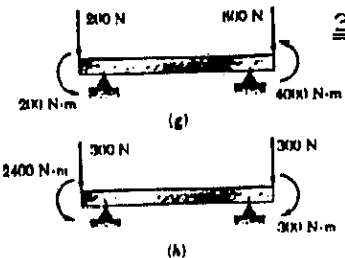
$$0.68 - 0.65 \sin 25^\circ = -0.6x \text{ OR } x = -0.75 \text{ m}$$

### 3.101 and 3.102



(CONTINUED)

### 3.101 and 3.102 CONTINUED



3.101

GIVEN: APPLIED LOADS  
 AND COUPLES

FIND: (a) EQUIVALENT  
 FORCE-COUPLE  
 $(R, M)$  AT A FOR  
 EACH LOADING  
 (b) EQUIVALENT  
 LOADINGS

(a) HAVE...

$$a. R_a = \Sigma F = -400 - 200 \text{ OR } R_a = 600 \text{ N} \downarrow$$

$$M_a = \Sigma M_A = 1800 \text{ N-m} - (4 \text{ m})(200 \text{ N})$$

$$\text{OR } M_a = 1000 \text{ N-m}$$

$$b. R_b = \Sigma F = -600$$

$$\text{OR } R_b = 600 \text{ N} \downarrow$$

$$M_b = \Sigma M_A = -900 \text{ N-m}$$

$$\text{OR } M_b = 900 \text{ N-m}$$

$$c. R_c = \Sigma F = 300 - 900$$

$$\text{OR } R_c = 600 \text{ N} \downarrow$$

$$M_c = \Sigma M_A = 4500 \text{ N-m} - (4 \text{ m})(900 \text{ N})$$

$$\text{OR } M_c = 900 \text{ N-m}$$

$$d. R_d = \Sigma F = -400 + 800$$

$$\text{OR } R_d = 400 \text{ N} \uparrow$$

$$M_d = \Sigma M_A = -2300 \text{ N-m} + (4 \text{ m})(800 \text{ N})$$

$$\text{OR } M_d = 900 \text{ N-m}$$

$$e. R_e = \Sigma F = -400 - 200$$

$$\text{OR } R_e = 600 \text{ N} \downarrow$$

$$M_e = \Sigma M_A = 200 \text{ N-m} + 400 \text{ N-m} - (4 \text{ m})(200 \text{ N})$$

$$\text{OR } M_e = 200 \text{ N-m}$$

$$f. R_f = \Sigma F = -800 + 200$$

$$\text{OR } R_f = 600 \text{ N} \downarrow$$

$$M_f = \Sigma M_A = -300 \text{ N-m} + 300 \text{ N-m} + (4 \text{ m})(200 \text{ N})$$

$$\text{OR } M_f = 800 \text{ N-m}$$

$$g. R_g = \Sigma F = -200 - 800$$

$$\text{OR } R_g = 1000 \text{ N} \downarrow$$

$$M_g = \Sigma M_A = 200 \text{ N-m} + 4000 \text{ N-m} - (4 \text{ m})(800 \text{ N})$$

$$\text{OR } M_g = 1000 \text{ N-m}$$

$$h. R_h = \Sigma F = -300 - 300$$

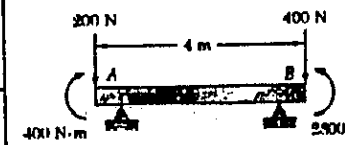
$$\text{OR } R_h = 600 \text{ N} \downarrow$$

$$M_h = \Sigma M_A = 2400 \text{ N-m} - 300 \text{ N-m} - (4 \text{ m})(300 \text{ N})$$

$$\text{OR } M_h = 900 \text{ N-m}$$

(b)  $\therefore$  LOADINGS (c) AND (h) ARE EQUIVALENT

### 3.102



GIVEN: APPLIED LOADS  
 AND COUPLES

FIND: LOADING OF  
 PROB. 3.101  
 EQUIVALENT TO  
 THE GIVEN LOADING

FIRST REPLACE THE GIVEN LOADING WITH AN  
 EQUIVALENT FORCE-COUPLE SYSTEM  $(R, M)$  AT  
 A. THUS..

$$R = \Sigma F = -200 - 400$$

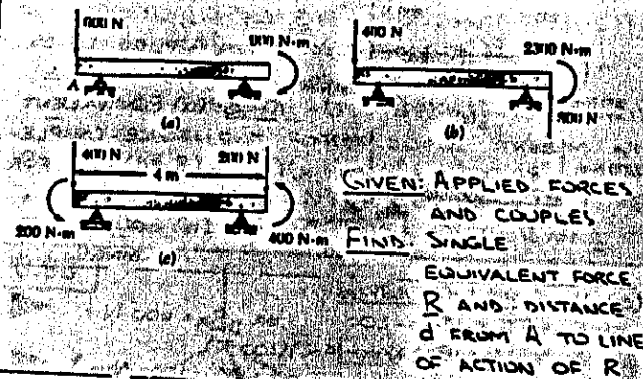
$$\text{OR } R = 600 \text{ N} \downarrow$$

$$\text{AND } M = \Sigma M_A = -400 \text{ N-m} + 2800 \text{ N-m} - (4 \text{ m})(400 \text{ N})$$

$$\text{OR } M = 800 \text{ N-m}$$

$\therefore$  THE GIVEN LOADING IS EQUIVALENT TO  
 LOADING (f)  
 OF PROB. 3.101.

3.103



FOR EACH LOADING, FIRST DETERMINE THE EQUIVALENT FORCE-COUPLE SYSTEM ( $R, M$ ) AT  $A$

(a)  $R = \sum F_i = -600$  OR  $R = 600 \text{ N}$   
AND  $M = \sum M_A = -900 \text{ N}\cdot\text{m}$

THEN FOR EQUIVALENCE:  
 $\sum M_A = -900 \text{ N}\cdot\text{m} = d(600 \text{ N})$   
OR  $d = 1.5 \text{ m}$

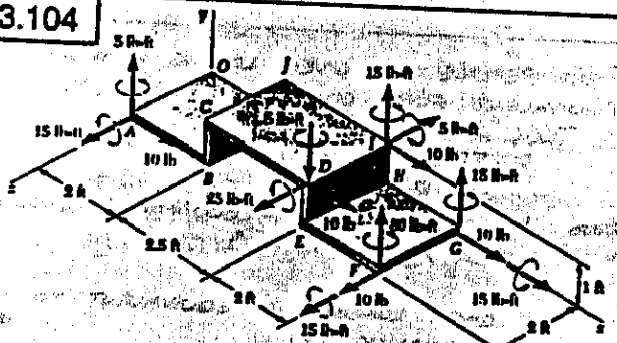
(b)  $R = \sum F_i = -400 + 800$  OR  $R = 400 \text{ N}$   
AND  $M = \sum M_A = -2300 \text{ N}\cdot\text{m} + (4 \text{ m})(800 \text{ N}) = 900 \text{ N}\cdot\text{m}$

THEN FOR EQUIVALENCE:  
 $\sum M_A = 900 \text{ N}\cdot\text{m} = d(400 \text{ N})$   
OR  $d = 2.25 \text{ m}$

(c)  $R = \sum F_i = -400 - 200$  OR  $R = 600 \text{ N}$   
AND  $M = \sum M_A = 200 \text{ N}\cdot\text{m} - 400 \text{ N}\cdot\text{m} - (4 \text{ m})(200 \text{ N}) = -200 \text{ N}\cdot\text{m}$

THEN FOR EQUIVALENCE:  
 $\sum M_A = -200 \text{ N}\cdot\text{m} = d(600 \text{ N})$   
OR  $d = 0.333 \text{ m}$

3.104



3.104 CONTINUED

FIRST NOTE THAT THE FORCE-COUPLE SYSTEM AT  $F$  CANNOT BE EQUIVALENT BECAUSE OF THE DIRECTION OF THE FORCE [THE FORCE OF THE OTHER FOUR SYSTEMS IS  $(10 \text{ lb})_i$ ]. NEXT MOVE EACH OF THE SYSTEMS TO THE ORIGIN  $O$ , THE FORCES REMAIN UNCHANGED.

A:  $M_A = \sum M_O = (5 \text{ lb}\cdot\text{ft})_j - (15 \text{ lb}\cdot\text{ft})_j - (2 \text{ ft})(10 \text{ lb})_i$   
 $= (25 \text{ lb}\cdot\text{ft})_j + (15 \text{ lb}\cdot\text{ft})_j$   
 $= (40 \text{ lb}\cdot\text{ft})_j$

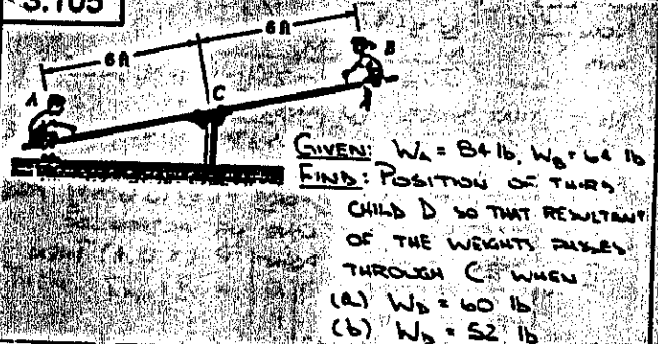
D:  $M_D = \sum M_O = -(4 \text{ lb}\cdot\text{ft})_j + (25 \text{ lb}\cdot\text{ft})_j$   
 $+ [(4.5 \text{ ft})_i + (1 \text{ ft})_j] \cdot (10 \text{ lb})_i$   
 $= (15 \text{ lb}\cdot\text{ft})_j + (15 \text{ lb}\cdot\text{ft})_j$   
 $= (30 \text{ lb}\cdot\text{ft})_j$

G:  $M_G = \sum M_O = (15 \text{ lb}\cdot\text{ft})_i + (15 \text{ lb}\cdot\text{ft})_j$

I:  $M_I = \sum M_O = (15 \text{ lb}\cdot\text{ft})_j - (5 \text{ lb}\cdot\text{ft})_j$   
 $= [(4.5 \text{ ft})_i + (1 \text{ ft})_j] \cdot (10 \text{ lb})_i$   
 $= (15 \text{ lb}\cdot\text{ft})_j - (15 \text{ lb}\cdot\text{ft})_j$   
 $= 0$

THE EQUIVALENT FORCE-COUPLE SYSTEM IS THE SYSTEM AT CORNER D

3.105



FROM THE STATEMENT OF THE PROBLEM IT FOLLOWS THAT THE THREE WEIGHTS ARE EQUIVALENT TO A SINGLE FORCE AT  $C$ ; THAT THE SEESAW WILL BE BALANCED THEN.

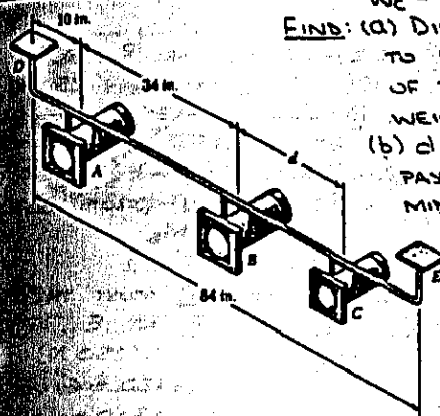
$84 \text{ lb} \cdot 4 \text{ ft} = W_D \cdot 6 \text{ ft}$   
AND  $\sum M_C: (6 \text{ ft})(84 \text{ lb}) - d(W_D \text{ lb}) - (6 \text{ ft})(64 \text{ lb}) = 0$   
OR  $d = \frac{120}{W_D} \text{ (ft)}$

(a)  $W_D = 60 \text{ lb}$ ,  $d = \frac{120}{60} = 2 \text{ ft}$   
 $\therefore$  THE THIRD CHILD SHOULD SIT 2 ft TO THE RIGHT OF  $C$ .

(b)  $W_D = 52 \text{ lb}$ ,  $d = \frac{120}{52} = 2.31 \text{ ft}$   
 $\therefore$  THE THIRD CHILD SHOULD SIT 2.31 ft TO THE RIGHT OF  $C$ .



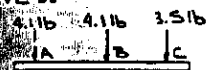
3.106



GIVEN:  $W_A = W_B = 4.1 \text{ lb}$   
 $W_C = 3.5 \text{ lb}$

FIND: (a) DISTANCE FROM D TO RESULTANT R OF THE THREE WEIGHTS IF  $d = 25 \text{ in}$   
 (b) d SO THAT R PASSES THROUGH MIDPOINT OF DPE

HAVE ..



FOR EQUIVALENCE ..

$$\sum F_y = 4.1 + 4.1 + 3.5 = -R \quad \text{OR} \quad R = 11.7 \text{ lb}$$

$$\sum M_D = -(10 \text{ in})(4.1 \text{ lb}) - (34 \text{ in})(4.1 \text{ lb}) - ((44 + d) \text{ in})(3.5 \text{ lb}) = -(L \text{ in})(11.7 \text{ lb})$$

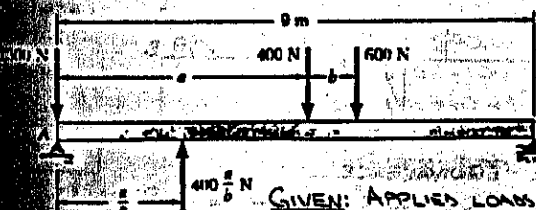
$$\text{OR} \quad 375.4 + 3.5d = 11.7L \quad (d, L \text{ IN IN.})$$

(a)  $d = 25 \text{ in.}$ HAVE  $375.4 + 3.5(25) = 11.7L$  OR  $L = 39.6 \text{ in.}$ 

THE RESULTANT PASSES THROUGH A POINT 39.6 IN. TO THE RIGHT OF D.

(b)  $L = 42 \text{ in.}$ HAVE  $375.4 + 3.5d = 11.7(42)$ OR  $d = 33.1 \text{ in.}$ 

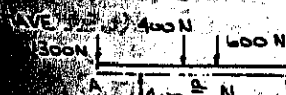
3.107



GIVEN: APPLIED LOADS,  $b = 1.5 \text{ m}$   
 LOADS ARE EQUIVALENT

FIND: (a) a SO THAT DISTANCE L FROM A TO R IS MAXIMUM

(b) R AND POINT OF APPLICATION ON THE BEAM



FOR EQUIVALENCE ..

$$\sum F_y = 1300 + 400 - 400 - 600 = -R$$

$$\text{OR} \quad R = (2300 - 400) \text{ N} \quad (1)$$

$$\sum M_A = \frac{1}{2}(400) - a(400) - (a+b)(600) = -LR$$

(CONTINUED)

3.107 CONTINUED

$$\text{OR} \quad L = \frac{1000a + 600b - 200}{2300 - 400}$$

$$\text{THEN WITH } b = 1.5 \text{ m} \quad L = \frac{100a + 9 - \frac{4}{3}a^2}{23 - \frac{4}{3}a} \quad (2)$$

WHERE a, L ARE IN m

(a) FIND VALUE OF a TO MAXIMIZE L ..

$$\frac{dL}{da} = \frac{(10 - \frac{4}{3}a)(23 - \frac{4}{3}a) - (100a + 9 - \frac{4}{3}a^2)(-\frac{4}{3})}{(23 - \frac{4}{3}a)^2}$$

$$\text{OR} \quad 230 - \frac{194}{3}a - \frac{80}{3}a + \frac{16}{9}a^2 - \frac{4}{3}a + 24 - \frac{32}{3}a^2 = 0$$

$$\text{OR} \quad 16a^2 - 276a + 1143 = 0$$

$$\text{THEN} \quad a = \frac{276 \pm \sqrt{(-276)^2 - 4(16)(1143)}}{2(16)}$$

OR  $a = 10.3435 \text{ m}$  AND  $a = 6.9065 \text{ m}$ SINCE  $AB = 9 \text{ m}$ , a MUST BE LESS THAN 9 m $\therefore a = 6.91 \text{ m}$ 

(b) USING Eq. (1) ..

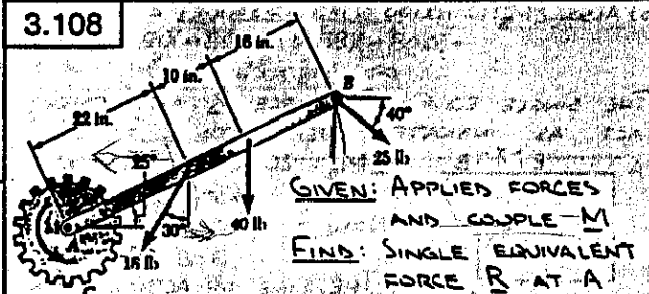
$$R = 2300 - 400 \frac{6.9065}{1.5} \quad \text{OR} \quad R = 458 \text{ N}$$

AND USING Eq. (2) ..

$$L = \frac{10(6.9065) + 9 - \frac{4}{3}(6.9065)^2}{23 - \frac{4}{3}(6.9065)} = 3.16 \text{ m}$$

 $\therefore R$  IS APPLIED 3.16 m TO THE RIGHT OF A.

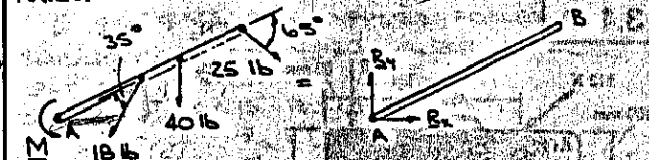
3.108



GIVEN: APPLIED FORCES AND COUPLE-M

FIND: SINGLE EQUIVALENT FORCE R AT A AND M

HAVE ..



FOR EQUIVALENCE ..

$$\sum F_x = -18 \sin 30^\circ + 25 \cos 40^\circ = R_x$$

$$\text{OR} \quad R_x = 10.1511 \text{ lb}$$

$$\sum F_y = -18 \cos 30^\circ - 40 - 25 \sin 40^\circ = R_y$$

$$\text{OR} \quad R_y = -71.658 \text{ lb}$$

$$\text{THEN} \quad R = \sqrt{(10.1511)^2 + (71.658)^2} \quad \tan \theta = \frac{71.658}{10.1511}$$

$$= 72.4 \text{ lb} \quad \text{OR} \quad \theta = 81.9^\circ$$

$$\therefore R = 72.4 \text{ lb} \quad \angle 81.9^\circ$$

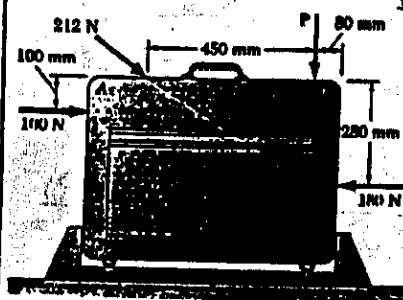
$$\text{ALSO} \quad \sum M_A = M - (22 \text{ in.})(18 \text{ lb}) \sin 30^\circ - (32 \text{ in.})(40 \text{ lb}) \cos 25^\circ - (18 \text{ in.})(25 \text{ lb}) \sin 65^\circ = 0$$

$$\text{OR} \quad M = 2474.8 \text{ lb-in.}$$

$$\text{OR} \quad M = 206 \text{ lb-ft}$$



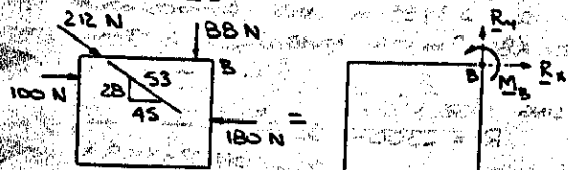
3.109

GIVEN:  $P = 88 \text{ N}$ 

FIND: (a) RESULTANT

 $R$  OF THE APPLIED FORCES

(b) POINTS WHERE

THE LINE OF ACTION OF  $R$  INTERSECTS SIDES OF THE SUITCASE(a) FIRST DETERMINE THE EQUIVALENT FORCE-COUPLE SYSTEM ( $R$ ,  $M_B$ ) AT B. HAVE..

THEN FOR EQUIVALENCE..

$$\sum F_x: 100 + \frac{45}{53}(212) - 180 = R_x \quad \text{OR } R_x = 100 \text{ N}$$

$$\sum F_y: -\frac{28}{53}(212) - 88 = R_y \quad \text{OR } R_y = -200 \text{ N}$$

$$\therefore R = (100 \text{ N})i - (200 \text{ N})j$$

$$\text{OR } R = 224 \text{ N} \angle 63.4^\circ$$

$$(b) \text{ ALSO.. } \sum M_B: (0.1 \text{ m} \times 100 \text{ N}) + (0.53 \text{ m} \times \frac{28}{53} \times 212 \text{ N}) + (0.08 \text{ m} \times 88 \text{ N}) - (0.28 \text{ m} \times 180 \text{ N}) = M_B$$

$$\text{OR } M_B = 26 \text{ N}\cdot\text{m}$$

THE SINGLE EQUIVALENT FORCE  $R$  MUST THEN ACT AS INDICATED. THEN WITH  $R$  AT E..

$$\sum M_B: 26 \text{ N}\cdot\text{m} = x(200 \text{ N})$$

$$\text{OR } x = 130 \text{ mm}$$

$$\text{NOW } \frac{y}{x} = \frac{2}{1} \Rightarrow y = 260 \text{ mm}$$

THE LINE OF ACTION OF  $R$  INTERSECTS TOP AB 130 mm TO THE LEFT OF B AND INTERSECTS SIDE BC 260 mm BELOW B.

3.110 CONTINUED

THEN FOR EQUIVALENCE..

$$\sum F_x: 100 - \frac{45}{53}(212) - 180 = R_x \quad \text{OR } R_x = 100 \text{ N}$$

$$\sum F_y: -\frac{28}{53}(212) - 138 = R_y \quad \text{OR } R_y = -250 \text{ N}$$

$$\therefore R = (100 \text{ N})i - (250 \text{ N})j$$

$$\text{OR } R = 269 \text{ N} \angle 68.2^\circ$$

$$(b) \text{ ALSO.. } \sum M_B: (0.1 \text{ m} \times 100 \text{ N}) + (0.53 \text{ m} \times \frac{28}{53} \times 212 \text{ N}) + (0.08 \text{ m} \times 138 \text{ N}) - (0.28 \text{ m} \times 180 \text{ N}) = M_B$$

$$\text{OR } M_B = 30 \text{ N}\cdot\text{m}$$

THE SINGLE EQUIVALENT FORCE  $R$  MUST THEN ACT AS INDICATED. THEN WITH  $R$  AT E..

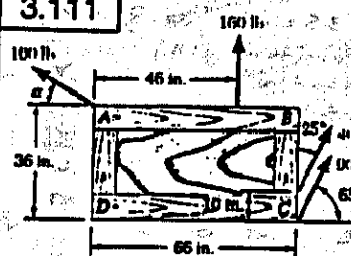
$$\sum M_B: 30 \text{ N}\cdot\text{m} = x(250 \text{ N})$$

$$\text{OR } x = 120 \text{ mm}$$

$$\text{NOW } \frac{y}{x} = \frac{3}{2} \Rightarrow y = 300 \text{ mm}$$

THE LINE OF ACTION OF  $R$  INTERSECTS TOP AB 120 mm TO THE LEFT OF B AND INTERSECTS SIDE BC 300 mm BELOW B.

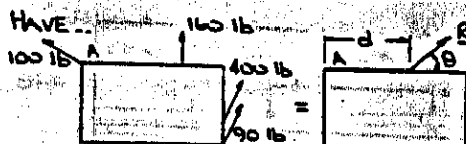
3.111



GIVEN: APPLIED FORCES

ARE EQUIVALENT TO A SINGLE FORCE  $R$  APPLIED ALONG ABFIND: (a)  $R$  AND DISTANCE FROM A TO ITS POINT OF APPLICATION IF  $\alpha = 30^\circ$ (b) IF  $R$  IS AT B

HAVE..



(a) FOR EQUIVALENCE..

$$\sum F_x: -100 \cos 30^\circ + 160 \cos 65^\circ + 90 \cos 65^\circ = R_x$$

$$\text{OR } R_x = 120.480 \text{ lb}$$

$$\sum F_y: 100 \sin 30^\circ + 160 + 400 \sin 65^\circ + 90 \sin 65^\circ = R_y$$

$$\text{OR } R_y = (604.09 + 100 \sin 30^\circ) \text{ lb} \quad (1)$$

$$\text{WITH } \alpha = 30^\circ \quad R_y = 654.09 \text{ lb}$$

$$\text{THEN.. } R = \sqrt{(120.480)^2 + (654.09)^2} \quad \tan \theta = \frac{654.09}{120.480}$$

$$= 665 \text{ lb}$$

$$\text{OR } \theta = 79.6^\circ$$

$$\text{ALSO.. } \sum M_A: (46 \text{ in.} \times 160 \text{ lb}) + (66 \text{ in.} \times 400 \text{ lb}) \sin 65^\circ + (26 \text{ in.} \times 400 \text{ lb}) \cos 65^\circ + (66 \text{ in.} \times 90 \text{ lb}) \sin 65^\circ + (36 \text{ in.} \times 90 \text{ lb}) \cos 65^\circ = d(654.09 \text{ lb})$$

$$\text{OR.. } \sum M_A = 42,435 \text{ lb}\cdot\text{in.} \quad \text{AND } d = 64.9 \text{ in.}$$

$$\therefore R = 665 \text{ lb} \angle 79.6^\circ$$

AND  $R$  IS APPLIED 64.9 in. TO THE RIGHT OF A.(b) HAVE..  $d = 66 \text{ in.}$ 

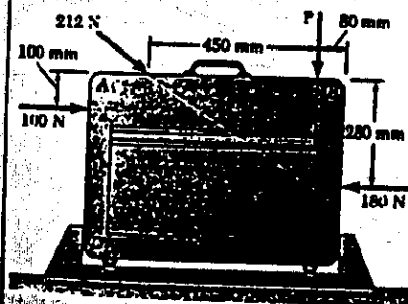
$$\text{THEN.. } \sum M_A: 42,435 \text{ lb}\cdot\text{in.} = (66 \text{ in.}) R_y$$

$$\text{OR } R_y = 642.95 \text{ lb}$$

$$\text{USING EQ (1) } 642.95 = 604.09 + 100 \sin \alpha$$

$$\text{OR } \alpha = 22.9^\circ$$

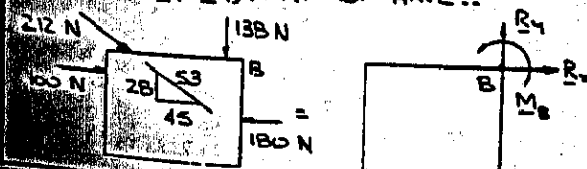
3.110

GIVEN:  $P = 138 \text{ N}$ 

FIND: (a) RESULTANT

 $R$  OF THE APPLIED FORCES

(b) POINTS WHERE

THE LINE OF ACTION OF  $R$  INTERSECTS SIDES OF THE SUITCASE(a) FIRST DETERMINE THE EQUIVALENT FORCE-COUPLE SYSTEM ( $R$ ,  $M_B$ ) AT B. HAVE..



### 3.114 and 3.115 CONTINUED

(b) THE RIVET HOLE IS 0.227 m TO THE RIGHT OF G.

3.115  $P = 60 \text{ N}$

HAVE  $R_x = (152.142 + 60) = 212.14 \text{ N}$

$M_G = -[55.544 + 0.13(60)] = -63.344 \text{ N}\cdot\text{m}$

THEN.. WITH  $R$  AT J..  $\Sigma M_G: -63.344 \text{ N}\cdot\text{m} = -a(244.53 \text{ N})$

OR  $a = 0.259 \text{ m}$

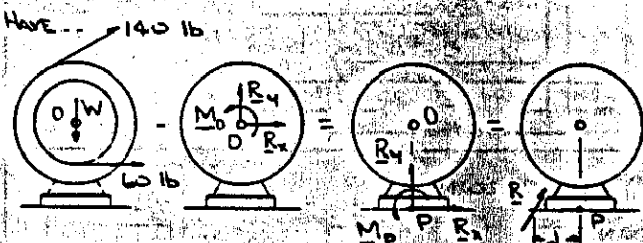
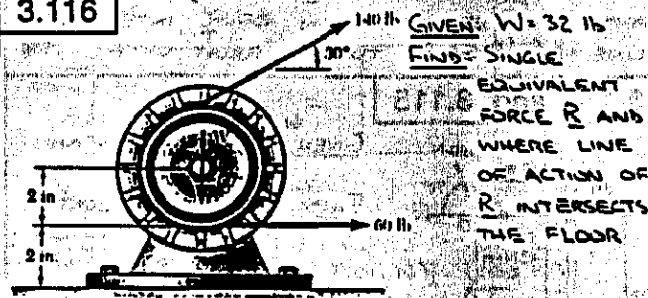
AND WITH  $R$  AT J..  $\Sigma M_G: -63.344 \text{ N}\cdot\text{m} = -b(212.14 \text{ N})$

OR  $b = 0.299 \text{ m}$

∴ (a) THE RIVET HOLE IS 0.299 m ABOVE G.

(b) THE RIVET HOLE IS 0.259 m TO THE RIGHT OF G.

### 3.116



FIRST REDUCE THE GIVEN FORCES TO AN EQUIVALENT FORCE-COUPLE SYSTEM AT D.

THEN FOR EQUIVALENCE..

$\Sigma F_x: 140 \cos 30^\circ + 60 = R_x$  OR  $R_x = 181.244 \text{ lb}$

$\Sigma F_y: 140 \sin 30^\circ - 32 = R_y$  OR  $R_y = 38 \text{ lb}$

$\Sigma M_D: -(2 \text{ in.} \times 140 \text{ lb}) + (2 \text{ in.} \times 60 \text{ lb}) = M_D$

OR  $M_D = -160 \text{ lb}\cdot\text{in.}$

NEXT MOVE THE EQUIVALENT FORCE-COUPLE SYSTEM TO THE POINT P WHICH LIES ON THE FLOOR DIRECTLY BELOW D. THUS..

AT P.  $R_x = 181.244 \text{ lb}$   $R_y = 38 \text{ lb}$

AND  $\Sigma M_P: -160 \text{ lb}\cdot\text{in.} - (4 \text{ in.} \times 181.244 \text{ lb}) = M_P$

OR  $M_P = -834.98 \text{ lb}\cdot\text{in.}$

FINALLY, REPLACE  $(R, M_P)$  WITH THE SINGLE EQUIVALENT FORCE  $R$ , WHERE..

$R = \sqrt{(181.244)^2 + (38)^2}$   $\tan \theta = \frac{38}{181.244}$

$= 185.2 \text{ lb}$

OR  $\theta = 11.84^\circ$

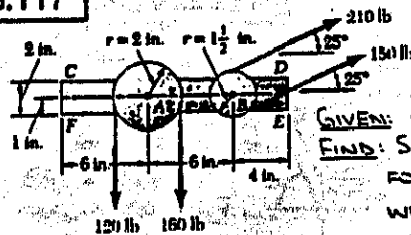
AND..  $\Sigma M_P: -834.98 \text{ lb}\cdot\text{in.} = d(38 \text{ lb})$

OR  $d = 23.3 \text{ in.}$

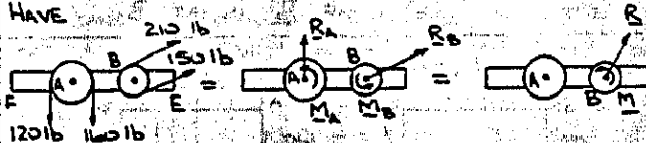
∴  $R = 185.2 \text{ lb}$   $\angle 11.84^\circ$

AND THE LINE OF ACTION OF  $R$  INTERSECTS THE FLOOR AT A POINT 23.3 in. TO THE LEFT OF THE VERTICAL CENTER LINE OF THE MOTOR.

### 3.117



HAVE



FIRST REPLACE THE FORCES ACTING ON EACH PULLEY WITH AN EQUIVALENT FORCE-COUPLE SYSTEM ACTING AT THE CENTER OF EACH PULLEY.

PULLEY A:  $\Sigma F_y: -120 - 160 = R_A$  OR  $R_A = -280 \text{ lb}$

$\Sigma M_A: (2 \text{ in.} \times 120 \text{ lb}) - (2 \text{ in.} \times 160 \text{ lb}) = M_A$

OR  $M_A = -80 \text{ lb}\cdot\text{in.}$

PULLEY B:  $\Sigma F_y: 210 - 150 = R_B$

OR  $R_B = 60 \text{ lb}$   $\angle 25^\circ$

$\Sigma M_B: (1.5 \text{ in.} \times 150 \text{ lb}) - (1.5 \text{ in.} \times 210 \text{ lb}) = M_B$

OR  $M_B = -90 \text{ lb}\cdot\text{in.}$

NEXT COMBINE  $(R_A, M_A)$  AND  $(R_B, M_B)$  INTO AN EQUIVALENT FORCE-COUPLE SYSTEM  $(R, M)$  AT B. HAVE..

$\Sigma F_x: 360 \cos 25^\circ = R_x$  OR  $R_x = 326.27 \text{ lb}$

$\Sigma F_y: -280 + 60 \sin 25^\circ = R_y$  OR  $R_y = -127.857 \text{ lb}$

$\Sigma M_B: -80 \text{ lb}\cdot\text{in.} + (6 \text{ in.} \times 280 \text{ lb}) - 90 \text{ lb}\cdot\text{in.} = M$

OR  $M = 1510 \text{ lb}\cdot\text{in.}$

FINALLY, REPLACE  $(R, M)$  WITH THE SINGLE EQUIVALENT FORCE  $R$ , WHERE..

$R = \sqrt{(326.27)^2 + (127.857)^2}$   $\tan \theta = \frac{127.857}{326.27}$

$= 350 \text{ lb}$  OR  $\theta = 21.4^\circ$

ALSO..

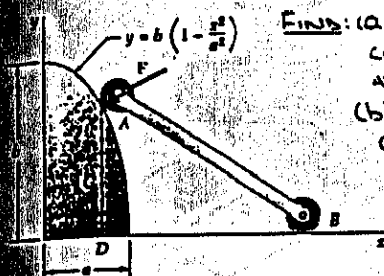
$\Sigma M_B: 1510 \text{ lb}\cdot\text{in.} = d(127.857 \text{ lb})$  OR  $d = 11.810 \text{ in.}$

AND  $a = \frac{1 \text{ in.}}{\tan 21.4^\circ} = 2.552 \text{ in.}$

∴  $R = 350 \text{ lb}$   $\angle 21.4^\circ$

AND THE LINE OF ACTION OF  $R$  INTERSECTS THE LOWER EDGE OF THE BRACKET  $(11.810 - 2.552) 9.26 \text{ in.}$  TO THE LEFT OF THE CENTER OF PULLEY B AND  $(12 - 9.26) 2.74 \text{ in.}$  TO THE RIGHT OF CORNER F.

3.118



GIVEN:  $F$  IS PERPENDICULAR TO THE SURFACE  
FIND: (a) EQUIVALENT FORCE-COUPLE SYSTEM ( $R, M$ ) AT D  
(b)  $x$  FOR  $M$  MAX IF  $a = 1 \text{ m}$ ,  $b = 2 \text{ m}$

(a) THE SLOPE AT ANY POINT ON THE SURFACE OF MEMBER C IS ..

$$\frac{dy}{dx} = \frac{d}{dx} \left[ b \left( 1 - \frac{x^2}{a^2} \right) \right] \\ = -\frac{2b}{a^2} x$$

SINCE  $F$  IS PERPENDICULAR TO THE SURFACE, IT FOLLOWS THAT

$$\tan \alpha = \frac{a^2}{2b} \frac{1}{x}$$

WHERE  $\alpha$  IS THE ANGLE THAT  $F$  FORMS WITH THE HORIZONTAL. THEN FOR EQUIVALENCE..

$$\sum F: F = R$$

$$\sum M_D: d_{DA} F \cos \alpha = M$$

SINCE A IS A POINT ON THE SURFACE HAVE

$$d_{DA} = y \text{ AT A}$$

ALSO,

$$\cos \alpha = \frac{2bx}{\sqrt{(a^2)^2 + (2bx)^2}}$$



THEN..

$$M = \left[ b \left( 1 - \frac{x^2}{a^2} \right) \right] \cdot F \cdot \frac{2bx}{\sqrt{a^4 + 4b^2x^2}} \\ = \frac{2Fb^2(x - \frac{x^3}{a^2})}{\sqrt{a^4 + 4b^2x^2}}$$

THE EQUIVALENT FORCE-COUPLE SYSTEM

$$\text{AT D IS: } R = F \quad \tan^{-1} \left( \frac{a^2}{2bx} \right) \\ M = \frac{2Fb^2(x - \frac{x^3}{a^2})}{\sqrt{a^4 + 4b^2x^2}}$$

(b) SUBSTITUTING  $a = 1 \text{ m}$ ,  $b = 2 \text{ m}$  IN THE EXPRESSION FOR  $M$  YIELDS..

$$M = \frac{8F(x - x^3)}{\sqrt{1 + 16x^2}}$$

TO FIND THE VALUE OF  $x$  TO MAXIMIZE  $M$ ..

$$\frac{dM}{dx} = \frac{8F(1 - 3x^2)\sqrt{1 + 16x^2} - (x - x^3) \cdot \frac{1}{2}(32x)(1 + 16x^2)^{-\frac{1}{2}}}{(1 + 16x^2)^2} = 0$$

$$\text{OR } (1 - 3x^2)(1 + 16x^2) - 16x(x - x^3) = 0$$

$$\text{OR } 32x^4 + 3x^2 - 1 = 0$$

$$\text{THEN: } x^2 = \frac{-3 \pm \sqrt{(3)^2 - 4(32)(-1)}}{2(32)}$$

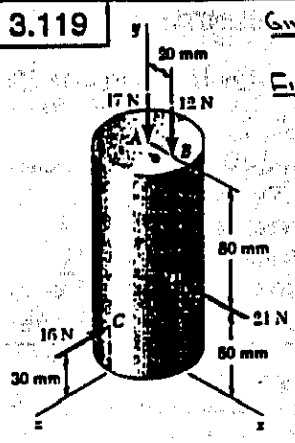
TAKING THE POSITIVE ROOT SINCE  $x^2 > 0$  YIELDS

$$x^2 = 0.136011 \text{ m}^2$$

AND THEN FOR  $M$  MAX.

$$x = 0.369 \text{ m}$$

3.119



GIVEN: DIAMETER = 60 mm,  
APPLIED FORCES  
FIND: EQUIVALENT FORCE-COUPLE SYSTEM ( $R, M$ ) AT C

FOR EQUIVALENCE..

$$\sum F: R = F_A + F_B + F_C + F_D \\ = -17j - 12j - 16j - 21j \\ = -(21 \text{ N})j - (29 \text{ N})j - (16 \text{ N})j$$

$$\sum M_C: M = \sum r_{AC} \times F_A + \sum r_{BC} \times F_B + \sum r_{DC} \times F_D$$

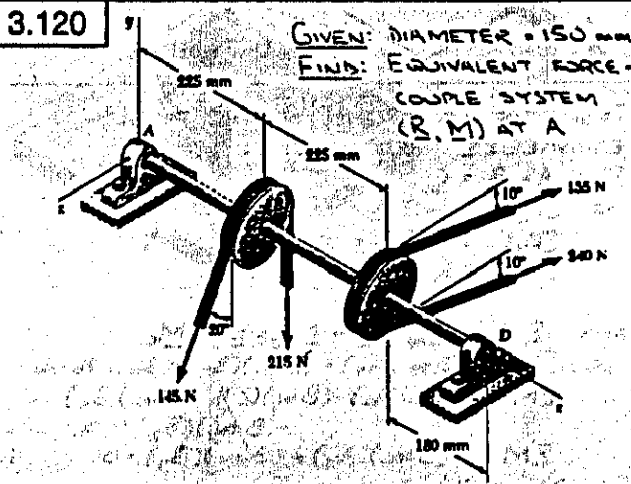
$$\text{OR } M = [(0.11 \text{ m})j - (0.03 \text{ m})k] \times [-17 \text{ N}]j \\ + [(0.02 \text{ m})j + (0.11 \text{ m})j - (0.03 \text{ m})k] \times [-12 \text{ N}]j \\ + [(0.03 \text{ m})j + (0.03 \text{ m})j - (0.03 \text{ m})k] \times [-21 \text{ N}]j \\ = -(0.51 \text{ N}\cdot\text{m})j - (0.24 \text{ N}\cdot\text{m})k - (0.36 \text{ N}\cdot\text{m})j \\ + [(0.63 \text{ N}\cdot\text{m})k + (0.63 \text{ N}\cdot\text{m})j]$$

$\therefore$  THE EQUIVALENT FORCE-COUPLE SYSTEM AT C IS ..

$$R = -(21 \text{ N})j - (29 \text{ N})j - (16 \text{ N})k$$

$$M = -(0.87 \text{ N}\cdot\text{m})j + (0.63 \text{ N}\cdot\text{m})j + (0.39 \text{ N}\cdot\text{m})k$$

3.120



GIVEN: DIAMETER = 150 mm  
FIND: EQUIVALENT FORCE-COUPLE SYSTEM ( $R, M$ ) AT A

FIRST, REPLACE THE BELT FORCES ON EACH PULLEY WITH AN EQUIVALENT FORCE-COUPLE SYSTEM AT THE CENTER OF THE PULLEY; THIS ELIMINATES THE NEED TO DETERMINE WHERE THE BELTS CONTACT THE PULLEYS.

$$\text{PULLEY B: } \sum F: R_B = -215j + 145(-\cos 20^\circ j - \sin 20^\circ k) \\ = -(351.26 \text{ N})j + (49.593 \text{ N})k$$

$$\sum M_B: M_B = [(0.075 \text{ m})(145 \text{ N}) - (0.075 \text{ m})(215 \text{ N})]j \\ = -(5.25 \text{ N}\cdot\text{m})j$$

$$\text{PULLEY C: } \sum F: R_C = (155 + 240 \sin 10^\circ)j - \cos 10^\circ k \\ = (168.591 \text{ N})j - (389.00 \text{ N})k$$

$$\sum M_C: M_C = [(0.075 \text{ m})(240 \text{ N}) - (0.075 \text{ m})(155 \text{ N})]j \\ = (6.375 \text{ N}\cdot\text{m})j$$

THE EQUIVALENT FORCE-COUPLE SYSTEM AT A IS THEN

$$\sum F: R = R_B + R_C \\ = (-351.26j + 49.593k) + (168.591j - 389.00k) \\ = -(420 \text{ N})j - (339 \text{ N})k$$

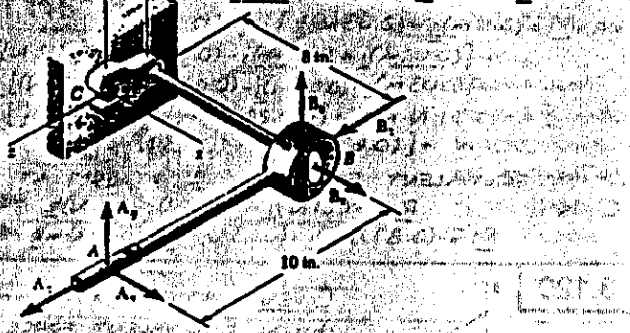
(CONTINUED)

### 3.120 CONTINUED

$$\begin{aligned}\text{AND } \Sigma \underline{M}_A = \underline{M} &= [(\underline{r}_{B/A} \times \underline{R}_B) + \underline{M}_B] + [(\underline{r}_{C/A} \times \underline{R}_C) + \underline{M}_C] \\ &= (0.22 \underline{i} \text{ m}) \times [-(351.2 \text{ N}) \underline{j} - (49.593 \text{ N}) \underline{k}] \\ &\quad + (5.25 \text{ N} \cdot \text{m}) \underline{i} \\ &\quad + (0.450 \text{ m}) \times [-(168.591 \text{ N}) \underline{j} - (389.00 \text{ N}) \underline{k}] \\ &\quad + (6.375 \text{ N} \cdot \text{m}) \underline{i} \\ &= [(-5.25 + 6.375) \text{ N} \cdot \text{m}] \underline{i} \\ &\quad + [(-11.584 + 175.05) \text{ N} \cdot \text{m}] \underline{j} \\ &\quad + [(-1.79,034 - 30.866) \text{ N} \cdot \text{m}] \underline{k} \\ &= (1.125 \text{ N} \cdot \text{m}) \underline{i} - (16.39 \text{ N} \cdot \text{m}) \underline{j} - (109.9 \text{ N} \cdot \text{m}) \underline{k}\end{aligned}$$

### 3.121

**GIVEN:**  $A_2 = 2 \text{ lb}$ , FORCES  $A$  AND  $B$  ARE EQUIVALENT TO  $\underline{C} = -(8 \text{ lb}) \underline{j} + (4 \text{ lb}) \underline{k}$ ,  $\underline{M}_C = (360 \text{ lb} \cdot \text{in}) \underline{j}$  AT  $C$   
**FIND:** FORCES  $A$  AND  $B$



FROM THE STATEMENT OF THE PROBLEM, EQUIVALENCE REQUIRES...

$$\begin{aligned}\Sigma \underline{F}: \underline{A} + \underline{B} &= \underline{C} \\ \text{OR } \Sigma F_x: A_x + B_x &= -8 \text{ lb} \quad (1) \\ \Sigma F_y: A_y + B_y &= 0 \quad (2) \\ \Sigma F_z: 2 \text{ lb} + B_z &= 4 \text{ lb} \quad (3)\end{aligned}$$

$$\begin{aligned}\text{AND } \Sigma \underline{M}_C: \underline{r}_{AC} \times \underline{A} + \underline{r}_{BC} \times \underline{B} &= \underline{M}_C \\ \text{OR } \Sigma M_x: -(8 \text{ in.})(A_y) + (2 \text{ in.})(B_y) &= 360 \text{ lb} \cdot \text{in.} \quad (4) \\ \Sigma M_y: (8 \text{ in.})(A_x) - (8 \text{ in.})(2 \text{ lb}) - (2 \text{ in.})(B_x) &= 0 \quad (5) \\ \Sigma M_z: (8 \text{ in.})(A_y) + (8 \text{ in.})(B_y) &= 0 \quad (6)\end{aligned}$$

$$\text{Eq. (2) OR (6)} \Rightarrow A_y = -B_y$$

$$\text{SUBSTITUTING INTO EQ. (4)} \Rightarrow -B(-B_y) + 2B_y = 360$$

$$\text{OR } B_y = 36 \text{ lb}$$

$$\text{AND } A_y = -36 \text{ lb}$$

$$\text{Eq. (3)} \Rightarrow B_z = 2 \text{ lb}$$

$$\text{Eq. (1)} \Rightarrow B_x = -(8 + A_x) \text{ lb}$$

SUBSTITUTING INTO EQ. (5)...

$$8A_x - 16 - 2[-(8 + A_x)] - 8(2) = 0$$

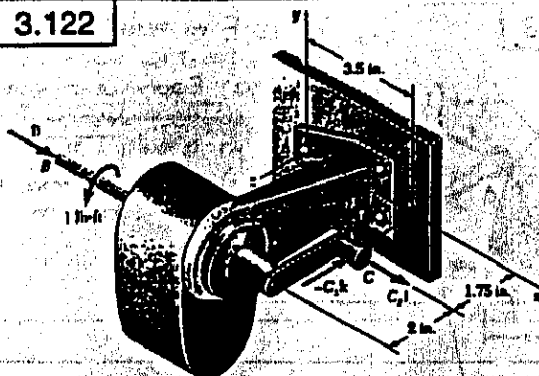
$$\text{OR } A_x = 1.6 \text{ lb}$$

$$\text{AND } B_x = -9.6 \text{ lb}$$

$$\underline{A} = (1.6 \text{ lb}) \underline{i} - (36 \text{ lb}) \underline{j} + (2 \text{ lb}) \underline{k}$$

$$\underline{B} = (-9.6 \text{ lb}) \underline{i} + (36 \text{ lb}) \underline{j} + (2 \text{ lb}) \underline{k}$$

### 3.122



**GIVEN:** FORCES  $B$  AND  $C$  ARE EQUIVALENT TO  $\underline{R} = (2.6 \text{ lb}) \underline{j} - R_y \underline{j} - (0.7 \text{ lb}) \underline{k}$ ,  $\underline{M}_A = \underline{M}_C = (1 \text{ lb} \cdot \text{ft}) \underline{j} - (0.72 \text{ lb} \cdot \text{ft}) \underline{k}$  AT  $A$   
**FIND:** (a) FORCES  $B$  AND  $C$   
(b)  $R_y$ ,  $M_x$

(a) FROM THE STATEMENT OF THE PROBLEM, EQUIVALENCE REQUIRES...

$$\begin{aligned}\Sigma \underline{F}: \underline{B} + \underline{C} &= \underline{R} \\ \text{OR } \Sigma F_x: B_x + C_x &= 2.6 \text{ lb} \quad (1) \\ \Sigma F_y: -C_y &= R_y \quad (2) \\ \Sigma F_z: -C_z &= -0.7 \text{ lb} \quad \text{OR } C_z = 0.7 \text{ lb}\end{aligned}$$

$$\text{AND } \Sigma \underline{M}_A: (\underline{r}_{BA} \times \underline{B}) + \underline{M}_B = \underline{M}_A$$

$$\text{OR } \Sigma M_x: (1 \text{ lb} \cdot \text{ft}) + (1.75 \text{ ft})(C_y) = M_x \quad (3)$$

$$\Sigma M_y: \left(\frac{3.75}{12} \text{ ft}\right)(B_x) + \left(\frac{1.75}{12} \text{ ft}\right)(C_x) + \left(\frac{3.5}{12} \text{ ft}\right)(0.7 \text{ lb}) = 1 \text{ lb} \cdot \text{ft}$$

$$\text{OR } 3.75 B_x + 1.75 C_x = 9.55$$

$$\text{USING EQ. (1)} \Rightarrow 3.75 B_x + 1.75(2.6 - B_x) = 9.55$$

$$\text{OR } B_x = 2.5 \text{ lb}$$

$$\text{AND } C_x = 0.1 \text{ lb}$$

$$\Sigma M_z: -\left(\frac{3.5}{12} \text{ ft}\right)(C_y) = -0.72 \text{ lb} \cdot \text{ft}$$

$$\text{OR } C_y = 2.4686 \text{ lb}$$

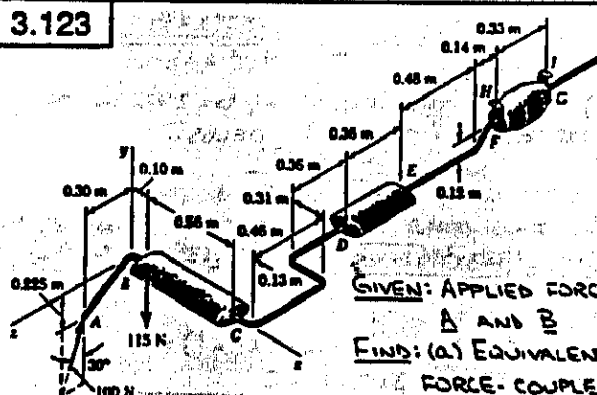
$$\therefore \underline{B} = (2.5 \text{ lb}) \underline{i} \quad \underline{C} = (0.1 \text{ lb}) \underline{i} - (2.47 \text{ lb}) \underline{j} - (0.7 \text{ lb}) \underline{k}$$

$$(b) \text{ Eq. (2)} \Rightarrow R_y = -2.47 \text{ lb}$$

$$\text{USING EQ. (3)} \Rightarrow 1 + \left(\frac{1.75}{12}\right)(2.4686) = M_x$$

$$\text{OR } M_x = 1.360 \text{ lb} \cdot \text{ft}$$

### 3.123



**GIVEN:** APPLIED FORCES  $A$  AND  $B$

**FIND:** (a) EQUIVALENT FORCE-COUPLE SYSTEM ( $\underline{R}$ ,  $\underline{M}_O$ ) AT  $D$

(b) DIRECTION OF ROTATION OF PIPE (D) RELATIVE TO MUFFLER DE

(CONTINUED)



### 3.123 CONTINUED

(a) EQUIVALENCE REQUIRES...

$$\sum \mathbf{F} = \mathbf{R} = \mathbf{A} + \mathbf{B}$$

$$= (100 \text{ N})(\cos 30^\circ \mathbf{j} - \sin 30^\circ \mathbf{k}) - (115 \text{ N})\mathbf{j}$$

$$= -(28.4 \text{ N})\mathbf{j} - (50 \text{ N})\mathbf{k}$$

AND  $\sum \mathbf{M}_O: \mathbf{M}_O = \mathbf{r}_{A/O} \times \mathbf{F}_A + \mathbf{r}_{B/O} \times \mathbf{F}_B$

WHERE  $\mathbf{r}_{A/O} = -(0.48 \text{ m})\mathbf{j} - (0.225 \text{ m})\mathbf{j} - (1.12 \text{ m})\mathbf{k}$

$\mathbf{r}_{B/O} = -(0.38 \text{ m})\mathbf{j} + (0.82 \text{ m})\mathbf{k}$

THEN...

$$\mathbf{M}_O = 100 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.48 & -0.225 & -1.12 \\ 0 & \cos 30^\circ & -\sin 30^\circ \end{vmatrix}$$

$$+ 115 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.38 & 0 & 0.82 \\ 0 & -1 & 0 \end{vmatrix}$$

$$= 100[(0.225 \sin 30^\circ - 1.12 \cos 30^\circ)\mathbf{i}$$

$$+ (-0.48 \cos 30^\circ)\mathbf{j} + (-0.48 \sin 30^\circ)\mathbf{k}]$$

$$+ 115[(0.82)\mathbf{j} + (0.38)\mathbf{k}]$$

$$= 8.56\mathbf{i} - 24.0\mathbf{j} + 2.13\mathbf{k}$$

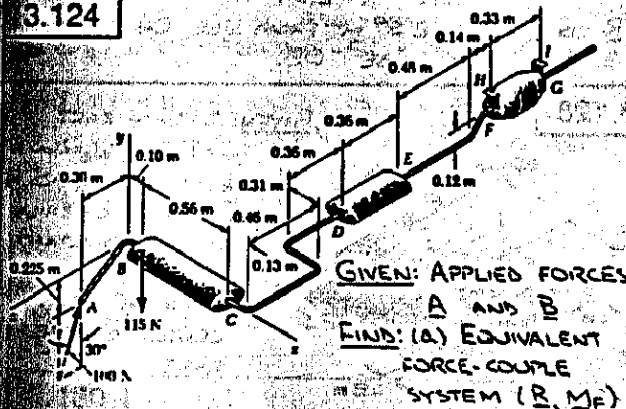
THE EQUIVALENT FORCE-COUPLE SYSTEM AT D IS...

$$\mathbf{R} = -(28.4 \text{ N})\mathbf{j} - (50 \text{ N})\mathbf{k}$$

$$\mathbf{M}_O = (8.56 \text{ N}\cdot\text{m})\mathbf{i} - (24.0 \text{ N}\cdot\text{m})\mathbf{j} - (2.13 \text{ N}\cdot\text{m})\mathbf{k}$$

(b) SINCE  $(M_O)_z$  IS POSITIVE, PIPE CD WILL TEND TO ROTATE COUNTERCLOCKWISE RELATIVE TO MUFFLER DE.

### 3.124



(b) DIRECTION OF ROTATION OF PIPE EF RELATIVE TO THE MECHANIC

(a) EQUIVALENCE REQUIRES...

$$\sum \mathbf{F} = \mathbf{R} = \mathbf{A} + \mathbf{B}$$

$$= (100 \text{ N})(\cos 30^\circ \mathbf{j} - \sin 30^\circ \mathbf{k}) - (115 \text{ N})\mathbf{j}$$

$$= -(28.4 \text{ N})\mathbf{j} - (50 \text{ N})\mathbf{k}$$

AND  $\mathbf{M}_F = \mathbf{M}_F = \mathbf{r}_{A/F} \times \mathbf{F}_A + \mathbf{r}_{B/F} \times \mathbf{F}_B$

WHERE  $\mathbf{r}_{A/F} = -(0.48 \text{ m})\mathbf{j} - (0.345 \text{ m})\mathbf{j} + (2.10 \text{ m})\mathbf{k}$

$\mathbf{r}_{B/F} = -(0.38 \text{ m})\mathbf{j} - (0.12 \text{ m})\mathbf{j} + (1.80 \text{ m})\mathbf{k}$

THEN...

$$\mathbf{M}_F = 100 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.48 & -0.345 & 2.10 \\ 0 & \cos 30^\circ & -\sin 30^\circ \end{vmatrix}$$

$$+ 115 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.38 & -0.12 & 1.80 \\ 0 & -1 & 0 \end{vmatrix}$$

(CONTINUED)

### 3.124 CONTINUED

$$\mathbf{M}_F = 100[(0.345 \sin 30^\circ - 2.10 \cos 30^\circ)\mathbf{i}$$

$$+ (-0.48 \sin 30^\circ)\mathbf{j} + (-0.48 \cos 30^\circ)\mathbf{k}]$$

$$+ 115[(1.80)\mathbf{j} - (0.38)\mathbf{k}]$$

$$= 42.4\mathbf{i} - 24.0\mathbf{j} + 2.13\mathbf{k}$$

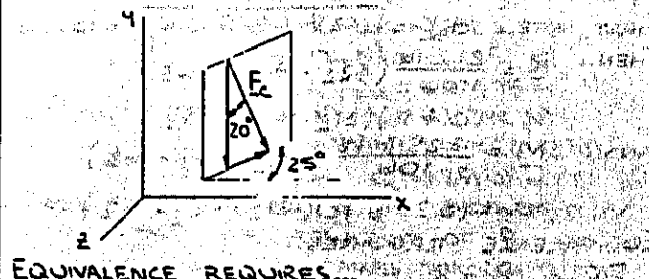
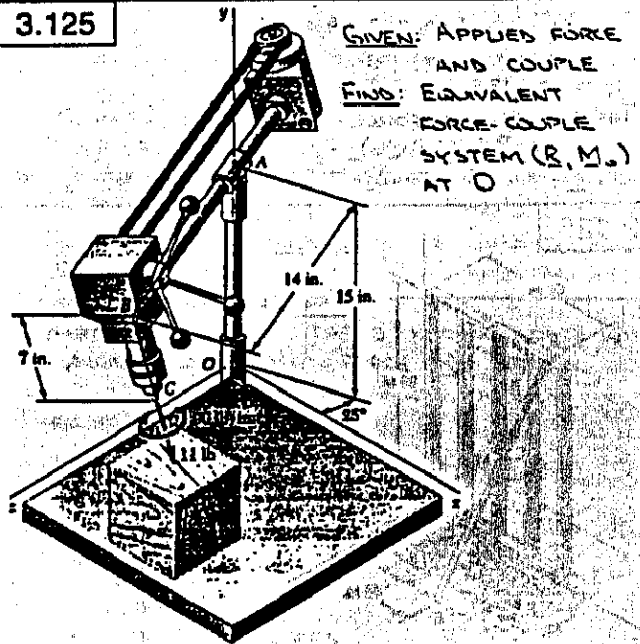
THE EQUIVALENT FORCE-COUPLE SYSTEM AT F IS...

$$\mathbf{R} = -(28.4 \text{ N})\mathbf{j} - (50 \text{ N})\mathbf{k}$$

$$\mathbf{M}_F = (42.4 \text{ N}\cdot\text{m})\mathbf{i} - (24.0 \text{ N}\cdot\text{m})\mathbf{j} + (2.13 \text{ N}\cdot\text{m})\mathbf{k}$$

(b) SINCE  $(M_F)_z$  IS POSITIVE, PIPE EF WILL TEND TO ROTATE COUNTERCLOCKWISE RELATIVE TO THE MECHANIC.

### 3.125



EQUIVALENCE REQUIRES...

$$\sum \mathbf{F} = \mathbf{R} = \mathbf{F}_C$$

$$= (11 \text{ lb})(\sin 20^\circ \cos 25^\circ \mathbf{i} - \cos 20^\circ \mathbf{j}$$

$$- \sin 20^\circ \sin 25^\circ \mathbf{k})$$

$$= (3.41 \text{ lb})\mathbf{i} - (10.34 \text{ lb})\mathbf{j} - (1.590 \text{ lb})\mathbf{k}$$

TO SIMPLIFY THE COMPUTATION OF  $\mathbf{M}_O$ , SLIDE FORCE  $\mathbf{F}_C$  ALONG ITS LINE OF ACTION TO B.

THEN...

$$\sum \mathbf{M}_O: \mathbf{M}_O = \mathbf{r}_{B/O} \times \mathbf{F}_C + \mathbf{M}_C$$

WHERE

$$\mathbf{r}_{B/O} = (14 \text{ in.})\sin 25^\circ \mathbf{j} + (15 \text{ in.})\mathbf{j}$$

$$+ (14 \text{ in.})\cos 25^\circ \mathbf{k}$$

$$\mathbf{M}_C = (90 \text{ lb}\cdot\text{in.})(\sin 20^\circ \cos 25^\circ \mathbf{i} - \cos 20^\circ \mathbf{j}$$

$$- \sin 20^\circ \sin 25^\circ \mathbf{k})$$

(CONTINUED)

### 3.125 CONTINUED

THEN  $\rightarrow$

$$\underline{M}_O = 11 \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 14 \sin 25^\circ & 15 & 14 \cos 25^\circ \\ \sin 20^\circ \cos 25^\circ & -\cos 20^\circ & -\sin 20^\circ \sin 25^\circ \\ 90(\sin 20^\circ \cos 25^\circ) & -\cos 20^\circ & -\sin 20^\circ \sin 25^\circ \end{vmatrix}$$

$$= [11(-15 \sin 20^\circ \sin 25^\circ + 14 \cos 25^\circ \cos 20^\circ) + 90(\sin 20^\circ \cos 25^\circ)] \underline{j}$$

$$= [11(14 \sin 20^\circ \cos 25^\circ + 14 \sin 20^\circ \sin 25^\circ) + 90(-\cos 20^\circ)] \underline{j}$$

$$= [11(-14 \sin 25^\circ \cos 20^\circ - 15 \sin 20^\circ \cos 25^\circ) + 90(-\sin 20^\circ \sin 25^\circ)] \underline{j}$$

$$= (-23.849 + 131.154 + 27.898) \underline{j}$$

$$= (43.203 + 9.407 - 64.572) \underline{j}$$

$$= (-6.158 - 51.146 - 13.095) \underline{j}$$

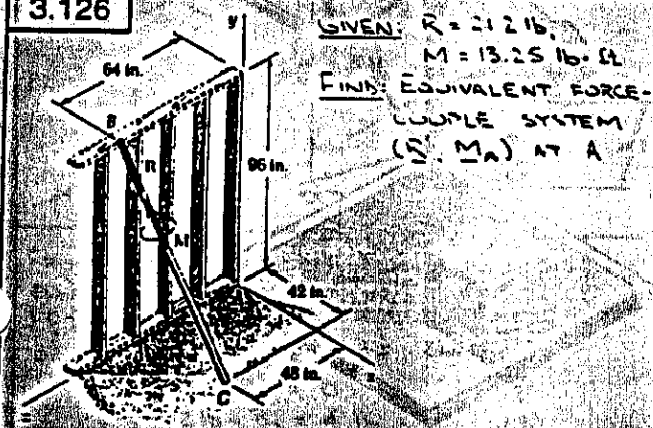
$$= -135.2 \underline{j} - 319 \underline{j} - 125.3 \underline{k}$$

THE EQUIVALENT FORCE-COUPLE SYSTEM AT O IS

$$\underline{R} = (3.41 \text{ lb}) \underline{i} - (10.34 \text{ lb}) \underline{j} - (1.590 \text{ lb}) \underline{k}$$

$$\underline{M}_O = (135.2 \text{ lb}\cdot\text{in}) \underline{j} - (319 \text{ lb}\cdot\text{in}) \underline{j} - (125.3 \text{ lb}\cdot\text{in}) \underline{k}$$

### 3.126



GIVEN:  $R = 21.2 \text{ lb}$   
 $M = 13.25 \text{ lb}\cdot\text{ft}$   
 FIND: EQUIVALENT FORCE-COUPLE SYSTEM ( $\underline{S}, \underline{M}_A$ ) AT A

FIRST NOTE:  $d_{BC} = \sqrt{(42)^2 + (-96)^2 + (-16)^2} = 106 \text{ in.}$

THEN:  $\underline{R} = \frac{21.2 \text{ lb}}{106} (42 \underline{i} - 96 \underline{j} - 16 \underline{k})$

$$= (0.4 \text{ lb}) (21 \underline{i} - 48 \underline{j} - 8 \underline{k})$$

AND  $\underline{M} = \frac{13.25 \text{ lb}\cdot\text{ft}}{106} (-42 \underline{i} - 96 \underline{j} - 16 \underline{k})$

$$= (0.25 \text{ lb}\cdot\text{ft}) (-21 \underline{i} - 48 \underline{j} - 8 \underline{k})$$

EQUIVALENCE REQUIRES:

$$\sum \underline{F}: \underline{R}' = \underline{R}$$

$$\sum \underline{M}_A: \underline{M}_A = \underline{r}_{CA} \times \underline{R} + \underline{M}$$

WHERE  $\underline{r}_{CA} = (42 \text{ in.}) \underline{i} + (48 \text{ in.}) \underline{j} + (35 \text{ ft.}) \underline{k} + (4 \text{ ft.}) \underline{k}$

THEN:

$$\underline{M}_A = 0.4 \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 3.5 & 0 & 4 \\ 21 & -48 & -8 \end{vmatrix} + (5.25 \underline{i} - 12 \underline{j} + 2 \underline{k})$$

$$= [0.4(192) - 5.25] \underline{i}$$

$$+ [0.4(84 + 28) + 12] \underline{j}$$

$$+ [0.4(-168) + 2] \underline{k}$$

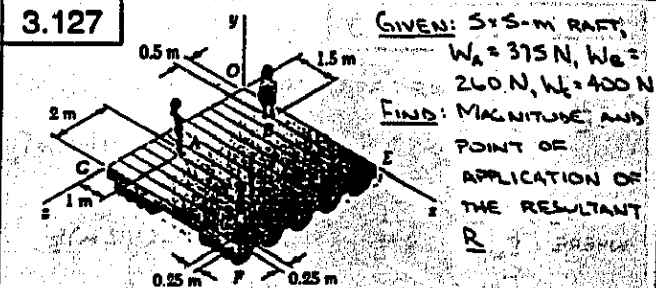
$$= 71.55 \underline{i} + 56.8 \underline{j} - 65.2 \underline{k}$$

THE EQUIVALENT FORCE-COUPLE SYSTEM AT A IS:

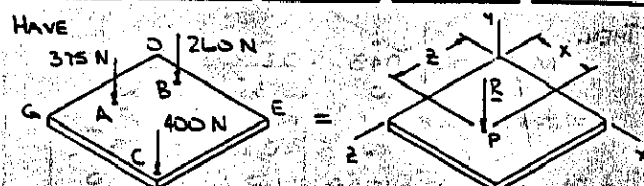
$$\underline{R}' = (8.4 \text{ lb}) \underline{i} - (19.2 \text{ lb}) \underline{j} - (3.2 \text{ lb}) \underline{k}$$

$$\underline{M}_A = (71.6 \text{ lb}\cdot\text{ft}) \underline{i} + (56.8 \text{ lb}\cdot\text{ft}) \underline{j} - (65.2 \text{ lb}\cdot\text{ft}) \underline{k}$$

### 3.127



GIVEN: S+S-M RAFT,  
 $W_A = 375 \text{ N}$ ,  $W_B = 260 \text{ N}$ ,  $W_C = 400 \text{ N}$   
 FIND: MAGNITUDE AND POINT OF APPLICATION OF THE RESULTANT  $\underline{R}$



EQUIVALENCE REQUIRES:

$$\sum F_y: -375 - 260 - 400 = -R$$

$$R = 1035 \text{ N}$$

LET  $\underline{R}$  BE APPLIED AT POINT P WHOSE COORDINATES ARE  $(x, 0, 2)$ . THEN:

$$\sum M_x: (3 \text{ m})(375 \text{ N}) + (0.5 \text{ m})(260 \text{ N}) + (4.75 \text{ m})(400 \text{ N})$$

$$= 2(1035 \text{ N})$$

$$\text{OR } 2 = 3.05 \text{ m}$$

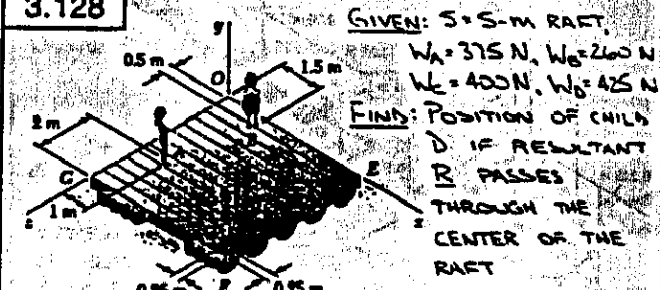
$$\sum M_z: -(1 \text{ m})(375 \text{ N}) - (1.5 \text{ m})(260 \text{ N}) - (4.75 \text{ m})(400 \text{ N})$$

$$= -x(1035 \text{ N})$$

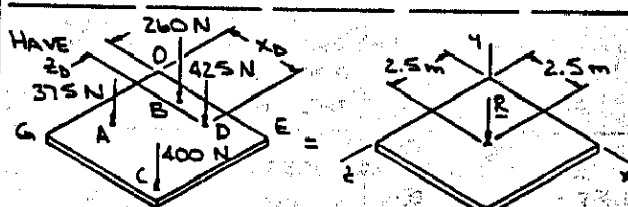
$$\text{OR } x = 2.57 \text{ m}$$

$\therefore \underline{R}$  IS APPLIED 2.57 m FROM SIDE OG AND 3.05 m FROM SIDE OE.

### 3.128



GIVEN: S+S-M RAFT,  
 $W_A = 375 \text{ N}$ ,  $W_B = 260 \text{ N}$ ,  $W_C = 400 \text{ N}$ ,  $W_D = 425 \text{ N}$   
 FIND: POSITION OF CHILD D IF RESULTANT  $\underline{R}$  PASSES THROUGH THE CENTER OF THE RAFT



EQUIVALENCE REQUIRES:

$$\sum F_y: -375 - 260 - 400 - 425 = -R \quad \text{OR } R = 1460 \text{ N}$$

$$\sum M_x: (3 \text{ m})(375 \text{ N}) + (0.5 \text{ m})(260 \text{ N}) + (4.75 \text{ m})(400 \text{ N})$$

$$+ 2.5(425 \text{ N}) = (2.5 \text{ m})(1460 \text{ N})$$

$$\text{OR } 2.5 = 1.165 \text{ m}$$

$$\sum M_z: -(1 \text{ m})(375 \text{ N}) - (1.5 \text{ m})(260 \text{ N}) - (4.75 \text{ m})(400 \text{ N})$$

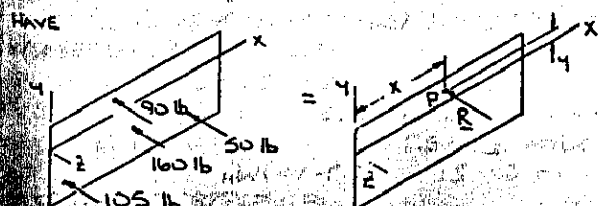
$$- x_D(425 \text{ N}) = -(2.5 \text{ m})(1460 \text{ N})$$

$$\text{OR } x_D = 2.32 \text{ m}$$

$\therefore$  THE CHILD SHOULD STAND 2.32 m FROM SIDE OG AND 1.165 m FROM SIDE OE

[illegible]

FIND: MAGNITUDE AND POINT OF APPLICATION OF RESULTANT  $R$


$$\Sigma F_x = -105 - 90 - 160 - 50 = -R$$

$$\Sigma M_A: (5 \text{ ft})(105 \text{ lb}) - (1 \text{ ft})(90 \text{ lb}) + (3 \text{ ft})(160 \text{ lb}) + (5.5 \text{ ft})(50 \text{ lb}) = -4(400 \text{ lb})$$

$$Q_2 \quad y = -2.94 \text{ ft}$$

$$\Sigma M_y: (5.5 \text{ ft})(105 \text{ lb}) + (12 \text{ ft})(90 \text{ lb}) + (14.5 \text{ ft})(160 \text{ lb}) + (22.5 \text{ ft})(50 \text{ lb}) - x(405 \text{ lb})$$

OR  $X = 12.63 \text{ ft}$

... R ACTS 12.60 £L TO THE RIGHT OF  
MEMBER AB AND 2.94 £L BELOW MEMBER  
BC

3.130

The diagram shows a beam structure with a horizontal beam of length 8 ft. A vertical support is at the left end. A force of 105 lb is applied vertically downwards at point B, which is 5 ft from the left end. A force of 100 lb is applied vertically downwards at point G, which is 8 ft from the left end. A force of 50 lb is applied vertically downwards at point H, which is 8 ft from the left end. A force of 50 lb is applied vertically downwards at point D, which is 8 ft from the left end. The beam is supported by a vertical support at the left end and a vertical support at point D. The beam is also supported by a vertical support at point G. The beam is also supported by a vertical support at point H. The beam is also supported by a vertical support at point B. The beam is also supported by a vertical support at point A. The beam is also supported by a vertical support at point C. The beam is also supported by a vertical support at point E. The beam is also supported by a vertical support at point F. The beam is also supported by a vertical support at point I. The beam is also supported by a vertical support at point J. The beam is also supported by a vertical support at point K. The beam is also supported by a vertical support at point L. The beam is also supported by a vertical support at point M. The beam is also supported by a vertical support at point N. The beam is also supported by a vertical support at point O. The beam is also supported by a vertical support at point P. The beam is also supported by a vertical support at point Q. The beam is also supported by a vertical support at point R. The beam is also supported by a vertical support at point S. The beam is also supported by a vertical support at point T. The beam is also supported by a vertical support at point U. The beam is also supported by a vertical support at point V. The beam is also supported by a vertical support at point W. The beam is also supported by a vertical support at point X. The beam is also supported by a vertical support at point Y. The beam is also supported by a vertical support at point Z.

GIVEN: APPLIED FORCES, RESULTANT  $R$  OF THE FOUR FORCES ACTS AT  $G$

FIND: DISTANCES  $Q$  AND  $b$

SINCE  $R$  ACTS AT  $G$  EQUILIBRIUM

GIVEN: APPLIED FORCES, RESULTANT  
R OF THE FOUR FORCES  
ACTS AT G

FIND: DISTANCES  $a$  AND  $b$

SINCE  $R$  ACTS AT G, EQUIVALENCE THEN  
REQUIRES THAT  $\Sigma M_G$  OF THE APPLIED  
(CONTINUED)

SYSTEM OF FORCES ALSO BE ZERO. THEN..

$$\text{AT G: } \Sigma M_x = -(2+3)(4) - (90 \text{ lb})(4) + (2 \text{ ft})(105 \text{ lb}) \\ + (2.5 \text{ ft})(50 \text{ lb}) = 0$$


or  $Q = 0.722 \text{ ft}$

$$\Sigma M_y: -(9 \text{ ft})(105 \text{ ft}) - (14.5 - b) \text{ ft} \cdot (90 \text{ lb})$$

$$F_{\text{spring}} + (84t)(50 \text{ lb}) = 0$$

OR  $b = 20.6 \text{ ft}$

• 3.131



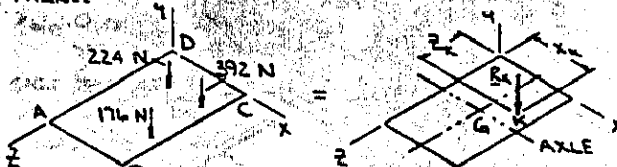
GIVEN:  $Z = 3.3\text{-m}$  TRAILER,  
 $0.66 + 0.66 = 0.66\text{-m}$   
AND  $0.66 + 0.66 =$   
 $1.2\text{ m}$  BOXES

FIND: SMALLEST LOAD  
AND LOCATION  
IF RESULTANT  
OF FOUR  
WEIGHTS PASSES  
THROUGH CENTER  
LINES OF AXLE  
AND TRAILER

GIVEN:  $2 \times 3.3\text{-m}$  TRAILER,  
 $0.66 \times 0.66 = 0.66\text{ m}$   
 AND  $0.66 \times 0.66 =$   
 $1.2\text{ m}$  BOXES

FIND: SMALLEST LOAD  
AND LOCATION  
IF RESULTANT  
OF FOUR  
WEIGHTS PASSES  
THROUGH CENTER  
LINES OF AXLE  
AND TRAILER

FIRST REPLACE THE THREE KNOWN LOADS WITH  
A SINGLE EQUIVALENT FORCE  $R_K$  APPLIED  
AT POINT K WHOSE COORDINATES ARE  $(x_k, y_k, z_k)$ .  
THEN



EQUIVALENCE REQUIRES...

$$\Sigma F_y: 0 - 224 - 392 - 176 = -R_r \quad \text{OR} \quad R_r = 792 \text{ N}$$

$$\Sigma M_x: (0.33 \text{ m})(224 \text{ N}) + (0.6 \text{ m})(392 \text{ N}) + (2 \text{ m})(176 \text{ N}) = Z_x(792 \text{ N})$$

$$\Sigma M_2: -(0.33 \text{ m})(224 \text{ N}) - (1.67 \text{ m})(392 \text{ N}) - (1.67 \text{ m})(176 \text{ N}) = -x_u(792 \text{ N})$$

OR  $x_k = 1.29101 \text{ m}$

FROM THE STATEMENT OF THE PROBLEM, IT IS KNOWN THAT THE RESULTANT OF  $R_K$  AND THE LIGHTEST LOAD  $W_L$  PASSES THROUGH G, THE POINT OF INTERSECTION OF THE TWO CENTER LINES. THUS,  $\sum M_G = 0$

FURTHER, SINCE  $W_L$  IS TO BE AS SMALL AS POSSIBLE, THE FURL BOX SHOULD BE PLACED AS FAR FROM G AS POSSIBLE. THESE TWO REQUIREMENTS IMPLY...

$$0.33 \text{ m} \leq x_1 \leq 1.0 \text{ m} \quad \text{AND} \quad 1.5 \text{ m} \leq x_2 \leq 2.97 \text{ m}$$

WHERE THE LOWER BOUND ON  $X$  AND THE UPPER BOUND ON  $Z$  ARE IMPOSED SO THAT THE BOX DOES NOT OVERHANG THE TRAILER. SINCE THE BOX IS TO BE AS FAR FROM  $G$  AS POSSIBLE, CONSIDER FIRST IF THESE BOUNDS ARE PHYSICALLY POSSIBLE.

(CONTINUED)



# 3.146 CONTINUED

WRENCH AND THE PRESCRIBED LINE OF ACTION (LINE AA') ARE KNOWN, IT FOLLOWS THAT THE DISTANCE  $Q$  CAN BE DETERMINED. IN THE FOLLOWING SOLUTION, IT IS ASSUMED THAT  $Q$  IS KNOWN. THEN, FOR EQUIVALENCE

$$\sum F_x: 0 = A\lambda_1 + B_1 \quad (1)$$

$$\sum F_y: R = A\lambda_1 + B_1 \quad (2)$$

$$\sum F_z: 0 = A\lambda_2 + B_2 \quad (3)$$

$$\sum M_x: 0 = -2B_1 \quad (4)$$

$$\sum M_y: M = -QA\lambda_2 + 2B_1 - \lambda_2 B_2 \quad (5)$$

$$\sum M_z: 0 = QA\lambda_1 + \lambda_1 B_1 \quad (6)$$

THUS, THERE ARE SIX UNKNOWN (A, B<sub>1</sub>, B<sub>2</sub>, λ<sub>1</sub>, λ<sub>2</sub>) AND SIX INDEPENDENT EQUATIONS. THEREFORE, IT WILL BE POSSIBLE TO OBTAIN A SOLUTION.

CASE 1: EQ (4) ⇒ z = 0

$$\text{NOW... EQ. (2) ⇒ } A\lambda_1 = R - B_1$$

$$\text{EQ. (3) ⇒ } B_2 = -A\lambda_2$$

$$\text{EQ. (6) ⇒ } x = -\frac{QA\lambda_1}{B_1} = -\frac{A}{B_1} (R - B_1)$$

SUBSTITUTING INTO EQ. (5)...

$$M = -QA\lambda_2 - \left[-\frac{A}{B_1} (R - B_1)\right] (-A\lambda_2)$$

$$\text{OR } A = -\frac{1}{\lambda_2} \frac{M}{QR} B_1$$

SUBSTITUTING INTO EQ (2)...

$$R = -\frac{1}{\lambda_2} \frac{M}{QR} B_1 (\lambda_1) + B_1$$

$$\text{OR } B_1 = \frac{\lambda_2 QR^2}{\lambda_2 QR - \lambda_1 M}$$

THEN...

$$A = \frac{MR}{\lambda_2 QR - \lambda_1 M}$$

$$B_2 = \frac{\lambda_1 MR}{\lambda_2 QR - \lambda_1 M}$$

$$B_1 = \frac{\lambda_2 MR}{\lambda_2 QR - \lambda_1 M}$$

IN SUMMARY...

$$A = \frac{R}{\lambda_1 - \frac{QR}{M} \lambda_2} \lambda_2$$

$$B = \frac{R}{\lambda_2 QR - \lambda_1 M} (\lambda_2 M i + \lambda_2 QR j + \lambda_2 M k)$$

$$\text{ALSO... } x = a(1 - \frac{R}{B_1}) = a(1 - \frac{R}{\frac{\lambda_2 QR^2}{\lambda_2 QR - \lambda_1 M}})$$

$$= \frac{\lambda_1 M}{\lambda_2 R}$$

NOTE THAT FOR THIS CASE, THE LINES OF ACTION OF BOTH A AND B INTERSECT THE x AXIS.

CASE 2: EQ (4) ⇒ B<sub>1</sub> = 0

$$\text{THEN EQ. (2) ⇒ } A = \frac{R}{\lambda_1}$$

(CONTINUED)

# 3.146 CONTINUED

$$\text{AND EQ. (1) ⇒ } B_2 = -\frac{R}{\lambda_1} \lambda_2$$

$$\text{EQ. (3) ⇒ } B_2 = -\frac{R}{\lambda_1} \lambda_2$$

$$\text{EQ. (6) ⇒ } QA\lambda_1 = 0 \dots \text{WHICH REQUIRES THAT } Q = 0$$

THEN, SUBSTITUTING INTO EQ. (5)...

$$M = 2(-\frac{R}{\lambda_1} \lambda_2) - \lambda_1 (-\frac{R}{\lambda_1} \lambda_2)$$

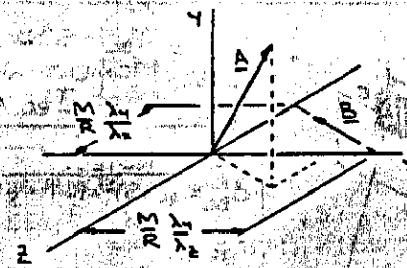
$$\text{OR } \lambda_2 x - \lambda_1 z = \frac{M}{R} \lambda_1$$

THIS LAST EXPRESSION IS THE EQUATION OF THE LINE OF ACTION OF FORCE B. IN SUMMARY...

$$A = \frac{R}{\lambda_1} \lambda_1$$

$$B = \frac{R}{\lambda_1} (-\lambda_2 i - \lambda_2 k)$$

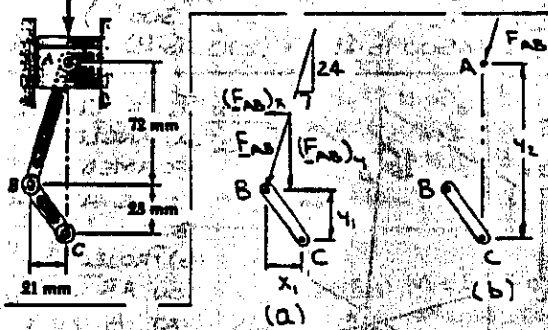
ASSUMING THAT λ<sub>1</sub>, λ<sub>2</sub> > 0, THE EQUIVALENT FORCE SYSTEM IS...



NOTE THAT THE COMPONENT OF A LYING IN THE xz PLANE IS PARALLEL TO B.

## 3.147

GIVEN: F<sub>AB</sub> = 1.5 kN  
FIND: MOMENT OF F<sub>AB</sub> ABOUT C



Using (a)

$$M_C = y_1 (F_{AB})_{x_1} + x_1 (F_{AB})_{y_1}$$

$$= (0.028 \text{ m}) (\frac{2}{25} \cdot 1500 \text{ N}) + (0.021 \text{ m}) (\frac{24}{25} \cdot 1500 \text{ N})$$

$$= 42 \text{ N}\cdot\text{m}$$

$$\text{OR } M_C = 42 \text{ N}\cdot\text{m}$$

Using (b)

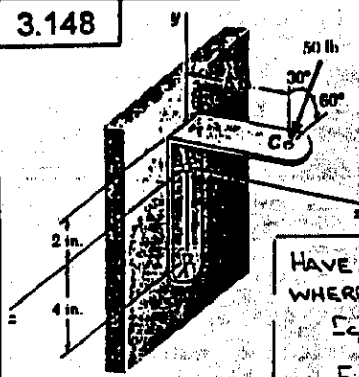
$$M_C = y_2 (F_{AB})_{x_2}$$

$$= (0.1 \text{ m}) (\frac{2}{25} \cdot 1500 \text{ N})$$

$$= 42 \text{ N}\cdot\text{m}$$

$$\text{OR } M_C = 42 \text{ N}\cdot\text{m}$$

3.148



GIVEN: FORCE  $F_C$   
FIND: MOMENT OF  
 $F_C$  ABOUT A

HAVE..  $M_A = r_{CA} \times F_C$

WHERE

$$r_{CA} = (5 \text{ in.})_i + (6 \text{ in.})_j$$

$$F_C = -(50 \text{ lb}) \cos 30^\circ j + (50 \text{ lb}) \sin 30^\circ k$$

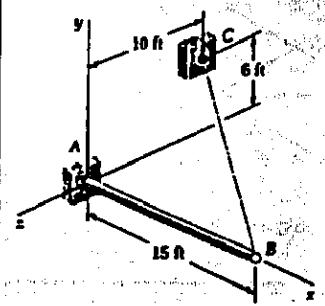
THEN..

$$M_A = 50 \begin{vmatrix} i & j & k \\ 5 & 6 & 0 \\ 0 & -\cos 30^\circ & \sin 30^\circ \end{vmatrix}$$

$$= 50 [(6 \sin 30^\circ)_i - (5 \sin 30^\circ)_j - (5 \cos 30^\circ)_k]$$

OR  $M_A = (150 \text{ lb}\cdot\text{in.})_i - (125 \text{ lb}\cdot\text{in.})_j - (217 \text{ lb}\cdot\text{in.})_k$

3.149



GIVEN:  $T_{BC} = 570 \text{ lb}$   
FIND: MOMENT ABOUT  
A OF  $T_{BC}$  AT B

FIRST NOTE..

$$d_{BC} = \sqrt{(-15)^2 + (-6)^2 + (-10)^2}$$

$$= 19 \text{ ft}$$

THEN..

$$T_{BC} = \frac{570 \text{ lb}}{19} (-15i - 6j - 10k)$$

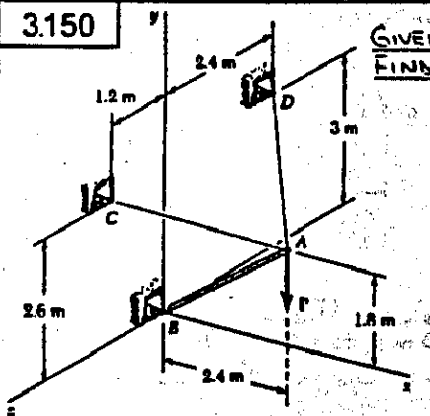
$$= -(450 \text{ lb})_i - (180 \text{ lb})_j - (300 \text{ lb})_k$$

HAVE..  $M_A = r_{BA} \times T_{BC}$  WHERE  $r_{BA} = (15 \text{ ft})_i$

THEN..  $M_A = 15i \times (-450j + 180j - 300k)$

OR  $M_A = (4500 \text{ lb}\cdot\text{ft})_j + (2700 \text{ lb}\cdot\text{ft})_k$

3.150



GIVEN:  $T_{AC} = 1260 \text{ N}$   
FIND: (a) ANGLE  $\theta$   
FORMED BY  
CABLE AC AND  
BOOM AB  
(b) PROJECTION  
ON AB OF  $T_{AC}$   
AT A

(a) FIRST NOTE..  $AC = \sqrt{(-2.4)^2 + (0.8)^2 + (1.2)^2}$   
 $= 2.8 \text{ m}$

$$AB = \sqrt{(-2.4)^2 + (-1.8)^2 + (0)^2}$$

$$= 3.0 \text{ m}$$

AND  $\vec{AC} = (-2.4 \text{ m})_i + (0.8 \text{ m})_j + (1.2 \text{ m})_k$   
 $\vec{AB} = (-2.4 \text{ m})_i - (1.8 \text{ m})_j$

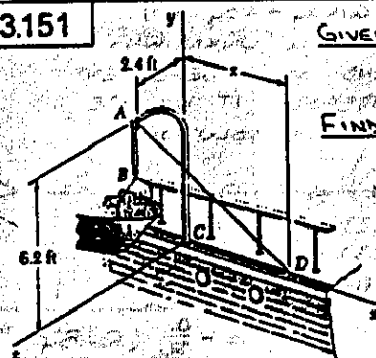
(CONTINUED)

3.150 CONTINUED

BY DEFINITION..  $\vec{AC} \cdot \vec{AB} = (AC)(AB) \cos \theta$   
OR  $(-2.4i + 0.8j + 1.2k) \cdot (-2.4i - 1.8j) = (2.8)(3.0) \cos \theta$   
OR  $(-2.4)(-2.4) + (0.8)(-1.8) + (1.2)(0) = 8.4 \cos \theta$   
OR  $\cos \theta = 0.51429$   
OR  $\theta = 59.0^\circ$

(b) HAVE..  $(T_{AC})_{AB} = T_{AC} \cdot \lambda_{AB}$   
 $= T_{AC} \cos \theta$   
 $= (1260 \text{ N})(0.51429)$   
OR  $(T_{AC})_{AB} = 648 \text{ N}$

3.151



GIVEN:  $M_{z1}$  OF  $R_A$  AT  
A  $= 160 \text{ lb}\cdot\text{ft}$   
 $x = 4.8 \text{ ft}$   
FIND:  $T_{max}$

FIRST NOTE THAT  $R_A = 2T_{AB} + T_{AD}$   
AND THEN OBSERVE THAT ONLY  $T_{AD}$  WILL  
CONTRIBUTE TO THE MOMENT ABOUT THE  
Z AXIS. NOW..

$$d_{AD} = \sqrt{(4.8)^2 + (-6.2)^2 + (-2.4)^2} = 8.2 \text{ ft}$$

THEN  $T_{AD} = \frac{T}{8.2} (4.8i - 6.2j - 2.4k)$   
 $= \frac{T}{41} (24i - 31j - 12k)$

NOW..  $M_z = k \cdot (r_{AC} \times T_{AD})$   
WHERE  $r_{AC} = (6.2 \text{ ft})_j + (2.4 \text{ ft})_k$

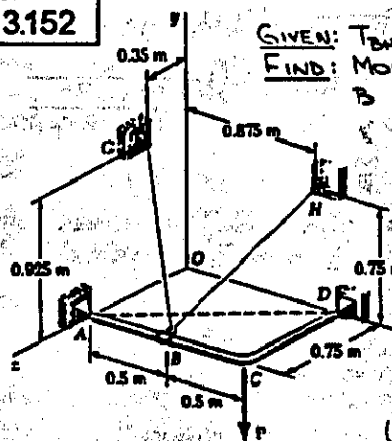
THEN FOR  $T_{max}$ ..

$$160 = \begin{vmatrix} T_{max} & 0 & 0 & 1 \\ 41 & 0 & 6.2 & 2.4 \\ 24 & -31 & -12 & 0 \end{vmatrix}$$

$$= \frac{T_{max}}{41} |-(1)(6.2)(24)|$$

OR  $T_{max} = 44.1 \text{ lb}$

3.152



GIVEN:  $T_{BH} = 450 \text{ N}$   
FIND: MOMENT OF  $T_{BH}$  AT  
B ABOUT DIAGONAL AD

(CONTINUED)

### 3.152 CONTINUED

$$M_{AD} = \lambda_{AD} \cdot (\Sigma F_{B/A} + I_{BH})$$

WHERE  $\lambda_{AD} = \frac{1}{3}(4\mathbf{i} - 3\mathbf{k})$   
 $\Sigma F_{B/A} = (0.5\text{ m})\mathbf{j}$

AND THEN  $d_{BH} = \sqrt{(0.375)^2 + (0.75)^2 + (0.75)^2} = 1.125\text{ m}$

$$I_{BH} = \frac{450\text{ N}}{1.125} (0.375\mathbf{j} + 0.75\mathbf{j} - 0.75\mathbf{k})$$

$$= (150\text{ N})\mathbf{j} + (300\text{ N})\mathbf{j} - (300\text{ N})\mathbf{k}$$

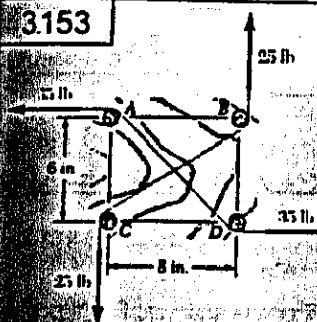
FINALLY..

$$M_{AD} = \frac{1}{3} \begin{vmatrix} 4 & 0 & -3 \\ 0.5 & 0 & 0 \\ 150 & 300 & -300 \end{vmatrix}$$

$$= \frac{1}{3} [(-3)(0.5)(300)]$$

OR  $M_{AD} = -90\text{ N}\cdot\text{m}$

3.153

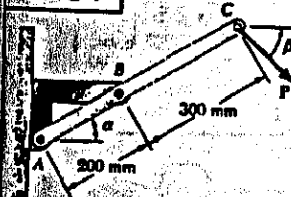


GIVEN: APPLIED FORCES,  
 $M = 425\text{ lb}\cdot\text{in.}$   
 FIND: DIAMETER  $d$  OF  
 THE PEGS

HAVE..  $M = M_{AD} = M_{BC}$

OR  $M = d_{AD} F_{AD} + d_{BC} F_{BC}$   
 OR  $425\text{ lb}\cdot\text{in.} = [(6+d)\text{ in.}](35\text{ lb})$   
 $+ [(8+d)\text{ in.}](25\text{ lb})$   
 OR  $d = 1.25\text{ in.}$

3.154



GIVEN:  $P = 250\text{ N}$ ,  
 $\alpha = 30^\circ$ ,  $\beta = 60^\circ$

FIND: (a) EQUIVALENT  
 FORCE-COUPLE  
 SYSTEM ( $F$ ,  $M$ )  
 AT B  
 (b) EQUIVALENT  
 SYSTEM ( $F_A$ ,  $F_B$ ),  
 WHERE  $F_A$  AND  
 $F_B$  ARE PARALLEL

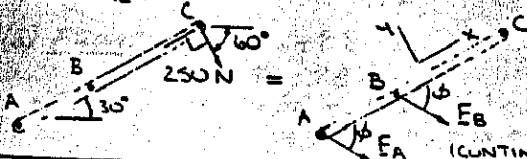
(a) EQUIVALENCE REQUIRES..

$$\Sigma F: E = P \quad \text{OR} \quad E = 250\text{ N} \angle 60^\circ$$

$$\Sigma M_B: M = -(0.3\text{ m})(250\text{ N}) = -75\text{ N}\cdot\text{m}$$

THE EQUIVALENT FORCE-COUPLE SYSTEM AT B  
 IS..  $F = 250\text{ N} \angle 60^\circ$ ,  $M = 75\text{ N}\cdot\text{m}$

(b) REQUIRE



(CONTINUED)

### 3.154 CONTINUED

EQUIVALENCE THEN REQUIRES..

$$\Sigma F_x: 0 = F_A \cos \phi + F_B \cos \phi$$

$$\therefore F_A = -F_B \quad \text{OR} \quad \cos \phi = 0$$

$$\Sigma F_y: -250 = -F_A \sin \phi - F_B \sin \phi$$

NOW.. IF  $F_A = -F_B \Rightarrow -250 = 0$  .. REJECT

$$\therefore \cos \phi = 0$$

$$\text{OR} \quad \phi = 90^\circ$$

$$\text{AND} \quad F_A + F_B = 250$$

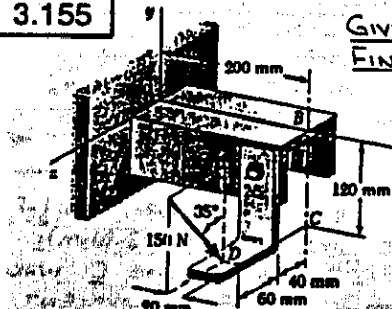
$$\text{ALSO.. } \Sigma M_B: -(0.3\text{ m})(250\text{ N}) = (0.2\text{ m})F_A$$

$$\text{OR} \quad F_A = -375\text{ N}$$

$$\text{AND} \quad F_B = 625\text{ N}$$

$$\therefore F_A = 375\text{ N} \angle 60^\circ, \quad F_B = 625\text{ N} \angle 60^\circ$$

3.155



GIVEN: 150-N FORCE  
 FIND: EQUIVALENT  
 FORCE-COUPLE  
 SYSTEM ( $F$ ,  $M$ )  
 AT A

EQUIVALENCE REQUIRES..

$$\Sigma F: E = (150\text{ N})(-\cos 35^\circ \mathbf{j} - \sin 35^\circ \mathbf{k})$$

$$\text{TOTAL} \quad E = -(122.873\text{ N})\mathbf{j} - (86.036\text{ N})\mathbf{k}$$

$$\Sigma M_A: M = \Sigma r_{A/A} \times F$$

WHERE  $\Sigma r_{A/A} = (0.18\text{ m})\mathbf{i} - (0.12\text{ m})\mathbf{j} + (0.1\text{ m})\mathbf{k}$

THEN..

$$M = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.18 & -0.12 & 0.1 \\ 0 & -122.873 & -86.036 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & -122.873 & -86.036 \\ 0.18 & -0.12 & 0.1 \end{vmatrix}$$

$$= [(-0.12)(-86.036) - (0.1)(-122.873)]\mathbf{j}$$

$$+ [(-0.18)(-86.036)]\mathbf{j}$$

$$+ [(0.18)(-122.873)]\mathbf{k}$$

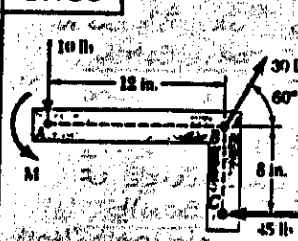
$$= (22.6\text{ N}\cdot\text{m})\mathbf{j} + (15.49\text{ N}\cdot\text{m})\mathbf{j} - (22.1\text{ N}\cdot\text{m})\mathbf{k}$$

THE EQUIVALENT FORCE-COUPLE SYSTEM AT A IS..

$$F = -(122.9\text{ N})\mathbf{j} - (86.0\text{ N})\mathbf{k}$$

$$M = (22.6\text{ N}\cdot\text{m})\mathbf{j} + (15.49\text{ N}\cdot\text{m})\mathbf{j} - (22.1\text{ N}\cdot\text{m})\mathbf{k}$$

3.156



GIVEN:  $M = 54\text{ lb}\cdot\text{in.}$ ,  
 APPLIED FORCES

FIND: (a) RESULTANT  $R$

(b) POINTS WHERE

LINE OF ACTION

OF  $R$  INTERSECTS

LINE AB AND

BC

(a) HAVE..  $\Sigma F: R = (-10\mathbf{j}) + (30\cos 60^\circ\mathbf{j})$

$$+ (30\sin 60^\circ\mathbf{j}) + (-45\mathbf{j})$$

$$= -(30\text{ lb})\mathbf{j} + (15.9808\text{ lb})\mathbf{j}$$

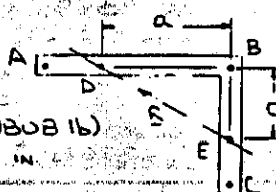
$$\text{OR} \quad R = 34.0\text{ lb} \angle 28.0^\circ$$

(CONTINUED)

### 3.156 CONTINUED

(b) FIRST REDUCE THE GIVEN FORCES AND COUPLE TO AN EQUIVALENT FORCE-COUPLE SYSTEM  $(R, M_B)$  AT B. HAVE

$$\Sigma M_B: M_B = (54 \text{ lb} \cdot \text{in.}) + (12 \text{ in.})(10 \text{ lb}) - (8 \text{ in.})(45 \text{ lb}) = -186 \text{ lb} \cdot \text{in.}$$



THEN WITH  $R$  AT D:

$$\Sigma M_B: -186 \text{ lb} \cdot \text{in.} = a(15.9308 \text{ lb})$$

$$\text{OR } a = 11.64 \text{ in.}$$

AND WITH  $R$  AT E:

$$\Sigma M_B: -186 \text{ lb} \cdot \text{in.} = c(30 \text{ lb})$$

$$\text{OR } c = 6.2 \text{ in.}$$

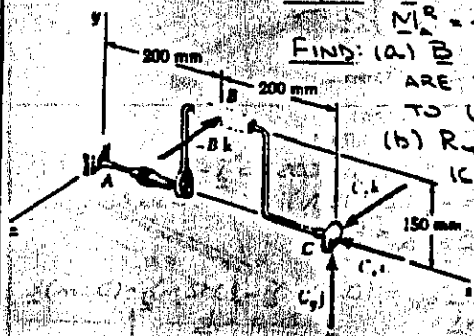
$\therefore$  THE LINE OF ACTION OF  $R$  INTERSECTS LINE AB 11.64 IN. TO THE LEFT OF B AND INTERSECTS LINE BC 6.2 IN. BELOW B.

### 3.157

GIVEN:  $K = (130 \text{ N})_i + R_y j + R_z k$   
 $M_A^R = -(12 \text{ N} \cdot \text{m})_i$

FIND: (a)  $B$  AND  $C$  WHICH ARE EQUIVALENT TO  $(R, M_A^R)$   
 (b)  $R_x$  AND  $R_z$

(c) ORIENTATION OF SLOT TO MINIMIZE SLIDING



(a) EQUIVALENCE REQUIRES...

$$\Sigma F: R = B + C$$

EQUATING THE  $i$  COEFFICIENTS...

$$i: -30 \text{ N} = -C_x \quad \text{OR } C_x = 30 \text{ N}$$

$$\text{ALSO } \Sigma M_A: M_A^R = r_{AB} \times B + r_{AC} \times C$$

$$\text{OR } -(12 \text{ N} \cdot \text{m})_i = [(0.2 \text{ m})_i + (0.15 \text{ m})_j] \times (-B)_k + [(0.4 \text{ m})_i + (-0.4 \text{ m})_j] \times [(-40 \text{ N})_i + C_y j + C_z k]$$

EQUATING COEFFICIENTS...

$$i: -12 \text{ N} \cdot \text{m} = -(0.15 \text{ m})B \quad \text{OR } B = 80 \text{ N}$$

$$j: 0 = (0.4 \text{ m})C_y \quad \text{OR } C_y = 0$$

$$k: 0 = (0.2 \text{ m})(80 \text{ N}) - (0.4 \text{ m})C_z \quad \text{OR } C_z = 40 \text{ N}$$

$$\therefore B = (80 \text{ N})_k \quad C = (-30 \text{ N})_i + (40 \text{ N})_k$$

(b) NOW HAVE FOR THE EQUIVALENCE OF FORCES...

$$-(30 \text{ N})_i + R_y j + R_z k = (80 \text{ N})_k + [(-30 \text{ N})_i + (40 \text{ N})_k]$$

EQUATING COEFFICIENTS...

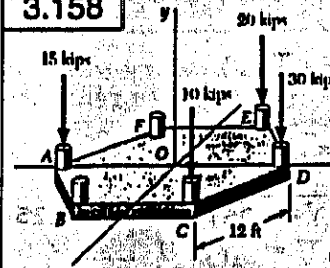
$$i: R_y = 0 \quad \text{OR } R_y = 0$$

$$k: R_z = -80 + 40 \quad \text{OR } R_z = -40 \text{ N}$$

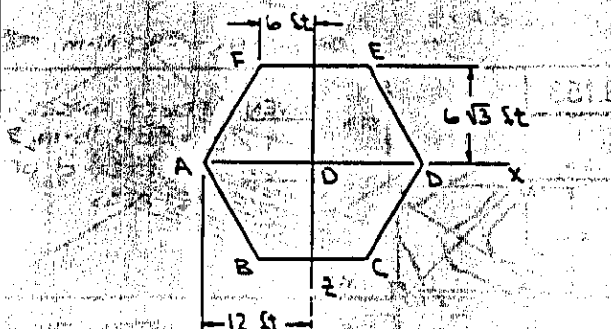
(c) FIRST NOTE THAT  $R = -(30 \text{ N})_i - (40 \text{ N})_k$ . THUS, THE SCREW IS BEST ABLE TO RESIST THE LATERAL FORCE  $R_z$  WHEN THE SLOT IN THE HEAD OF THE SCREW IS VERTICAL.

### 3.158

GIVEN: RESULTANT  $R$  PASSES THROUGH POINT O  
 FIND: VERTICAL FORCES  $E_B$  AND  $E_C$



FROM THE STATEMENT OF THE PROBLEM IT CAN BE CONCLUDED THAT THE SIX APPLIED LOADS ARE EQUIVALENT TO THE RESULTANT  $R$  AT O. IT THEN FOLLOWS THAT  $\Sigma M_O = 0$  OR  $\Sigma M_x = 0, \Sigma M_z = 0$  FOR THE APPLIED LOADS.



$$\Sigma M_x = 0: (6\sqrt{3} \text{ ft})F_B + (6\sqrt{3} \text{ ft})(10 \text{ kips}) - (6\sqrt{3} \text{ ft})(20 \text{ kips}) - (6\sqrt{3} \text{ ft})F_C = 0$$

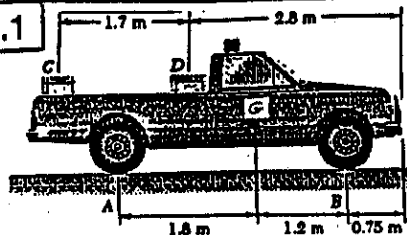
$$\text{OR } F_B - F_C = 10 \quad (1)$$

$$\Sigma M_z = 0: (12 \text{ ft})(15 \text{ kips}) + (6 \text{ ft})F_B - (6 \text{ ft})(10 \text{ kips}) - (12 \text{ ft})(30 \text{ kips}) - (6 \text{ ft})(20 \text{ kips}) - (6 \text{ ft})F_C = 0$$

$$\text{OR } F_B + F_C = 60 \quad (2)$$

$$\text{THEN } (1) + (2) \Rightarrow F_B = 35 \text{ kips} \quad \text{AND} \quad F_C = 25 \text{ kips}$$

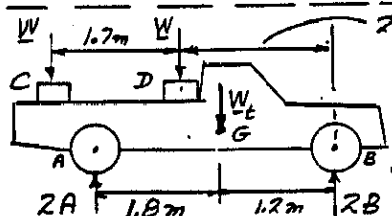
4.1



GIVEN:

TRUCK, 1400 kg  
CRATES,  
350 kg (EACH)

FIND:  
REACTIONS AT  
EACH WHEEL



$$2.8 - 0.75 = 2.05 \text{ m}$$

FREE BODY  
DIAGRAM

$$W = (350 \text{ kg})(9.81 \text{ m/s}^2) = 3.434 \text{ kN}$$

$$W_c = (1400 \text{ kg})(9.81 \text{ m/s}^2) = 13.734 \text{ kN}$$

(a) REAR WHEELS,  $\sum M_B = 0$

$$W(1.7 \text{ m} + 2.05 \text{ m}) + W_c(2.05 \text{ m}) + W_c(1.2 \text{ m}) - 2A(3 \text{ m}) = 0$$

$$(3.434 \text{ kN})(3.75 \text{ m}) + (3.434 \text{ kN})(2.05 \text{ m}) + (13.734 \text{ kN})(1.2 \text{ m}) - 2A(3 \text{ m}) = 0$$

$$A = +6.066 \text{ kN} \quad A = 6.07 \text{ kN} \uparrow$$

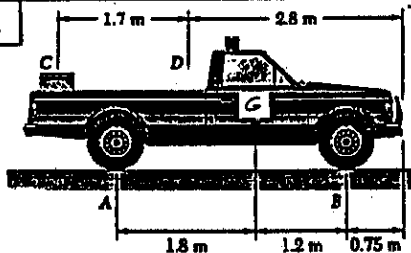
(b) FRONT WHEELS,  $\sum F_y = 0$

$$-W - W_c - W_c + 2A + 2B = 0$$

$$-3.434 \text{ kN} - 3.434 \text{ kN} - 13.734 \text{ kN} + 2(6.066 \text{ kN}) + 2B = 0$$

$$B = +4.235 \text{ kN} \quad B = 4.24 \text{ kN} \uparrow$$

4.2

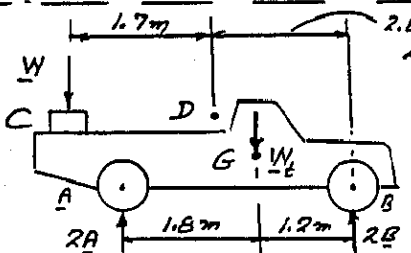


GIVEN:

TRUCK: 1400 kg  
CRATE C:  
350 kg

(CRATE D HAS  
BEEN REMOVED)

FIND: REACTIONS  
AT EACH WHEEL



$$2.8 \text{ m} - 0.75 \text{ m} = 2.05 \text{ m}$$

FREE-BODY  
DIAGRAM

$$W = (350 \text{ kg})(9.81 \text{ m/s}^2) = 3.434 \text{ kN}$$

$$W_c = (1400 \text{ kg})(9.81 \text{ m/s}^2) = 13.734 \text{ kN}$$

(a) REAR WHEELS,  $\sum M_B = 0$

$$W(1.7 \text{ m} + 2.05 \text{ m}) + W_c(1.2 \text{ m}) - 2A(3 \text{ m}) = 0$$

$$(3.434 \text{ kN})(3.75 \text{ m}) + (13.734 \text{ kN})(1.2 \text{ m}) - 2A(3 \text{ m}) = 0$$

$$A = +4.893 \text{ kN} \quad A = 4.89 \text{ kN} \uparrow$$

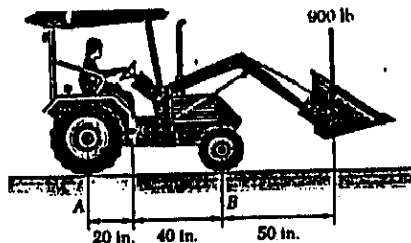
(b) FRONT WHEELS,  $\sum F_y = 0$

$$-W - W_c + 2A + 2B = 0$$

$$-3.434 \text{ kN} - 13.734 \text{ kN} + 2(4.893 \text{ kN}) + 2B = 0$$

$$B = +3.691 \text{ kN} \quad B = 3.69 \text{ kN} \uparrow$$

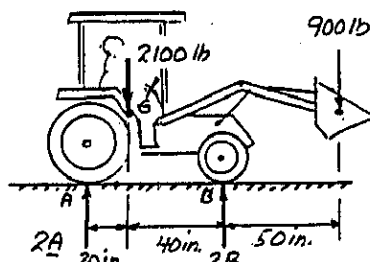
4.3



GIVEN:

2100-lb  
TRACTOR

FIND:  
REACTIONS AT  
EACH WHEEL



(a) REAR WHEELS,  $\sum M_B = 0$

$$+(2100 \text{ lb})(40 \text{ in}) - (900 \text{ lb})(50 \text{ in}) + 2A(60 \text{ in}) = 0$$

$$A = +325 \text{ lb} \quad A = 325 \text{ lb} \uparrow$$

(b) FRONT WHEELS,  $\sum M_A = 0$

$$(2100 \text{ lb})(20 \text{ in}) - (900 \text{ lb})(110 \text{ in}) - 2B(60 \text{ in}) = 0$$

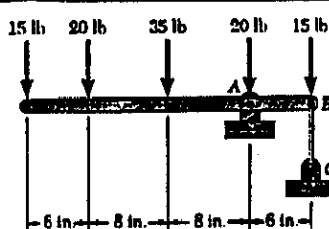
$$B = +1175 \text{ lb} \quad B = 1175 \text{ lb} \uparrow$$

CHECK:  $\sum F_y = 0$ ,  $2A + 2B - 2100 \text{ lb} - 900 \text{ lb} = 0$

$$2(325 \text{ lb}) + 2(1175 \text{ lb}) - 2100 \text{ lb} - 900 \text{ lb} = 0$$

$$0 = 0 \quad (\text{CHECKS})$$

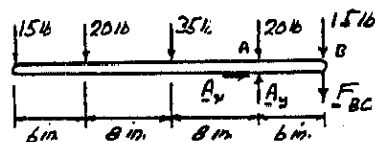
4.4



FIND:

(a) REACTION AT A  
(b) TENSION IN  
CABLE BC

FREE-BODY DIAGRAM



(a) REACTION AT A:  $\sum F_x = 0$ ,  $A_x = 0$

$\sum M_B = 0$ :

$$(15 \text{ lb})(28 \text{ in}) + (20 \text{ lb})(22 \text{ in}) + (35 \text{ lb})(14 \text{ in}) + (20 \text{ lb})(6 \text{ in}) - A_y(6 \text{ in}) = 0$$

$$A_y = +245 \text{ lb}$$

$$A = 245 \text{ lb} \uparrow$$

(b) TENSION IN BC

$\sum M_A = 0$

$$(15 \text{ lb})(22 \text{ in}) + (20 \text{ lb})(16 \text{ in}) + (35 \text{ lb})(8 \text{ in}) - (15 \text{ lb})(6 \text{ in}) - F_{BC}(6 \text{ in}) = 0$$

$$F_{BC} = +140 \text{ lb}$$

$$F_{BC} = 140 \text{ lb}$$

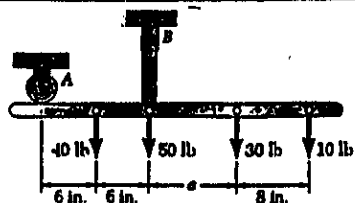
CHECK:  $\sum F_y = 0$ :

$$-15 \text{ lb} - 20 \text{ lb} - 35 \text{ lb} - 15 \text{ lb} + A - F_{BC} = 0$$

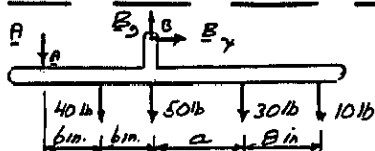
$$-105 \text{ lb} + 245 \text{ lb} - 140 \text{ lb} = 0$$

$$0 = 0 \quad (\text{CHECKS})$$

4.5



FIND:  
REACTIONS  
AT A AND B



FREE-BODY  
DIAGRAM

$$\sum F_x = 0: B_x = 0$$

$$+\sum M_B = 0: (40\text{ lb})(6\text{ in}) - (30\text{ lb})a - (10\text{ lb})(a + 8\text{ in}) + (12\text{ in})A = 0$$

$$A = (40a - 160)/12 \quad (1)$$

$$+\sum M_A = 0:$$

$$-(40\text{ lb})(6\text{ in}) - (50\text{ lb})(12\text{ in}) - (30\text{ lb})(a + 12\text{ in}) - (10\text{ lb})(a + 20\text{ in}) + (12\text{ in})B_y = 0$$

$$B_y = (1400 + 40a)/12$$

$$\text{Since } B_x = 0, \quad B = (1400 + 40a)/12 \quad (2)$$

(a) For  $a = 10\text{ in}$ :

$$\text{Eq. (1): } A = (40 \times 10 - 160)/12 = +20\text{ lb} \quad A = 20\text{ lb} \uparrow$$

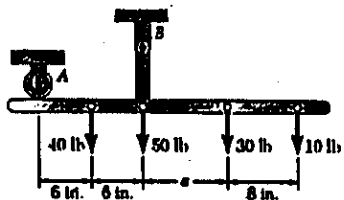
$$\text{Eq. (2): } B = (1400 + 40 \times 10)/12 = +150\text{ lb} \quad B = 150\text{ lb} \uparrow$$

(b) For  $a = 7\text{ in}$ :

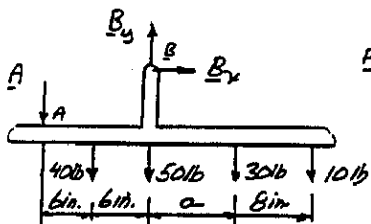
$$\text{Eq. (1): } A = (40 \times 7 - 160)/12 = +10\text{ lb} \quad A = 10\text{ lb} \uparrow$$

$$\text{Eq. (2): } B = (1400 + 40 \times 7)/12 = +140\text{ lb} \quad B = 140\text{ lb} \uparrow$$

4.6



FIND:  
SMALLEST  
DISTANCE  $a$   
FOR NO MOTION



FREE-BODY  
DIAGRAM

FOR NO MOTION REACTION AT A  
MUST BE DOWNWARD OR ZERO  
SMALLEST DISTANCE  $a$  FOR NO MOTION  
CORRESPONDS TO  $A = 0$

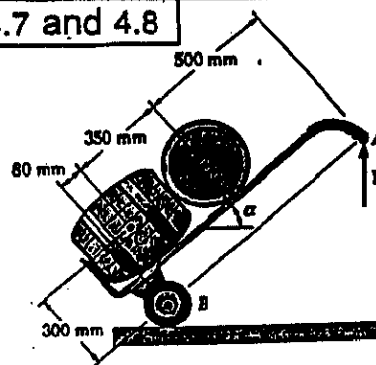
$$+\sum M_B = 0$$

$$(40\text{ lb})(6\text{ in}) - (30\text{ lb})a - (10\text{ lb})(a + 8\text{ in}) + (12\text{ in})A = 0$$

$$A = (40a - 160)/12$$

$$A = 0: (40a - 160) = 0 \quad a = 4\text{ in.}$$

4.7 and 4.8

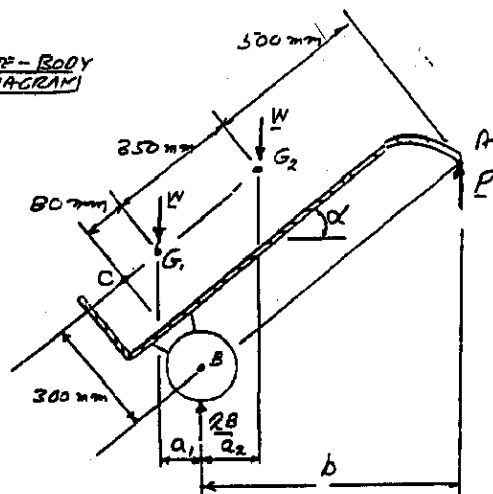


MASS OF EACH  
WHEEL IS 40 kg

FIND:  
(a)  $P$   
(b) REACTION AT  
EACH WHEEL

$$\text{FOR EACH WHEEL: } W = mg = (40\text{ kg})(9.81\text{ m/s}^2) = 392.4\text{ N}$$

FREE-BODY  
DIAGRAM



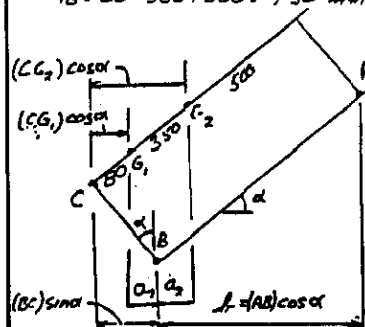
$$+\sum M_B = 0: W a_1 - W a_2 + P b = 0 \quad P = W(a_2 - a_1)/b \quad (1)$$

$$+\sum F_y = 0: -W - W + P + 2B_y = 0 \quad B_y = W - \frac{1}{2}P \quad (2)$$

GEOMETRY FIND  $a_1$  AND  $a_2$  IN TERMS OF  $\alpha$

$$BC = 300\text{ mm}, CG_1 = 80\text{ mm}, CG_2 = 80 + 300 = 430\text{ mm}$$

$$AB = 80 + 350 + 500 = 930\text{ mm}$$



$$a_1 = (BC) \sin \alpha - (CG_1) \cos \alpha$$

$$a_1 = 300 \sin \alpha - 80 \cos \alpha$$

$$a_2 = (CG_2) \cos \alpha - (BC) \sin \alpha$$

$$= 430 \cos \alpha - 300 \sin \alpha$$

$$b = (AB) \cos \alpha = 930 \cos \alpha$$

$$\text{Eq. (1): } P = W(a_2 - a_1)/b$$

$$P = W[(430 \cos \alpha - 300 \sin \alpha) - (300 \sin \alpha - 80 \cos \alpha)] / 930 \cos \alpha$$

$$= (392.4\text{ N})(510 \cos \alpha - 600 \sin \alpha) / 930 \cos \alpha$$

$$P = (392.4)(0.5484 - 0.6452 \tan \alpha)$$

$$\text{PROB. 4.7 } \alpha = 35^\circ;$$

$$P = 392.4(0.5484 - 0.6452 \tan 35^\circ) = +37.9\text{ N} \quad P = 37.9\text{ N} \uparrow$$

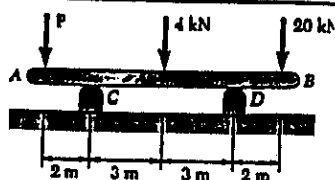
$$\text{Eq. (2): } B_y = W - \frac{1}{2}P = 392.4\text{ N} - \frac{1}{2}(37.9\text{ N}) = +374\text{ N} \quad B_y = 374\text{ N} \uparrow$$

$$\text{PROB. 4.8 } \alpha = 40^\circ;$$

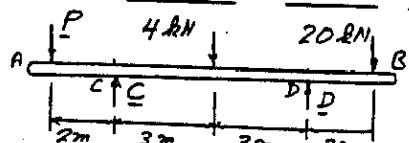
$$P = 392.4(0.5484 - 0.6452 \tan 40^\circ) = +2.76\text{ N} \quad P = 2.76\text{ N} \uparrow$$

$$\text{Eq. (2): } B_y = W - \frac{1}{2}P = 392.4\text{ N} - \frac{1}{2}(2.76\text{ N}) = +391\text{ N} \quad B_y = 391\text{ N} \uparrow$$

4.9



FIND:  
RANGE OF  
VALUES OF  $P$   
FOR  
EQUILIBRIUM



FREE-BODY  
DIAGRAM

$$+\circlearrowleft \Sigma M_C = 0: P(2m) - (4kN)(3m) - (20kN)(5m) + D(6m) = 0$$

$$P = 86kN - 3D \quad (1)$$

$$+\circlearrowleft \Sigma M_D = 0: P(8m) + (4kN)(3m) - (20kN)(2m) - C(6m) = 0$$

$$P = 3.5kN + 0.75C \quad (2)$$

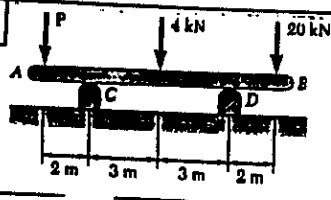
FOR NO MOTION  $C \geq 0$  AND  $D \geq 0$

FOR  $C \geq 0$  FROM (2)  $P \leq 3.5kN$

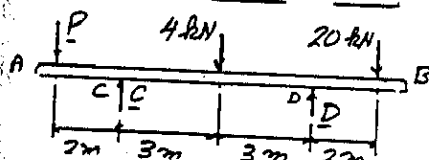
FOR  $D \geq 0$  FROM (1)  $P \leq 86kN$

RANGE OF  $P$  FOR NO MOTION:  $3.5kN \leq P \leq 86kN$

4.10



FIND: RANGE OF  
VALUES OF  $P$  IF  
REACTIONS MUST  
BE  $\leq 50kN$  AND  
BE DIRECTED  
UPWARD



FREE-BODY  
DIAGRAM

$$+\circlearrowleft \Sigma M_C = 0: P(2m) - (4kN)(3m) - (20kN)(5m) + D(6m) = 0$$

$$P = 86kN - 3D \quad (1)$$

$$+\circlearrowleft \Sigma M_D = 0: P(8m) + (4kN)(3m) - (20kN)(2m) - C(6m) = 0$$

$$P = 3.5kN + 0.75C \quad (2)$$

FOR  $C \geq 0$ , FROM (2):  $P \geq 3.5kN$

FOR  $D \geq 0$ , FROM (1):  $P \leq 86kN$

FOR  $C \leq 50kN$ , FROM (2):

$$P \leq 3.5kN + 0.75(50kN)$$

$$P \leq 41kN$$

FOR  $D \leq 50kN$ , FROM (1):

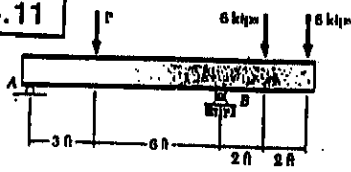
$$P \geq 86kN - 3(50kN)$$

$$P \geq -64kN$$

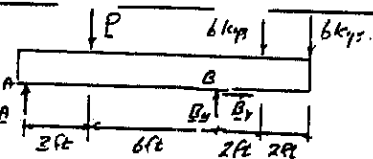
COMPARING THE FOUR CRITERIA, WE FIND

$$3.5kN \leq P \leq 41kN$$

4.11



FIND:  $P_{MAX}$  FOR  
REACTIONS  $< 30kips$   
AND REACTION  
AT A UPWARD



$$\Sigma F_x = 0: B_x = 0$$

$$\therefore B = B_y \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: -P(3ft) + B(9ft) - (6kips)(11ft) - (6kips)(13ft) = 0$$

$$P = 3B - 48kips \quad (1)$$

$$+\circlearrowleft \Sigma M_B = 0: -A(9ft) + P(6ft) - (6kips)(2ft) - (6kips)(4ft) = 0$$

$$P = 1.5A + 6kips \quad (2)$$

SINCE  $B \leq 30kips$ , EQ. (1) YIELDS

$$P \leq (3)(30kips) - 48kips$$

$$P \leq 42kips \quad \triangleleft$$

SINCE  $0 \leq A \leq 30kips$ , EQ. (2) YIELDS

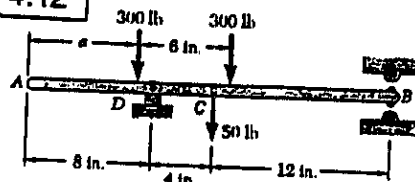
$$0 + 6kips \leq P \leq (1.5)(30kips) + 6kips$$

$$6kips \leq P \leq 51kips$$

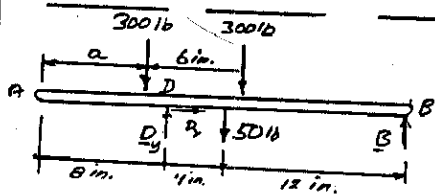
RANGE OF VALUES OF  $P$  FOR WHICH BEAM WILL BE SAFE:

$$6kips \leq P \leq 42kips \quad \triangleleft$$

4.12

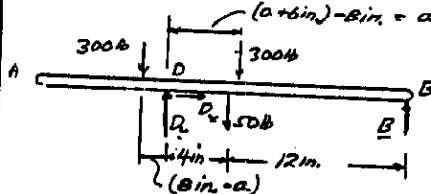


FIND: RANGE OF  
DISTANCE  $a$   
FOR WHICH  
 $B \leq 100lb \downarrow$  AND  
 $B \leq 200lb \uparrow$



ASSUME  $\uparrow$  IS  
POSITIVE WHEN  
DIRECTED  $\uparrow$

SKETCH SHOWING DISTANCE FROM  $D$  TO FORCES.



$$+\circlearrowleft \Sigma M_D = 0$$

$$(300lb)(8in - a) - (300lb)(a - 2in) - (50lb)(4in) - 16B = 0$$

$$-600a + 2800 + 16B = 0$$

$$a = (2800 + 16B)/600 \quad (1)$$

FOR  $B = 100lb \downarrow = -100lb$ , EQ. (1) YIELDS:

$$a \geq [2800 + 16(-100)]/600 = \frac{1200}{600} = 2in$$

$$a \geq 2in \quad \triangleleft$$

FOR  $B = 200lb \uparrow = 200lb$ , EQ. (1) YIELDS:

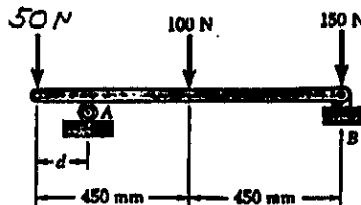
$$a \leq [2800 + 16(200)]/600 = \frac{6000}{600} = 10in$$

$$a \leq 10in \quad \triangleleft$$

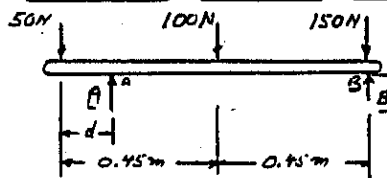
REQUIRED RANGE:  $2in \leq a \leq 10in \quad \triangleleft$



4.13



FIND: RANGE OF DISTANCE  $d$  FOR WHICH THE REACTIONS ARE  $\leq 180\text{N}$



$$\sum F_x = 0: B_x = 0 \\ \therefore B = B_y$$

$$+\circlearrowleft \sum M_A = 0:$$

$$(50\text{N})d - (100\text{N})(0.45\text{m} - d) - (150\text{N})(0.9\text{m} - d) + B(0.9\text{m} - d) = 0$$

$$50d - 45 + 100d - 135 + 150d + 0.9B - Bd = 0$$

$$d = \frac{180\text{N}\cdot\text{m} - (0.9\text{m})B}{300\text{N} - B} \quad (1)$$

$$+\circlearrowleft \sum M_B = 0:$$

$$(50\text{N})(0.9\text{m}) - A(0.9\text{m} - d) + (100\text{N})(0.45\text{m}) = 0$$

$$45 - 0.9A + Ad + 45 = 0$$

$$d = \frac{(0.9\text{m})A - 90\text{N}\cdot\text{m}}{A} \quad (2)$$

SINCE  $B \leq 180\text{N}$ , EQ. (1) YIELDS.

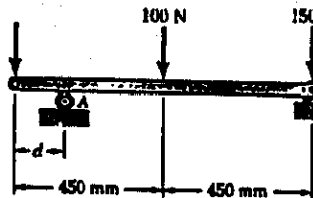
$$d \geq \frac{180 - (0.9)(180)}{300 - 180} = \frac{18}{120} = 0.15\text{m} \quad d \geq 150\text{mm}$$

SINCE  $A \leq 180\text{N}$ , EQ. (2) YIELDS.

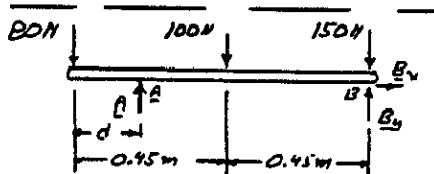
$$d \leq \frac{(0.9)(180) - 90}{180} = \frac{72}{180} = 0.40\text{m} \quad d \leq 400\text{mm}$$

$$\text{RANGE: } 150\text{mm} \leq d \leq 400\text{mm}$$

4.14



FIND: RANGE OF DISTANCE  $d$  FOR WHICH REACTIONS ARE  $\leq 180\text{N}$



$$\sum F_x = 0: B_x = 0 \\ \therefore B = B_y$$

$$+\circlearrowleft \sum M_A = 0: (180\text{N})d - (100\text{N})(0.45\text{m} - d) - (150\text{N})(0.9\text{m} - d) + B(0.9\text{m} - d) = 0$$

$$180d - 45 + 100d - 135 + 150d + 0.9B - Bd = 0$$

$$d = \frac{180\text{N}\cdot\text{m} - 0.9B}{330\text{N} - B} \quad (1)$$

$$+\circlearrowleft \sum M_B = 0: (180\text{N})(0.9\text{m}) - A(0.9\text{m} - d) + (100\text{N})(0.45\text{m}) = 0$$

$$d = \frac{0.9A - 117}{A} \quad (2)$$

SINCE  $B \leq 180\text{N}$ , EQ. (1) YIELDS.

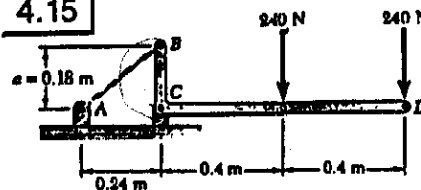
$$d \geq \frac{(180 - 0.9)(180)}{(330 - 180)} = \frac{18}{150} = 0.12\text{m} \quad d \geq 120\text{mm}$$

SINCE  $A \leq 180\text{N}$ , EQ. (2) YIELDS.

$$d \leq \frac{(0.9)(180) - 117}{180} = \frac{45}{180} = 0.25\text{m} \quad d \leq 250\text{mm}$$

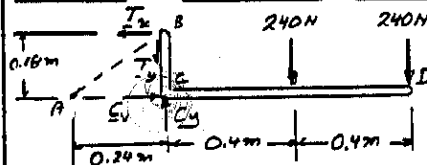
$$\text{RANGE: } 120\text{mm} \leq d \leq 250\text{mm}$$

4.15



FIND:

- (a) TENSION IN AB  
(b) REACTION AT C



AT B:

$$\frac{T_x}{T_y} = \frac{0.18\text{m}}{0.24\text{m}}$$

$$T_y = \frac{3}{4} T_x \quad (1)$$

$$+\circlearrowleft \sum M_C = 0: T_x(0.18\text{m}) - (240\text{N})(0.4\text{m}) - (240\text{N})(0.8\text{m}) = 0$$

$$T_x = +1600\text{N}$$

$$\text{EQ. (1)} \quad T_y = \frac{3}{4}(1600\text{N}) = 1200\text{N}$$

$$T = \sqrt{T_x^2 + T_y^2} = \sqrt{1600^2 + 1200^2} = 2000\text{N}$$

$$T = 2.0\text{kN}$$

$$\pm \sum F_x = 0: C_x - T_x = 0$$

$$C_x - 1600\text{N} = 0 \quad C_x = +1600\text{N}$$

$$C_y = 1600\text{N} \rightarrow$$

$$+\uparrow \sum F_y = 0: C_y - T_y - 240\text{N} - 240\text{N} = 0$$

$$C_y - 1200\text{N} - 480\text{N} = 0$$

$$C_y = +1680\text{N}$$

$$C_y = 1680\text{N} \uparrow$$

$$C_x = 1600\text{N}$$

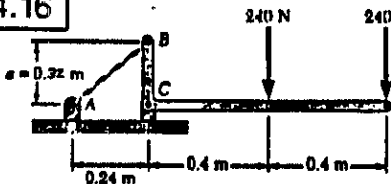
$$\alpha = 46.4^\circ$$

$$C = 2230\text{N}$$

$$C_x = 1600\text{N}$$

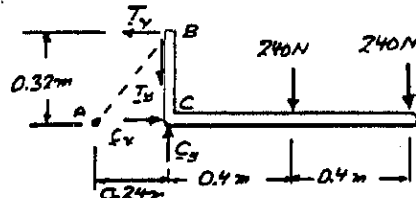
$$C = 2.23\text{kN} \angle 46.4^\circ$$

4.16



FIND:

- (a) TENSION IN AB  
(b) REACTION AT C



AT B:

$$\frac{T_y}{T_x} = \frac{0.32\text{m}}{0.24\text{m}}$$

$$T_y = \frac{4}{3} T_x$$

$$+\circlearrowleft \sum M_C = 0: T_x(0.32\text{m}) - (240\text{N})(0.4\text{m}) - (240\text{N})(0.8\text{m}) = 0$$

$$T_x = 900\text{N}$$

$$\text{EQ. (1)} \quad T_y = \frac{4}{3}(900\text{N}) = 1200\text{N}$$

$$T = \sqrt{T_x^2 + T_y^2} = \sqrt{900^2 + 1200^2} = 1500\text{N}$$

$$T = 1.5\text{kN}$$

$$\pm \sum F_x = 0: C_x - T_x = 0$$

$$C_x - 900\text{N} = 0 \quad C_x = +900\text{N}$$

$$C_x = 900\text{N} \rightarrow$$

$$+\uparrow \sum F_y = 0: C_y - T_y - 240\text{N} - 240\text{N} = 0$$

$$C_y - 1200\text{N} - 480\text{N} = 0$$

$$C_y = +1680\text{N}$$

$$C_y = 1680\text{N} \uparrow$$

$$C_y = 1680\text{N}$$

$$\alpha = 61.8^\circ$$

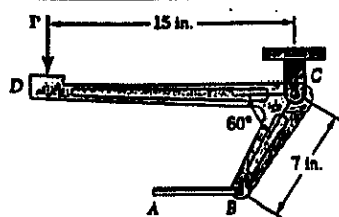
$$C = 1906\text{N}$$

$$C_x = 900\text{N}$$

$$C = 1.906\text{kN} \angle 61.8^\circ$$



4.17

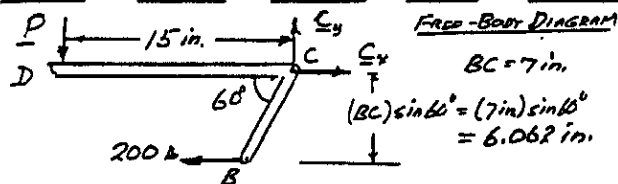


GIVEN:

$$T_{AB} = 200 \text{ lb}$$

FIND:

- (a) FORCE  $P$   
(b) REACTION AT C



FREE-BODY DIAGRAM

$$BC = 7 \text{ in.}$$

$$(BC) \sin 60^\circ = (7 \text{ in.}) \sin 60^\circ = 6.062 \text{ in.}$$

$$+\circlearrowleft \Sigma M_C = 0: P(15 \text{ in.}) - (200 \text{ lb})(6.062 \text{ in.}) = 0$$

$$P = 80.83 \text{ lb}$$

$$P = 80.83 \text{ lb} \downarrow$$

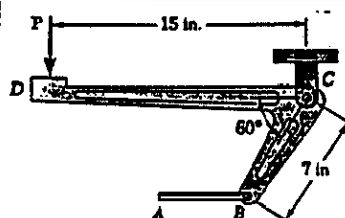
$$+\rightarrow \Sigma F_x = 0: C_x - 200 \text{ lb} = 0 \quad C_x = 200 \text{ lb} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: C_y - P = 0; C_y - 80.83 \text{ lb} = 0 \quad C_y = 80.83 \text{ lb} \uparrow$$

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(200 \text{ lb})^2 + (80.83 \text{ lb})^2} = 215.3 \text{ lb}$$

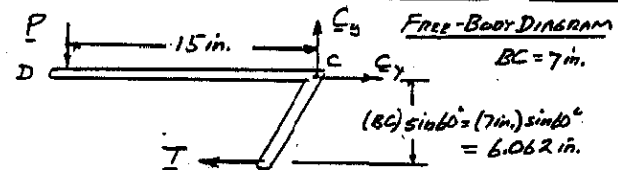
$$C = 215.3 \text{ lb} \angle 22.0^\circ$$

4.18



GIVEN: REACTION AT C = 250 lb

FIND: MAXIMUM ALLOWABLE TENSION IN AB



FREE-BODY DIAGRAM

$$BC = 7 \text{ in.}$$

$$(BC) \sin 60^\circ = (7 \text{ in.}) \sin 60^\circ = 6.062 \text{ in.}$$

$$+\circlearrowleft \Sigma M_C = 0: P(15 \text{ in.}) - T(6.062 \text{ in.}) = 0 \quad P = 0.40415 T$$

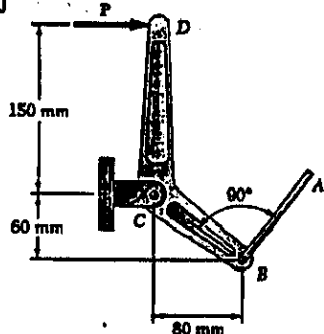
$$+\uparrow \Sigma F_y = 0: -P + C_y = 0; -0.40415 T + C_y = 0 \quad C_y = 0.40415 T$$

$$+\rightarrow \Sigma F_x = 0: -T + C_x = 0 \quad C_x = T$$

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{T^2 + (0.40415 T)^2} = 1.0786 T$$

$$\text{For } C = 250 \text{ lb, } 250 \text{ lb} = 1.0786 T \quad T = 232 \text{ lb}$$

4.19



GIVEN:

$$P = 400 \text{ N}$$

FIND:

- (a) TENSION IN ROD AB  
(b) REACTION AT C

4.19 CONTINUED

FREE-BODY DIAGRAM

$$BC = \sqrt{60^2 + 80^2} = 100 \text{ mm}$$

$$+\circlearrowleft \Sigma M_C = 0:$$

$$T(BC) - (400 \text{ N})(CD) = 0$$

$$T(100 \text{ mm}) - (400 \text{ N})(150 \text{ mm}) = 0$$

$$T = 600 \text{ N}$$

$$+\rightarrow \Sigma F_x = 0:$$

$$400 \text{ N} + C_x + \frac{3}{5} T = 0$$

$$400 \text{ N} + C_x + \frac{3}{5}(600 \text{ N}) = 0$$

$$C_x = -760 \text{ N} \quad C_x = 760 \text{ N} \leftarrow$$

$$+\uparrow \Sigma F_y = 0: C_y + \frac{4}{5} T = 0$$

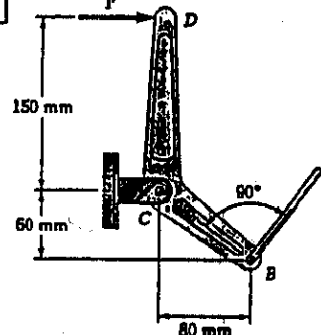
$$C_y + \frac{4}{5}(600 \text{ N}) = 0; C_y = -480 \text{ N} \quad C_y = 480 \text{ N} \downarrow$$

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(760 \text{ N})^2 + (480 \text{ N})^2} = 899.9 \text{ N}$$

$$\alpha = 32.28^\circ$$

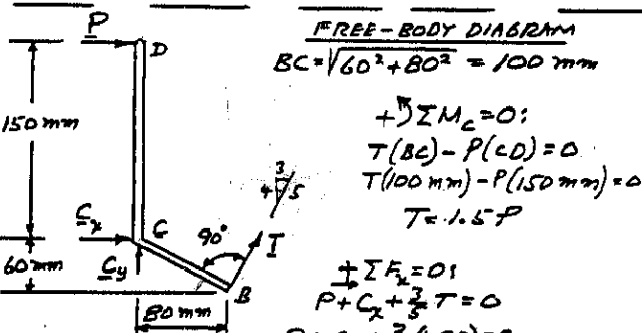
$$C = 899.9 \text{ N} \angle 32.3^\circ$$

4.20



GIVEN: REACTION AT C = 1000 N

FIND: MAXIMUM ALLOWABLE FORCE P



FREE-BODY DIAGRAM

$$BC = \sqrt{60^2 + 80^2} = 100 \text{ mm}$$

$$+\circlearrowleft \Sigma M_C = 0:$$

$$T(BC) - P(CD) = 0$$

$$T(100 \text{ mm}) - P(150 \text{ mm}) = 0$$

$$T = 1.5 P$$

$$+\rightarrow \Sigma F_x = 0:$$

$$P + C_x + \frac{3}{5} T = 0$$

$$P + C_x + \frac{3}{5}(1.5 P) = 0$$

$$C_x = -1.9 P \quad C_x = 1.9 P \leftarrow$$

$$+\uparrow \Sigma F_y = 0: C_y + \frac{4}{5} T = 0$$

$$C_y + \frac{4}{5}(1.5 P) = 0$$

$$C_y = -1.2 P \quad C_y = 1.2 P \downarrow$$

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(1.9 P)^2 + (1.2 P)^2} = 2.2472 P$$

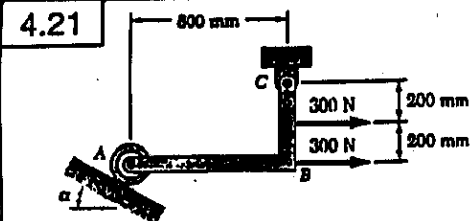
$$\text{For } C = 1000 \text{ N,}$$

$$1000 \text{ N} = 2.2472 P$$

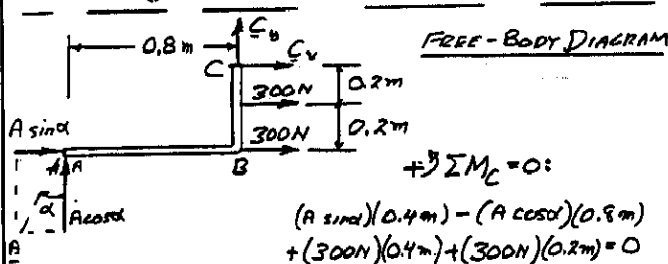
$$P = 444.99 \text{ N} \quad P = 445 \text{ lb}$$

(CONTINUED)

4.21



**FIND:**  
REACTIONS  
AT A AND C  
WHEN  
(a)  $\alpha = 0$   
(b)  $\alpha = 30^\circ$



$$+\circlearrowleft \Sigma M_C = 0:$$

$$(A \sin \alpha)(0.4 \text{ m}) - (A \cos \alpha)(0.8 \text{ m}) + (300 \text{ N})(0.4 \text{ m}) + (300 \text{ N})(0.2 \text{ m}) = 0$$

$$A = \frac{180}{0.8 \cos \alpha - 0.4 \sin \alpha} \quad (1)$$

$$+\rightarrow \Sigma F_x = 0: C_x + 300 \text{ N} + 300 \text{ N} + A \sin \alpha = 0$$

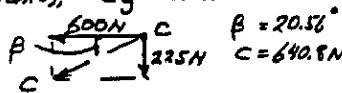
$$C_x = -600 - A \sin \alpha \quad (2)$$

$$+\uparrow \Sigma F_y = 0: C_y + A \cos \alpha = 0 \quad C_y = -A \cos \alpha \quad (3)$$

(a) WHEN  $\alpha = 0$ : EQ.(1),  $A = \frac{180}{0.8} = 225 \text{ N}$   $A = 225 \text{ N} \uparrow$

EQ.(2),  $C_x = -600 \text{ N}$

EQ.(3),  $C_y = -225 \text{ N}$



$$C = 640.8 \text{ N} \angle 20.6^\circ$$

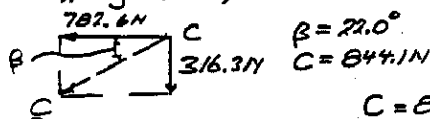
(b) WHEN  $\alpha = 30^\circ$ :

EQ.(1),  $A = \frac{180}{0.8 \cos 30^\circ - 0.4 \sin 30^\circ} = 365.2 \text{ N}$

$$A = 365.2 \text{ N} \angle 60^\circ$$

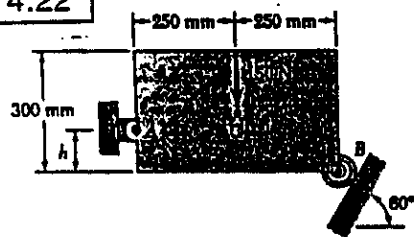
EQ.(2),  $C_x = -600 - (365.2) \sin 30^\circ = -782.6 \text{ N}$

EQ.(3),  $C_y = -(365.2) \cos 30^\circ = -316.3 \text{ N}$

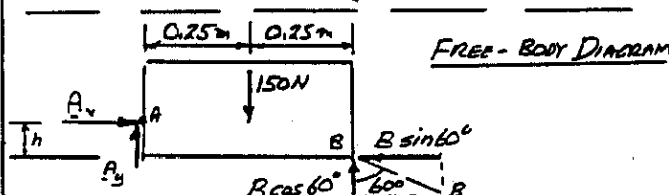


$$C = 844.1 \text{ N} \angle 22.0^\circ$$

4.22



**FIND:**  
REACTIONS  
AT A AND B  
WHEN  
(a)  $h = 0$   
(b)  $h = 200 \text{ mm}$



$$+\circlearrowleft \Sigma M_A = 0: (B \cos 60^\circ)(0.5 \text{ m}) - (B \sin 60^\circ)h - (150 \text{ N})(0.25 \text{ m}) = 0$$

$$B = \frac{37.5}{0.25 - 0.866 h} \quad (1)$$

(CONTINUED)

4.22 CONTINUED

(a) WHEN  $h = 0$ :

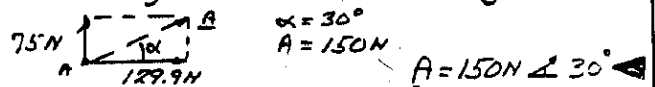
EQ.(1):  $B = \frac{37.5}{0.25} = 150 \text{ N}$   $B = 150 \text{ N} \angle 30^\circ$

$$+\rightarrow \Sigma F_x = 0: A_x - B \sin 60^\circ = 0$$

$$A_x = (150) \sin 60^\circ = 129.9 \text{ N} \quad A_x = 129.9 \text{ N} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: A_y - 150 + B \cos 60^\circ = 0$$

$$A_y = 150 - (150) \cos 60^\circ = 75 \text{ N} \quad A_y = 75 \text{ N} \uparrow$$



(b) WHEN  $h = 200 \text{ mm} = 0.2 \text{ m}$

EQ.(1):  $B = \frac{37.5}{0.25 - 0.866(0.2)} = 488.3 \text{ N}$

$$B = 488.3 \text{ N} \angle 30^\circ$$

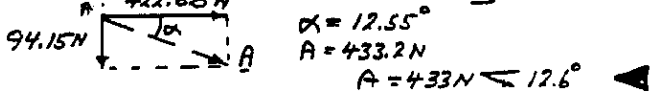
$$+\rightarrow \Sigma F_x = 0: A_x - B \sin 60^\circ = 0$$

$$A_x = (488.3) \sin 60^\circ = 422.88 \text{ N} \quad A_x = 422.88 \text{ N} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: A_y - 150 + B \cos 60^\circ = 0$$

$$A_y = 150 - (488.3) \cos 60^\circ = -94.15 \text{ N}$$

$$A_y = 94.15 \text{ N} \downarrow$$



$$A = 433.2 \text{ N} \angle 12.6^\circ$$

4.23

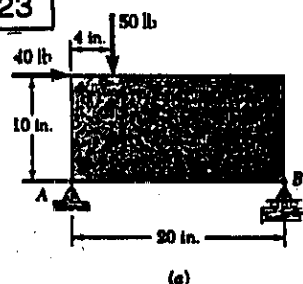
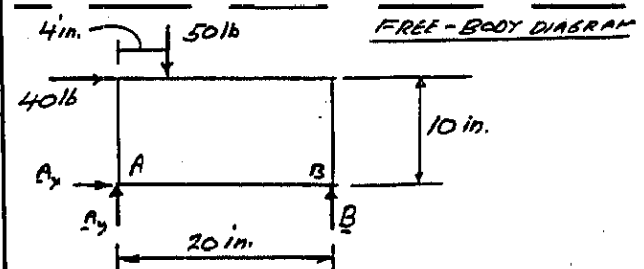


PLATE a

**FIND:**  
REACTIONS  
AT A AND B



$$+\circlearrowleft \Sigma M_A = 0:$$

$$B(20 \text{ in.}) - (50 \text{ lb})(16 \text{ in.}) - (40 \text{ lb})(10 \text{ in.}) = 0$$

$$B = +30 \text{ lb}$$

$$B = 30 \text{ lb} \uparrow$$

$$+\rightarrow \Sigma F_x = 0: A_x + 40 \text{ lb} = 0$$

$$A_x = -40 \text{ lb}$$

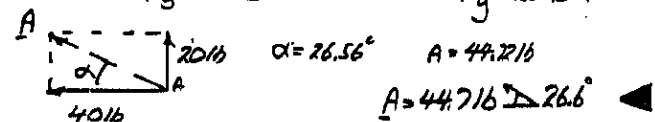
$$A_x = 40 \text{ lb} \leftarrow$$

$$+\uparrow \Sigma F_y = 0: A_y + B - 50 \text{ lb} = 0$$

$$A_y + 30 \text{ lb} - 50 \text{ lb} = 0$$

$$A_y = +20 \text{ lb}$$

$$A_y = 20 \text{ lb} \uparrow$$

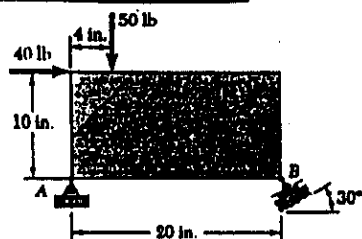


$$A = 44.72 \text{ lb} \angle 26.6^\circ$$

(CONTINUED)

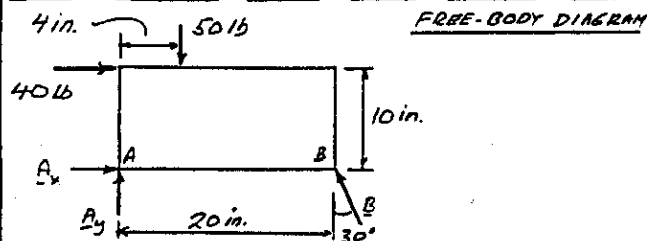
## 4.23 CONTINUED

PLATE b:



(b)

FIND:  
REACTIONS  
AT A AND B



FREE-BODY DIAGRAM

$$+\circlearrowleft \Sigma M_A = 0: (B \cos 30^\circ)(20 \text{ in}) - (40 \text{ lb})(10 \text{ in}) - (50 \text{ lb})(4 \text{ in}) = 0$$

$$B = 34.64 \text{ lb} \quad B = 34.6 \text{ lb} \nearrow 60^\circ$$

$$\pm \Sigma F_x = 0: A_x - B \sin 30^\circ + 40 \text{ lb} = 0$$

$$A_x - (34.64 \text{ lb}) \sin 30^\circ + 40 \text{ lb} = 0$$

$$A_x = -22.68 \text{ lb} \quad A_x = 22.68 \text{ lb} \leftarrow$$

$$+\uparrow \Sigma F_y = 0: A_y + B \cos 30^\circ - 50 \text{ lb} = 0$$

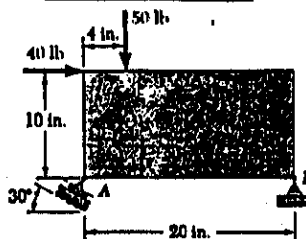
$$A_y + (34.64 \text{ lb}) \cos 30^\circ - 50 \text{ lb} = 0$$

$$A_y = +20 \text{ lb} \quad A_y = 20 \text{ lb} \uparrow$$

$$A = 22.68 \text{ lb} \quad \alpha = 41.4^\circ \quad A = 30.24 \text{ lb} \nearrow 41.4^\circ$$

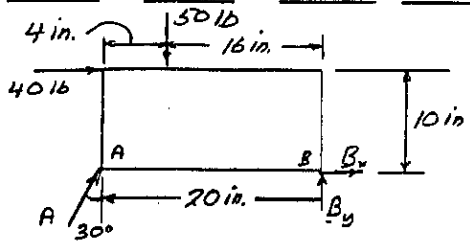
## 4.24 CONTINUED

PLATE b:



(b)

FIND:  
REACTIONS  
AT A AND B



$$+\circlearrowleft \Sigma M_B = 0: -(A \cos 30^\circ)(20 \text{ in}) - (40 \text{ lb})(10 \text{ in}) + (50 \text{ lb})(16 \text{ in}) = 0$$

$$A = 23.09 \text{ lb} \quad A = 23.1 \text{ lb} \nearrow 60^\circ$$

$$\pm \Sigma F_x = 0: A \sin 30^\circ + 40 \text{ lb} + B_x = 0$$

$$(23.09 \text{ lb}) \sin 30^\circ + 40 \text{ lb} + B_x = 0$$

$$B_x = -51.55 \text{ lb} \quad B_x = 51.55 \text{ lb} \leftarrow$$

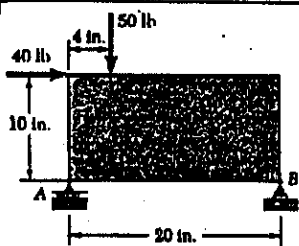
$$+\uparrow \Sigma F_y = 0: A \cos 30^\circ + B_y - 50 \text{ lb} = 0$$

$$(23.09 \text{ lb}) \cos 30^\circ + B_y - 50 \text{ lb} = 0$$

$$B_y = +30 \text{ lb} \quad B_y = 30 \text{ lb} \uparrow$$

$$B = 30 \text{ lb} \quad \alpha = 30.2^\circ \quad B = 59.6 \text{ lb} \nearrow 30.2^\circ$$

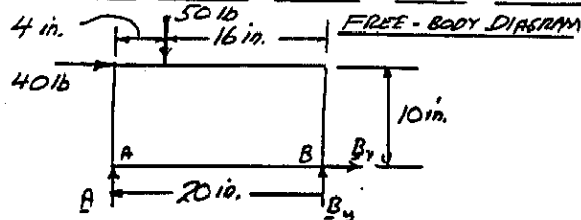
## 4.24



(a)

PLATE a:

FIND:  
REACTIONS  
AT A AND B



FREE-BODY DIAGRAM

$$+\circlearrowleft \Sigma M_B = 0: -A(20 \text{ in}) + (50 \text{ lb})(16 \text{ in}) - (40 \text{ lb})(10 \text{ in}) = 0$$

$$A = +20 \text{ lb}$$

$$A = 20 \text{ lb} \uparrow$$

$$\pm \Sigma F_x = 0: 40 \text{ lb} + B_x = 0$$

$$B_x = -40 \text{ lb}$$

$$B_x = 40 \text{ lb} \leftarrow$$

$$+\uparrow \Sigma F_y = 0: A + B_y - 50 \text{ lb} = 0$$

$$20 \text{ lb} + B_y - 50 \text{ lb} = 0$$

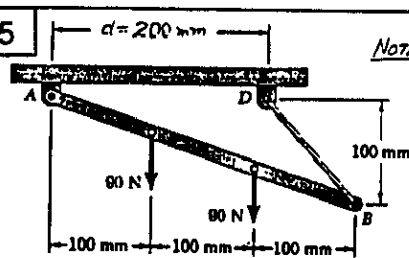
$$B_y = +30 \text{ lb}$$

$$B_y = 30 \text{ lb} \uparrow$$

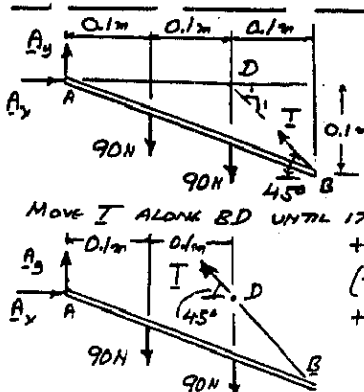
$$B = 50 \text{ lb} \quad \alpha = 36.87^\circ \quad B = 50 \text{ lb} \nearrow 36.9^\circ$$

(CONTINUED)

## 4.25

NOTE:  $d = 200 \text{ mm}$ 

FIND:  
(a) TENSION  
IN CABLE BD  
(b) REACTION  
AT A



FREE-BODY DIAGRAM

MOVE  $T$  ALONG  $BD$  UNTIL IT ACTS AT POINT  $D$ .

$$+\circlearrowleft \Sigma M_A = 0:$$

$$(T \sin 45^\circ)(0.2 \text{ m})$$

$$+ (90 \text{ N})(0.1 \text{ m}) + (90 \text{ N})(0.2 \text{ m}) = 0$$

$$T = 190.92 \text{ N}$$

$$T = 190.9 \text{ N}$$

$$\pm \Sigma F_x = 0: A_x - (190.92 \text{ N}) \cos 45^\circ = 0$$

$$A_x = +135 \text{ N}$$

$$A_x = 135 \text{ N} \rightarrow$$

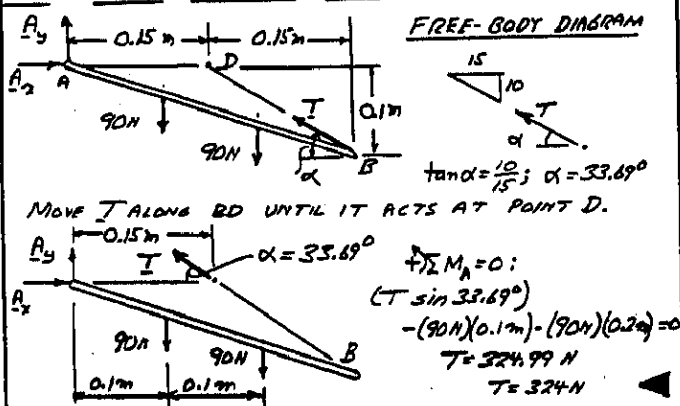
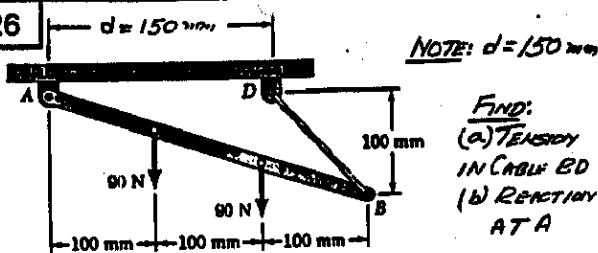
$$+\uparrow \Sigma F_y = 0: A_y - 90 \text{ N} - 90 \text{ N} + (190.92 \text{ N}) \sin 45^\circ = 0$$

$$A_y = +45 \text{ N}$$

$$A_y = 45 \text{ N} \uparrow$$

$$A = 142.3 \text{ N} \nearrow 18.4^\circ$$

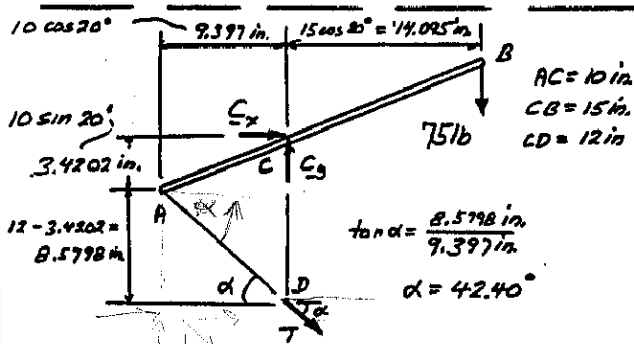
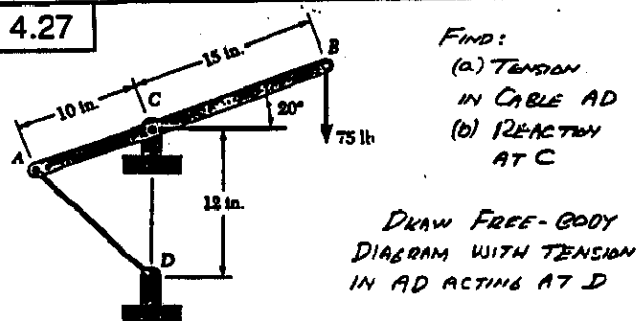
4.26



$$\begin{aligned} \pm \sum F_x = 0: A_x - (324.99 \text{ N}) \cos 33.69^\circ &= 0 \\ A_x &= +270 \text{ N} \quad A_x = 270 \text{ N} \rightarrow \\ + \sum F_y = 0: A_y - 90 \text{ N} - 90 \text{ N} + (324.99 \text{ N}) \sin 33.69^\circ &= 0 \\ A_y &= 0 \quad A_y = 0 \end{aligned}$$

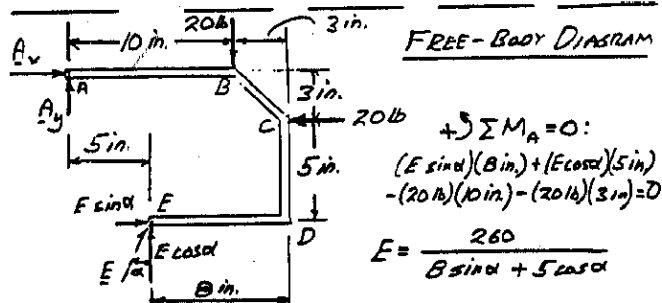
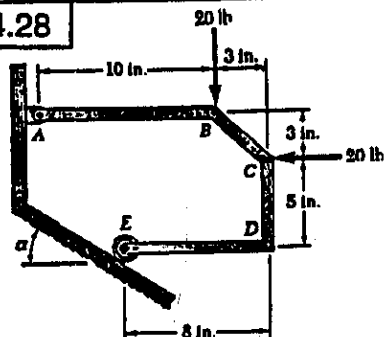
$A = 270 \text{ N} \rightarrow$

4.27



$$\begin{aligned} + \sum M_C = 0: (T \cos \alpha)(12 \text{ m}) - (75 \text{ lb})(14.095 \text{ m}) &= 0 \\ (T \cos 42.40^\circ)(12 \text{ m}) - (75 \text{ lb})(14.095 \text{ m}) &= 0 \\ T &= 119.29 \text{ lb} \quad T = 119.3 \text{ lb} \\ \pm \sum F_x = 0: C_x + (119.29 \text{ lb}) \cos 42.40^\circ &= 0 \\ C_x &= -88.094 \text{ lb} \quad C_x = 88.094 \text{ lb} \leftarrow \\ + \sum F_y = 0: C_y - 75 \text{ lb} - (119.29 \text{ lb}) \sin 42.40^\circ &= 0 \\ C_y &= +155.44 \text{ lb} \quad C_y = 155.44 \text{ lb} \uparrow \\ C &= 178.7 \text{ lb} \nearrow 60.5^\circ \end{aligned}$$

4.28



$$(a) \text{ WHEN } \alpha = 30^\circ: E = \frac{260}{8 \sin 30^\circ + 5 \cos 30^\circ} = 31.212 \text{ lb}$$

$E = 31.2 \text{ lb} \nearrow 60^\circ$

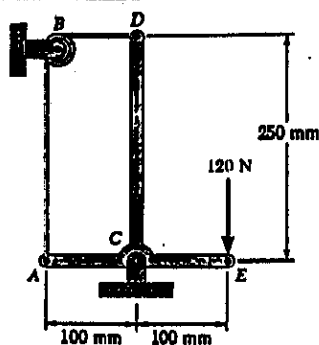
$$\begin{aligned} \pm \sum F_x = 0: A_x - 20 \text{ lb} + (31.212 \text{ lb}) \sin 30^\circ &= 0 \\ A_x &= +4.394 \text{ lb} \quad A_x = 4.394 \text{ lb} \rightarrow \\ + \sum F_y = 0: A_y - 20 + (31.212 \text{ lb}) \cos 30^\circ &= 0 \\ A_y &= -7.03 \text{ lb} \quad A_y = 7.03 \text{ lb} \downarrow \\ A &= 8.29 \text{ lb} \searrow 58.0^\circ \end{aligned}$$

$$(b) \text{ WHEN } \alpha = 45^\circ: E = \frac{260}{8 \sin 45^\circ + 5 \cos 45^\circ} = 28.28 \text{ lb}$$

$E = 28.3 \text{ lb} \nearrow 45^\circ$

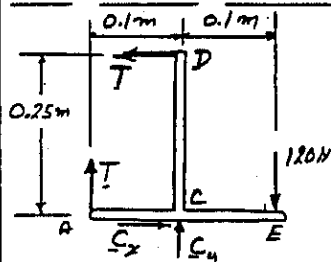
$$\begin{aligned} \pm \sum F_x = 0: A_x - 20 \text{ lb} + (28.28 \text{ lb}) \sin 45^\circ &= 0 \\ A_x &= 0 \quad A_x = 0 \\ + \sum F_y = 0: A_y - 20 \text{ lb} + (28.28 \text{ lb}) \cos 45^\circ &= 0 \\ A_y &= 0 \quad A_y = 0 \\ A &= 0 \end{aligned}$$

4.29



FIND:  
TENSION IN  
CABLE ABD.  
REACTION  
AT C.

LET  $T$  EQUAL  
TENSION IN CABLE



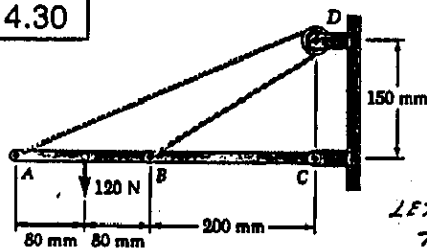
FREE-BODY DIAGRAM

$$\begin{aligned} +\sum M_C = 0 \\ T(0.25\text{ m}) - T(0.1\text{ m}) - (120\text{ N})(0.1\text{ m}) &= 0 \\ T &= 80\text{ N} \end{aligned}$$

$$\begin{aligned} +\sum F_x = 0: C_x - 80\text{ N} &= 0; C_x = +80\text{ N}; C_x = 80\text{ N} \rightarrow \\ +\sum F_y = 0: C_y - 120\text{ N} + 80\text{ N} &= 0; C_y = +40\text{ N}; C_y = 40\text{ N} \uparrow \end{aligned}$$

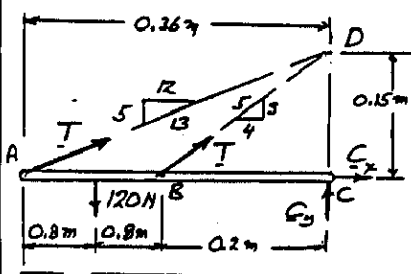
$$C = 89.4\text{ N} \angle 26.6^\circ$$

4.30

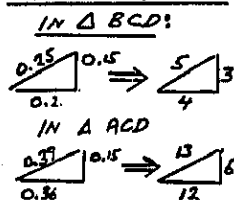


FIND:  
(a) TENSION IN  
CABLE ABD.  
(b) REACTION  
AT C.

LET  $T$  EQUAL  
TENSION IN CABLE.



FREE-BODY DIAGRAM



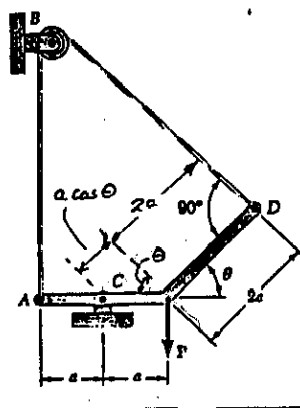
$$\begin{aligned} +\sum M_C = 0: (120\text{ N})(0.28\text{ m}) - \left(\frac{5}{13}T\right)(0.36\text{ m}) - \left(\frac{3}{5}T\right)(0.2\text{ m}) &= 0 \\ 33.6 - T(0.13846 + 0.12) &= 0 \\ T &= 130.00\text{ N} \end{aligned}$$

$$\begin{aligned} \pm\sum F_x = 0: C_x + \frac{12}{13}(130\text{ N}) + \frac{4}{5}(130\text{ N}) &= 0 \\ C_x &= -224\text{ N} \end{aligned}$$

$$\begin{aligned} +\sum F_y = 0: C_y - 120\text{ N} + \frac{5}{13}(130\text{ N}) + \frac{3}{5}(130\text{ N}) &= 0 \\ C_y &= -8.00\text{ N} \end{aligned}$$

$$\begin{aligned} C_x &= 224\text{ N} \\ C_y &= 8\text{ N} \\ C &= 224.14\text{ N} \\ \alpha &= 2.045^\circ \\ C &= 224\text{ N} \angle 2.0^\circ \end{aligned}$$

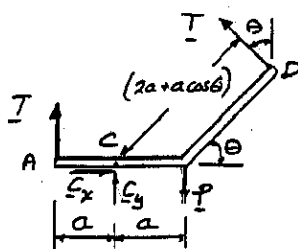
4.31 and 4.32



FIND:  
TENSION IN CABLE ABD  
REACTION AT C

WE NOTE THAT THE  
PERPENDICULAR DISTANCE  
FROM POINT C TO  
PORTION BD OF CABLE IS  
 $2a + a\cos\theta$

LET  $T$  EQUAL THE  
TENSION IN CABLE



$$\begin{aligned} +\sum M_C = 0: \\ T(2a + a\cos\theta) - Ta\sin\theta &= 0 \\ T &= \frac{P}{1 + \cos\theta} \end{aligned} \quad (1)$$

$$\begin{aligned} \pm\sum F_x = 0: C_x - T\sin\theta &= 0 \\ C_x &= T\sin\theta = \frac{P\sin\theta}{1 + \cos\theta} \end{aligned}$$

$$\begin{aligned} +\sum F_y = 0: C_y + T + T\cos\theta - P &= 0 \\ C_y &= P - T(1 + \cos\theta) = P - P\frac{1 + \cos\theta}{1 + \cos\theta}; C_y = 0 \end{aligned}$$

$$\text{SINCE } C_y = 0, C = C_x \quad C = P \frac{\sin\theta}{1 + \cos\theta} \quad (2)$$

FOR PROB 4.31  $\theta = 60^\circ$ :

$$\text{EQ(1): } T = \frac{P}{1 + \cos 60^\circ} = \frac{P}{1 + \frac{1}{2}} \quad T = \frac{2}{3}P$$

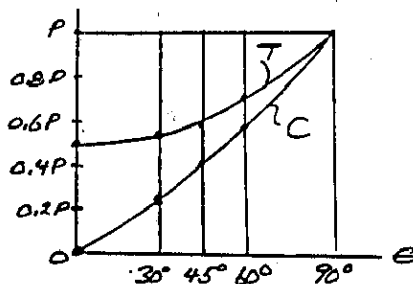
$$\text{EQ(2): } C = P \frac{\sin 60^\circ}{1 + \cos 60^\circ} = P \frac{0.866}{1 + \frac{1}{2}} \quad C = 0.577P$$

FOR PROB 4.32  $\theta = 45^\circ$ :

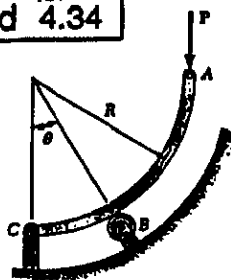
$$\text{EQ(1): } T = \frac{P}{1 + \cos 45^\circ} = \frac{P}{1.7071} \quad T = 0.588P$$

$$\text{EQ(2): } C = P \frac{\sin 45^\circ}{1 + \cos 45^\circ} = P \frac{0.7071}{1.7071} \quad C = 0.414P$$

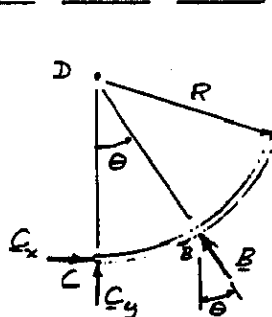
THE FOLLOWING IS A PLOT OF  
 $T$  AND  $C$  FOR  $0 \leq \theta \leq 90^\circ$



# 4.33 and 4.34



FIND:  
REACTION  
(a) AT B  
(b) AT C



FREE-BODY DIAGRAM

$$\begin{aligned} +\sum M_D = 0: \\ C_x(R) - P(R) = 0 \\ C_x = +P \end{aligned}$$

$$\begin{aligned} +\sum F_x = 0: C_x - B \sin \theta = 0 \\ P - B \sin \theta = 0 \\ B = P / \sin \theta \\ B = \frac{P}{\sin \theta} \quad \checkmark \end{aligned}$$

$$\begin{aligned} +\sum F_y = 0: C_y + B \cos \theta - P = 0 \\ C_y + (P / \sin \theta) \cos \theta - P = 0 \\ C_y = P(1 - \frac{1}{\tan \theta}) \end{aligned}$$

FOR PROB. 4.33  $\theta = 30^\circ$

$$(a) B = P / \sin 30^\circ = 2P \quad B = 2P \quad \Delta 60^\circ$$

$$(b) C_x = +P \quad C_x = P \rightarrow \\ C_y = P(1 - \frac{1}{\tan 30^\circ}) = -0.7321P \quad C_y = 0.7321P \downarrow$$

$$C_y = 0.7321P \quad C_x = P \quad C = 1.239P \quad \Delta 36.2^\circ$$

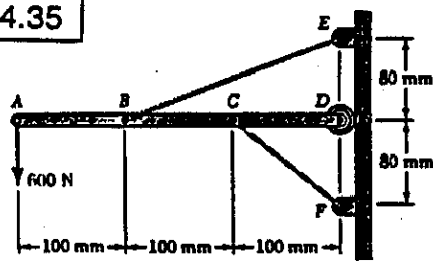
FOR PROB. 4.34  $\theta = 60^\circ$

$$(a) B = P / \sin 60^\circ = 1.1547P \quad B = 1.155P \quad \Delta 30^\circ$$

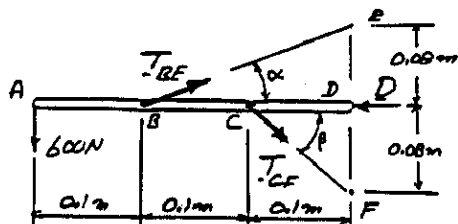
$$(b) C_x = +P \quad C_x = P \rightarrow \\ C_y = P(1 - \frac{1}{\tan 60^\circ}) = +0.4226P \quad C_y = 0.4226P \downarrow$$

$$C_y = 0.4226P \quad C_x = P \quad C = 1.086P \quad \Delta 22.9^\circ$$

# 4.35



FIND:  
TENSION IN  
EACH CABLE  
REACTION AT D



$$\tan \alpha = \frac{0.08 \text{ m}}{0.2 \text{ m}}$$

$$\alpha = 21.80^\circ$$

$$\tan \beta = \frac{0.08 \text{ m}}{0.1 \text{ m}}$$

$$\beta = 38.66^\circ$$

(CONTINUED)

# 4.35 CONTINUED

$$\sum M_B = 0: (600 \text{ N})(0.1 \text{ m}) - (T_{CF} \sin 38.66^\circ)(0.1 \text{ m}) = 0$$

$$T_{CF} = 960.47 \text{ N} \quad T_{CF} = 960 \text{ N}$$

$$+\sum M_C = 0: (600 \text{ N})(0.2 \text{ m}) - (T_{BE} \sin 21.80^\circ)(0.1 \text{ m}) = 0$$

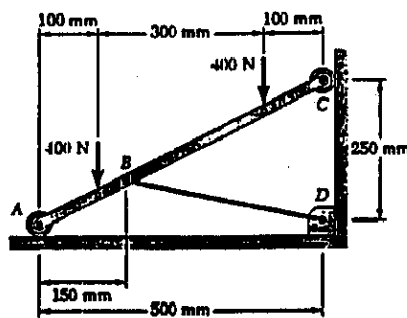
$$T_{BE} = 3231.1 \text{ N} \quad T_{BE} = 3230 \text{ N}$$

$$+\sum F_y = 0: T_{BE} \cos \alpha + T_{CF} \cos \beta - D = 0$$

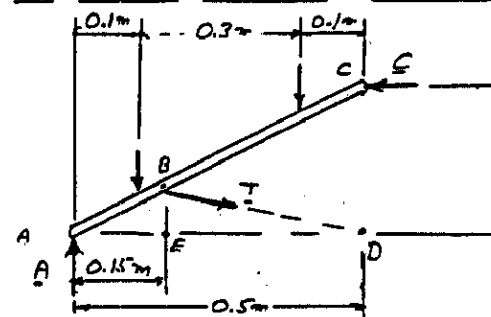
$$(3231.1 \text{ N}) \cos 21.80^\circ + (960.47 \text{ N}) \cos 38.66^\circ - D = 0$$

$$D = 3750.03 \text{ N} \quad D = 3750 \text{ N}$$

# 4.36

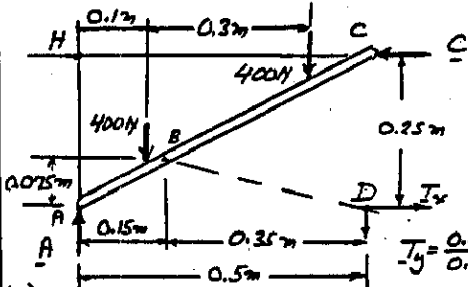


FIND:  
(a) TENSION IN  
CABLE BE,  
(b) REACTION  
AT A,  
(c) REACTION  
AT C.



SIMILAR TRIANGLES: ABE AND ACD

$$\frac{AE}{AD} = \frac{BE}{CD}; \frac{0.15 \text{ m}}{0.5 \text{ m}} = \frac{BE}{0.25 \text{ m}}; BE = 0.075 \text{ m}$$



$$+\sum M_A = 0: T_x(0.25 \text{ m}) - (\frac{0.075}{0.35} T_x)(0.5 \text{ m}) - (400 \text{ N})(0.1 \text{ m}) - (400 \text{ N})(0.4 \text{ m}) = 0$$

$$T_y = \frac{0.075}{0.35} (1400 \text{ N}) = 300 \text{ N}$$

$$T_x = 300 \text{ N} \quad T_x = 1400 \text{ N}$$

$$T = 1432 \text{ lb}$$

$$+\sum F_y = 0: A - 300 \text{ N} - 400 \text{ N} - 400 \text{ N} = 0$$

$$A = +1100 \text{ N}$$

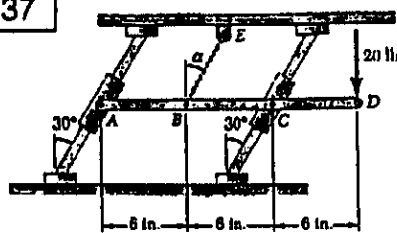
$$A = 1100 \text{ N} \uparrow$$

$$+\sum F_x = 0: -C + 1400 \text{ N} = 0$$

$$C = +1400 \text{ N}$$

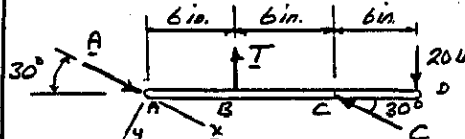
$$C = 1400 \text{ N} \leftarrow$$

4.37



IF CORD BE  
IS VERTICAL  
 $\alpha = 0$ ,  
FIND:  
TENSION IN BE  
REACTIONS  
AT A AND C

FREE-BODY  
DIAGRAM



$$+\uparrow \Sigma F_y = 0: -T \cos 30^\circ + (20 \text{ lb}) \cos 30^\circ = 0 \quad T = 20 \text{ lb}$$

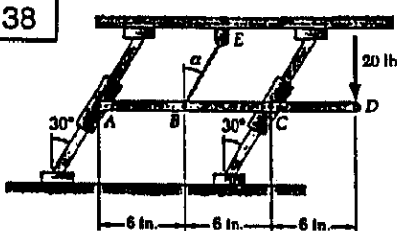
$$+\circlearrowleft \Sigma M_C = 0: (A \sin 30^\circ)(12 \text{ in}) - (20 \text{ lb})(6 \text{ in}) - (20 \text{ lb})(6 \text{ in}) = 0$$

$$A = +40 \text{ lb} \quad A = 40 \text{ lb} \angle 30^\circ$$

$$+\circlearrowleft \Sigma M_A = 0: (20 \text{ lb})(6 \text{ in}) - (20 \text{ lb})(18 \text{ in}) + (C \sin 30^\circ)(12 \text{ in}) = 0$$

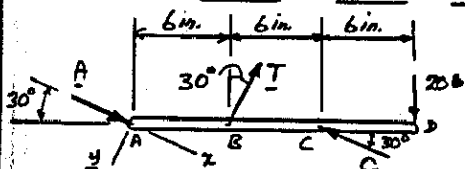
$$C = +40 \text{ lb} \quad C = 40 \text{ lb} \angle 30^\circ$$

4.38



IF  $\alpha = 30^\circ$ ,  
FIND:  
TENSION IN BE  
REACTIONS  
AT A AND C

FREE-BODY  
DIAGRAM



$$+\uparrow \Sigma F_y = 0: -T + (20 \text{ lb}) \cos 30^\circ = 0 \quad T = 17.32 \text{ lb}$$

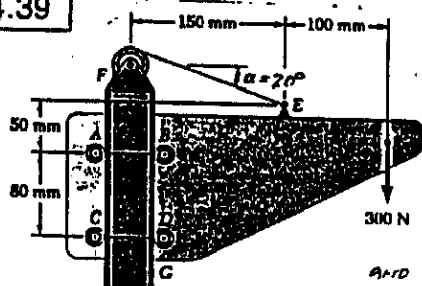
$$+\circlearrowleft \Sigma M_C = 0: -(17.32 \text{ lb}) \cos 30^\circ (6 \text{ in}) - (20 \text{ lb})(6 \text{ in}) - A \sin 30^\circ (12 \text{ in}) = 0$$

$$A = +35 \text{ lb} \quad A = 35 \text{ lb} \angle 30^\circ$$

$$+\circlearrowleft \Sigma M_A = 0: +(17.32 \text{ lb}) \cos 30^\circ (6 \text{ in}) - (20 \text{ lb})(18 \text{ in}) + (C \sin 30^\circ)(12 \text{ in}) = 0$$

$$C = +45 \text{ lb} \quad C = 45 \text{ lb} \angle 30^\circ$$

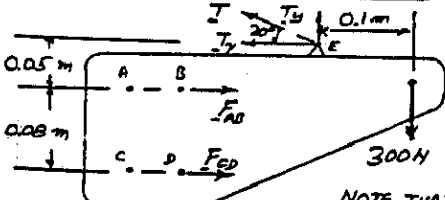
4.39



FIND: FORCE  
EXERTED ON  
POST BY  
EACH ROLLER.

Denote force  
at A + B by  $F_{AB}$   
and at C + D by  $F_{CD}$

FREE-BODY DIAGRAM



NOTE THAT  $T_y$  AND 300 N  
FORM A COUPLE:  $(300 \text{ N})(0.1 \text{ m}) = 30 \text{ N}\cdot\text{m}$   
(CONTINUED)

4.39 CONTINUED

"COUPLE"

$$+\circlearrowleft \Sigma M_C = 0: -F_{AB}(0.08 \text{ m}) + (T \cos 20^\circ)(0.130 \text{ m}) - 30 \text{ N}\cdot\text{m} = 0$$

$$\text{FROM EQ. (1)} \quad T = T_y / \sin 20^\circ = (300 \text{ N}) / \sin 20^\circ$$

$$-F_{AB}(0.08 \text{ m}) + (300 \text{ N}) \frac{\cos 20^\circ}{\sin 20^\circ} (0.130 \text{ m}) - 30 \text{ N}\cdot\text{m} = 0$$

$$F_{AB} = +964 \text{ lb} \quad F_{AB} = 964 \text{ lb} \rightarrow$$

THUS  $F_{AB}$  ACTS AT B. ON BRACKET:  $B = 964 \text{ lb} \rightarrow$ ,  $A = 0$   
ON POST:  $B = 964 \text{ lb} \leftarrow$ ;  $A = 0$

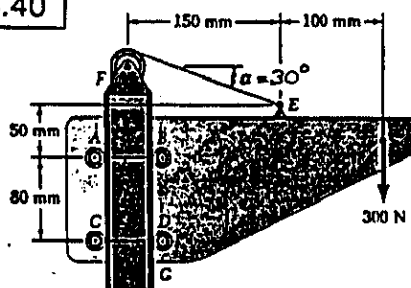
$$+\circlearrowleft \Sigma M_A = 0: +F_{CD}(0.08 \text{ m}) + (T \cos 20^\circ)(0.05 \text{ m}) - 30 \text{ N}\cdot\text{m} = 0$$

$$F_{CD}(0.08 \text{ m}) + (300 \text{ N}) \frac{\cos 20^\circ}{\sin 20^\circ} (0.05 \text{ m}) - 30 \text{ N}\cdot\text{m} = 0$$

$$F_{CD} = -140.2 \text{ N} \quad F_{CD} = 140.2 \text{ N} \leftarrow$$

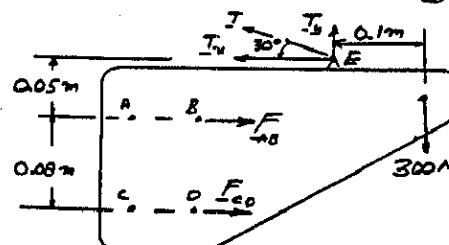
THUS  $F_{CD}$  ACTS AT C. ON BRACKET:  $C = 140.2 \text{ N} \leftarrow$ ,  $D = 0$   
ON POST:  $C = 140.2 \text{ N} \rightarrow$ ;  $D = 0$

4.40



FIND: FORCE  
EXERTED ON  
POST BY  
EACH ROLLER.

Denote force at A and B by  $F_{AB}$  and  
force at C and D by  $F_{CD}$



FREE-BODY  
DIAGRAM

$$+\uparrow \Sigma F_y = 0: T_y - 300 \text{ N} = 0 \quad T_y = 300 \text{ N} \uparrow$$

$$T_y = T_x \tan 20^\circ; 300 \text{ N} = T_x \tan 20^\circ \quad T_x = 579.62 \text{ N} \leftarrow$$

$$\text{NOTE THAT } T_y \text{ AND } 300\text{-N LOAD}$$

$$\text{FORM A COUPLE: } (300 \text{ N})(0.1 \text{ m}) = 30 \text{ N}\cdot\text{m}$$

$$+\circlearrowleft \Sigma M_C = 0: -F_{AB}(0.08 \text{ m}) + T_x(0.130 \text{ m}) - 30 \text{ N}\cdot\text{m} = 0$$

$$-F_{AB}(0.08 \text{ m}) + (579.62 \text{ N})(0.130 \text{ m}) - 30 \text{ N}\cdot\text{m} = 0$$

$$F_{AB} = +469.4 \text{ lb} \quad F_{AB} = 469.4 \text{ N} \rightarrow$$

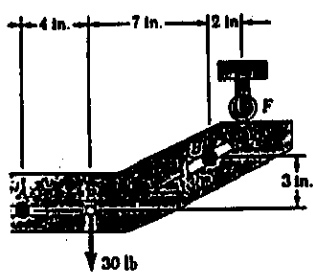
THUS  $F_{AB}$  ACTS AT B. ON BRACKET:  $B = 469 \text{ N} \rightarrow$ ,  $A = 0$   
ON POST:  $B = 469 \text{ N} \leftarrow$ ;  $A = 0$

$$+\circlearrowleft \Sigma M_A = 0: F_{CD}(0.08 \text{ m}) + T_x(0.05 \text{ m}) - 30 \text{ N}\cdot\text{m} = 0$$

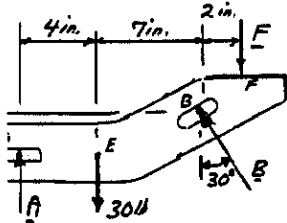
$$F_{CD}(0.08 \text{ m}) + (579.62 \text{ N})(0.05 \text{ m}) - 30 \text{ N}\cdot\text{m} = 0$$

$$F_{CD} = +50.2 \text{ N} \quad F_{CD} = 50.2 \text{ N} \rightarrow$$

THUS  $F_{CD}$  ACTS AT D. ON BRACKET:  $C = 0$ ;  $D = 50.2 \text{ N} \rightarrow$   
ON POST:  $C = 0$ ;  $D = 50.2 \text{ N} \leftarrow$

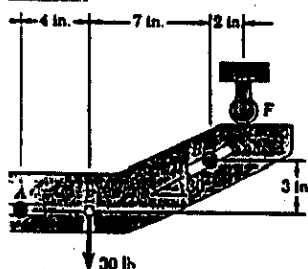


GIVEN:  
 $P = 15 \text{ lb}$   
 FIND:  
 REACTIONS  
 AT A AND B  
 REACTION  
 AT F

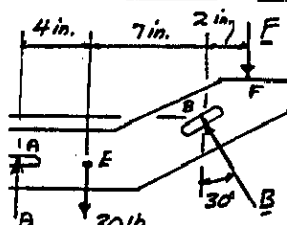


FREE-BODY  
 DIAGRAM

$$\begin{aligned} \sum F_y = 0: & \quad B - 30 \text{ lb} \sin 30^\circ = 0 \quad B = 30 \text{ lb} \nearrow 60^\circ \\ 4(4 \text{ in}) + B \sin 30^\circ(3 \text{ in}) + B \cos 30^\circ(11 \text{ in}) - F(13 \text{ in}) &= 0 \\ + (30 \text{ lb}) \sin 30^\circ(3 \text{ in}) + (30 \text{ lb}) \cos 30^\circ(11 \text{ in}) - F(13 \text{ in}) &= 0 \\ + 16.2145 \text{ lb} & \quad F = 16.21 \text{ lb} \uparrow \\ 30 \text{ lb} + B \cos 30^\circ - F = 0 & \\ 30 \text{ lb} + (30 \text{ lb}) \cos 30^\circ - 16.2145 \text{ lb} = 0 & \\ 20.23 \text{ lb} & \quad A = 20.2 \text{ lb} \uparrow \end{aligned}$$



FIND:  
 RANGE OF P  
 FOR  
 $0 \leq F \leq 20 \text{ lb}$

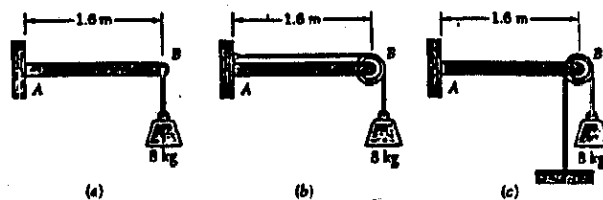


FREE-BODY  
 DIAGRAM

$$\begin{aligned} \sum F_y = 0: & \quad B - 2P \sin 30^\circ = 0 \quad B = 2P \nearrow 60^\circ \\ 16(4 \text{ in}) + B \sin 30^\circ(3 \text{ in}) + B \cos 30^\circ(11 \text{ in}) - F(13 \text{ in}) &= 0 \\ 16 + 2P \sin 30^\circ(3 \text{ in}) + 2P \cos 30^\circ(11 \text{ in}) - F(13 \text{ in}) &= 0 \\ 16 + 19.0525P - 13F = 0 & \\ \frac{13F + 120}{22.0525} & \quad (1) \\ \therefore P = \frac{13(0) + 120}{22.0525} = 5.442 \text{ lb} & \\ \text{b: } P = \frac{13(20) + 120}{22.0525} = 17.232 \text{ lb} & \\ 20 \text{ lb: } & \quad 5.44 \text{ lb} \leq P \leq 17.23 \text{ lb} \end{aligned}$$

4.43

FIND: REACTION AT A IN EACH CASE.



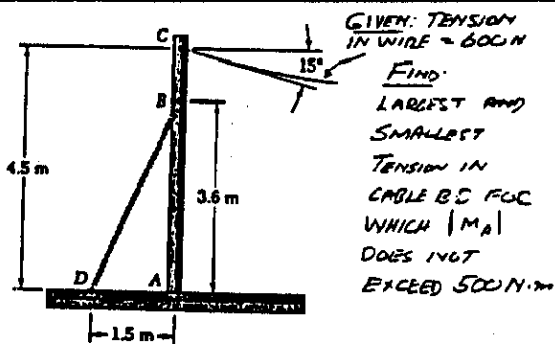
$$W = mg = (8 \text{ kg})(9.81 \text{ m/s}^2) = 78.48 \text{ N}$$

$$\begin{aligned} \text{(a)} \quad \sum F_x = 0: & \quad A_x = 0 \\ \uparrow \sum F_y = 0: & \quad A_y - W = 0 \quad A_y = 78.48 \text{ N} \uparrow \\ \rightarrow \sum M_A = 0: & \quad M_A - W(1.6 \text{ m}) = 0 \\ & \quad M_A = (78.48 \text{ N})(1.6 \text{ m}) \quad M_A = 125.56 \text{ N}\cdot\text{m} \\ & \quad A = 78.5 \text{ N} \uparrow; \quad M_A = 125.6 \text{ N}\cdot\text{m} \end{aligned}$$

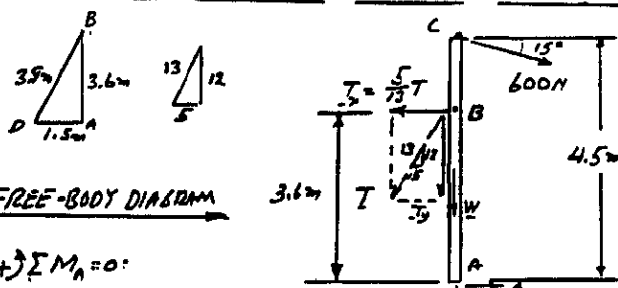
$$\begin{aligned} \text{(b)} \quad \sum F_x = 0: & \quad A_x - W = 0 \quad A_x = 78.48 \text{ N} \uparrow \\ \uparrow \sum F_y = 0: & \quad A_y - W = 0 \quad A_y = 78.48 \text{ N} \uparrow \\ & \quad A = (78.48 \text{ N})\sqrt{2} = 110.99 \text{ N} \angle 45^\circ \\ \rightarrow \sum M_A = 0: & \quad M_A - W(1.6 \text{ m}) = 0 \\ & \quad M_A = (78.48 \text{ N})(1.6 \text{ m}) \quad M_A = 125.56 \text{ N}\cdot\text{m} \\ & \quad A = 111.0 \text{ N} \angle 45^\circ; \quad M_A = 125.6 \text{ N}\cdot\text{m} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad \sum F_x = 0: & \quad A_x = 0 \\ \uparrow \sum F_y = 0: & \quad A_y - 2W = 0 \\ & \quad A_y = 2W = 2(78.48 \text{ N}) = 156.96 \text{ N} \uparrow \\ \rightarrow \sum M_A = 0: & \quad M_A - 2W(1.6 \text{ m}) = 0 \\ & \quad M_A = 2(78.48 \text{ N})(1.6 \text{ m}) \quad M_A = 251.1 \text{ N}\cdot\text{m} \\ & \quad A = 157.0 \text{ N} \uparrow; \quad M_A = 251 \text{ N}\cdot\text{m} \end{aligned}$$

4.44



GIVEN: TENSION  
 IN WIRE = 600 N  
 FIND:  
 LARGEST AND  
 SMALLEST  
 TENSION IN  
 CABLE BC FOR  
 WHICH  $|M_A|$   
 DOES NOT  
 EXCEED 500 N·m



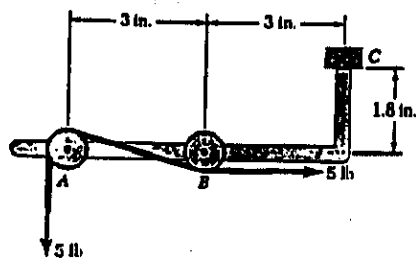
FREE-BODY DIAGRAM

$$\begin{aligned} \rightarrow \sum M_A = 0: & \quad \frac{5}{13}T(3.6 \text{ m}) - (600 \text{ N}) \cos 15^\circ(4.5 \text{ m}) + M_A = 0 \\ (1.3846 \text{ m})T - 2608 \text{ N}\cdot\text{m} + M_A &= 0 \\ T = \frac{2608 \text{ N}\cdot\text{m} + M_A}{1.3846 \text{ m}} & \\ \text{FOR } M_A = +500 \text{ N}\cdot\text{m: } T = \frac{2608 + 500}{1.3846} = 2244.7 \text{ N} & \\ \text{FOR } M_A = -500 \text{ N}\cdot\text{m: } T = \frac{2608 - 500}{1.3846} = 1522.4 \text{ N} & \\ T_{\max} = 2240 \text{ N}; \quad T_{\min} = 1522 \text{ N} & \end{aligned}$$

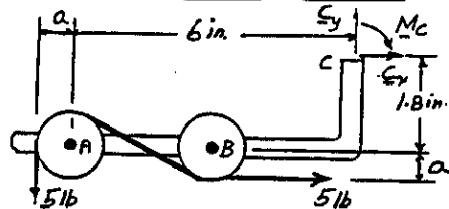


# 4.45 and 4.46

DEFINITE: RADII OF PULLEYS BY  $\alpha$ .



FIND:  
REACTION AT C  
PROB. 4.45:  $\alpha = 0.4$  in.  
PROB. 4.46:  $\alpha = 0.6$  in.



FREE-BODY DIAGRAM

$$\begin{aligned} \pm \sum F_x = 0: C_x + 5 \text{ lb} = 0; C_x = -5 \text{ lb} & \quad C_x = 5 \text{ lb} \leftarrow \\ + \sum F_y = 0: C_y - 5 \text{ lb} = 0; C_y = +5 \text{ lb} & \quad C_y = 5 \text{ lb} \uparrow \end{aligned}$$

$$C_x = 5 \text{ lb} \quad C_y = 5 \text{ lb} \quad C = 7.071 \text{ lb} \angle 45^\circ$$

$$+\sum M_C = 0: (5 \text{ lb})(6 \text{ in.} + \alpha) + (5 \text{ lb})(1.8 \text{ in.} + \alpha) - M_C = 0$$

$$M_C = 39 \text{ lb} \cdot \text{in.} + (10 \text{ lb})\alpha \quad (1)$$

PROB. 4.45 WITH  $\alpha = 0.4$  in.

$$\text{EQ(1): } M_C = 39 \text{ lb} \cdot \text{in.} + (10 \text{ lb})(0.4 \text{ in.}) = +43.0 \text{ lb} \cdot \text{in.}$$

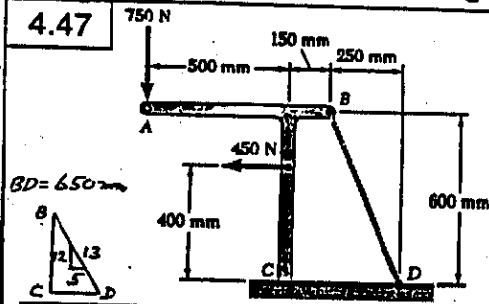
$$C = 7.071 \text{ lb} \angle 45^\circ; M_C = +43 \text{ lb} \cdot \text{in.}$$

PROB. 4.46 WITH  $\alpha = 0.6$  in.

$$\text{EQ(1): } M_C = 39 \text{ lb} \cdot \text{in.} + (10 \text{ lb})(0.6 \text{ in.}) = +45 \text{ lb} \cdot \text{in.}$$

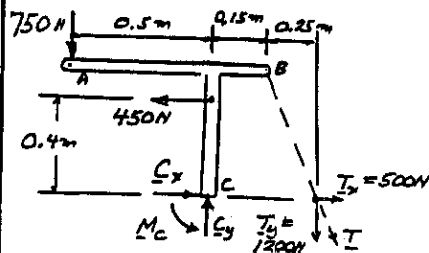
$$C = 7.071 \text{ lb} \angle 45^\circ; M_C = +45 \text{ lb} \cdot \text{in.}$$

# 4.47



GIVEN:  
 $T_{BD} = 1300 \text{ N}$

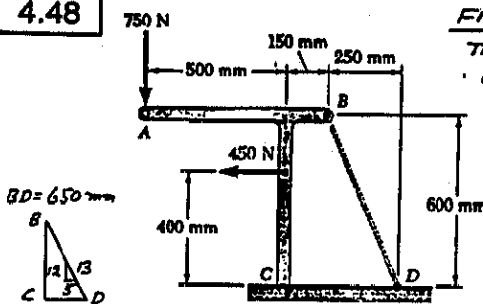
FIND:  
REACTION AT C



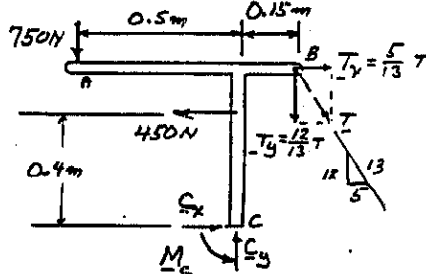
$$\begin{aligned} T &= 1300 \text{ N} \\ T_x &= \frac{5}{13} T = 500 \text{ N} \\ T_y &= \frac{12}{13} T = 1200 \text{ N} \end{aligned}$$

$$\begin{aligned} \pm \sum F_x = 0: C_x - 450 \text{ N} + 1300 \text{ N} = 0; C_x &= -850 \text{ N}; C_x = 850 \text{ N} \leftarrow \\ + \sum F_y = 0: C_y - 750 \text{ N} - 1200 \text{ N} = 0; C_y &= +1950 \text{ N}; C_y = 1950 \text{ N} \uparrow \\ + \sum M_C = 0: M_C + (750 \text{ N})(0.5 \text{ m}) + (450 \text{ N})(0.4 \text{ m}) - (1200 \text{ N})(0.4 \text{ m}) &= 0 \\ M_C &= -75 \text{ N} \cdot \text{m} \quad M_C = 75 \text{ N} \cdot \text{m} \end{aligned}$$

# 4.48



FIND RANGE OF TENSION IN CABLE BD FOR WHICH  $|M_C| \leq 100 \text{ N} \cdot \text{m}$



$$+\sum M_C = 0: (750 \text{ N})(0.5 \text{ m}) + (450 \text{ N})(0.4 \text{ m}) - \left(\frac{5}{13} T\right)(0.6 \text{ m}) - \left(\frac{12}{13} T\right)(0.4 \text{ m}) + M_C = 0$$

$$375 \text{ N} \cdot \text{m} + 180 \text{ N} \cdot \text{m} - \left(\frac{4.8}{13}\right) T + M_C = 0$$

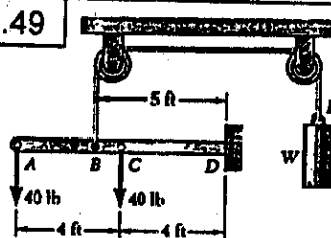
$$T = \frac{13}{4.8} (555 - M_C)$$

$$\text{FOR } M_C = +100 \text{ N} \cdot \text{m}: T = \frac{13}{4.8} (555 - 100) = 1232 \text{ N}$$

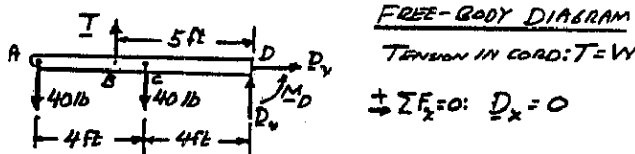
$$\text{FOR } M_C = -100 \text{ N} \cdot \text{m}: T = \frac{13}{4.8} (555 - (-100)) = 1774 \text{ N}$$

$$\text{FOR } |M_C| \leq 100 \text{ N} \cdot \text{m}: 1232 \text{ N} \leq T \leq 1774 \text{ N}$$

# 4.49



FIND:  
REACTION AT D  
(a) WHEN  $W = 100 \text{ lb}$   
(b) WHEN  $W = 90 \text{ lb}$



FREE-BODY DIAGRAM  
TENSION IN CORD:  $T = W$

$$\pm \sum F_x = 0: D_x = 0$$

$$+\sum F_y = 0: D_y - 40 \text{ lb} - 40 \text{ lb} + T = 0$$

$$D_y = 80 \text{ lb} - T \quad (1)$$

$$+\sum M_D = 0: M_D + (40 \text{ lb})(8 \text{ ft}) + (40 \text{ lb})(4 \text{ ft}) - T(5 \text{ ft}) = 0$$

$$M_D = -480 \text{ lb} \cdot \text{ft} + T(5 \text{ ft}) \quad (2)$$

a. WHEN  $W = 100 \text{ lb}$ :  $T = 100 \text{ lb}$ ,  $D_x = 0$

$$\text{EQ(1): } D_y = 80 \text{ lb} - 100 \text{ lb} = -20 \text{ lb} \quad D_y = 20 \text{ lb} \downarrow$$

$$\text{EQ(2): } M_D = -480 \text{ lb} \cdot \text{ft} + (100 \text{ lb})(5 \text{ ft})$$

$$M_D = +20 \text{ lb} \cdot \text{ft} \quad M_D = 20 \text{ lb} \cdot \text{ft}$$

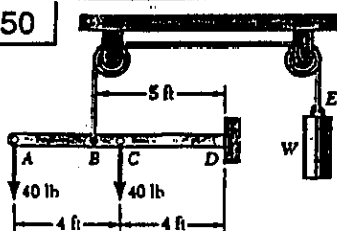
b. WHEN  $W = 90 \text{ lb}$ :  $T = 90 \text{ lb}$ ,  $D_x = 0$

$$\text{EQ(1): } D_y = 80 \text{ lb} - 90 \text{ lb} = -10 \text{ lb} \quad D_y = 10 \text{ lb} \downarrow$$

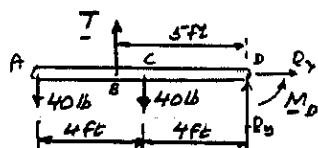
$$\text{EQ(2): } M_D = -480 \text{ lb} \cdot \text{ft} + (90 \text{ lb})(5 \text{ ft})$$

$$M_D = -30 \text{ lb} \cdot \text{ft} \quad M_D = 30 \text{ lb} \cdot \text{ft}$$

4.50



FIND: RANGE OF  
W FOR WHICH  
 $|M_D| \leq 40 \text{ lb}\cdot\text{ft}$



FREE-BODY DIAGRAM

TENSION IN CORD:  $T = W$ 

$$+\circlearrowleft \sum M_D = 0: M_D + (40 \text{ lb})(8 \text{ ft}) + (40 \text{ lb})(4 \text{ ft}) - T(5 \text{ ft}) = 0$$

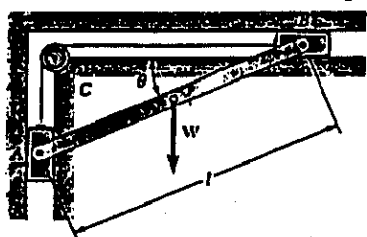
$$T = \frac{1}{5 \text{ ft}} (480 \text{ lb}\cdot\text{ft} + M_D)$$

FOR  $M_D = +40 \text{ lb}\cdot\text{ft}$ :  $T = \frac{1}{5} (480 + 40) = 104 \text{ lb}$

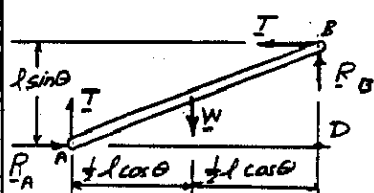
FOR  $M_D = -40 \text{ lb}\cdot\text{ft}$ :  $T = \frac{1}{5} (480 - 40) = 88 \text{ lb}$

RECALL THAT WEIGHT  $W = \text{TENSION } T$ , WE HAVE  
FOR  $|M_D| \leq 40 \text{ lb}\cdot\text{ft}$ :  $88 \text{ lb} \leq W \leq 104 \text{ lb}$

4.51



FIND:  
(a) TENSION  
IN CORD  
IN TERMS  
OF W AND  $\theta$   
(b) VALUE OF  
 $\theta$  FOR  $T = 3W$



$$(a) +\circlearrowleft \sum M_D = 0:$$

$$T(l \sin \theta) - T(l \cos \theta) + W(\frac{1}{2} l \cos \theta) = 0$$

$$T = \frac{1}{2} W \frac{\cos \theta}{\cos \theta - \sin \theta}$$

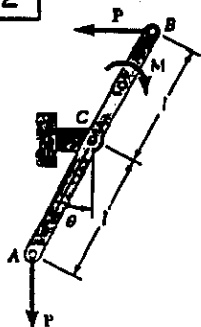
$$T = \frac{1}{2} W / (1 - \tan \theta)$$

(b) FOR  $T = 3W$ :  $3W = \frac{1}{2} W / (1 - \tan \theta)$

$$3 - 3 \tan \theta = \frac{1}{2}$$

$$\tan \theta = \frac{2.5}{3} = \frac{5}{6} \quad \theta = 39.8^\circ$$

4.52



FOR EQUILIBRIUM  
FIND:

(a) EQUATION IN  $\theta, P, M$ , AND  $l$

(b) VALUE OF  $\theta$ ,  
FOR  $M = 150 \text{ N}\cdot\text{m}$   
 $P = 200 \text{ N}$   
 $l = 600 \text{ mm}$

(CONTINUED)

4.52 CONTINUED

FREE-BODY DIAGRAM

$$(a) +\circlearrowleft \sum M_C = 0:$$

$$Pl \cos \theta + Pl \sin \theta - M = 0$$

$$\sin \theta + \cos \theta = \frac{M}{Pl}$$

(b) FOR  $M = 150 \text{ N}\cdot\text{m}$ ,  
 $P = 200 \text{ N}$ , AND  $l = 600 \text{ mm}$

$$\sin \theta + \cos \theta = \frac{150 \text{ N}\cdot\text{m}}{(200 \text{ N})(0.6 \text{ m})}$$

$$\sin \theta + \cos \theta = 1.25$$

$$\sin \theta + (1 - \sin^2 \theta)^{1/2} = 1.25$$

$$(1 - \sin^2 \theta)^{1/2} = 1.25 - \sin \theta$$

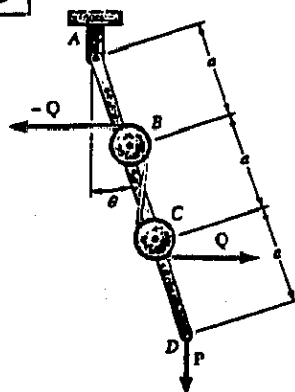
$$1 - \sin^2 \theta = 1.5625 - 2.5 \sin \theta + \sin^2 \theta$$

$$2 \sin^2 \theta - 2.5 \sin \theta + 0.5625 = 0$$

$$\sin \theta = 0.2943 \text{ and } \sin \theta = 0.9557$$

$$\theta = 17.1^\circ \text{ and } \theta = 72.9^\circ$$

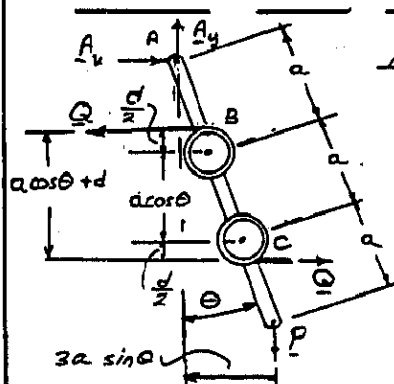
4.53



FOR EQUILIBRIUM  
FIND:

$$(a) P = f(Q, a, d, \theta)$$

(b) MAGNITUDE  
OF  $P$  FOR  
 $Q = 10 \text{ lb}$ ,  
 $a = 5 \text{ in}$ ,  
 $d = 0.8 \text{ in}$ , AND  
 $\theta = 30^\circ$



FREE-BODY DIAGRAM

$$(a) +\circlearrowleft \sum M_A = 0$$

$$Q(a \cos \theta + d) - P(3a \sin \theta) = 0$$

$$P = \frac{Q}{3} \cdot \frac{a \cos \theta + d}{a \sin \theta}$$

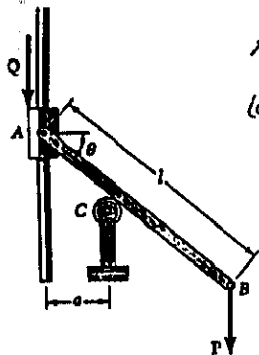
(b) FOR  $Q = 10 \text{ lb}$ ,  $a = 5 \text{ in}$ ,  $d = 0.8 \text{ in}$ ,  $\theta = 30^\circ$

$$P = \frac{10 \text{ lb}}{3} \cdot \frac{(5 \text{ in}) \cos 30^\circ + 0.8 \text{ in}}{(5 \text{ in}) \sin 30^\circ}$$

$$P = 6.840 \text{ lb}$$

$$P = 6.84 \text{ lb}$$

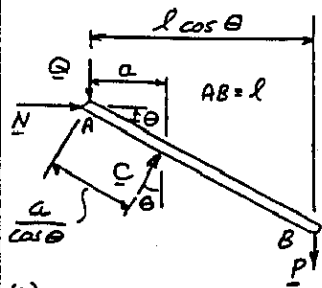
4.54



FOR EQUILIBRIUM  
FIND:

(a) EQUATION  
IN  $P, Q, a, l$ , AND  $\theta$ .

(b) VALUE OF  $\theta$ ,  
FOR  $P=16\text{ lb}$ ,  
 $Q=12\text{ lb}$ ,  $l=20\text{ in}$ ,  
AND  $a=5\text{ in}$ .



FREE-BODY DIAGRAM

$$+\uparrow \Sigma F_y = 0:$$

$$C \cos \theta - P - Q = 0$$

$$C = \frac{P+Q}{\cos \theta}$$

(a)

$$+\uparrow \Sigma M_A = 0: C \frac{a}{\cos \theta} - P l \cos \theta = 0$$

$$\frac{P+Q}{\cos \theta} \cdot \frac{a}{\cos \theta} - P l \cos \theta = 0$$

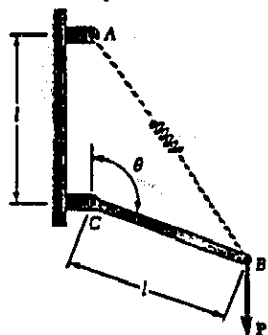
$$\cos^3 \theta = \frac{a(P+Q)}{Pl}$$

(b) For  $P=16\text{ lb}$ ,  $Q=12\text{ lb}$ ,  $l=20\text{ in}$ , AND  $a=5\text{ in}$ :

$$\cos^3 \theta = \frac{(5\text{ in})(16\text{ lb} + 12\text{ lb})}{(16\text{ lb})(20\text{ in})} = 0.4375$$

$$\cos \theta = 0.75915 \quad \theta = 40.6^\circ$$

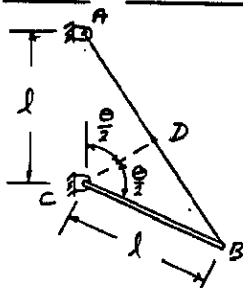
4.55



FOR EQUILIBRIUM  
FIND:

(a)  $\theta = f(P, k, l)$ .

(b) VALUE OF  $\theta$   
WHEN  $P = \frac{1}{4}kl$ .



GEOMETRY  
 $AB = 2l \sin \frac{\theta}{2}$

$$CD = l \cos \frac{\theta}{2}$$

LET ELONGATION OF SPRING =  $S$

$$S = (AB)_\theta - (AB)_{\theta=90^\circ}$$

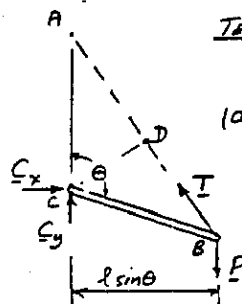
$$S = 2l \sin \frac{\theta}{2} - 2l \sin 45^\circ$$

$$S = 2l \left( \sin \frac{\theta}{2} - \frac{1}{\sqrt{2}} \right)$$

(CONTINUED)

4.55 CONTINUED

FREE-BODY DIAGRAM



TENSION IN SPRING

$$T = kS = 2kl \left( \sin \frac{\theta}{2} - \frac{1}{\sqrt{2}} \right)$$

(a)  $+\uparrow \Sigma M_C = 0:$

$$T(CD) - Pl \sin \theta = 0$$

$$2kl \left( \sin \frac{\theta}{2} - \frac{1}{\sqrt{2}} \right) (l \cos \frac{\theta}{2})$$

$$- Pl \sin \theta = 0$$

$$2kl^2 \left( \sin \frac{\theta}{2} - \frac{1}{\sqrt{2}} \right) \cos \frac{\theta}{2} - Pl \left( 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \right) = 0$$

$$\cos \frac{\theta}{2} \left[ 2(kl - P) \sin \frac{\theta}{2} - \frac{2}{\sqrt{2}} kl \right] = 0$$

$$\cos \frac{\theta}{2} = 0 \quad \text{OR} \quad \sin \frac{\theta}{2} = \frac{1}{\sqrt{2}} \cdot \frac{kl}{kl - P}$$

(TRIVIAL)

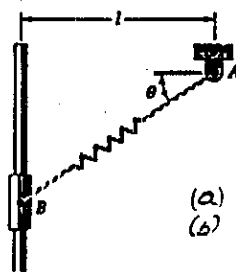
$$\theta = 2 \sin^{-1} \left[ \frac{1}{\sqrt{2}} \frac{kl}{(kl - P)} \right]$$

(b) For  $P = \frac{1}{4}kl$ :

$$\theta = 2 \sin^{-1} \left[ \frac{1}{\sqrt{2}} \frac{kl}{(kl - \frac{1}{4}kl)} \right] = 2 \sin^{-1} (0.97979)$$

$$\theta = 2(70.529^\circ) = 141.06^\circ \quad \theta = 141.1^\circ$$

4.56



GIVEN:

$W$  = WEIGHT OF LEVER,  
SPRING IS

UNDEFORMED FOR  $\theta = 0$

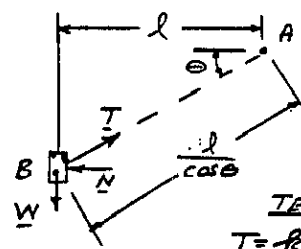
FIND: FOR EQUILIBRIUM

(a) EQUATION IN  $\theta, W, k, l$

(b) VALUE OF  $\theta$  WHEN

$W = 300\text{ N}$ ,  $l = 500\text{ mm}$ ,

AND  $k = 800\text{ N/m}$ .



FREE-BODY DIAGRAM

LET  $S$  = ELONGATION OF SPRING

$$S = \frac{l}{\cos \theta} - l$$

TENSION IN SPRING  
 $T = kS = kl \left( \frac{1}{\cos \theta} - 1 \right)$

(a)

$$+\uparrow \Sigma F_y = 0: T \sin \theta - W = 0$$

$$kl \left( \frac{1}{\cos \theta} - 1 \right) \sin \theta - W = 0$$

$$\frac{\sin \theta}{\cos \theta} - \sin \theta = \frac{W}{kl}$$

$$\tan \theta - \sin \theta = \frac{W}{kl}$$

(b)  $W = 300\text{ N}$ ,  $l = 500\text{ mm}$ ,  $k = 800\text{ N/m}$

$$\tan \theta - \sin \theta = \frac{300\text{ N}}{(800\text{ N/m})(0.5\text{ m})}$$

$$\tan \theta - \sin \theta = 0.75$$

SOLVE BY TRIAL + ERROR:  $\theta = 57.96^\circ$

$$\theta = 58.0^\circ$$

4.57

**GIVEN:** SPRING IS UNDEFORMED WHEN  $\theta = 90^\circ$ .

**FIND:** POSITION OF EQUILIBRIUM

NOTE: LET  $\theta = 90^\circ + \beta$

UNDEFORMED POSITION

$F = kS$

$S = r\beta$

$F = krs = kr\beta$

$\sum M_O = 0$

$Wl \cos \beta - (kr\beta)r = 0$

$\cos \beta = \frac{kr^2}{Wl} = \frac{(250 \text{ lb/in.})(3 \text{ in.})^2}{(400 \text{ lb})(8 \text{ in.})} = 0.703125$

**SOLVE BY TRIAL AND ERROR:**  $\beta = 0.87245 \text{ rad}$   $\beta = 51.1^\circ$

$\theta = 90^\circ + \beta = 90^\circ + 51.1^\circ$   $\theta = 141.1^\circ$

4.58

**GIVEN:** SPRING IS UNDEFORMED WHEN  $\theta = 0$ .

**FIND:** FOR EQUILIBRIUM, (a) EQUATION IN  $W, l, R, \theta$  (b) VALUE OF  $\theta$  WHEN  $W = 75 \text{ lb}$ ,  $l = 30 \text{ in.}$ , AND  $R = 3 \text{ lb/in.}$

**FREE-BODY DIAGRAM**

SPRING DEFORMATION  $s = l - l \cos \theta = l(1 - \cos \theta)$

$F = kS = Rl(1 - \cos \theta)$

(a)  $\sum M_D = 0$ :  $F l \sin \theta - W \left( \frac{l}{2} \cos \theta \right) = 0$

$Rl(1 - \cos \theta) \sin \theta - \frac{1}{2} W l \cos \theta = 0$

$(1 - \cos \theta) \tan \theta = \frac{W}{2Rl}$

(b) WHEN  $W = 75 \text{ lb}$ ,  $l = 30 \text{ in.}$ , AND  $R = 3 \text{ lb/in.}$

$(1 - \cos \theta) \tan \theta = \frac{75 \text{ lb}}{2(3 \text{ lb/in.})(30 \text{ in.})}$

$(1 - \cos \theta) \tan \theta = 0.41667$

**SOLVE BY TRIAL AND ERROR**

$\theta = 49.71^\circ$   $\theta = 49.7^\circ$

4.59

**DETERMINE WHETHER (a) PLATE IS CONSTRAINED, (b) REACTIONS ARE DETERMINATE, (c) IF POSSIBLE, FIND REACTIONS.**  $m = 40 \text{ kg}$ ;  $W = (40 \text{ kg})(9.81 \text{ m/s}^2) = 392 \text{ N}$

1. THREE NON-COINCIDENT, NON-PARALLEL REACTIONS:

(a) PLATE: COMPLETELY CONSTRAINED

(b) REACTIONS: DETERMINATE

(c) EQUILIBRIUM MAINTAINED

$A = C = 196.2 \text{ N} \uparrow$

2. THREE NON-COINCIDENT, NON-PARALLEL REACTIONS:

(a) PLATE: COMPLETELY CONSTRAINED

(b) REACTIONS: DETERMINATE

(c) EQUILIBRIUM MAINTAINED

$B = 0$ ,  $C = D = 196.2 \text{ N} \uparrow$

3. FOUR NON-COINCIDENT, NON-PARALLEL REACTIONS:

(a) PLATE: COMPLETELY CONSTRAINED

(b) REACTIONS: INDETERMINATE

(c) EQUILIBRIUM MAINTAINED

$A_x = 294 \text{ N} \rightarrow$ ,  $D_x = 294 \text{ N} \leftarrow$

$(A_y + D_y = 392 \text{ N} \uparrow)$

4. THREE CONCURRENT REACTIONS (IMPROPERLY CONSTRAINED):

(a) PLATE: IMPROPERLY CONSTRAINED

(b) REACTIONS: INDETERMINATE

(c) NO EQUILIBRIUM ( $\sum M_C \neq 0$ )

5. TWO REACTIONS

(a) PLATE: IMPROPERLY CONSTRAINED

(b) REACTIONS: DETERMINATE

(c) EQUILIBRIUM MAINTAINED

$C = D = 196.2 \text{ N} \uparrow$

6. THREE NON-COINCIDENT, NON-PARALLEL REACTIONS:

(a) PLATE: COMPLETELY CONSTRAINED

(b) REACTIONS: DETERMINATE

(c) EQUILIBRIUM MAINTAINED

$B = 294 \text{ N} \rightarrow$ ,  $D = 491 \text{ N} \angle 53.1^\circ$

7. TWO REACTIONS

(a) PLATE: IMPROPERLY CONSTRAINED

(b) REACTIONS DETERMINED BY DYNAMICS

(c) NO EQUILIBRIUM ( $\sum F_y \neq 0$ )

8. FOUR NON-COINCIDENT, NON-PARALLEL REACTIONS

(a) PLATE: COMPLETELY CONSTRAINED

(b) REACTIONS: INDETERMINATE

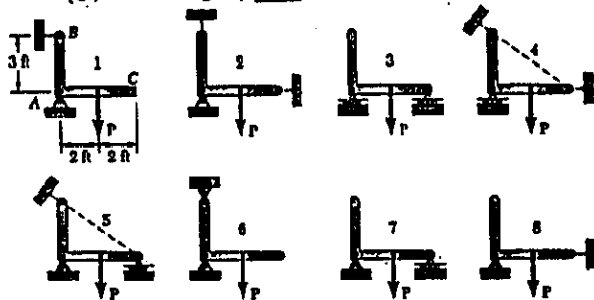
(c) EQUILIBRIUM MAINTAINED

$B = D_y = 196.2 \text{ N} \uparrow$

$(C + D_x = 0)$

4.60

DETERMINE WHETHER (a) BRACKET IS  
CONSTRAINED, (b) REACTIONS ARE DETERMINATE,  
(c) IF POSSIBLE, FIND REACTIONS.  $P = 100 \text{ lb}$ .



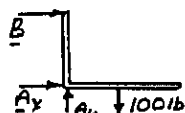
1. THREE NON-CONCURRENT, NON-PARALLEL REACTIONS

(a) BRACKET: COMPLETE CONSTRAINT

(b) REACTIONS: DETERMINATE

(c) EQUILIBRIUM MAINTAINED

$$A = 120.2 \text{ lb} \angle 56.3^\circ, B = 66.7 \text{ lb} \rightarrow$$

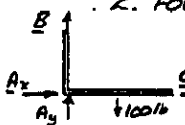


2. FOUR CONCURRENT REACTIONS (THROUGH A)

(a) BRACKET: IMPROPER CONSTRAINT

(b) REACTIONS: INDETERMINATE

(c) NO EQUILIBRIUM ( $\sum M_A \neq 0$ )



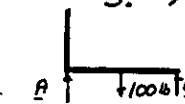
3. TWO REACTIONS

(a) BRACKET: PARTIAL CONSTRAINT

(b) REACTIONS: DETERMINATE

(c) EQUILIBRIUM MAINTAINED

$$A = 50 \text{ lb} \uparrow, C = 50 \text{ lb} \uparrow$$



4. THREE NON-CONCURRENT, NON-PARALLEL REACTIONS

(a) BRACKET: COMPLETE CONSTRAINT

(b) REACTIONS: DETERMINATE

(c) EQUILIBRIUM MAINTAINED

$$A = 50 \text{ lb} \uparrow, B = 83.3 \text{ lb} \angle 36.9^\circ, C = 66.7 \text{ lb} \rightarrow$$



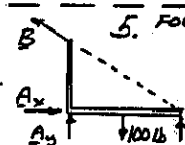
5. FOUR NON-CONCURRENT, NON-PARALLEL REACTIONS

(a) BRACKET: COMPLETE CONSTRAINT

(b) REACTIONS: INDETERMINATE

(c) EQUILIBRIUM MAINTAINED

$$(\sum M_C = 0) A_y = 50 \text{ lb} \uparrow$$



6. FOUR NON-CONCURRENT, NON-PARALLEL REACTIONS

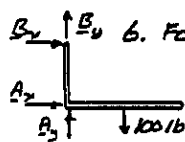
(a) BRACKET: COMPLETE CONSTRAINT

(b) REACTIONS: INDETERMINATE

(c) EQUILIBRIUM MAINTAINED

$$A_x = 66.7 \text{ lb} \rightarrow, B_x = 66.7 \text{ lb} \leftarrow$$

$$(A_y + B_y = 100 \text{ lb} \uparrow)$$



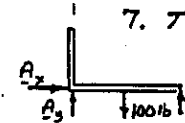
7. THREE NON-CONCURRENT, NON-PARALLEL REACTIONS

(a) BRACKET: COMPLETE CONSTRAINT

(b) REACTIONS: DETERMINATE

(c) EQUILIBRIUM MAINTAINED

$$A = C = 50 \text{ lb} \uparrow$$

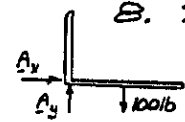


8. THREE CONCURRENT REACTIONS (THROUGH B)

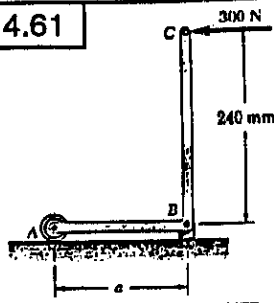
(a) BRACKET: IMPROPER CONSTRAINT

(b) REACTIONS: INDETERMINATE

(c) NO EQUILIBRIUM ( $\sum M_A \neq 0$ )



4.61



GIVEN:  $a = 180 \text{ mm}$

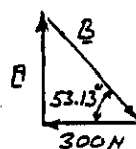
FIND: REACTIONS

FREE-BODY DIAGRAM

(THREE-FORCE MEMBER)  
REACTION AT  $B$  MUST  
PASS THROUGH  $D$  WHERE  
 $A$  AND  $300\text{-N}$  LOAD  
INTERSECT.

$$\Delta BCD: \tan \beta = \frac{240}{180}; \beta = 53.13^\circ$$

FORCE TRIANGLE



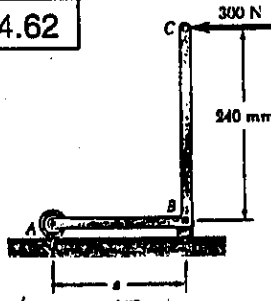
$$A = (300 \text{ N}) \tan 53.13^\circ = 400 \text{ N}$$

$$A = 400 \text{ N} \leftarrow$$

$$B = \frac{300 \text{ N}}{\cos 53.13^\circ} = 500 \text{ N}$$

$$B = 500 \text{ N} \angle 53.1^\circ \leftarrow$$

4.62

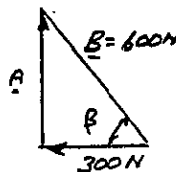


FIND: RANGE OF  
DISTANCE  $a$  FOR  
WHICH  
 $B \leq 600 \text{ N}$

FREE-BODY DIAGRAM  
(THREE-FORCE MEMBER)  
REACTION AT  $B$  MUST  
PASS THROUGH  $D$  WHERE  
 $A$  AND  $300\text{-N}$  LOAD  
INTERSECT.

$$a = \frac{240 \text{ mm}}{\sin \beta} \quad (1)$$

FORCE TRIANGLE  
(WITH  $B = 600 \text{ N}$ )



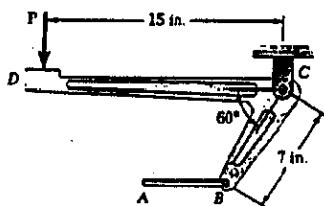
$$\cos \beta = \frac{300 \text{ N}}{600 \text{ N}} = 0.5$$

$$\beta = 60^\circ$$

$$\text{EQ. (1)} \quad a = \frac{240 \text{ mm}}{\tan 60^\circ} = 138.56 \text{ mm}$$

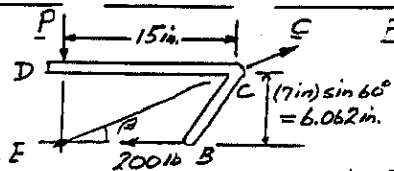
$$\text{For } B \leq 600 \text{ N}; \quad a \geq 138.6 \text{ mm}$$

4.63



GIVEN: TENSION  
IN AB = 200 lb.

FIND:  
(a) FORCE P.  
(b) REACTION  
AT C.



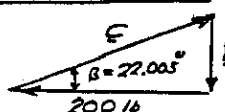
FREE-BODY DIAGRAM  
(3-FORCE BODY)  
REACTION AT  
C MUST PASS  
THROUGH E,

WHERE D AND 200 lb.

$$\tan \beta = \frac{6.062 \text{ in.}}{15 \text{ in.}}; \beta = 22.005^\circ$$

FORCE INTERSECT

FORCE TRIANGLE



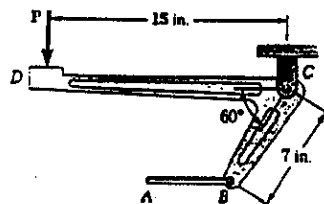
$$(a) P = (200 \text{ lb}) \tan 22.005^\circ$$

$$P = 80.83 \text{ lb} \quad P = 80.8 \text{ lb} \quad \blacktriangleleft$$

$$(b) C = \frac{200 \text{ lb}}{\cos 22.005^\circ} = 215.7 \text{ lb}$$

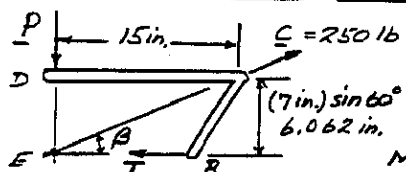
$$C = 216 \text{ lb} \angle 22.0^\circ \quad \blacktriangleleft$$

4.64



GIVEN:  
REACTION  
AT C = 250 lb.

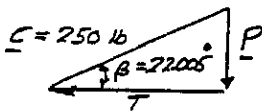
FIND: TENSION  
IN CABLE AB.



FREE-BODY  
DIAGRAM  
(3-FORCE BODY)  
REACTION AT C  
MUST PASS THROUGH  
E, WHERE D AND THE  
FORCE T INTERSECT

$$\tan \beta = \frac{6.062 \text{ in.}}{15 \text{ in.}}; \beta = 22.005^\circ$$

FORCE TRIANGLE

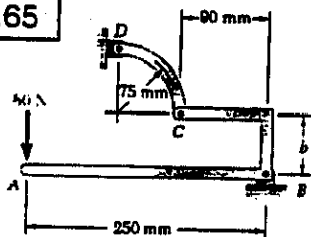


$$T = (250 \text{ lb}) \cos 22.005^\circ$$

$$T = 231.8 \text{ lb}$$

$$T = 232 \text{ lb} \quad \blacktriangleleft$$

4.65



GIVEN:  $b = 60 \text{ mm}$

FIND: REACTIONS  
AT B AND D.

SINCE CD IS A TWO-FORCE  
MEMBER, THE LINE OF ACTION OF  
REACTION AT D MUST PASS THROUGH  
POINTS C AND D.

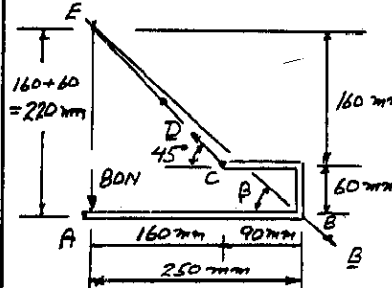


(CONTINUED)

4.65 CONTINUED

FREE-BODY DIAGRAM  
(3-FORCE BODY)

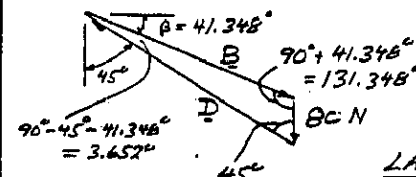
REACTION AT B  
MUST PASS THROUGH  
E, WHERE THE  
REACTION AT D AND  
80-N FORCE INTERSECT.



$$\tan \beta = \frac{220 \text{ mm}}{250 \text{ mm}}$$

$$\beta = 41.348^\circ$$

FORCE TRIANGLE



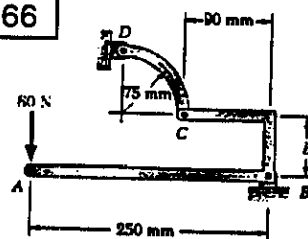
LAW OF SINES

$$\frac{80 \text{ N}}{\sin 3.652^\circ} = \frac{B}{\sin 45^\circ} = \frac{D}{\sin 131.348^\circ}$$

$$B = 888.0 \text{ N} \quad D = 942.8 \text{ N}$$

$$B = 888 \text{ N} \angle 41.3^\circ, D = 943 \text{ N} \angle 45^\circ \quad \blacktriangleleft$$

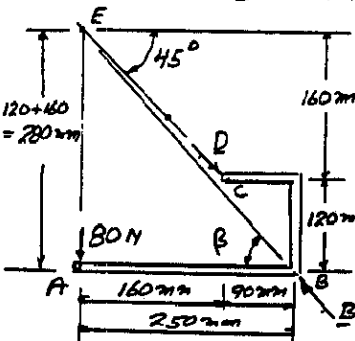
4.66



GIVEN:  
 $b = 120 \text{ mm}$

FIND: REACTIONS  
AT B AND D

SINCE CD IS A 2-FORCE MEMBER, LINE OF ACTION  
OF REACTION AT D MUST PASS THROUGH C & D



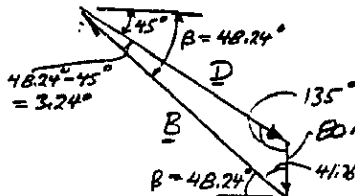
FREE-BODY DIAGRAM  
(3-FORCE BODY)

REACTION AT B  
MUST PASS THROUGH  
E, WHERE THE  
REACTION AT D AND  
80-N FORCE INTERSECT

$$\tan \beta = \frac{180 \text{ mm}}{250 \text{ mm}}$$

$$\beta = 48.24^\circ$$

FORCE TRIANGLE



LAW OF SINES

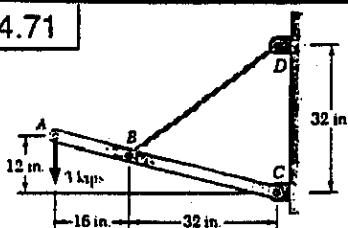
$$\frac{80 \text{ N}}{\sin 3.24^\circ} = \frac{B}{\sin 135^\circ} = \frac{D}{\sin 41.76^\circ}$$

$$B = 1000.9 \text{ N} \quad D = 942.8 \text{ N}$$

$$B = 1001 \text{ N} \angle 48.2^\circ, D = 943 \text{ N} \angle 45^\circ \quad \blacktriangleleft$$

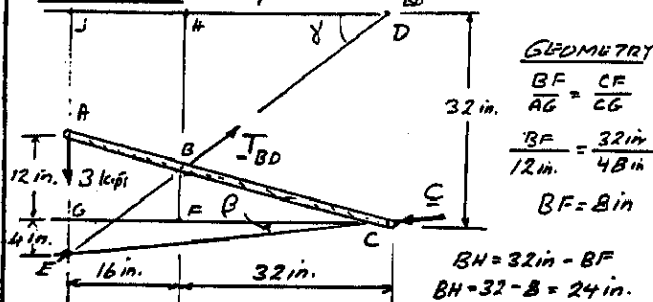


4.71



FIND:  
(a) TENSION IN CORD BD  
(b) REACTION AT C

3-FORCE BODY: 3-kip load AND  $T_{BD}$  INTERSECT AT E



GEOMETRY

$$\frac{BF}{AG} = \frac{CF}{CG}$$

$$\frac{BF}{12 \text{ in.}} = \frac{32 \text{ in.}}{48 \text{ in.}}$$

$$BF = 8 \text{ in.}$$

$$BH = 32 \text{ in.} - BF = 24 \text{ in.}$$

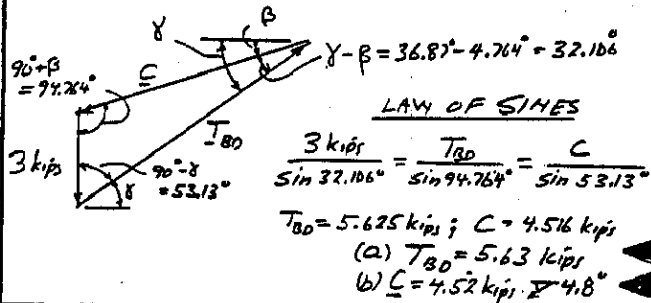
$$\frac{JE}{BH} = \frac{DJ}{DH}; \frac{JE}{24 \text{ in.}} = \frac{48 \text{ in.}}{32 \text{ in.}}; JE = 36 \text{ in.}$$

$$EG = JE - JG = 36 \text{ in.} - 32 \text{ in.} = 4 \text{ in.}$$

$$\text{IN } \triangle CEG: \tan \beta = \frac{EG}{CG} = \frac{4 \text{ in.}}{48 \text{ in.}}; \beta = 4.764^\circ$$

$$\text{IN } \triangle BDH: \tan \gamma = \frac{BH}{DH} = \frac{24 \text{ in.}}{32 \text{ in.}}; \gamma = 36.87^\circ$$

FORCE TRIANGLE FOR 3 FORCES INTERSECTING AT E



LAW OF SINES

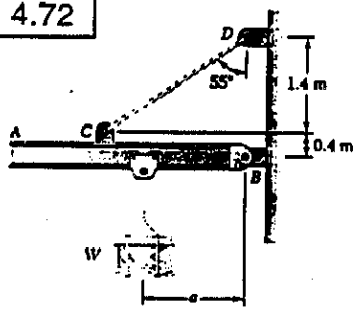
$$\frac{3 \text{ kips}}{\sin 32.106^\circ} = \frac{T_{BD}}{\sin 94.764^\circ} = \frac{C}{\sin 53.13^\circ}$$

$$T_{BD} = 5.625 \text{ kips}; C = 4.516 \text{ kips}$$

$$(a) T_{BD} = 5.63 \text{ kips}$$

$$(b) C = 4.52 \text{ kips} \angle 4.8^\circ$$

4.72

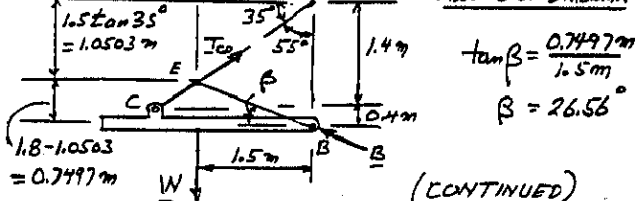


GIVEN:  
 $\alpha = 1.5 \text{ m}$   
 $W = 50 \text{ lb}$

FIND:  
(a) TENSION IN CABLE CD  
(b) REACTION AT B

3-FORCE BODY:  $W$  AND  $T_{CD}$  INTERSECT AT E

FREE-BODY DIAGRAM

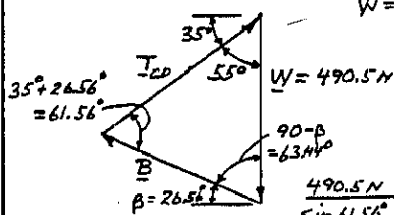


(CONTINUED)

4.72 CONTINUED

FORCE TRIANGLE

3 FORCES INTERSECT AT E  
 $W = (50 \text{ lb}) 9.81 \text{ m/s}^2 = 490.5 \text{ N}$



LAW OF SINES

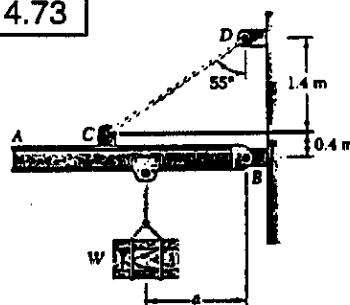
$$\frac{490.5 \text{ N}}{\sin 61.56^\circ} = \frac{T_{CD}}{\sin 63.44^\circ} = \frac{B}{\sin 55^\circ}$$

$$T_{CD} = 490.9 \text{ N}; B = 456.9 \text{ N}$$

$$(a) T_{CD} = 499 \text{ N}$$

$$(b) B = 457 \text{ N} \angle 76.6^\circ$$

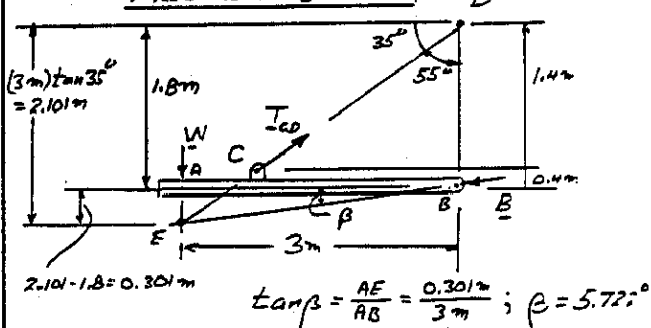
4.73



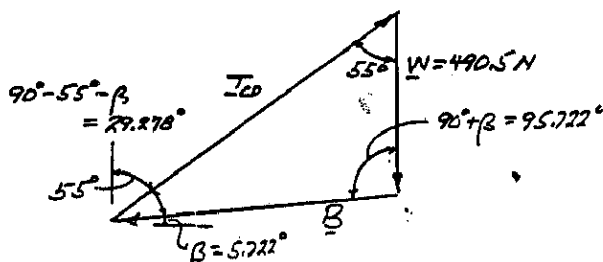
GIVEN:  
 $\alpha = 3 \text{ m}$   
 $W = 50 \text{ lb}$

FIND:  
(a) TENSION IN CABLE CD  
(b) REACTION AT B

3-FORCE BODY  $W$  AND  $T_{CD}$  INTERSECT AT E  
FREE-BODY DIAGRAM



FORCE TRIANGLE (3 FORCES INTERSECT AT E)  
 $W = (50 \text{ lb}) 9.81 \text{ m/s}^2 = 490.5 \text{ N}$



LAW OF SINES

$$\frac{490.5 \text{ N}}{\sin 29.278^\circ} = \frac{T_{CD}}{\sin 95.722^\circ} = \frac{B}{\sin 55^\circ}$$

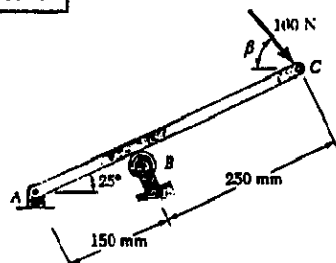
$$T_{CD} = 992.99 \text{ N}; B = 821.59 \text{ N}$$

$$(a) T_{CD} = 998 \text{ N}$$

$$(b) B = 822 \text{ N} \angle 5.7^\circ$$



4.74



GIVEN:  
 $\beta = 50^\circ$

FIND: REACTIONS  
AT A AND B.

FREE-BODY DIAGRAM (3-FORCE BODY)

REACTION A MUST PASS THROUGH  
POINT D WHERE 100-N FORCE  
AND B INTERSECT

IN RIGHT  $\triangle BCD$ :

$$\alpha = 90^\circ - 75^\circ = 15^\circ$$

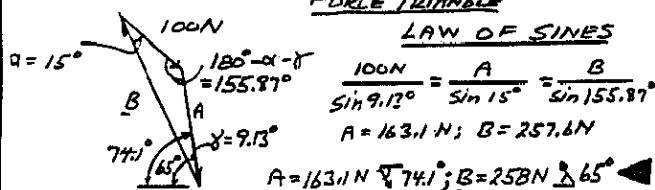
$$BD = 250 \tan 75^\circ = 933.0 \text{ mm}$$

IN RIGHT  $\triangle ABD$ :

$$\tan \gamma = \frac{AB}{BD} = \frac{150 \text{ mm}}{933 \text{ mm}}$$

$$\gamma = 9.13^\circ$$

DIMENSIONS  
IN mm.



FORCE TRIANGLE

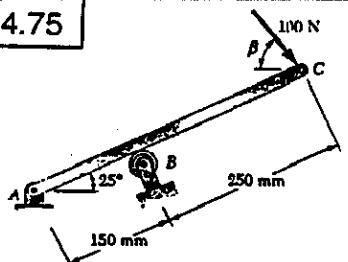
LAW OF SINES

$$\frac{100 \text{ N}}{\sin 9.13^\circ} = \frac{A}{\sin 15^\circ} = \frac{B}{\sin 155.87^\circ}$$

$$A = 163.1 \text{ N}; B = 257.6 \text{ N}$$

$$A = 163.1 \text{ N} \angle 74.1^\circ; B = 258 \text{ N} \angle 65^\circ$$

4.75



GIVEN:  
 $\beta = 80^\circ$

FIND:  
REACTIONS  
AT A AND B

FREE-BODY DIAGRAM (3-FORCE BODY)

REACTION A MUST  
PASS THROUGH POINT  
D WHERE 100-N  
FORCE AND B  
INTERSECT

IN RIGHT  $\triangle BCD$ :

$$\alpha = 90^\circ - 75^\circ = 15^\circ$$

$$BD = BC \tan 75^\circ = 250 \tan 75^\circ$$

$$BD = 933.0 \text{ mm}$$

IN RIGHT  $\triangle ABD$ :

$$\tan \gamma = \frac{AB}{BD} = \frac{150 \text{ mm}}{933 \text{ mm}}$$

$$\gamma = 9.13^\circ$$

(CONTINUED)

4.75 CONTINUED

FORCE TRIANGLE

LAW OF SINES

$$\frac{100 \text{ N}}{\sin 9.13^\circ} = \frac{A}{\sin 15^\circ} = \frac{B}{\sin 155.87^\circ}$$

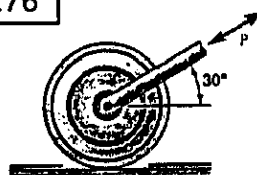
$$A = 163.1 \text{ N}$$

$$B = 257.6 \text{ N}$$

$$A = 163.1 \text{ N} \angle 55.9^\circ$$

$$B = 258 \text{ N} \angle 65^\circ$$

4.76



GIVEN: 40-lb ROLLER  
OF DIAMETER B in.  
THICKNESS OF  
TILE IS 0.3 in.

FIND: FORCE P TO MOVE  
ROLLER ON TO TILES IF  
ROLLER IS (a) PUSHED  $\leftarrow$ ,  
(b) PULLED  $\rightarrow$

GEOMETRY FOR EACH

CASE AS ROLLER COMES  
INTO CONTACT WITH TILE

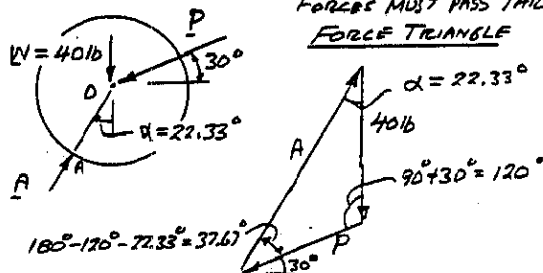
$$\alpha = \cos^{-1} \frac{2.7 \text{ in.}}{3 \text{ in.}}$$

$$\alpha = 22.33^\circ$$

(a) ROLLER PUSHED TO LEFT (3-FORCE BODY)

FORCES MUST PASS THROUGH O.

FORCE TRIANGLE



LAW OF SINES

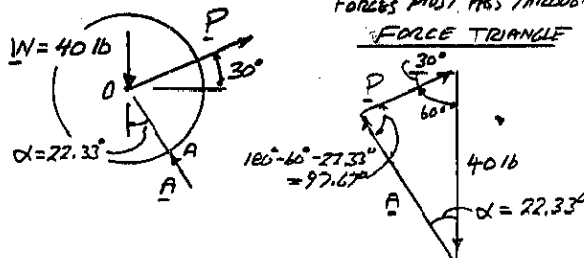
$$\frac{40 \text{ lb}}{\sin 37.67^\circ} = \frac{P}{\sin 22.33^\circ}; P = 24.86 \text{ lb}$$

$$P = 24.9 \text{ lb} \angle 30^\circ$$

(b) ROLLER PULLED TO RIGHT (3-FORCE BODY)

FORCES MUST PASS THROUGH C

FORCE TRIANGLE

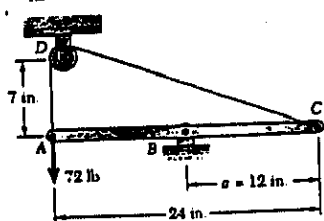


LAW OF SINES

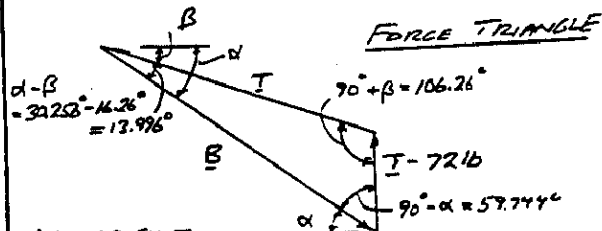
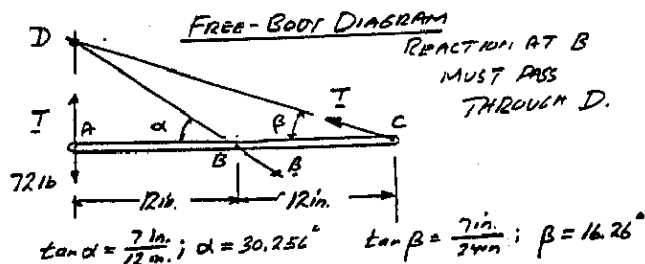
$$\frac{40 \text{ lb}}{\sin 97.67^\circ} = \frac{P}{\sin 22.33^\circ}; P = 15.33 \text{ lb}$$

$$P = 15.33 \text{ lb} \angle 30^\circ$$

4.77



FIND:  
TENSION  
IN CORD  
REACTION AT B



LAW OF SINES

$$\frac{T}{\sin 59.744^\circ} = \frac{T-72 \text{ lb}}{\sin 13.976^\circ} = \frac{B}{\sin 106.26^\circ}$$

$$T(\sin 13.976^\circ) = (T-72)(\sin 59.744^\circ)$$

$$T(0.24185) = (T-72)(0.86378)$$

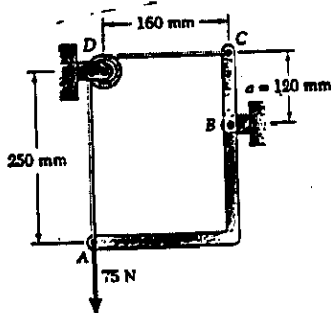
$$T = 100.00 \text{ lb}$$

$$B = (100 \text{ lb}) \frac{\sin 106.26^\circ}{\sin 59.744^\circ} = 111.14 \text{ lb}$$

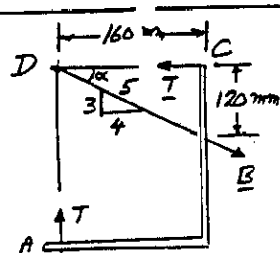
$$T = 100 \text{ lb}$$

$$B = 111 \text{ lb} \angle 30.3^\circ$$

4.78



FIND:  
TENSION  
IN CORD  
REACTION  
AT B



FORCE TRIANGLE

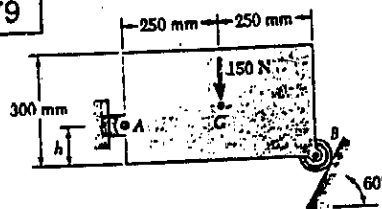
$$\frac{T}{4} = \frac{T-75 \text{ lb}}{3} = \frac{B}{5}$$

$$3T = 4T - 300; T = 300 \text{ lb}$$

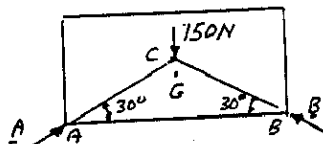
$$B = \frac{5}{4}T = \frac{5}{4}(300 \text{ lb}) = 375 \text{ lb}$$

$$B = 375 \text{ lb} \angle 36.9^\circ$$

4.79



FIND:  
REACTIONS AT  
A AND B WHEN  
(a)  $h = 0$   
(b)  $h = 200 \text{ mm}$

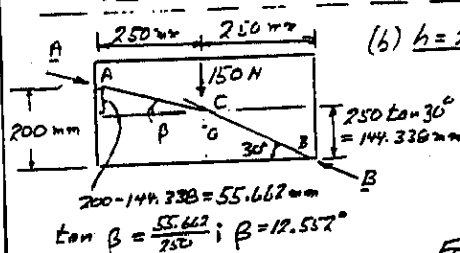


FREE-BODY DIAGRAM  
REACTION A MUST PASS  
THROUGH C WHERE 150 N  
WEIGHT AND B INTERSECT

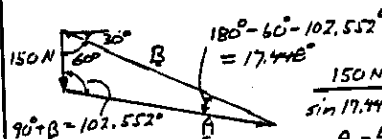
FORCE TRIANGLE IS EQUILATERAL

$$A = 150 \text{ N} \angle 30^\circ$$

$$B = 150 \text{ N} \angle 30^\circ$$



FORCE TRIANGLE  
LAW OF SINE



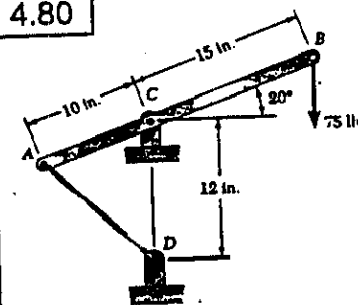
$$\frac{150 \text{ N}}{\sin 17.448^\circ} = \frac{A}{\sin 102.552^\circ} = \frac{B}{\sin 102.552^\circ}$$

$$A = 433.247 \text{ N}; B = 488.31 \text{ N}$$

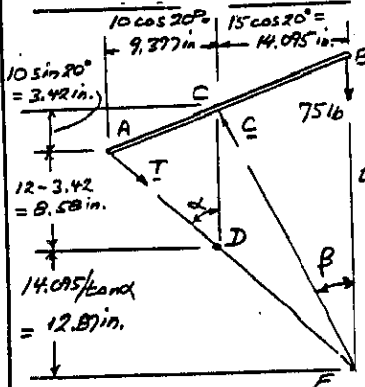
$$A = 433 \text{ N} \angle 12.6^\circ$$

$$B = 488 \text{ N} \angle 30^\circ$$

4.80



FIND:  
(a) TENSION  
IN CABLE AD  
(b) REACTION AT C



FREE-BODY DIAGRAM  
REACTION C MUST PASS  
THROUGH E WHERE 75 N  
FORCE AND T INTERSECT

$$\tan \alpha = \frac{9.377 \text{ in.}}{8.58 \text{ in.}}; \alpha = 47.2^\circ$$

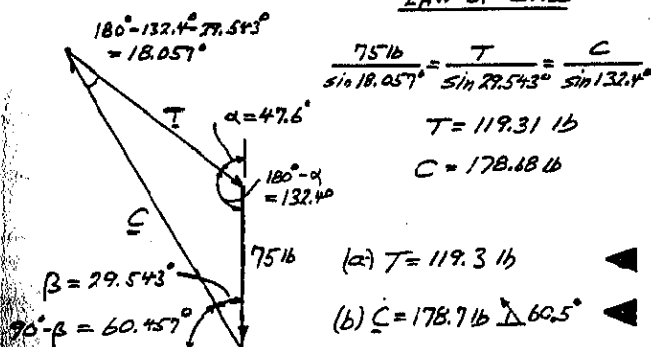
$$\tan \beta = \frac{14.095 \text{ in.}}{12.87 \text{ in.}} = \frac{14.095}{12.87}$$

$$\beta = 29.543^\circ$$

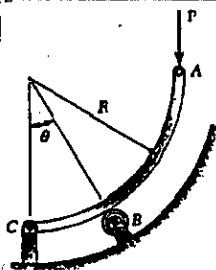
(CONTINUED)

# 4.80 CONTINUED

## FORCE TRIANGLE LAW OF SINES



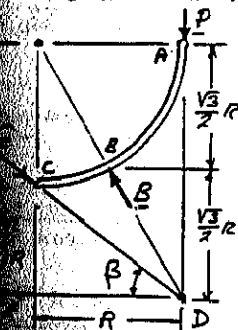
# 4.81



GIVEN:  
 $\theta = 30^\circ$

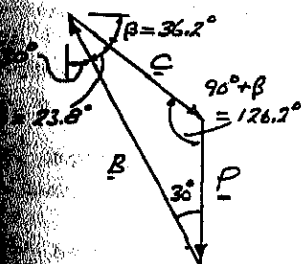
FIND: REACTION  
(a) AT B  
(b) AT C

FREE-BODY DIAGRAM (3-FORCE BODY)  
REACTION AT C MUST PASS THROUGH D WHERE  
FORCE P AND REACTION AT B INTERSECT



IN  $\Delta CDE$ :  
 $\tan \beta = \frac{(\sqrt{3}-1)R}{R} = \sqrt{3}-1$   
 $\beta = 36.2^\circ$

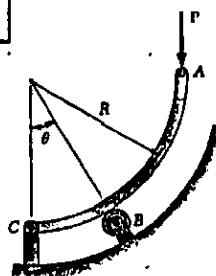
## FORCE TRIANGLE



$\frac{P}{\sin 36.2^\circ} = \frac{B}{\sin 126.2^\circ} = \frac{C}{\sin 23.8^\circ}$

$B = 2.00 P$ ;  $C = 1.239 P$   
 (a)  $B = 2 P \angle 60^\circ$   
 (b)  $C = 1.239 P \angle 36.2^\circ$

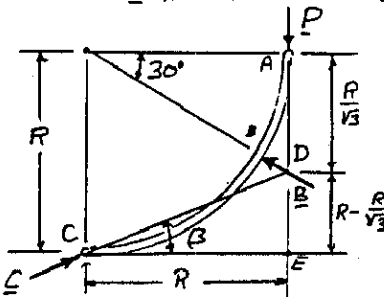
# 4.82



GIVEN:  
 $\theta = 60^\circ$

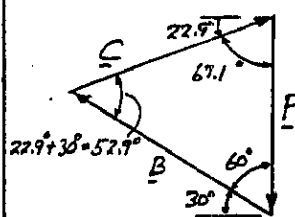
FIND: REACTION  
(a) AT B  
(b) AT C

FREE-BODY DIAGRAM (3-FORCE BODY)  
REACTION AT C MUST PASS THROUGH D WHERE  
FORCE P AND REACTION AT B INTERSECT.



IN  $\Delta CDE$ :  
 $\tan \beta = \frac{R - R/\sqrt{3}}{R} = 1 - \frac{1}{\sqrt{3}}$   
 $\beta = 22.9^\circ$

## FORCE TRIANGLE LAW OF SINES



$\frac{P}{\sin 57.9^\circ} = \frac{B}{\sin 67.1^\circ} = \frac{C}{\sin 22.9^\circ}$   
 $B = 1.155 P$ ;  $C = 1.086 P$   
 (a)  $B = 1.155 P \angle 30^\circ$   
 (b)  $C = 1.086 P \angle 22.9^\circ$

# 4.83 and 4.84

FOR EQUILIBRIUM,  
PROB. 4.83:

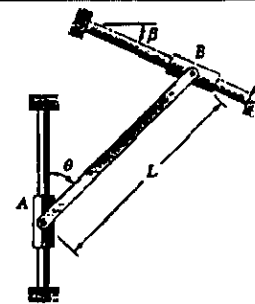
FIND:  $\theta = f(\beta)$ .

PROB. 4.84:

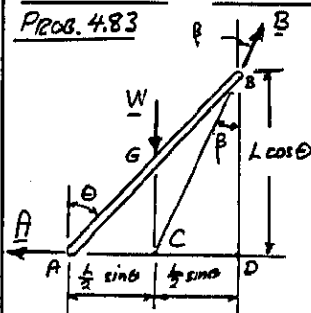
GIVEN:  $m = 8.829$ ,  $\beta = 30^\circ$ .

FIND: (a) ANGLE  $\theta$ .

(b) REACTIONS  
AT A AND B.



## PROB. 4.83

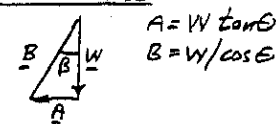


FREE-BODY DIAGRAM (3-FORCE BODY) FORCES INTERSECT AT C.

IN  $\Delta BCD$   
 $\tan \beta = \frac{\frac{1}{2} L \sin \theta}{L \cos \theta} = \frac{1}{2} \frac{\sin \theta}{\cos \theta}$

$\tan \theta = 2 \tan \beta$

## FORCE TRIANGLE

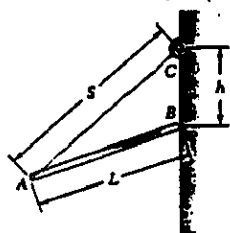


## PROB. 4.84

GIVEN:  $m = 8.829$ ;  $W = (8.829)(9.81 \text{ m/s}^2) = 78.48 \text{ N}$ ,  $\beta = 30^\circ$

(a)  $\tan \theta = 2 \tan 30^\circ = 1.1547$   $\theta = 49.1^\circ$   
 (b)  $A = W \tan \theta = (78.48 \text{ N}) \tan 30^\circ$   $A = 45.3 \text{ N}$   
 $B = W / \cos \beta = (78.48 \text{ N}) / \cos 30^\circ$   $B = 90.6 \text{ N} \angle 60^\circ$

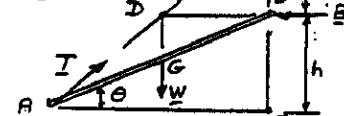
# 4.85 and 4.86



PROB. 4.85:

$$AC = S$$

$$AB = L$$



$$\text{IN } \triangle ACE: (2h)^2 + (AE)^2 = S^2 \quad (1)$$

$$\text{IN } \triangle ABE: h^2 + (AE)^2 = L^2 \quad (2)$$

$$EQ(1) - EQ(2): 3h^2 = S^2 - L^2 \quad (3)$$

FREE-BODY DIAGRAM  
(3-FORCE BODY)

THE FORCES  $W$  AND  $B$   
MUST INTERSECT AT  $D$   
ON LINE OF ACTION OF  $T$ .

$$h = \sqrt{(S^2 - L^2)/3}$$

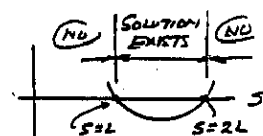
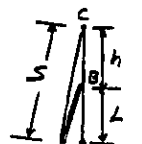
AS LENGTH  $S$  INCREASES  
RELATIVE TO  $L$ , ANGLE  $\theta$   
INCREASES UNTIL ROD  
AB IS VERTICAL AND  
 $h \geq S - L$

$$\sqrt{(S^2 - L^2)/3} \geq S - L$$

$$S^2 - L^2 \geq 3(S^2 - 2SL + L^2)$$

$$0 \geq 2S^2 - 6SL + 4L^2$$

$$0 \geq 2(S - L)(S - 2L)$$



$\therefore$  NO SOLUTION FOR  $S > 2L$

PROB 4.86

$$L = 20 \text{ in.}, S = 30 \text{ in.}, W = 10 \text{ lb}$$

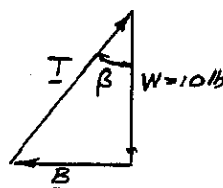
$$h = \sqrt{(S^2 - L^2)/3} = \sqrt{(30^2 - 20^2)/3} = \sqrt{500/3}$$

$$(a) \quad h = 12.91 \text{ in.}$$

$$\text{IN } \triangle ACE: \cos \beta = \frac{2h}{S} = \frac{2(12.91 \text{ in.})}{30 \text{ in.}} = 0.8607$$

$$\beta = 30.609^\circ$$

FORCE TRIANGLE



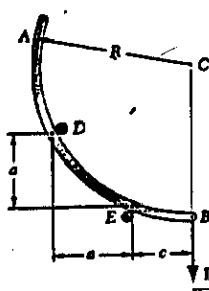
$$T = \frac{W}{\cos \beta} = \frac{10 \text{ lb}}{\cos 30.609^\circ}$$

$$(b) \quad T = 11.62 \text{ lb}$$

$$B = W \tan \beta = (10 \text{ lb}) \tan 30.609^\circ$$

$$(c) \quad B = 5.92 \text{ lb}$$

# 4.87

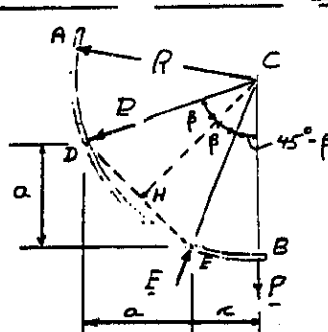


GIVEN:

$$a = 20 \text{ mm}$$

$$R = 100 \text{ mm}$$

FIND: DISTANCE  $C$   
CORRESPONDING  
TO EQUILIBRIUM



SLOPE OF DE IS  $\triangle 45^\circ$   
 $\therefore$  SLOPE OF CH IS  $\triangle 45^\circ$

$$DE = \sqrt{2} a$$

$$DH = HE = \frac{1}{2} DE = \frac{\sqrt{2}}{2} a$$

IN  $\triangle DHC$  AND IN  $\triangle CEN$ :

$$\sin \beta = \frac{\frac{\sqrt{2}}{2} a}{R} = \frac{a}{\sqrt{2} R}$$

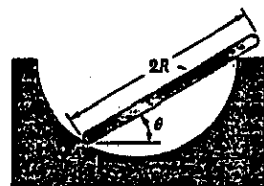
$$C = R \sin(45^\circ - \beta)$$

FOR  $a = 20 \text{ mm}, R = 100 \text{ mm}$

$$\sin \beta = \frac{20 \text{ mm}}{\sqrt{2}(100 \text{ mm})} \quad \beta = 8.13^\circ$$

$$C = (100 \text{ mm}) \sin(45^\circ - 8.13^\circ) \quad C = 60.0 \text{ mm}$$

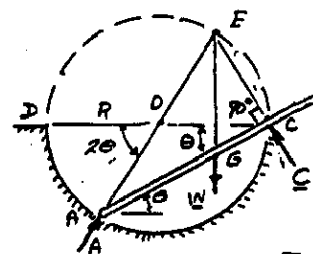
# 4.88



GIVEN: RADIUS  
OF BOWL IS  $R$ .

FIND: ANGLE  $\theta$   
FOR EQUILIBRIUM

FREE-BODY DIAGRAM  
(3-FORCE BODY)



POINT E IS POINT OF  
INTERSECTION OF  
A AND C.

SINCE A PASSES  
THROUGH O AND SINCE  
C IS PERPENDICULAR

TO ROD, TRIANGLE ACE IS A  
RIGHT TRIANGLE INSCRIBED IN  
THE CIRCLE. THUS E IS A POINT  
ON THE CIRCLE.

NOTE THAT  $\angle DOA$  IS THE CENTRAL ANGLE  
CORRESPONDING TO THE INSCRIBED ANGLE  $\angle DCA$ .

THUS  $\angle DOA = 2\theta$

HORIZONTAL PROJECTIONS OF AE AND AC ARE EQUAL.

$$(AE) \cos 2\theta = (AC) \cos \theta$$

$$(2R) \cos 2\theta = (R) \cos \theta$$

$$\text{SFT: } \cos 2\theta = 2 \cos^2 \theta - 1$$

$$4 \cos^2 \theta - 2 = \cos \theta$$

$$4 \cos^2 \theta - \cos \theta - 2 = 0$$

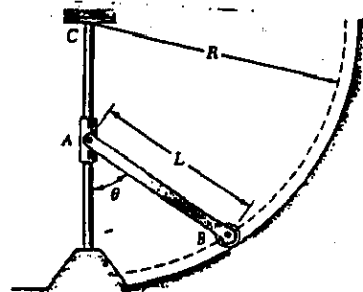
$$\cos \theta = 0.84307$$

$$\theta = 32.5^\circ$$

$$\cos \theta = -0.59307$$

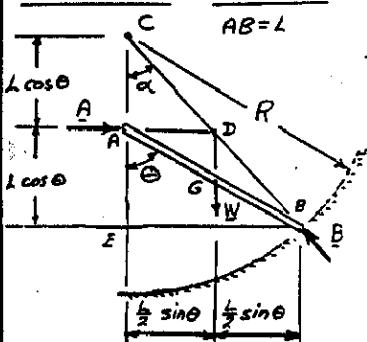
$$\theta = 126.4^\circ \text{ (DISCARD)}$$

# 4.89 and 4.90



PROB. 4.89:  
DERIVE EQUATION  
IN  $\theta$ ,  $L$ , AND  $R$   
FOR POSITION OF  
EQUILIBRIUM

PROB. 4.90:  
GIVEN:  $L = 15$  in.,  
 $R = 20$  in. AND  $W = 10$  lb  
FIND: ANGLE  $\theta$  FOR  
EQUILIBRIUM



FREE-BODY DIAGRAM  
(3-FORCE BODY)

REACTION  $B$  MUST  
PASS THROUGH  $D$   
WHERE  $B$  AND  $W$   
INTERSECT.

NOTE THAT  $\triangle ABC$  AND  
 $\triangle BGD$  ARE SIMILAR.  
 $\therefore AC = AE = L \cos \theta$

PROB. 4.89

$$\begin{aligned} \text{IN } \triangle ABC: (CE)^2 + (BE)^2 &= (BC)^2 \\ (2L \cos \theta)^2 + (L \sin \theta)^2 &= L^2 \\ (RL)^2 &= 4 \cos^2 \theta + \sin^2 \theta \\ (RL)^2 &= 4 \cos^2 \theta + 1 - \cos^2 \theta \\ (RL)^2 &= 3 \cos^2 \theta + 1 \end{aligned}$$

$$\cos^2 \theta = \frac{1}{3} \left[ \left( \frac{R}{L} \right)^2 - 1 \right]$$

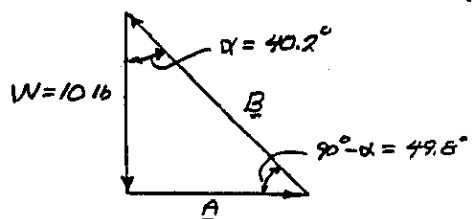
PROB. 4.90. FOR  $L = 15$  in.,  $R = 20$  in., AND  $W = 10$  lb.

$$\cos^2 \theta = \frac{1}{3} \left[ \left( \frac{20}{15} \right)^2 - 1 \right]; \cos \theta = 57.39^\circ; \theta = 57.4^\circ$$

$$\text{IN } \triangle ABC: \tan \alpha = \frac{BE}{CE} = \frac{L \sin \theta}{2L \cos \theta} = \frac{1}{2} \tan \theta$$

$$\tan \alpha = \frac{1}{2} \tan 57.39^\circ = 0.8452; \alpha = 40.2^\circ$$

FORCE TRIANGLE



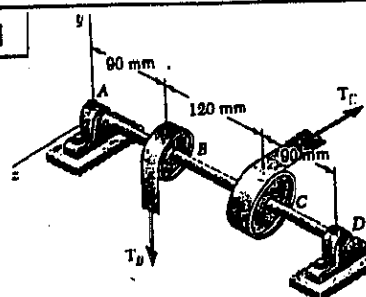
$$A = W \tan \alpha = (10 \text{ lb}) \tan 40.2^\circ = 8.45 \text{ lb}$$

$$B = W / \cos \alpha = (10 \text{ lb}) / \cos 40.2^\circ = 13.09 \text{ lb}$$

$$A = 8.45 \text{ lb}$$

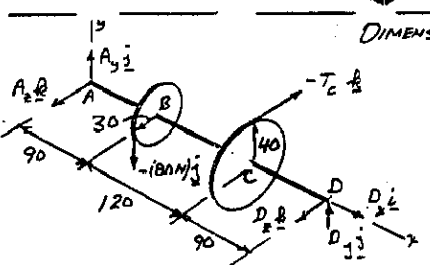
$$B = 13.09 \text{ lb} \angle 49.8^\circ$$

# 4.91



GIVEN:  $T_B = 80$  N  
 $r_B = 30$  mm  
 $r_C = 40$  mm

FIND:  
REACTIONS  
AT A AND D.



DIMENSIONS IN mm

WE HAVE 6 UNKNOWN  
AND 6 EGS. OF  
EQUILIBRIUM.

$$\begin{aligned} \Sigma M_A = 0: (90 \text{ i} + 30 \text{ j}) \times (-80 \text{ j}) + (210 \text{ i} + 40 \text{ j}) \times (-T_C \text{ j}) \\ + (300 \text{ i}) \times (D_x \text{ i} + D_y \text{ j} + D_z \text{ k}) = 0 \\ -7200 \text{ k} + 2400 \text{ i} + 210 T_C \text{ j} - 40 T_C \text{ i} + 300 D_x \text{ j} - 300 D_y \text{ i} = 0 \end{aligned}$$

EQUATE COEFFICIENTS OF UNIT VECTORS TO ZERO:

$$\textcircled{1} 2400 - 40 T_C = 0 \quad T_C = 60 \text{ N}$$

$$\textcircled{2} 210 T_C - 300 D_x = 0; (210 \times 60) - 300 D_x = 0; D_x = 42 \text{ N}$$

$$\textcircled{3} -7200 + 300 D_y = 0 \quad D_y = 24 \text{ N}$$

$$\Sigma F_x = 0: D_x = 0$$

$$\Sigma F_y = 0: A_y + D_y - 80 \text{ N} = 0$$

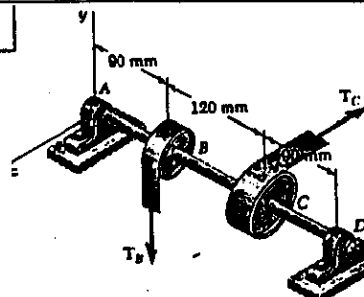
$$A_y = 80 - 24 = 56 \text{ N}$$

$$\Sigma F_z = 0: A_z + D_z - 60 \text{ N} = 0$$

$$A_z = 60 - 42 = 18 \text{ N}$$

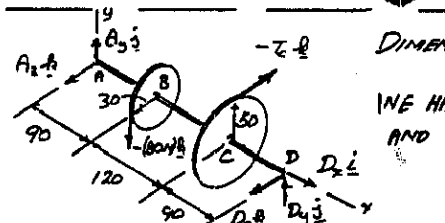
$$A = (56 \text{ N}) \text{ j} + (18 \text{ N}) \text{ k}; D = (24 \text{ N}) \text{ j} + (42 \text{ N}) \text{ k}$$

# 4.92



GIVEN:  $T_B = 80$  N  
 $r_B = 30$  mm  
 $r_C = 50$  mm

FIND:  
REACTIONS  
AT A AND D



DIMENSIONS IN mm

WE HAVE 6 UNKNOWN  
AND 6 EGS. OF  
EQUILIBRIUM

$$\begin{aligned} \Sigma M_A = 0: (90 \text{ i} + 30 \text{ j}) \times (-80 \text{ j}) + (210 \text{ i} + 50 \text{ j}) \times (-T_C \text{ j}) + (300 \text{ i}) \times (D_x \text{ i} + D_y \text{ j} + D_z \text{ k}) = 0 \\ -7200 \text{ k} + 2400 \text{ i} + 210 T_C \text{ j} - 50 T_C \text{ i} + 300 D_x \text{ j} - 300 D_y \text{ i} = 0 \end{aligned}$$

EQUATE COEFFICIENTS OF UNIT VECTORS TO ZERO:

$$\textcircled{1} 2400 - 50 T_C = 0 \quad T_C = 48 \text{ N}$$

$$\textcircled{2} 210 T_C - 300 D_x = 0; (210 \times 48) - 300 D_x = 0; D_x = 33.6 \text{ N}$$

$$\textcircled{3} -7200 + 300 D_y = 0 \quad D_y = 24 \text{ N}$$

$$\Sigma F_x = 0: D_x = 0$$

$$\Sigma F_y = 0: A_y + D_y - 80 \text{ N} = 0;$$

$$A_y = 80 - 24 = 56 \text{ N}$$

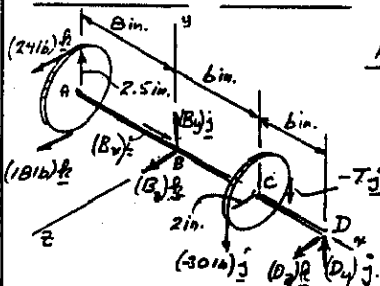
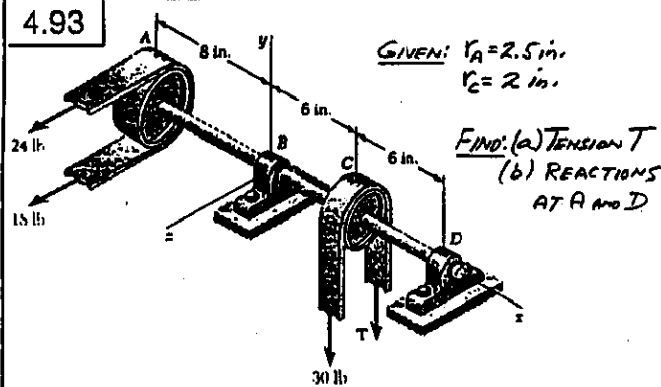
$$\Sigma F_z = 0: A_z + D_z - 48 = 0;$$

$$A_z = 48 - 33.6 = 14.4 \text{ N}$$

$$A = (56 \text{ N}) \text{ j} + (14.4 \text{ N}) \text{ k}$$

$$D = (24 \text{ N}) \text{ j} + (33.6 \text{ N}) \text{ k}$$

4.93



FREE-BODY DIAGRAM  
 WE HAVE 6 UNKNOWN  
 AND 6 EQUATIONS OF  
 EQUILIBRIUM

$$\sum M_B = 0: (-8\hat{i} + 2.5\hat{j}) \times (24\hat{j}) + (-8\hat{i} - 2.5\hat{j}) \times (15\hat{j}) + (6\hat{i} + 2\hat{j}) \times (-30\hat{j}) + (6\hat{i} - 2\hat{j}) \times (-T\hat{j}) + (12\hat{i}) \times (D_y\hat{j} + D_z\hat{k}) = 0$$

$$192\hat{j} + 60\hat{k} + 144\hat{j} - 45\hat{k} - 180\hat{k} + 60\hat{k} - 6T\hat{k} - 2T\hat{k} + 12D_y\hat{k} - 12D_z\hat{j} = 0$$

EQUATING TO ZERO THE COEFFICIENTS OF UNIT VECTORS:

$$\begin{aligned} \textcircled{1} \quad 60 - 45 + 60 - 27 &= 0 & T &= 37.5 \text{ lb} \\ \textcircled{2} \quad 192 + 144 - 12D_z &= 0 & D_z &= 28 \text{ lb} \\ \textcircled{3} \quad -180 - 6(37.5) + 12D_y &= 0 & D_y &= 33.75 \text{ lb} \end{aligned}$$

$$\sum F_x = 0: B_x = 0$$

$$B_x = 0$$

$$\sum F_y = 0: B_y - 30 - 37.5 + 33.75 = 0$$

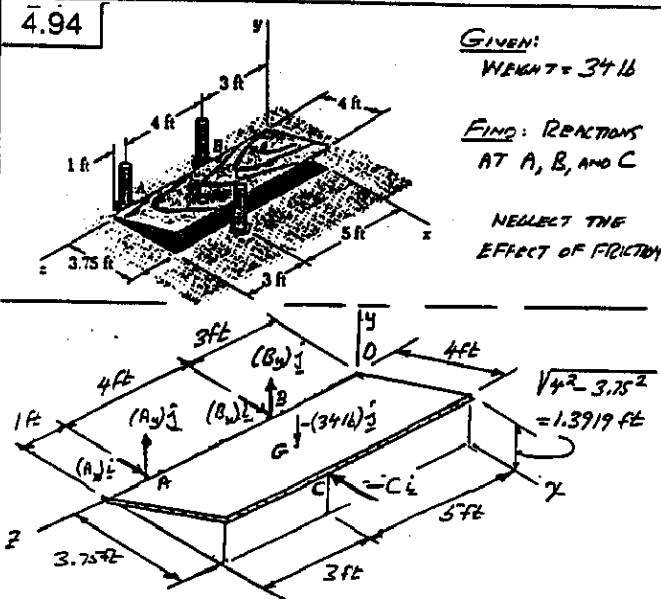
$$B_y = 33.75 \text{ lb}$$

$$\sum F_z = 0: B_z + 24 + 15 + 28 = 0$$

$$B_z = -70 \text{ lb}$$

$$\begin{aligned} \vec{B} &= (33.75\hat{j}) - (70\hat{k}) \\ \vec{D} &= (33.75\hat{j}) + (28\hat{k}) \end{aligned}$$

4.94



(CONTINUED)

$$\vec{r}_{AB} = \frac{3.75}{2}\hat{i} + \frac{1.399}{2}\hat{j} + \hat{k}$$

4.94 CONTINUED

WE HAVE 5 UNKNOWN AND  
 6 EQS. OF EQUILIBRIUM.

PLYWOOD SHEET IS FREE TO MOVE IN Z DIRECTION,  
 BUT EQUILIBRIUM IS MAINTAINED ( $\sum F_z = 0$ )

$$\sum M_B = 0: r_{BA} \times (A_x\hat{i} + A_y\hat{j}) + r_{CB} \times (-C\hat{i}) + r_{GB} \times (-W\hat{j}) = 0$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 4 \\ 0 & 0 & 4 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3.75 & 1.399 & 2 \\ 0 & 0 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1.875 & 0.699 & 1 \\ 0 & -34 & 0 \end{vmatrix} = 0$$

$$-4A_x\hat{i} + 4A_y\hat{j} - 2C\hat{i} + 1.399C\hat{k} + 34\hat{i} - 63.75\hat{k} = 0$$

EQUATING COEFFICIENTS OF UNIT VECTORS TO ZERO:

$$\textcircled{1} \quad -4A_x + 34 = 0$$

$$A_x = 8.5 \text{ lb}$$

$$\textcircled{2} \quad -2C + 4A_y = 0; \quad A_y = \frac{1}{2}C = \frac{1}{2}(45.80) = 22.9 \text{ lb}$$

$$\textcircled{3} \quad 1.399C - 63.75 = 0; \quad C = 45.80 \text{ lb} \quad C = 45.8 \text{ lb}$$

$$\sum F_z = 0: A_z + B_z - C = 0; \quad B_z = 45.8 - 22.9 = 22.9 \text{ lb}$$

$$\sum F_y = 0: A_y + B_y - W = 0; \quad B_y = 34 - 22.9 = 11.1 \text{ lb}$$

$$\vec{A} = (8.5\hat{i}) + (11.1\hat{j}) + (22.9\hat{k}) \quad \vec{B} = (22.9\hat{i}) + (11.1\hat{j}) + (22.9\hat{k}) \quad \vec{C} = (45.8\hat{i})$$

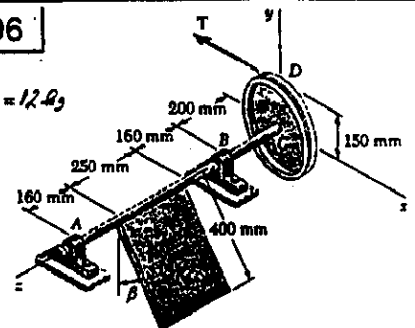
4.95 and 4.96

GIVEN: MASS OF PLATE = 12 kg

FIND: (a) TENSION  $T$   
 (b) REACTIONS AT A AND B

PROB. 4.95:  $\beta = 30^\circ$

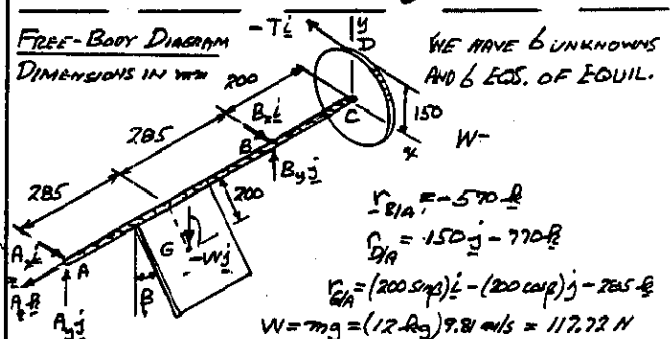
PROB. 4.96:  $\beta = 60^\circ$



FREE-BODY DIAGRAM

DIMENSIONS IN mm

WE HAVE 6 UNKNOWN  
 AND 6 EQS. OF EQUIL.



$$\vec{r}_{BA} = -570\hat{k}$$

$$\vec{r}_{BA} = 150\hat{j} - 770\hat{k}$$

$$\vec{r}_{BA} = (200\sin\beta)\hat{i} - (200\cos\beta)\hat{j} - 285\hat{k}$$

$$W = mg = (12 \text{ kg})(9.81 \text{ m/s}^2) = 117.72 \text{ N}$$

$$\sum M_A = 0: r_{GA} \times (W\hat{j}) + r_{BA} \times (-T\hat{i}) + r_{BA} \times (B_x\hat{i} + B_y\hat{j}) = 0$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 200\sin\beta & -200\cos\beta & -285 \\ 0 & 0 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 150 & -770 \\ 0 & 0 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & -570 \\ B_x & B_y & 0 \end{vmatrix} = 0$$

$$-285W\hat{i} - (200\sin\beta)W\hat{j} + 770T\hat{i} + 150T\hat{j} + 570B_x\hat{i} - 570B_y\hat{j} = 0$$

EQUATING COEFFICIENTS OF UNIT VECTORS

$$\textcircled{1} \quad -285W + 570B_x = 0; \quad B_x = (285/570)W \quad (1)$$

$$\textcircled{2} \quad 770T - 570B_y = 0; \quad B_y = (770/570)T \quad (2)$$

$$\textcircled{3} \quad -(200\sin\beta)W + 150T = 0; \quad T = (200/150)\sin\beta W \quad (3)$$

$$\sum F_x = 0: A_x + B_x - T = 0$$

$$A_x = T - B_x \quad (4)$$

$$\sum F_y = 0: A_y + B_y - W = 0$$

$$A_y = W - B_y \quad (5)$$

$$\sum F_z = 0: A_z = 0$$

$$A_z = 0 \quad (6)$$

(CONTINUED)

# 4.95 and 4.96 CONTINUED

PROB 4.95:  $\beta = 30^\circ$ ,  $W = 117.72 \text{ N}$

$$\begin{aligned} \text{EQ. (1): } B_y &= (285/570)117.72 \text{ N} = 58.86 \text{ N} \\ \text{EQ. (2): } T &= (200/150)(\sin 30^\circ)117.72 \text{ N} = 78.48 \text{ N} \\ \text{EQ. (3): } B_x &= (770/570)78.48 \text{ N} = 106.02 \text{ N} \\ \text{EQ. (4): } A_x &= 78.48 \text{ N} - 106.02 \text{ N} = -27.54 \text{ N} \\ \text{EQ. (5): } A_y &= 117.72 \text{ N} - 58.86 \text{ N} = 58.86 \text{ N} \\ \text{EQ. (6): } A_z &= 0 \end{aligned}$$

$$(a) T = 78.5 \text{ N}$$

$$(b) A = -(27.5 \text{ N})\hat{i} + (58.9 \text{ N})\hat{j}; B = (106.0 \text{ N})\hat{i} + (58.9 \text{ N})\hat{j}$$

PROB 4.96:  $\beta = 60^\circ$ ,  $W = 117.72 \text{ N}$

$$\begin{aligned} \text{EQ. (1): } B_y &= (285/570)117.72 \text{ N} = 58.86 \text{ N} \\ \text{EQ. (2): } T &= (200/150)(\sin 60^\circ)117.72 \text{ N} = 135.93 \text{ N} \\ \text{EQ. (3): } B_x &= (770/570)135.93 \text{ N} = 183.63 \text{ N} \\ \text{EQ. (4): } A_x &= 135.93 \text{ N} - 183.63 \text{ N} = -47.70 \text{ N} \\ \text{EQ. (5): } A_y &= 117.72 \text{ N} - 58.86 \text{ N} = 58.86 \text{ N} \\ \text{EQ. (6): } A_z &= 0 \end{aligned}$$

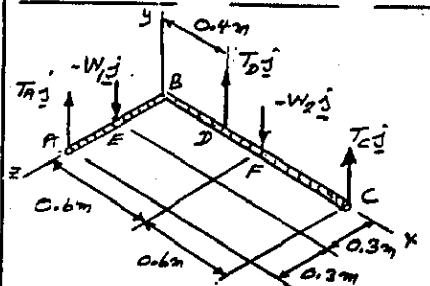
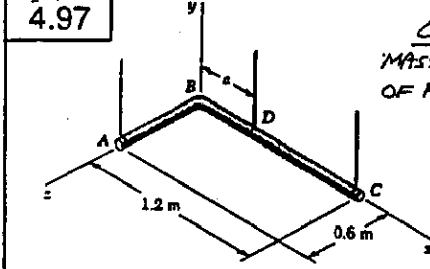
$$(a) T = 135.9 \text{ N}$$

$$(b) A = -(47.7 \text{ N})\hat{i} + (58.9 \text{ N})\hat{j}; B = (183.6 \text{ N})\hat{i} + (58.9 \text{ N})\hat{j}$$

4.97

GIVEN:  $a = 0.4 \text{ m}$ ,  
MASS PER UNIT LENGTH  
OF PIPES:  $m' = 8.8 \text{ kg/m}$

FIND: TENSION  
IN WIRES AT  
A, B, AND C.



$$\begin{aligned} W_1 &= 0.6 \text{ m}'g \\ W_2 &= 1.2 \text{ m}'g \end{aligned}$$

$$\begin{aligned} \Sigma M_D = 0: & \quad \hat{i} \hat{n}_D \times T_A \hat{j} + \hat{r}_{ED} \times (-W_1 \hat{j}) + \hat{r}_{FD} \times (-W_2 \hat{j}) + \hat{r}_{CD} \times T_C \hat{j} = 0 \\ & \quad (-0.4\hat{i} + 0.6\hat{j}) \times T_A \hat{j} + (-0.4\hat{i} + 0.3\hat{j}) \times (-W_1 \hat{j}) + 0.2\hat{i} \times (-W_2 \hat{j}) \\ & \quad + 0.8\hat{i} \times T_C \hat{j} = 0 \end{aligned}$$

$$-0.4T_A \hat{k} - 0.6T_A \hat{i} + 0.4W_1 \hat{k} + 0.3W_1 \hat{i} - 0.2W_2 \hat{k} + 0.8T_C \hat{k} = 0$$

EQUATE COEFFICIENTS OF UNIT VECTORS TO ZERO.

$$\textcircled{C} \quad -0.6T_A + 0.3W_1 = 0; \quad T_A = \frac{1}{2}W_1 = \frac{1}{2}(0.6 \text{ m})g = 0.3 \text{ m}'g$$

$$\textcircled{A} \quad -0.4T_A + 0.4W_1 - 0.2W_2 + 0.8T_C = 0$$

$$-0.4(0.3 \text{ m}'g) + 0.4(0.6 \text{ m}'g) - 0.2(1.2 \text{ m}'g) + 0.8T_C = 0$$

$$T_C = (0.12 - 0.24 + 0.24) \text{ m}'g / 0.8 = 0.15 \text{ m}'g$$

$$\Sigma F_y = 0: T_A + T_C + T_B - W_1 - W_2 = 0$$

$$0.3 \text{ m}'g + 0.15 \text{ m}'g + T_B - 0.6 \text{ m}'g - 1.2 \text{ m}'g = 0$$

$$T_B = 1.35 \text{ m}'g$$

$$m'g = (8.8 \text{ kg/m})(9.81 \text{ m/s}^2) = 78.48 \text{ N/m}$$

$$T_A = 0.3 \text{ m}'g = 0.3 \times 78.48 \quad T_A = 23.5 \text{ N}$$

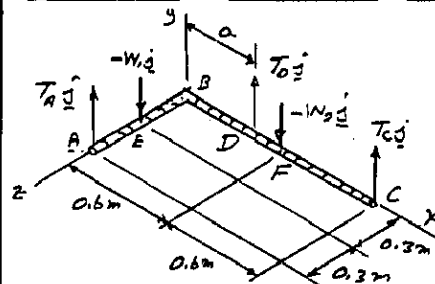
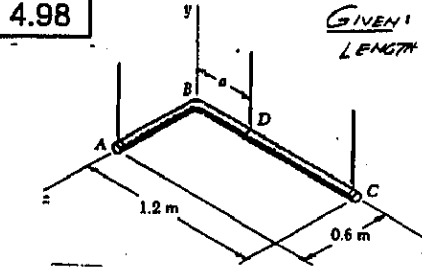
$$T_B = 1.35 \text{ m}'g = 1.35 \times 78.48 \quad T_B = 117.7 \text{ N}$$

$$T_C = 1.35 \text{ m}'g = 1.35 \times 78.48 \quad T_C = 105.9 \text{ N}$$

4.98

GIVEN: MASS PER UNIT  
LENGTH OF PIPES:  $m' = 8.8 \text{ kg/m}$

FIND: (a) LARGEST  
VALUE OF  $a$  FOR  
NO TIPPING  
(b) CORRESPONDING  
TENSION  
IN EACH WIRE



$$\begin{aligned} W_1 &= 0.6 \text{ m}'g \\ W_2 &= 1.2 \text{ m}'g \end{aligned}$$

$$\Sigma M_D = 0: \hat{i} \hat{n}_D \times T_A \hat{j} + \hat{r}_{ED} \times (-W_1 \hat{j}) + \hat{r}_{FD} \times (-W_2 \hat{j}) + \hat{r}_{CD} \times T_C \hat{j} = 0$$

$$(-a\hat{i} + 0.6\hat{j}) \times T_A \hat{j} + (-a\hat{i} + 0.3\hat{j}) \times (-W_1 \hat{j}) + (0.6 - a)\hat{i} \times (-W_2 \hat{j}) + (1.2 - a)\hat{i} \times T_C \hat{j} = 0$$

$$-Ta \hat{k} - 0.6Ta \hat{i} + W_1 a \hat{k} + 0.3W_1 \hat{i} - W_2(0.6 - a)\hat{k} + T_C(1.2 - a)\hat{k} = 0$$

$$\textcircled{C} \quad -0.6Ta + 0.3W_1 = 0; \quad T_A = \frac{1}{2}W_1 = \frac{1}{2}(0.6 \text{ m})g = 0.3 \text{ m}'g$$

$$\textcircled{A} \quad -Ta + W_1 a - W_2(0.6 - a) + T_C(1.2 - a) = 0$$

$$-0.3 \text{ m}'g a + 0.6 \text{ m}'g a - 1.2 \text{ m}'g(0.6 - a) + T_C(1.2 - a) = 0$$

$$T_C = \frac{0.3a - 0.6a + 1.2(0.6 - a)}{1.2 - a} \quad \text{FOR MAX } a \text{ AND NO TIPPING, } T_C = 0$$

$$(a) \quad -0.3a + 1.2(0.6 - a) = 0$$

$$-0.3a + 0.72 - 1.2a = 0$$

$$1.5a = 0.72$$

$$a = 0.48 \text{ m}$$

$$(b) \text{ REACTIONS: } m'g = (8.8 \text{ kg/m})(9.81 \text{ m/s}^2) = 78.48 \text{ N/m}$$

$$T_A = 0.3 \text{ m}'g = 0.3 \times 78.48 = 23.535 \text{ N}$$

$$T_A = 23.5 \text{ N}$$

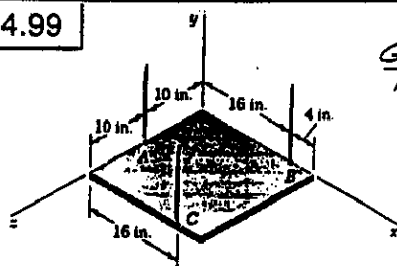
$$\Sigma F_y = 0: T_A + T_C + T_B - W_1 - W_2 = 0$$

$$T_A + 0 + T_B - 0.6 \text{ m}'g - 1.2 \text{ m}'g = 0$$

$$T_B = 1.8 \text{ m}'g - T_A = 1.8 \times 78.48 - 23.535 = 117.67 \text{ N}$$

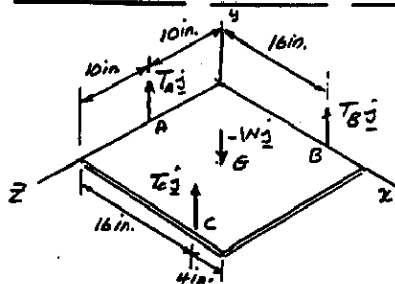
$$T_B = 117.7 \text{ N}$$

4.99



GIVEN: WEIGHT OF  
PLATE  $W = 56 \text{ lb}$

FIND: TENSION  
IN EACH WIRE



$$\sum M_A = 0: \sum \mathbf{r}_{GA} \times \mathbf{T}_B \mathbf{j} + \sum \mathbf{r}_{CA} \times \mathbf{T}_C \mathbf{j} + \sum \mathbf{r}_{GA} \times (-W \mathbf{j}) = 0$$

$$(16\mathbf{i} - 10\mathbf{j}) \times T_B \mathbf{j} + (16\mathbf{i} + 10\mathbf{j}) \times T_C \mathbf{j} + 10\mathbf{i} \times (-W \mathbf{j}) = 0$$

$$16T_B \mathbf{k} + 10T_C \mathbf{k} - 10W \mathbf{k} = 0$$

EQUATE COEFFICIENTS OF UNIT VECTORS TO ZERO

$$\textcircled{1} 10T_B - 10T_C = 0; T_B = T_C$$

$$\textcircled{2} 16T_B + 16T_C - 10W = 0$$

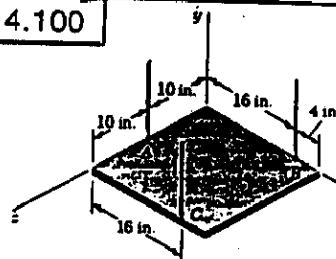
$$16T_B + 16T_B - 10(56 \text{ lb}) = 0; T_B = T_C = 17.5 \text{ lb}$$

$$\sum F_y = 0: T_A + T_B + T_C - W = 0$$

$$T_A + 17.5 + 17.5 - 56 = 0$$

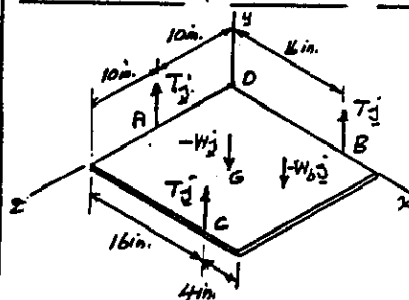
$$T_A = 21.0 \text{ lb}$$

4.100



GIVEN: WEIGHT OF  
PLATE  $W = 56 \text{ lb}$

FIND: LONGEST BLOCK  
TO P. - ON PLATE  
FOR WHICH TENSIONS  
IN THE 3 WIRES  
ARE DOWN



LET  $-W_b \mathbf{j}$  BE THE  
WEIGHT OF BLOCK AND  
 $x$  AND  $z$  THE BLOCK'S  
COORDINATES.

SINCE TENSIONS  
IN WIRES ARE EQUAL,  
LET  $T_A = T_B = T_C = T$

$$\sum M_G = 0: \sum \mathbf{r}_{GA} \times \mathbf{T}_B \mathbf{j} + \sum \mathbf{r}_{GB} \times \mathbf{T}_C \mathbf{j} + \sum \mathbf{r}_{GP} \times (-W_b \mathbf{j}) + (\mathbf{r}_{GA} + \mathbf{r}_{GB}) \times (-W \mathbf{j}) = 0$$

$$(10\mathbf{i} - 10\mathbf{j}) \times T \mathbf{j} + (16\mathbf{i} - 10\mathbf{j}) \times T \mathbf{j} + (16\mathbf{i} + 20\mathbf{j}) \times (-W_b \mathbf{j})$$

$$+ (10\mathbf{i} + 10\mathbf{j}) \times (-W \mathbf{j}) + (\mathbf{r}_{GA} + \mathbf{r}_{GB}) \times (-W \mathbf{j}) = 0$$

$$-10T \mathbf{k} + 16T \mathbf{k} + 16T \mathbf{k} - 20T \mathbf{k} - 10W_b \mathbf{k} + 10W_b \mathbf{k} - W_b x \mathbf{k} + W_b z \mathbf{k} = 0$$

EQUATE COEFFICIENTS OF UNIT VECTORS TO ZERO

$$\textcircled{1} -30T + 10W_b + W_b z = 0 \quad (1)$$

$$\textcircled{2} 32T - 10W - W_b x = 0 \quad (2)$$

ALSO

$$\sum F_y = 0: 3T - W - W_b = 0 \quad (3)$$

(CONTINUED)

4.100 CONTINUED

NOW, ELIMINATE  $T$ :

$$(EQ.1) + 10(EQ.3): (2-10)W_b = 0 \quad (4)$$

$$3(EQ.2) - 32(EQ.3): 2W + (-2x + 32)W_b = 0 \quad (5)$$

NOTE THAT  $x \leq 20 \text{ in.}$  AND  $z \leq 20 \text{ in.}$

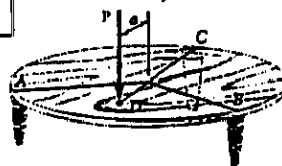
FROM EQ.(4):  $z = 10 \text{ in.}$  OR

$$\text{FROM EQ.(5): } \frac{W_b}{W} = \frac{2}{3x - 32} \geq \frac{2}{2(20) - 32} = \frac{2}{8} = \frac{1}{4}$$

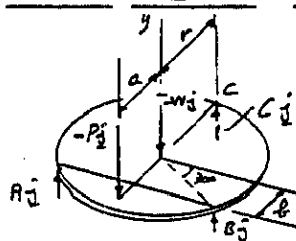
$$\therefore W_b = \frac{1}{4}W = \frac{1}{4}(56 \text{ lb}) = 14 \text{ lb}$$

$$W = 4 \text{ lb AT } x = 20 \text{ in.}, z = 10 \text{ in.}$$

4.101



GIVEN:  $P = 100 \text{ lb}$ ,  
TABLE,  $W = 30 \text{ lb}$ ,  $r = 2 \text{ ft}$ .  
FIND: MINIMUM  $a$   
FOR NO TIPPING



$r = 2 \text{ ft}$   $b = r \sin 30^\circ = 1 \text{ ft}$   
WE SHALL SUM MOMENTS  
ABOUT AB

$$(b+r)C + (a-b)P - bW = 0$$

$$(1+2)C + (a-1)100 - (1)30 = 0$$

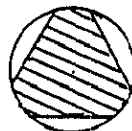
$$C = \frac{1}{3}[30 - (a-1)100]$$

IF TABLE IS NOT TO TIP,  $C \geq 0$

$$[30 - (a-1)100] \geq 0$$

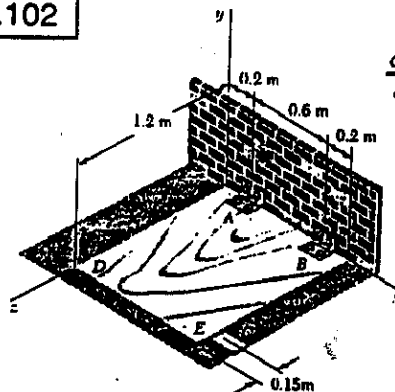
$$30 \geq (a-1)100$$

$$a - 1 \leq 0.1 \quad a \leq 1.3 \text{ ft} \quad a = 1.300 \text{ ft}$$



ONLY  $\perp$  DISTANCE FROM P TO AB  
MATTERS. SAME CONDITION MUST BE  
SATISFIED FOR EACH LEG.  $\therefore$  P MUST BE  
LOCATED IN SHADED AREA FOR NO TIPPING

4.102



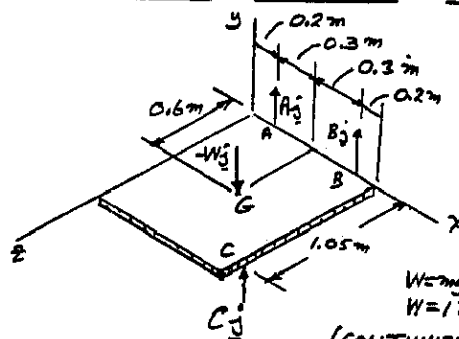
GIVEN: MASS  
OF PLYWOOD  
SHEET,  $m = 18 \text{ kg}$

FIND: REACTIONS

(a) AT A.

(b) AT B.

(c) AT C.



$$\sum \mathbf{r}_{GA} \times \mathbf{T}_B \mathbf{j} + \sum \mathbf{r}_{GB} \times \mathbf{T}_C \mathbf{j} + \sum \mathbf{r}_{GP} \times (-W \mathbf{j}) + (\mathbf{r}_{GA} + \mathbf{r}_{GB}) \times (-W \mathbf{j}) = 0$$

$$(0.6\mathbf{i} - 0.3\mathbf{j}) \times T \mathbf{j} + (1.05\mathbf{i} - 0.3\mathbf{j}) \times T \mathbf{j} + (0.6\mathbf{i} + 0.3\mathbf{j}) \times (-W \mathbf{j})$$

$$+ (0.6\mathbf{i} + 0.3\mathbf{j}) \times (-W \mathbf{j}) = 0$$

$$W = mg = (18 \text{ kg})(9.81)$$

$$W = 176.58 \text{ N}$$

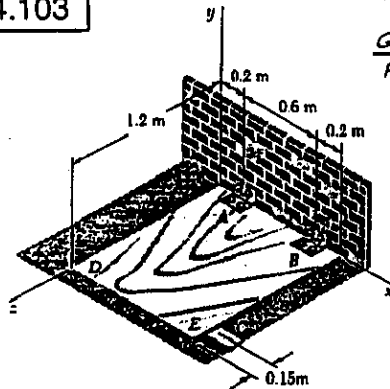
(CONTINUED)



# 4.102 CONTINUED

$$\begin{aligned}\Sigma M_A = 0: & \quad r_{BA} \times B_j + r_{CA} \times C_j + r_{GA} \times (-W_j) = 0 \\ & (0.6i) \times B_j + (0.2i + 1.05j) \times C_j + (0.3i + 0.6j) \times (-W_j) = 0 \\ & 0.6B_k - 0.6WC_k - 1.05C_k - 0.3W_k + 0.6W_k = 0 \\ & \text{Equate coefficients of unit vectors to zero.} \\ & \textcircled{1} \quad -1.05C + 0.6W = 0; \quad C = (0.6/1.05)(176.58N) = 100.90N \\ & \textcircled{2} \quad 0.6B + 0.6C - 0.3W = 0 \\ & \quad 0.6B + 0.6(100.90N) - 0.3(176.58N) = 0; \quad B = -46.24N \\ \Sigma F_y = 0: & \quad A + B + C - W = 0 \\ & \quad A - 46.24N + 100.90N + 176.58N = 0; \quad A = 121.92N \\ & \text{(a) } A = 121.9N \quad \text{(b) } B = -46.2N \quad \text{(c) } C = 100.9N\end{aligned}$$

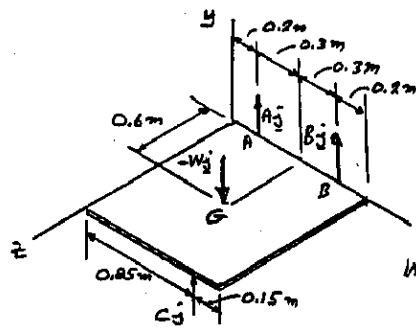
# 4.103



GIVEN: MASS OF  
PLYWOOD SHEET  
 $m = 10 \text{ kg}$

FIND: REACTIONS

- (a) AT A.  
(b) AT B.  
(c) AT C.



$$\begin{aligned}r_{BA} &= 0.6i \\ r_{CA} &= 0.65i + 1.2j \\ r_{GA} &= 0.3i + 0.6j\end{aligned}$$

$$\begin{aligned}W &= mg = (10 \text{ kg})(9.81 \text{ m/s}^2) \\ W &= 98.1N\end{aligned}$$

$$\begin{aligned}\Sigma M_A = 0: & \quad r_{BA} \times B_j + r_{CA} \times C_j + r_{GA} \times (-W_j) = 0 \\ & (0.6i) \times B_j + (0.65i + 1.2j) \times C_j + (0.3i + 0.6j) \times (-W_j) = 0 \\ & 0.6B_k + 0.65C_k - 1.2C_k - 0.3W_k + 0.6W_k = 0\end{aligned}$$

Equate coefficients of unit vectors to zero.

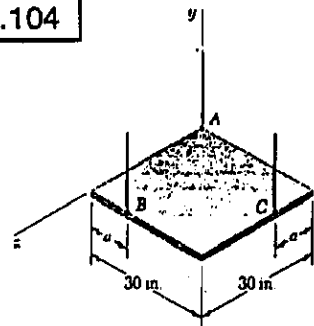
$$\textcircled{1} \quad -1.2C + 0.6W = 0; \quad C = (0.6/1.2)(98.1N) = 49.05N$$

$$\begin{aligned}\textcircled{2} \quad 0.6B + 0.65C - 0.3W &= 0 \\ 0.6B + 0.65(49.05N) - 0.3(98.1N) &= 0 \\ B &= -7.36N\end{aligned}$$

$$\begin{aligned}\Sigma F_y = 0: & \quad A + B + C - W = 0 \\ A - 7.36N + 49.05N - 98.1N &= 0 \\ A &= 56.31N\end{aligned}$$

$$\text{(a) } A = 56.3N \quad \text{(b) } -7.36N \quad \text{(c) } 49.05N$$

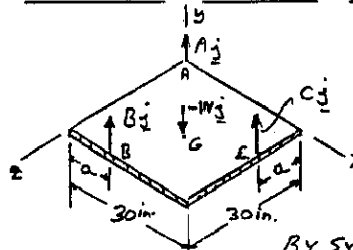
# 4.104



GIVEN: WEIGHT OF  
PLATE  $W = 24 \text{ lb}$

FIND: (a) WIRE  
TENSIONS WHEN  
 $a = 10 \text{ in.}$

(b) VALUE OF  $a$   
FOR WHEN  
TENSION IN  
EACH WIRE IS  $8 \text{ lb}$



$$\begin{aligned}r_{BA} &= a_i + 30j \\ r_{CA} &= 30i + a_j \\ r_{GA} &= 15i + 15j\end{aligned}$$

BY SYMMETRY:  $B = C$

$$\begin{aligned}\Sigma M_A = 0: & \quad r_{BA} \times B_j + r_{CA} \times C_j + r_{GA} \times (-W_j) = 0 \\ & (a_i + 30j) \times B_j + (30i + a_j) \times C_j + (15i + 15j) \times (-W_j) = 0 \\ & Ba_k - 30B_k + 30C_k - Ba_k - 15W_k + 15W_k = 0 \\ & \text{Equate coefficients of unit vectors to zero.} \\ & \textcircled{1} \quad -30B - Ba + 15W = 0\end{aligned}$$

$$B = \frac{15W}{30+a} \quad C = B = \frac{15W}{30+a} \quad (1)$$

$$\begin{aligned}\Sigma F_y = 0: & \quad A + B + C - W = 0 \\ A + 2\left[\frac{15W}{30+a}\right] - W &= 0; \quad A = \frac{aW}{30+a} \quad (2)\end{aligned}$$

(a) For  $a = 10 \text{ in.}$

$$\text{EQ. (1)} \quad C = B = \frac{15(24 \text{ lb})}{30+10} = 9 \text{ lb}$$

$$\text{EQ. (2)} \quad A = \frac{10(24 \text{ lb})}{30+10} = 6 \text{ lb}$$

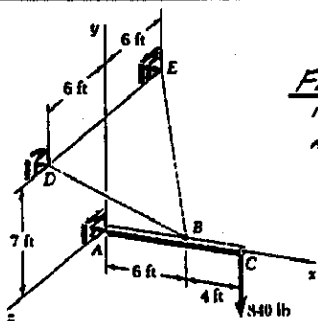
$$A = 6 \text{ lb}; \quad B = C = 9 \text{ lb}$$

(b) FOR TENSION IN EACH WIRE =  $8 \text{ lb}$

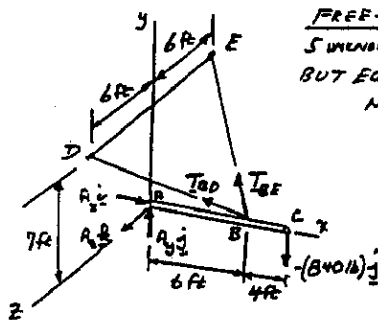
$$\text{EQ. (1)} \quad 8 \text{ lb} = \frac{15(24 \text{ lb})}{30+a}$$

$$\begin{aligned}30 \text{ in.} + a &= 45 \\ a &= 15 \text{ in.}\end{aligned}$$

4.105



FIND: TENSION  
IN EACH CABLE  
AND REACTION  
AT A.



FREE-BODY DIAGRAM WE HAVE  
5 UNKNOWN AND 6 EQS. OF EQUIL.,  
BUT EQUILIBRIUM IS  
MAINTAINED ( $\Sigma M_x = 0$ )

$$\vec{BD} = (-6\hat{i})\hat{i} + (7\hat{j})\hat{j} + (6\hat{k})\hat{k} \quad BD = 11 \text{ ft}$$

$$\vec{BE} = (-6\hat{i})\hat{i} + (7\hat{j})\hat{j} - (6\hat{k})\hat{k} \quad BE = 11 \text{ ft}$$

$$T_{BD} = T_{BD} \frac{\vec{BD}}{BD} = \frac{T_{BD}}{11} (-6\hat{i} + 7\hat{j} + 6\hat{k})$$

$$T_{BE} = T_{BE} \frac{\vec{BE}}{BE} = \frac{T_{BE}}{11} (-6\hat{i} + 7\hat{j} - 6\hat{k})$$

$$\Sigma M_A = 0: \hat{j} \times T_{BD} + \hat{j} \times T_{BE} + \hat{k} \times (-340\hat{j}) = 0$$

$$6\hat{i} \times \frac{T_{BD}}{11} (-6\hat{i} + 7\hat{j} + 6\hat{k}) + 6\hat{i} \times \frac{T_{BE}}{11} (-6\hat{i} + 7\hat{j} - 6\hat{k}) + 10\hat{i} \times (-340\hat{j}) = 0$$

$$\frac{42}{11} T_{BD} \hat{k} - \frac{36}{11} T_{BD} \hat{j} + \frac{42}{11} T_{BE} \hat{k} + \frac{36}{11} T_{BE} \hat{j} - 3400\hat{k} = 0$$

EQUATE COEFFICIENTS OF UNIT VECTORS TO ZERO.

$$\textcircled{1} -\frac{36}{11} T_{BD} + \frac{36}{11} T_{BE} = 0; \quad T_{BE} = T_{BD}$$

$$\textcircled{2} \frac{42}{11} T_{BD} + \frac{42}{11} T_{BE} - 3400 = 0$$

$$2\left(\frac{42}{11} T_{BD}\right) = 3400; \quad T_{BD} = 1100 \text{ lb}$$

$$T_{BE} = 1100 \text{ lb}$$

$$\Sigma F_x = 0: A_x - \frac{6}{11}(1100 \text{ lb}) - \frac{6}{11}(1100 \text{ lb}) = 0$$

$$A_x = 1200 \text{ lb}$$

$$\Sigma F_y = 0: A_y + \frac{7}{11}(1100 \text{ lb}) + \frac{7}{11}(1100 \text{ lb}) - 340 \text{ lb} = 0$$

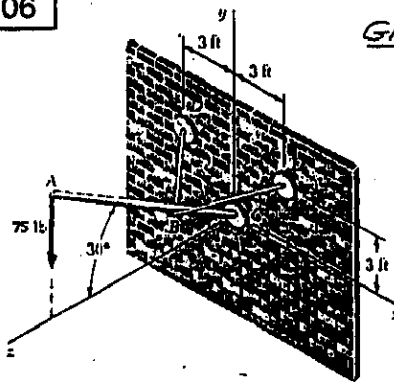
$$A_y = -560 \text{ lb}$$

$$\Sigma F_z = 0: A_z + \frac{6}{11}(1100 \text{ lb}) - \frac{6}{11}(1100 \text{ lb}) = 0$$

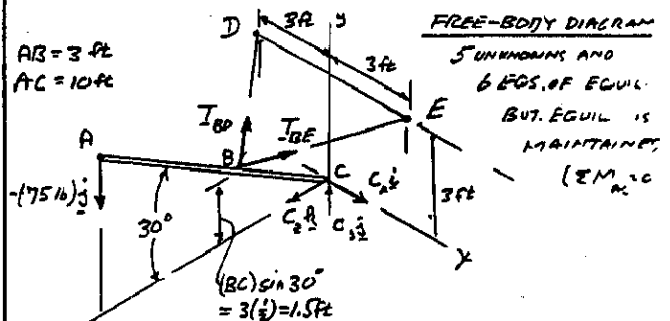
$$A_z = 0$$

$$\underline{A} = (1200 \text{ lb})\hat{i} - (560 \text{ lb})\hat{j}$$

4.106



GIVEN: BC = 3 ft  
AC = 10 ft  
FIND: TENSION  
IN EACH BRACE  
AND REACTION  
AT C.



FREE-BODY DIAGRAM  
5 UNKNOWN AND  
6 EQS. OF EQUIL.  
BUT EQUIL IS  
MAINTAINED  
( $\Sigma M_x = 0$ )

$$\vec{BD} = (3\hat{i})\hat{i} + (3\hat{j})\hat{j} = 3\sqrt{2} \quad BD = 4.243 \text{ ft}$$

$$\vec{BE} = (3\hat{i})\hat{i} + (3\hat{j})\hat{j} - (10\hat{k})\hat{k} = 10.44 \text{ ft}$$

$$T_{BD} = T_{BD} \frac{\vec{BD}}{BD} = \frac{T_{BD}}{4.243} (3\hat{i} + 3\hat{j})$$

$$\vec{BD} = \vec{r}_D - \vec{r}_B = -3\hat{i} + 3\hat{j} - 1.5\hat{j} - 2.598\hat{k}$$

$$\vec{BD} = -3\hat{i} + 1.5\hat{j} - 2.598\hat{k} \quad BD = 4.243 \text{ ft}$$

$$\vec{BE} = 3\hat{i} + 1.5\hat{j} - 2.598\hat{k} \quad BE = 4.243 \text{ ft}$$

$$T_{BD} = T_{BD} \frac{\vec{BD}}{BD} = T_{BD} (-0.707\hat{i} + 0.3535\hat{j} - 0.6123\hat{k})$$

$$T_{BE} = T_{BE} \frac{\vec{BE}}{BE} = T_{BE} (0.707\hat{i} + 0.3535\hat{j} - 0.6123\hat{k})$$

$$\Sigma M_C = 0: \hat{j} \times T_{BD} + \hat{j} \times T_{BE} + (1.5\hat{j} + 8.66\hat{k}) \times (-75\hat{j}) = 0$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1.5 & 2.598 \\ -0.707 & 0.3535 & -0.6123 \end{vmatrix} T_{BD} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1.5 & 2.598 \\ 0.707 & 0.3535 & -0.6123 \end{vmatrix} T_{BE} + 649.5\hat{i} = 0$$

EQUATE COEFFICIENTS OF UNIT VECTORS TO ZERO

$$\textcircled{1} -1.837 T_{BD} + 1.837 T_{BE} = 0; \quad T_{BE} = T_{BD}$$

$$\textcircled{2} -1.837 T_{BD} - 1.837 T_{BE} + 649.5 = 0; \quad T_{BD} = 176.8 \text{ lb}$$

$$T_{BE} = 176.8 \text{ lb}$$

$$\Sigma F_x = 0: C_x + (176.8)(-0.707) + (176.8)(0.707) = 0$$

$$C_x = 0$$

$$\Sigma F_y = 0: C_y + (176.8)(0.3535) + (176.8)(0.3535) - 75 \text{ lb} = 0$$

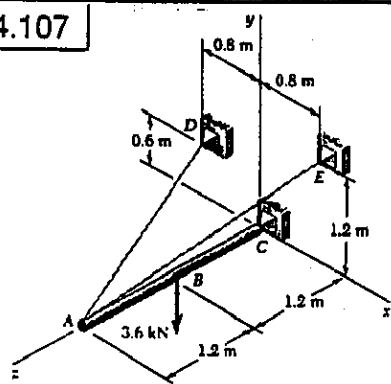
$$C_y = -50$$

$$\Sigma F_z = 0: C_z + (176.8)(-0.6123) + (176.8)(-0.6123) = 0$$

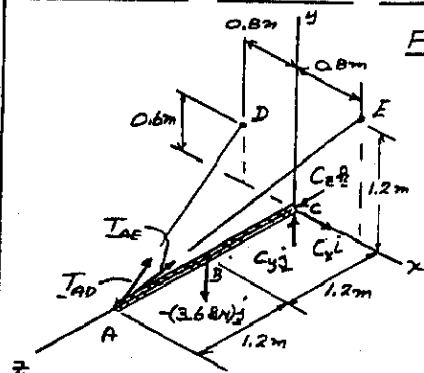
$$C_z = 216 \text{ lb}$$

$$\underline{C} = (-50 \text{ lb})\hat{j} + (216 \text{ lb})\hat{k}$$

4.107



FIND: TENSION  
IN EACH CABLE  
AND REACTION  
AT C.



FREE-BODY DIAGRAM

5 UNKNOWNS AND  
6 EQS. OF EQUIL.  
EQUILIBRIUM  
MAINTAINED ( $\sum M_{AC} = 0$ )

$$\begin{aligned} r_B &= 1.2\hat{j} \\ r_A &= 2.4\hat{i} \end{aligned}$$

$$\vec{AD} = -0.8\hat{i} + 0.6\hat{j} - 2.4\hat{k}$$

$$\vec{AE} = 0.8\hat{i} + 1.2\hat{j} - 2.4\hat{k}$$

$$AD = 2.6 \text{ m}$$

$$AE = 2.8 \text{ m}$$

$$T_{AD} = \frac{\vec{AD}}{AD} = \frac{T_{AD}}{2.6} (-0.8\hat{i} + 0.6\hat{j} - 2.4\hat{k})$$

$$T_{AE} = \frac{\vec{AE}}{AE} = \frac{T_{AE}}{2.8} (0.8\hat{i} + 1.2\hat{j} - 2.4\hat{k})$$

$$\sum M_C = 0: r_A \times T_{AD} + r_A \times T_{AE} + r_B \times (-3.6\hat{j}) = 0$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 2.4 \\ 0 & 0 & 2.4 \end{vmatrix} \frac{T_{AD}}{2.6} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 2.4 \\ 0.8 & 1.2 & -2.4 \end{vmatrix} \frac{T_{AE}}{2.8} + 1.2\hat{k} \times (-3.6\hat{j}) = 0$$

Equate coefficients of unit vectors to zero

$$\textcircled{1} -0.55385 T_{AD} - 1.02857 T_{AE} + 4.32 = 0 \quad (1)$$

$$\textcircled{2} -0.73846 T_{AD} + 0.08571 T_{AE} = 0$$

$$T_{AD} = 0.92857 T_{AE} \quad (2)$$

$$\text{EQ (1): } -0.55385(0.92857 T_{AE}) - 1.02857 T_{AE} + 4.32 = 0$$

$$1.54296 T_{AE} = 4.32$$

$$T_{AE} = 2.800 \text{ kN}$$

$$T_{AE} = 2.80 \text{ kN}$$

$$\text{EQ (2): } T_{AD} = 0.92857(2.80) = 2.60 \text{ kN}$$

$$T_{AD} = 2.60 \text{ kN}$$

$$\sum F_x = 0: C_x - \frac{0.8}{2.6}(2.6 \text{ kN}) + \frac{0.8}{2.8}(2.8 \text{ kN}) = 0; C_x = 0$$

$$\sum F_y = 0: C_y + \frac{0.6}{2.6}(2.6 \text{ kN}) + \frac{1.2}{2.8}(2.8 \text{ kN}) - (3.6 \text{ kN}) = 0$$

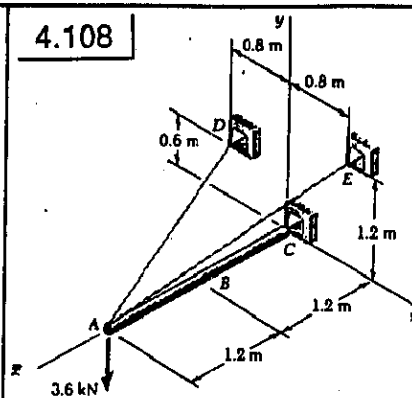
$$C_y = 1.800 \text{ kN}$$

$$\sum F_z = 0: C_z - \frac{2.4}{2.6}(2.6 \text{ kN}) - \frac{2.4}{2.8}(2.8 \text{ kN}) = 0$$

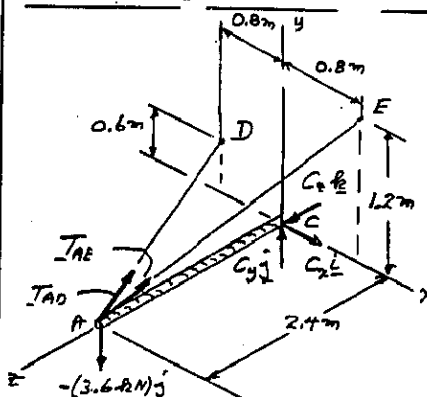
$$C_z = 4.80 \text{ kN}$$

$$\underline{C} = (1.800 \text{ kN})\hat{j} + (4.80 \text{ kN})\hat{k}$$

4.108



FIND: TENSION  
IN EACH CABLE  
AND REACTION  
AT C.



FREE-BODY DIAGRAM

5 UNKNOWNS AND  
6 EQS. OF EQUIL.  
EQUILIBRIUM  
MAINTAINED  
( $\sum M_{AC} = 0$ )

$$\vec{AD} = -0.8\hat{i} + 0.6\hat{j} - 2.4\hat{k}$$

$$\vec{AE} = 0.8\hat{i} + 1.2\hat{j} - 2.4\hat{k}$$

$$AD = 2.6 \text{ m}$$

$$AE = 2.8 \text{ m}$$

$$T_{AD} = \frac{\vec{AD}}{AD} = \frac{T_{AD}}{2.6} (-0.8\hat{i} + 0.6\hat{j} - 2.4\hat{k})$$

$$T_{AE} = \frac{\vec{AE}}{AE} = \frac{T_{AE}}{2.8} (0.8\hat{i} + 1.2\hat{j} - 2.4\hat{k})$$

$$\sum M_C = 0: r_A \times T_{AD} + r_A \times T_{AE} + r_B \times (-3.6\hat{j}) = 0$$

$$\text{Factor } r_A: r_A \times (T_{AD} + T_{AE} - (3.6 \text{ kN})\hat{j}) = 0$$

$$\text{OR: } T_{AD} + T_{AE} - (3 \text{ kN})\hat{j} = 0 \quad (\text{FORCES CONCURRENT AT A})$$

$$\text{Coeff. of } \hat{i}: -\frac{T_{AD}}{2.6}(0.8) + \frac{T_{AE}}{2.8}(0.8) = 0$$

$$T_{AD} = \frac{2.6}{2.8} T_{AE} \quad (1)$$

$$\text{Coeff. of } \hat{j}: \frac{T_{AD}}{2.6}(0.6) + \frac{T_{AE}}{2.8}(1.2) - 3.6 \text{ kN} = 0$$

$$\frac{2.6}{2.8} T_{AE} \left( \frac{0.6}{2.6} \right) + \frac{1.2}{2.8} T_{AE} - 3.6 \text{ kN} = 0$$

$$T_{AE} \left( \frac{0.6 + 1.2}{2.8} \right) = 3.6 \text{ kN}$$

$$T_{AE} = 5.600 \text{ kN}$$

$$T_{AE} = 5.60 \text{ kN}$$

$$\text{EQ (1): } T_{AD} = \frac{2.6}{2.8}(5.6) = 5.200 \text{ kN}$$

$$T_{AD} = 5.20 \text{ kN}$$

$$\sum F_x = 0: C_x - \frac{0.8}{2.6}(5.2 \text{ kN}) + \frac{0.8}{2.8}(5.6 \text{ kN}) = 0; C_x = 0$$

$$\sum F_y = 0: C_y + \frac{0.6}{2.6}(5.2 \text{ kN}) + \frac{1.2}{2.8}(5.6 \text{ kN}) - 3.6 \text{ kN} = 0$$

$$C_y = 0$$

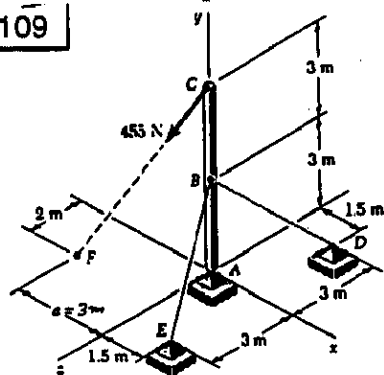
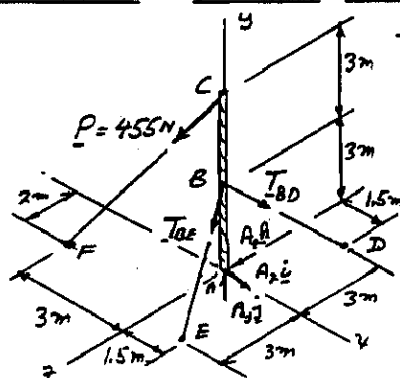
$$\sum F_z = 0: C_z - \frac{2.4}{2.6}(5.2 \text{ kN}) - \frac{2.4}{2.8}(5.6 \text{ kN}) = 0$$

$$C_z = 9.6 \text{ kN}$$

$$\underline{C} = (9.6 \text{ kN})\hat{k}$$

NOTE: SINCE FORCES + REACTION ARE CONCURRENT AT A, WE COULD HAVE USED THE METHODS OF CHAPTER 2.

4.109

GIVEN:  $a = 3\text{ m}$ FIND: TENSION  
IN EACH CABLE  
AND REACTION  
AT A.FREE-BODY DIAGRAM  
5 UNKNOWNS AND  
6 EQS. OF EQUIL.  
BUT, EQUILIBRIUM  
MAINTAINED  
( $\sum M_{AC} = 0$ )

$$\mathbf{r}_B = 3\mathbf{j}$$

$$\mathbf{r}_E = 6\mathbf{j}$$

$$\overline{CF} = -3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}$$

$$CF = 7\text{ m}$$

$$\overline{BD} = 1.5\mathbf{i} - 3\mathbf{j} - 3\mathbf{k}$$

$$BD = 4.5\text{ m}$$

$$\overline{BE} = 1.5\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$$

$$BE = 4.5\text{ m}$$

$$\mathbf{P} = P \frac{\overline{CF}}{CF} = \frac{P}{7}(-3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k})$$

$$T_{BD} = T_{BD} \frac{\overline{BD}}{BD} = \frac{T_{BD}}{4.5}(1.5\mathbf{i} - 3\mathbf{j} - 3\mathbf{k}) = \frac{T_{BD}}{3}(\mathbf{i} - 2\mathbf{j} - 2\mathbf{k})$$

$$T_{BE} = T_{BE} \frac{\overline{BE}}{BE} = \frac{T_{BE}}{3}(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$$

$$\sum M_A = 0: \mathbf{r}_B \times T_{BD} + \mathbf{r}_E \times T_{BE} + \mathbf{r}_C \times \mathbf{P} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & 0 \\ 1 & -2 & -2 \end{vmatrix} \frac{T_{BD}}{3} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 6 & 0 \\ 1 & -2 & 2 \end{vmatrix} \frac{T_{BE}}{3} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -6 & 2 \end{vmatrix} \frac{P}{7} = 0$$

$$\text{COEFF. OF } \mathbf{i}: -2T_{BD} + 2T_{BE} + \frac{12}{7}P = 0 \quad (1)$$

$$\text{COEFF. OF } \mathbf{j}: -T_{BD} - T_{BE} + \frac{18}{7}P = 0 \quad (2)$$

$$\text{EQ. (1)} + 2\text{EQ. (2)}: -4T_{BD} + \frac{48}{7}P = 0 \quad T_{BD} = \frac{12}{7}P$$

$$\text{EQ. (2)}: -\frac{12}{7}P - T_{BE} + \frac{18}{7}P = 0 \quad T_{BE} = \frac{6}{7}P$$

$$\text{SINCE } P = 445\text{ N}, \quad T_{BD} = \frac{12}{7}(445) \quad T_{BD} = 780\text{ N}$$

$$T_{BE} = \frac{6}{7}(445) \quad T_{BE} = 390\text{ N}$$

$$\sum \mathbf{F} = 0: T_{BD} + T_{BE} + \mathbf{P} + \mathbf{A} = 0$$

$$\text{COEFF. OF } \mathbf{i}: \frac{780}{3} + \frac{390}{3} - \frac{445}{7}(3) + A_x = 0$$

$$260 + 130 - 195 + A_x = 0; \quad A_x = -195\text{ N}$$

$$\text{COEFF. OF } \mathbf{j}: -\frac{780}{3}(2) - \frac{390}{3}(2) - \frac{445}{7}(6) + A_y = 0$$

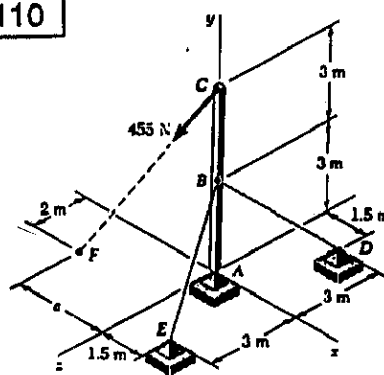
$$-520 - 260 - 390 + A_y = 0; \quad A_y = 1170\text{ N}$$

$$\text{COEFF. OF } \mathbf{k}: -\frac{780}{3}(2) + \frac{390}{3}(2) + \frac{445}{7}(2) + A_z = 0$$

$$-520 + 260 + 130 + A_z = 0; \quad A_z = +130\text{ N}$$

$$\mathbf{A} = -(195\text{ N})\mathbf{i} + (1170\text{ N})\mathbf{j} + (130\text{ N})\mathbf{k}$$

4.110

FREE-BODY DIAGRAM  
5 UNKNOWNS AND  
6 EQS. OF EQUIL.  
BUT, EQUILIBRIUM  
MAINTAINED  
( $\sum M_{AC} = 0$ )

$$\mathbf{r}_B = 3\mathbf{j}$$

$$\mathbf{r}_E = 6\mathbf{j}$$

$$\overline{CF} = -1.5\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}$$

$$CF = 6.5\text{ m}$$

$$\overline{BD} = 1.5\mathbf{i} - 3\mathbf{j} - 3\mathbf{k}$$

$$BD = 4.5\text{ m}$$

$$\overline{BE} = 1.5\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$$

$$BE = 4.5\text{ m}$$

$$\mathbf{P} = P \frac{\overline{CF}}{CF} = \frac{P}{6.5}(-1.5\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}) = \frac{P}{13}(-3\mathbf{i} - 12\mathbf{j} + 4\mathbf{k})$$

$$T_{BD} = T_{BD} \frac{\overline{BD}}{BD} = \frac{T_{BD}}{4.5}(1.5\mathbf{i} - 3\mathbf{j} - 3\mathbf{k}) = \frac{T_{BD}}{3}(\mathbf{i} - 2\mathbf{j} - 2\mathbf{k})$$

$$T_{BE} = T_{BE} \frac{\overline{BE}}{BE} = \frac{T_{BE}}{3}(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$$

$$\sum M_A = 0: \mathbf{r}_B \times T_{BD} + \mathbf{r}_E \times T_{BE} + \mathbf{r}_C \times \mathbf{P} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & 0 \\ 1 & -2 & -2 \end{vmatrix} \frac{T_{BD}}{3} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 6 & 0 \\ 1 & -2 & 2 \end{vmatrix} \frac{T_{BE}}{3} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -12 & 4 \end{vmatrix} \frac{P}{13} = 0$$

$$\text{COEFF. OF } \mathbf{i}: -2T_{BD} + 2T_{BE} + \frac{24}{13}P = 0 \quad (1)$$

$$\text{COEFF. OF } \mathbf{j}: -T_{BD} - T_{BE} + \frac{18}{13}P = 0 \quad (2)$$

$$\text{EQ. (1)} + 2\text{EQ. (2)}: -4T_{BD} + \frac{60}{13}P = 0 \quad T_{BD} = \frac{15}{13}P$$

$$\text{EQ. (2)}: -\frac{15}{13}P - T_{BE} + \frac{18}{13}P = 0 \quad T_{BE} = \frac{3}{13}P$$

$$\text{SINCE } P = 455\text{ N}, \quad T_{BD} = \frac{15}{13}(455) \quad T_{BD} = 525\text{ N}$$

$$T_{BE} = \frac{3}{13}(455) \quad T_{BE} = 105\text{ N}$$

$$\sum \mathbf{F} = 0: T_{BD} + T_{BE} + \mathbf{P} + \mathbf{A} = 0$$

$$\text{COEFF. OF } \mathbf{i}: \frac{525}{3} + \frac{105}{3} - \frac{455}{13}(3) + A_x = 0$$

$$175 + 35 - 105 + A_x = 0; \quad A_x = -105\text{ N}$$

$$\text{COEFF. OF } \mathbf{j}: -\frac{525}{3}(2) - \frac{105}{3}(2) - \frac{455}{13}(12) + A_y = 0$$

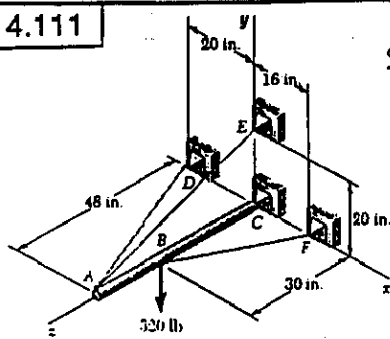
$$-350 - 70 - 420 + A_y = 0; \quad A_y = 840\text{ N}$$

$$\text{COEFF. OF } \mathbf{k}: -\frac{525}{3}(2) + \frac{105}{3}(2) + \frac{455}{13}(4) + A_z = 0$$

$$-350 + 70 + 140 + A_z = 0; \quad A_z = 140\text{ N}$$

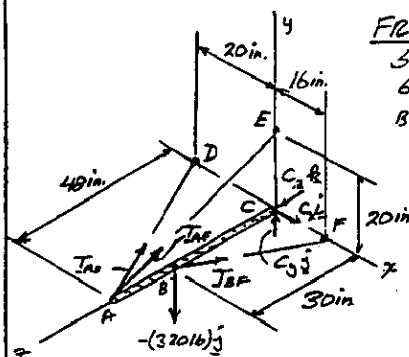
$$\mathbf{A} = -(105\text{ N})\mathbf{i} + (840\text{ N})\mathbf{j} + (140\text{ N})\mathbf{k}$$

4.111



GIVEN: CABLE DAE  
PASSES OVER A  
PULLEY AT A

FIND: TENSION  
IN EACH CABLE  
AND REACTION  
AT C.



FREE-BODY DIAGRAM

5 UNKNOWN AND  
6 EQS. OF EQUIL.  
BUT, EQUILIBRIUM  
MAINTAINED ( $\sum M_A = 0$ )

T = TENSION IN  
BOTH PARTS OF  
CABLE DAE.

$$r_B = 30\hat{i} - 48\hat{j}$$

$$r_A = 48\hat{i}$$

$$\vec{AD} = -20\hat{i} - 48\hat{j} \quad AD = 52\text{ in.}$$

$$\vec{AE} = 20\hat{i} - 48\hat{j} \quad AE = 52\text{ in.}$$

$$\vec{BF} = 16\hat{i} - 30\hat{j} \quad BF = 34\text{ in.}$$

$$T_{AD} = T \frac{\vec{AD}}{AD} = \frac{T}{52} (-20\hat{i} - 48\hat{j}) = \frac{T}{13} (-5\hat{i} - 12\hat{j})$$

$$T_{AE} = T \frac{\vec{AE}}{AE} = \frac{T}{52} (20\hat{i} - 48\hat{j}) = \frac{T}{13} (5\hat{i} - 12\hat{j})$$

$$T_{BF} = T \frac{\vec{BF}}{BF} = \frac{T}{34} (16\hat{i} - 30\hat{j}) = \frac{T}{17} (8\hat{i} - 15\hat{j})$$

$$\sum M_C = 0: r_A \times T_{AD} + r_A \times T_{AE} + r_B \times T_{BF} + r_B \times (-320\hat{j}) = 0$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 48 \\ 0 & 0 & 48 \end{vmatrix} \frac{T}{13} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 48 \\ 0 & 0 & 48 \end{vmatrix} \frac{T}{13} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 30 \\ 0 & 0 & 30 \end{vmatrix} \frac{T}{17} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 30 \\ 0 & 0 & 30 \end{vmatrix} (-320) = 0$$

$$\text{COEFF. OF } \hat{i}: -\frac{240}{13} T + 9600 = 0 \quad T = 520\text{ lb}$$

$$\text{COEFF. OF } \hat{j}: -\frac{240}{13} T + \frac{240}{17} T_{BD} = 0$$

$$T_{BD} = \frac{17}{13} T = \frac{17}{13} (520); T_{BD} = 680\text{ lb}$$

$$\sum F = 0: T_{AD} + T_{AE} + T_{BF} - 320\hat{j} + C = 0$$

$$\text{COEFF. OF } \hat{i}: -\frac{20}{52} (520) + \frac{20}{52} (520) + C_x = 0$$

$$-200 + 320 + C_x = 0 \quad C_x = -120\text{ lb}$$

$$\text{COEFF. OF } \hat{j}: \frac{20}{52} (520) - 320 + C_y = 0$$

$$200 - 320 + C_y = 0 \quad C_y = 120\text{ lb}$$

$$\text{COEFF. OF } \hat{k}: -\frac{48}{52} (520) - \frac{48}{52} (520) - \frac{30}{34} (680) + C_z = 0$$

$$-480 - 480 - 600 + C_z = 0 \quad C_z = 1560\text{ lb}$$

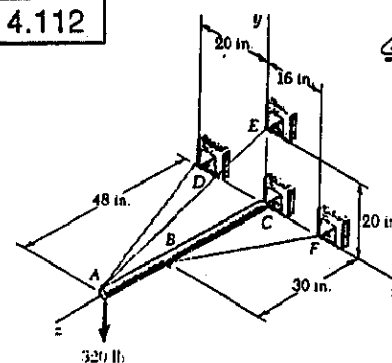
$$\text{ANSWERS: } T_{DAE} = T$$

$$T_{DAE} = 520\text{ lb}$$

$$T_{BD} = 680\text{ lb}$$

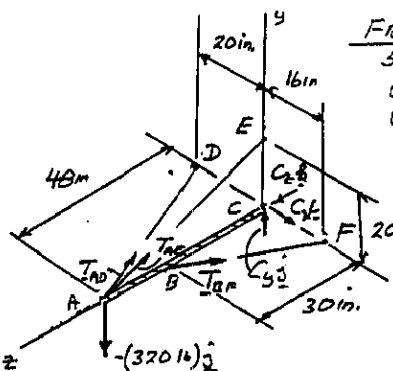
$$C = -(120\text{ lb})\hat{i} + (120\text{ lb})\hat{j} + (1560\text{ lb})\hat{k}$$

4.112



GIVEN: CABLE DAE  
PASSES OVER A  
PULLEY AT A

FIND: TENSION  
IN EACH CABLE  
AND REACTION  
AT C.



FREE-BODY DIAGRAM

5 UNKNOWN AND  
6 EQS. OF EQUIL.  
BUT, EQUILIBRIUM  
MAINTAINED  
( $\sum M_A = 0$ )

T = TENSION IN  
BOTH PARTS OF  
CABLE DAE.

$$r_B = 30\hat{i} - 48\hat{j}$$

$$r_A = 48\hat{i}$$

$$\vec{AD} = -20\hat{i} - 48\hat{j} \quad AD = 52\text{ in.}$$

$$\vec{AE} = 20\hat{i} - 48\hat{j} \quad AE = 52\text{ in.}$$

$$\vec{BF} = 16\hat{i} - 30\hat{j} \quad BF = 34\text{ in.}$$

$$T_{AD} = T \frac{\vec{AD}}{AD} = \frac{T}{52} (-20\hat{i} - 48\hat{j}) = \frac{T}{13} (-5\hat{i} - 12\hat{j})$$

$$T_{AE} = T \frac{\vec{AE}}{AE} = \frac{T}{52} (20\hat{i} - 48\hat{j}) = \frac{T}{13} (5\hat{i} - 12\hat{j})$$

$$T_{BF} = T \frac{\vec{BF}}{BF} = \frac{T}{34} (16\hat{i} - 30\hat{j}) = \frac{T}{17} (8\hat{i} - 15\hat{j})$$

$$\sum M_C = 0: r_A \times T_{AD} + r_A \times T_{AE} + r_B \times T_{BF} + r_B \times (-320\hat{j}) = 0$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 48 \\ 0 & 0 & 48 \end{vmatrix} \frac{T}{13} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 48 \\ 0 & 0 & 48 \end{vmatrix} \frac{T}{13} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 30 \\ 0 & 0 & 30 \end{vmatrix} \frac{T}{17} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 30 \\ 0 & 0 & 30 \end{vmatrix} (-320) = 0$$

$$\text{COEFF. OF } \hat{i}: -\frac{240}{13} T + 15360 = 0 \quad T = 832\text{ lb}$$

$$\text{COEFF. OF } \hat{j}: -\frac{240}{13} T + \frac{240}{17} T_{BD} = 0$$

$$T_{BD} = \frac{17}{13} T = \frac{17}{13} (832) \quad T_{BD} = 1088\text{ lb}$$

$$\sum F = 0: T_{AD} + T_{AE} + T_{BF} - 320\hat{j} + C = 0$$

$$\text{COEFF. OF } \hat{i}: -\frac{20}{52} (832) + \frac{20}{52} (832) + C_x = 0$$

$$-320 + 512 + C_x = 0 \quad C_x = -192\text{ lb}$$

$$\text{COEFF. OF } \hat{j}: \frac{20}{52} (832) - 320 + C_y = 0$$

$$320 - 320 + C_y = 0 \quad C_y = 0$$

$$\text{COEFF. OF } \hat{k}: -\frac{48}{52} (832) - \frac{48}{52} (832) - \frac{30}{34} (1088) + C_z = 0$$

$$-768 - 768 - 960 + C_z = 0 \quad C_z = 2496\text{ lb}$$

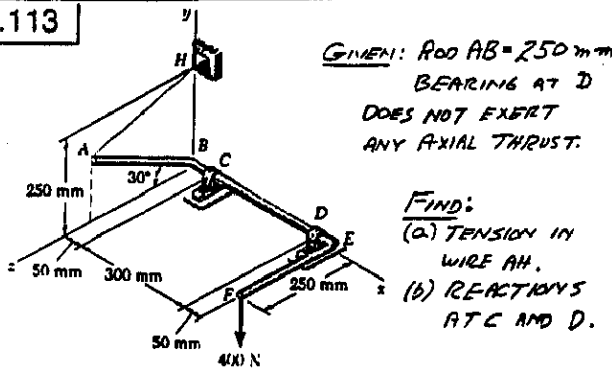
$$\text{ANSWERS: } T_{DAE} = T$$

$$T_{DAE} = 832\text{ lb}$$

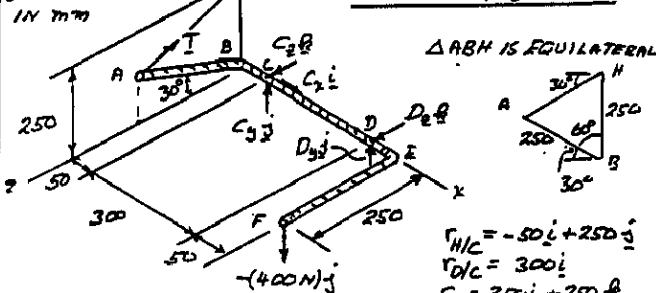
$$T_{BD} = 1088\text{ lb}$$

$$C = -(192\text{ lb})\hat{i} + (2496\text{ lb})\hat{k}$$

4.113

DIMENSIONS  
IN mm

FREE-BODY DIAGRAM



$$T = T(\sin 30^\circ)\hat{j} - T(\cos 30^\circ)\hat{k} = T(0.5\hat{j} - 0.866\hat{k})$$

$$\sum M_C = 0: r_{H/C} \times T + r_{D/C} \times D + r_{E/C} \times (-400\hat{j}) = 0$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -50 & 250 & 0 \\ T & 300 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 350 & 250 & 0 \\ 0 & 0 & -400 \end{vmatrix} = 0$$

$$\text{COEFF. OF } \hat{i}: -216.5T + 100 \times 10^3 = 0$$

$$T = 461.9 \text{ N} \quad T = 462 \text{ N}$$

$$\text{COEFF. OF } \hat{j}: -43.3T - 300D_z = 0$$

$$-43.3(461.9) - 300D_z = 0; \quad D_z = -66.67 \text{ N}$$

$$\text{COEFF. OF } \hat{k}: -25T + 300D_y - 140 \times 10^3 = 0$$

$$-25(461.9) + 300D_y - 140 \times 10^3 = 0; \quad D_y = 505.1 \text{ N}$$

$$D = (505.1\hat{j} - 66.7\hat{k}) \text{ N}$$

$$\sum F = 0: C + D + T - 400\hat{j} = 0$$

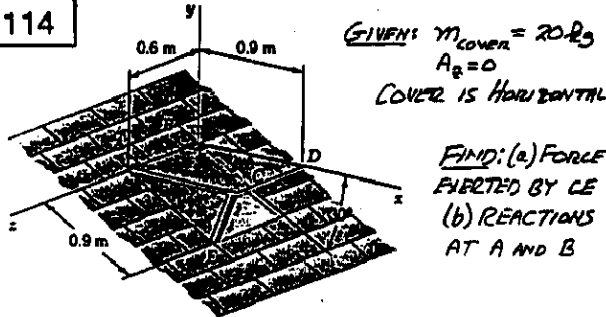
$$\text{COEFF. OF } \hat{i}: C_x = 0 \quad C_x = 0$$

$$\text{COEFF. OF } \hat{j}: C_y + (461.9)(0.5) + 505.1 - 400 = 0; \quad C_y = -336 \text{ N}$$

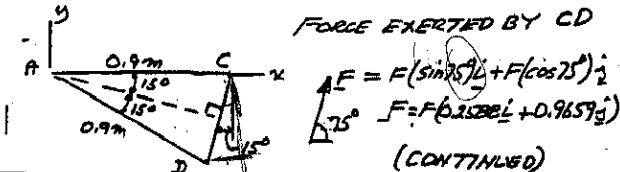
$$\text{COEFF. OF } \hat{k}: C_z - (461.9)(0.866) - 66.67 = 0 \quad C_z = 467 \text{ N}$$

$$C = (-336\hat{j} + 467\hat{k}) \text{ N}$$

4.114



FORCE EXERTED BY CD



4.114 CONTINUED

$$W = mg = 20 \text{ kg}(9.81 \text{ m/s}^2) = 196.2 \text{ N}$$

$$r_{A/B} = 0.6\hat{k}$$

$$r_{C/B} = 0.9\hat{i} + 0.6\hat{j}$$

$$r_{G/B} = 0.45\hat{i} + 0.3\hat{j}$$

$$F = F(0.9659\hat{i} + 0.2598\hat{j})$$

$$\sum M_B = 0: r_{G/B} \times (-196.2\hat{j}) + r_{C/B} \times F + r_{A/B} \times A = 0$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.45 & 0 & 0.3 \\ 0 & -196.2 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.9 & 0 & 0.6 \\ F & 0 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 0.6 \\ A_x & A_y & 0 \end{vmatrix} = 0$$

$$\text{COEFF. OF } \hat{i}: +58.86 - 0.5796F - 0.6A_y = 0$$

$$\text{COEFF. OF } \hat{j}: -0.1553F + 0.6A_x = 0$$

$$\text{COEFF. OF } \hat{k}: -88.29 + 0.8693F = 0; \quad F = 101.56 \text{ N}$$

$$\text{EQ. (1)}: +58.86 - 0.5796(101.56) - 0.6A_y = 0; \quad A_y = 0$$

$$\text{EQ. (2)}: -0.1553(101.56) + 0.6A_x = 0; \quad A_x = 26.29$$

$$F = 101.6 \text{ N}; \quad A = (26.3\hat{x}) \text{ N}$$

$$\sum F = 0: A + B + F - W\hat{j} = 0$$

$$\text{COEFF. OF } \hat{i}: 26.29 + B_x + 0.9659(101.56) = 0; \quad B_x = 0$$

$$\text{COEFF. OF } \hat{j}: B_y + 0.2598(101.56) - 196.2 = 0; \quad B_y = 98.1 \text{ N}$$

$$\text{COEFF. OF } \hat{k}: B_z = 0 \quad B = 98.1 \text{ N}$$

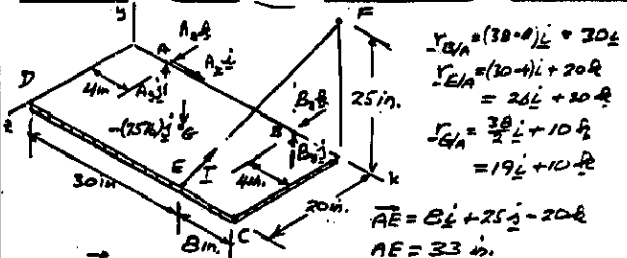
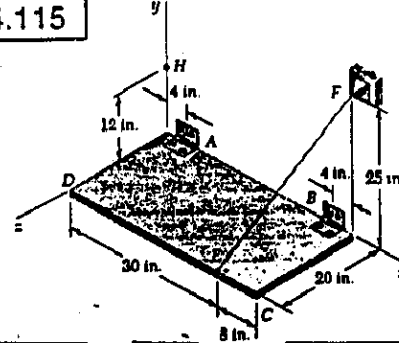
4.115

GIVEN:  $W = 100 \text{ N}$ 

$$B_z = 0$$

FIND

(a) TENSION  
IN CABLE  
(b) REACTIONS  
AT A AND B



$$T = T \frac{r_{E/A}}{r_{E/A}} = T \frac{(30\hat{i} + 30\hat{j})}{\sqrt{30^2 + 30^2}} = T \frac{(30\hat{i} + 30\hat{j})}{42.43}$$

$$\sum M_A = 0: r_{E/A} \times T + r_{G/A} \times (-75\hat{j}) + r_{H/A} \times B = 0$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 26 & 0 & 20 \\ T & 33 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 19 & 0 & 10 \\ 0 & -75 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 30 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

$$\text{COEFF. OF } \hat{i}: -(25)(20) \frac{T}{42.43} + 750 = 0; \quad T = 49.5 \text{ lb}$$

$$\text{COEFF. OF } \hat{j}: (160 + 520) \frac{49.5}{42.43} - 30B_z = 0; \quad B_z = 34 \text{ lb}$$

$$\text{COEFF. OF } \hat{k}: (26)(25) \frac{49.5}{42.43} - 1425 + 30B_y = 0; \quad B_y = 15 \text{ lb}$$

$$B = (15\hat{j} + 34\hat{k}) \text{ lb}$$

$$\sum F = 0: A + B + T - (75\hat{j}) = 0$$

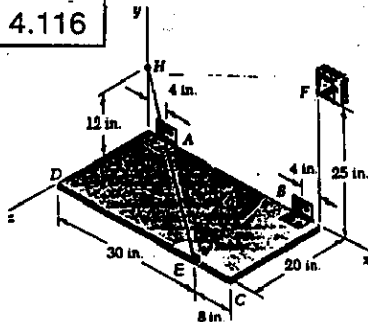
$$\text{COEFF. OF } \hat{i}: A_x + \frac{33}{42.43}(49.5) = 0 \quad A_x = -12 \text{ lb}$$

$$\text{COEFF. OF } \hat{j}: A_y + 15 + \frac{33}{42.43}(49.5) - 75 = 0 \quad A_y = 22.5 \text{ lb}$$

$$\text{COEFF. OF } \hat{k}: A_z + 34 - \frac{33}{42.43}(49.5) = 0 \quad A_z = -4 \text{ lb}$$

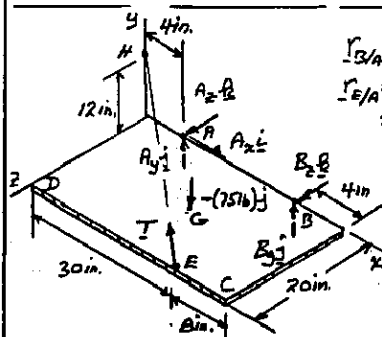
$$A = (-12\hat{i} + 22.5\hat{j} - 4\hat{k}) \text{ lb}$$

4.116



GIVEN:  $W = 75 \text{ lb}$   
 $B_x = 0$

FIND:  
 (a) TENSION IN CABLE EH.  
 (b) REACTIONS AT A AND B.



$$r_{B/A} = (30-8)\hat{i} = 22\hat{i}$$

$$r_{E/A} = (30-8)\hat{i} + 20\hat{j} = 22\hat{i} + 20\hat{j}$$

$$r_{G/A} = \frac{30}{2}\hat{i} + 10\hat{j} = 15\hat{i} + 10\hat{j}$$

$$\vec{EH} = -30\hat{i} + 4\hat{j} - 20\hat{j} = -30\hat{i} - 16\hat{j}$$

$$T = T \frac{\vec{EH}}{EH} = \frac{T}{38} (-30\hat{i} - 16\hat{j})$$

$$\sum M_A = 0: r_{E/A} \times T + r_{G/A} \times (-75\hat{j}) + r_{B/A} \times B = 0$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 22 & 0 & 20 \\ -30 & -16 & -20 \end{vmatrix} \frac{T}{38} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 15 & 10 & 0 \\ 0 & 0 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 22 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

$$\text{COEFF. OF } \hat{i}: -(12)(20)\frac{T}{38} + 750 = 0; T = 118.75; T = 118.8 \text{ lb}$$

$$\text{COEFF. OF } \hat{j}: (-600 + 520)\frac{118.75}{38} - 30B_z = 0; B_z = -8.33 \text{ lb}$$

$$\text{COEFF. OF } \hat{k}: (26)(12)\frac{118.75}{38} - 1425 + 30B_y = 0; B_y = 15.00 \text{ lb}$$

$$\sum F = 0: A + B + T - (75\hat{j}) = 0$$

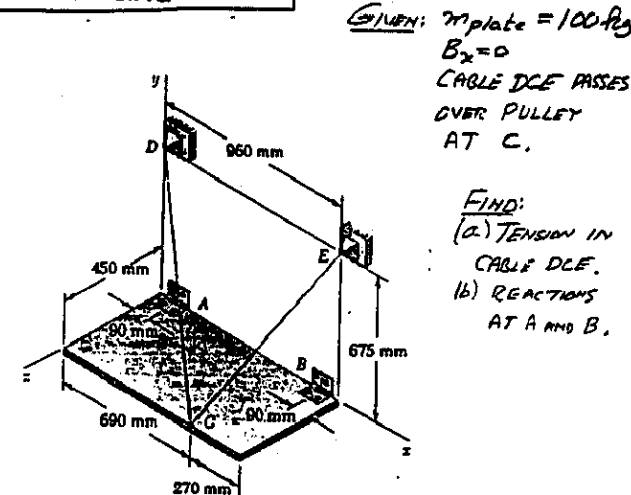
$$\text{COEFF. OF } \hat{i}: A_x - \frac{30}{38}(118.75) = 0; A_x = 93.75 \text{ lb}$$

$$\text{COEFF. OF } \hat{j}: A_y + 15 + \frac{16}{38}(118.75) - 75 = 0; A_y = 22.5 \text{ lb}$$

$$\text{COEFF. OF } \hat{k}: A_z - 8.33 - \frac{20}{38}(118.75) = 0; A_z = 70.83 \text{ lb}$$

$$A = (93.8 \text{ lb})\hat{i} + (22.5 \text{ lb})\hat{j} + (70.8 \text{ lb})\hat{k}$$

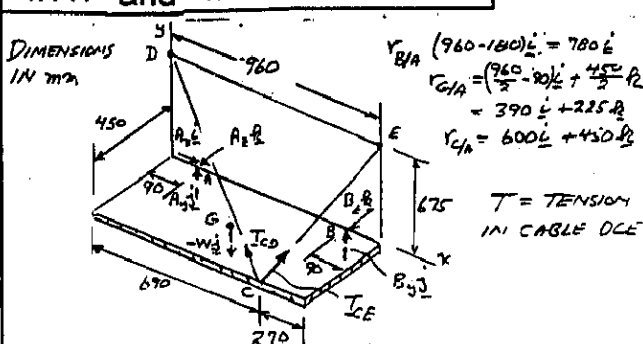
4.117 and 4.118



GIVEN:  $m_{\text{plate}} = 100 \text{ kg}$   
 $B_x = 0$   
 CABLE DCE PASSES OVER PULLEY AT C.

FIND:  
 (a) TENSION IN CABLE DCE.  
 (b) REACTIONS AT A AND B.

4.117 and 4.118 CONTINUED



$$\vec{CD} = -690\hat{i} + 675\hat{j} - 450\hat{k}$$

$$\vec{CE} = 270\hat{i} + 675\hat{j} - 450\hat{k}$$

$$T_{CD} = \frac{T}{1065} (-690\hat{i} + 675\hat{j} - 450\hat{k})$$

$$T_{CE} = \frac{T}{855} (270\hat{i} + 675\hat{j} - 450\hat{k})$$

$$W = -mg\hat{j} = -(100 \text{ kg})(9.81 \text{ m/s}^2)\hat{j} = -(981 \text{ N})\hat{j}$$

PROB. 4.117

$$\sum M_A = 0: r_{C/A} \times T_{CD} + r_{C/A} \times T_{CE} + r_{B/A} \times (-W\hat{j}) + r_{B/A} \times B = 0$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 600 & 0 & 450 \\ -670 & 675 & -450 \end{vmatrix} \frac{T}{1065} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 600 & 0 & 450 \\ 270 & 675 & -450 \end{vmatrix} \frac{T}{855} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 600 & 0 & 450 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

$$+ \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 780 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

$$\text{COEFF. OF } \hat{i}: -(450)(675)\frac{T}{1065} - (450)(675)\frac{T}{855} + 220.725 \times 10^3 = 0$$

$$T = 344.6 \text{ N} \quad T = 345 \text{ N}$$

$$\text{COEFF. OF } \hat{j}: (-670)(450) + 600(450)\frac{344.6}{1065} + (270)(450) + 600(450)\frac{344.6}{855} - 780B_z = 0$$

$$B_z = 185.79 \text{ N}$$

$$\text{COEFF. OF } \hat{k}: (600)(675)\frac{344.6}{1065} + (600)(675)\frac{344.6}{855} - 382.57 \times 10^3 + 780B_y = 0$$

$$B_y = 113.2 \text{ N}$$

$$\sum F = 0: A + B + T_{CD} + T_{CE} + W = 0$$

$$\text{COEFF. OF } \hat{i}: A_x - \frac{690}{1065}(344.6) + \frac{270}{855}(344.6) = 0; A_x = 114.4 \text{ N}$$

$$\text{COEFF. OF } \hat{j}: A_y + 113.2 + \frac{675}{1065}(344.6) + \frac{675}{855}(344.6) - 981 = 0; A_y = 377 \text{ N}$$

$$\text{COEFF. OF } \hat{k}: A_z + 185.79 - \frac{450}{1065}(344.6) - \frac{450}{855}(344.6) = 0; A_z = 111.5 \text{ N}$$

$$A = (114.4 \text{ N})\hat{i} + (377 \text{ N})\hat{j} + (111.5 \text{ N})\hat{k}$$

PROB. 4.118

$$\sum M_A = 0: r_{C/A} \times T_{CD} + r_{C/A} \times (-W\hat{j}) + r_{B/A} \times B = 0$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 600 & 0 & 450 \\ -670 & 675 & -450 \end{vmatrix} \frac{T}{855} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 600 & 0 & 450 \\ 270 & 675 & -450 \end{vmatrix} \frac{T}{855} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 600 & 0 & 450 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

$$\text{COEFF. OF } \hat{i}: -(450)(675)\frac{T}{855} + 220.725 \times 10^3 = 0$$

$$T = 621.3 \text{ N} \quad T = 621 \text{ N}$$

$$\text{COEFF. OF } \hat{j}: (270)(450) + 600(450)\frac{621.3}{855} - 780B_z = 0; B_z = 364.7 \text{ N}$$

$$\text{COEFF. OF } \hat{k}: (600)(675)\frac{621.3}{855} - 382.57 \times 10^3 + 780B_y = 0; B_y = 113.2 \text{ N}$$

$$B = (113.2 \text{ N})\hat{j} + (364.7 \text{ N})\hat{k}$$

$$\sum F = 0: A + B + T_{CD} + W = 0$$

$$\text{COEFF. OF } \hat{i}: A_x + \frac{270}{855}(621.3) = 0$$

$$A_x = -196.2 \text{ N}$$

$$\text{COEFF. OF } \hat{j}: A_y + 113.2 + \frac{675}{855}(621.3) - 981 = 0$$

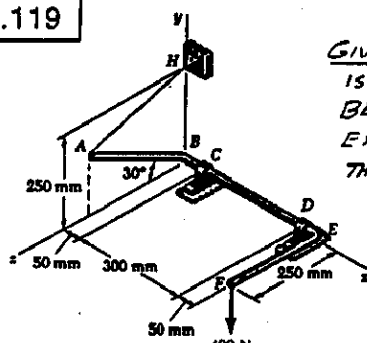
$$A_y = 377.3 \text{ N}$$

$$\text{COEFF. OF } \hat{k}: A_z + 364.7 - \frac{450}{855}(621.3) = 0$$

$$A_z = -377 \text{ N}$$

$$A = (-196.2 \text{ N})\hat{i} + (377 \text{ N})\hat{j} - (377 \text{ N})\hat{k}$$

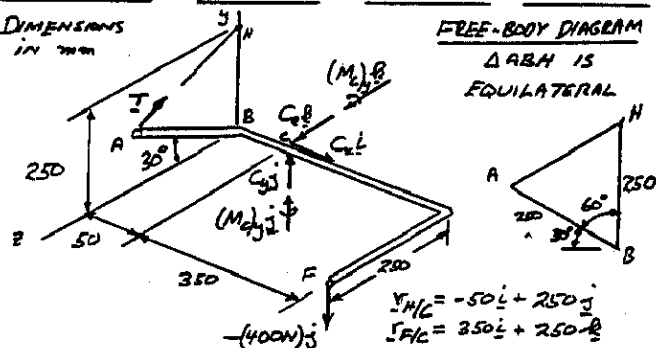
4.119



GIVEN: BEARING AT D IS REMOVED AND BEARING AT C CAN EXERT COUPLES ABOUT THE Y AND Z AXES.

FIND: TENSION IN WIRE AH AND REACTION AT C.

DIMENSIONS IN mm



FREE-BODY DIAGRAM  
△ABH IS  
EQUILATERAL

$$T = T(\sin 30^\circ)\mathbf{j} - T(\cos 30^\circ)\mathbf{k} = T(0.5\mathbf{j} - 0.866\mathbf{k})$$

$$\sum M_C = 0: \mathbf{r}_{F/C} \times (-400\mathbf{j}) + \mathbf{r}_{H/C} \times T + (M_C)_y\mathbf{j} + (M_C)_z\mathbf{k} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 350 & 0 & 250 \\ 0 & -400 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -50 & 250 & 0 \\ 0 & 0.5 & -0.866 \end{vmatrix} T + (M_C)_y\mathbf{j} + (M_C)_z\mathbf{k} = 0$$

$$\text{COEFF. OF } \mathbf{i}: +100,000 - 216.5T = 0; T = 461.9\text{ N}; T = 462\text{ N}$$

$$\text{COEFF. OF } \mathbf{j}: -43.3(461.9) + (M_C)_y = 0$$

$$(M_C)_y = 20 \times 10^3 \text{ N}\cdot\text{mm}; (M_C)_y = 20 \text{ N}\cdot\text{m}$$

$$\text{COEFF. OF } \mathbf{k}: -140,000 - 26(461.9) + (M_C)_z = 0$$

$$(M_C)_z = 151.57 \times 10^3 \text{ N}\cdot\text{mm}; (M_C)_z = 151.57 \text{ N}\cdot\text{m}$$

$$\sum F = 0: C_x + T = 0 \Rightarrow C_x = 0$$

$$\text{COEFF. OF } \mathbf{j}: C_y + 0.5(461.9) - 400 = 0$$

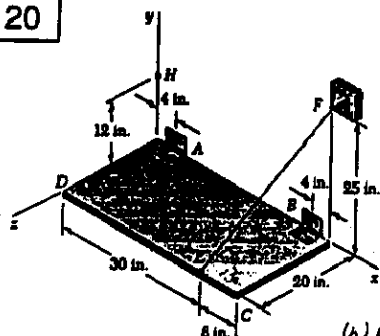
$$C_y = 169.1\text{ N}$$

$$\text{COEFF. OF } \mathbf{k}: C_z - 0.866(461.9) = 0$$

$$C_z = 400\text{ N}$$

$$\mathbf{C} = (169.1\text{ N})\mathbf{j} + (400\text{ N})\mathbf{k}$$

4.120



GIVEN:  $W = 75\text{ lb}$   
HINGE AT B IS REMOVED  
HINGE AT A CAN EXERT COUPLES PARALLEL TO y AND z AXES

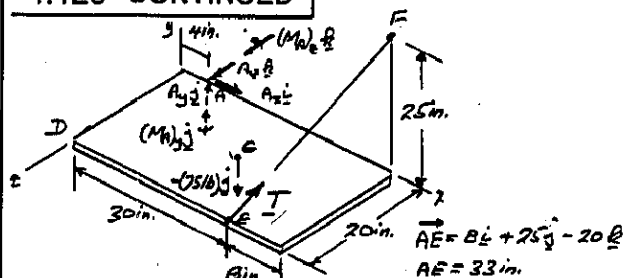
FIND:  
(a) TENSION IN CABLE EF  
(b) REACTION AT A

$$\mathbf{r}_{F/A} = (30-4)\mathbf{i} + 20\mathbf{j} = 26\mathbf{i} + 20\mathbf{j}$$

$$\mathbf{r}_{G/A} = (0.5 \times 38)\mathbf{i} + 10\mathbf{j} = 19\mathbf{i} + 10\mathbf{j}$$

(CONTINUED)

4.120 CONTINUED



$$T = T \frac{\mathbf{r}_{F/A}}{AE} = \frac{T}{33} (26\mathbf{i} + 25\mathbf{j} - 20\mathbf{k})$$

$$\sum M_A = 0: \mathbf{r}_{F/A} \times T + \mathbf{r}_{G/A} \times (-75\mathbf{j}) + (M_A)_y\mathbf{j} + (M_A)_z\mathbf{k} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 26 & 0 & 20 \\ 8 & 25 & -10 \end{vmatrix} \frac{T}{33} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 19 & 0 & 10 \\ 0 & -75 & 0 \end{vmatrix} + (M_A)_y\mathbf{j} + (M_A)_z\mathbf{k} = 0$$

$$\text{COEFF. OF } \mathbf{i}: -(20 \times 25) \frac{T}{33} + 750 = 0$$

$$T = 49.51\text{ lb}$$

$$\text{COEFF. OF } \mathbf{j}: (160 + 520) \frac{49.5}{33} + (M_A)_y = 0; (M_A)_y = -1020\text{ lb}\cdot\text{in.}$$

$$\text{COEFF. OF } \mathbf{k}: (24 \times 25) \frac{49.5}{33} - 1425 + (M_A)_z = 0; (M_A)_z = 450\text{ lb}\cdot\text{in.}$$

$$\sum F = 0: A + T - 75\mathbf{j} = 0 \Rightarrow \mathbf{A} = (1020\text{ lb}\cdot\text{in.})\mathbf{j} + (450\text{ lb}\cdot\text{in.})\mathbf{k}$$

$$\text{COEFF. OF } \mathbf{i}: A_x + \frac{26}{33}(49.5) = 0$$

$$A_x = -12\text{ lb}$$

$$\text{COEFF. OF } \mathbf{j}: A_y + \frac{25}{33}(49.5) - 75 = 0$$

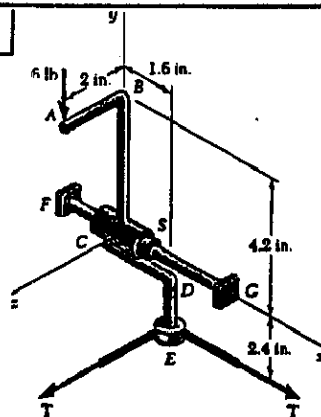
$$A_y = 37.5\text{ lb}$$

$$\text{COEFF. OF } \mathbf{k}: A_z - \frac{20}{33}(49.5) = 0$$

$$A_z = 30\text{ lb}$$

$$\mathbf{A} = (-12\text{ lb})\mathbf{i} + (37.5\text{ lb})\mathbf{j} + (30\text{ lb})\mathbf{k}$$

4.121



FIND:  
(a) TENSION IN TAPE  
(b) REACTION AT C

FREE-BODY DIAGRAM

$$\mathbf{r}_{A/C} = 4.2\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{r}_{E/C} = 1.6\mathbf{i} - 2.4\mathbf{j}$$

$$\sum M_C = 0$$

$$\mathbf{r}_{A/C} \times (-6\mathbf{j}) + \mathbf{r}_{E/C} \times T(\mathbf{i} + \mathbf{j}) + (M_C)_y\mathbf{j} + (M_C)_z\mathbf{k} = 0$$

$$(4.2\mathbf{j} + 2\mathbf{k}) \times (-6\mathbf{j}) + (1.6\mathbf{i} - 2.4\mathbf{j}) \times T(\mathbf{i} + \mathbf{j}) + (M_C)_y\mathbf{j} + (M_C)_z\mathbf{k} = 0$$

$$\text{COEFF. OF } \mathbf{i}: 12 - 2.4T = 0; T = 5\text{ lb}$$

$$\text{COEFF. OF } \mathbf{j}: -1.6(5) + (M_C)_y = 0; (M_C)_y = 8\text{ lb}\cdot\text{in.}$$

$$\text{COEFF. OF } \mathbf{k}: 2.4(5) + (M_C)_z = 0; (M_C)_z = -12\text{ lb}\cdot\text{in.}$$

$$\mathbf{M}_C = (8\text{ lb}\cdot\text{in.})\mathbf{j} - (12\text{ lb}\cdot\text{in.})\mathbf{k}$$

$$\sum F = 0: C_x\mathbf{i} + C_y\mathbf{j} + C_z\mathbf{k} - (6\text{ lb})\mathbf{j} + (5\text{ lb})\mathbf{i} + (5\text{ lb})\mathbf{j} = 0$$

$$\text{EQUATE COEFFICIENTS OF UNIT VECTORS TO ZERO}$$

$$C_x = -5\text{ lb}$$

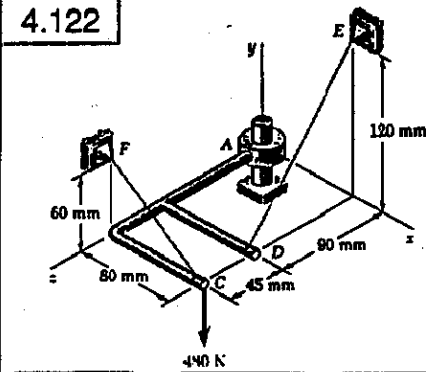
$$C_y = 6\text{ lb}$$

$$C_z = -5\text{ lb}$$

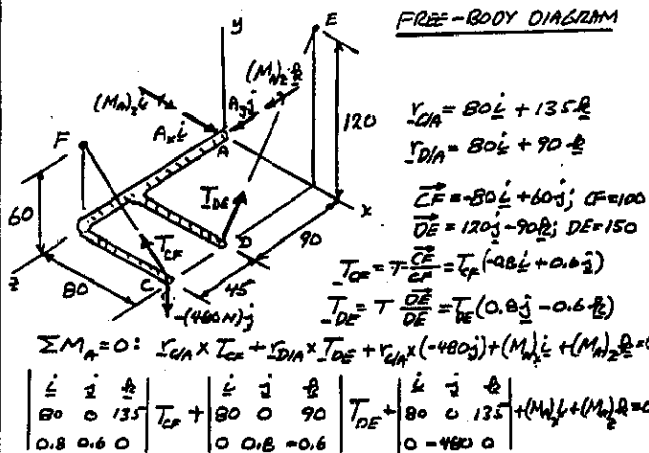
$$\mathbf{C} = (-5\text{ lb})\mathbf{i} + (6\text{ lb})\mathbf{j} - (5\text{ lb})\mathbf{k}$$



## 4.122



FIND:  
TENSION IN  
EACH CABLE,  
REACTION  
AT A.



FREE-BODY DIAGRAM

$$r_{CA} = 80\mathbf{i} + 135\mathbf{j}$$

$$r_{DA} = 80\mathbf{i} + 90\mathbf{j}$$

$$r_{CE} = -80\mathbf{i} + 60\mathbf{j}; CE = 100$$

$$r_{DE} = 120\mathbf{j} - 90\mathbf{k}; DE = 150$$

$$T_{CE} = T_{CE} \frac{r_{CE}}{CE} = T_{CE} (-0.8\mathbf{i} + 0.6\mathbf{j})$$

$$T_{DE} = T_{DE} \frac{r_{DE}}{DE} = T_{DE} (0.8\mathbf{j} - 0.6\mathbf{k})$$

$$\Sigma M_A = 0: r_{CA} \times T_{CE} + r_{DA} \times T_{DE} + r_{EA} \times (-480\mathbf{j}) + (M_A)_x \mathbf{i} + (M_A)_y \mathbf{j} + (M_A)_z \mathbf{k} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 80 & 0 & 135 \\ T_{CE} & -0.8T_{CE} & 0.6T_{CE} \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 80 & 0 & 90 \\ T_{DE} & 0.8T_{DE} & -0.6T_{DE} \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

$$\text{COEFF. OF } \mathbf{i}: -81T_{CE} - 72T_{DE} + 64(480) + (M_A)_x = 0 \quad (1)$$

$$\text{COEFF. OF } \mathbf{j}: 108T_{CE} + 72T_{DE} = 0; T_{CE} = \frac{4}{9}T_{DE} \quad (2)$$

$$\text{COEFF. OF } \mathbf{k}: 48T_{CE} + 64T_{DE} - 38.4(480) + (M_A)_z = 0 \quad (3)$$

$$\Sigma F = 0: T_{CE} + T_{DE} - 480\mathbf{j} + \mathbf{A} = 0$$

$$\text{COEFF. OF } \mathbf{j}: 0.6T_{CE} + 0.8T_{DE} - 480 = 0$$

$$\text{USE EQ. (2)}: 0.6(\frac{4}{9}T_{DE}) + 0.8T_{DE} = 480; T_{DE} = 450 \text{ N}$$

$$T_{CE} = \frac{4}{9}T_{DE} = \frac{4}{9}(450) = 200 \text{ N}$$

$$\text{COEFF. OF } \mathbf{i}: A_x - 0.8(200) = 0; A_x = 160 \text{ N}$$

$$\text{COEFF. OF } \mathbf{k}: A_z - 0.6(450) = 0; A_z = 270 \text{ N}$$

$$\mathbf{A} = (160 \text{ N})\mathbf{i} + (270 \text{ N})\mathbf{k}$$

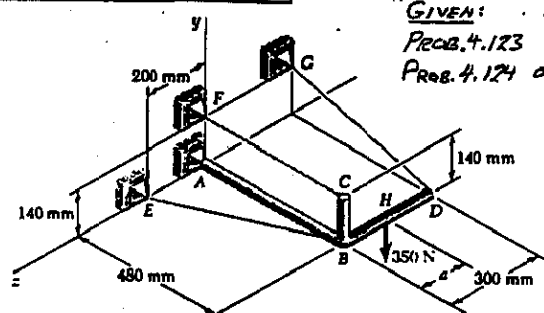
$$EG(1): -81(200) - 72(450) + 64(480) + (M_A)_x = 0$$

$$(M_A)_x = -16.2(10^3) \text{ N}\cdot\text{mm}; (M_A)_y = -16.2(10^3) \text{ N}\cdot\text{mm}$$

$$EG(3): 48(200) + 64(450) - 38.4(480) + (M_A)_z = 0; (M_A)_z = 0$$

$$M_A = -(16.2 \text{ N}\cdot\text{m})\mathbf{i}$$

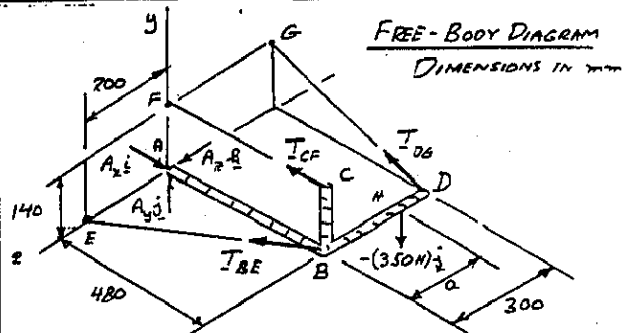
## 4.123 and 4.124



FIND: TENSION IN CABLES,  
REACTION AT A.

(CONTINUED)

## 4.123 and 4.124 CONTINUED



FREE-BODY DIAGRAM

DIMENSIONS IN mm

$$r_{BE} = -480\mathbf{i} + 200\mathbf{j}; BE = 520 \text{ mm}; \frac{r_{BE}}{BE} = \frac{1}{52}(-12\mathbf{i} + 5\mathbf{j})$$

$$r_{DE} = -480\mathbf{i} + 140\mathbf{j}; DE = 500 \text{ mm}; \frac{r_{DE}}{DE} = \frac{1}{25}(-24\mathbf{i} + 7\mathbf{j})$$

$$T_{BE} = T_{BE} \frac{r_{BE}}{BE} = \frac{T_{BE}}{52}(-12\mathbf{i} + 5\mathbf{j}); T_{CF} = -T_{CF}\mathbf{i}$$

$$T_{DG} = T_{DG} \frac{r_{DG}}{DG} = \frac{T_{DG}}{25}(-24\mathbf{i} + 7\mathbf{j})$$

$$\text{PROB. 4.123 } a = 150 \text{ mm } r_{HA} = 480\mathbf{i} - 150\mathbf{k}$$

$$\Sigma M_A = 0: r_{BA} \times T_{BE} + r_{FA} \times T_{CF} + r_{GA} \times T_{DG} + r_{HA} \times (-350\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 480 & 0 & 0 \\ T_{BE} & -12T_{BE}/52 & 5T_{BE}/52 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 140 & 0 \\ T_{CF} & -T_{CF} & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 140 & -300 \\ T_{DG} & -24T_{DG}/25 & 7T_{DG}/25 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 480 & 0 & -150 \\ 0 & -350 & 0 \end{vmatrix} = 0$$

$$\text{COEFF. OF } \mathbf{i}: 2100 \frac{T_{BE}}{52} - (150 \times 350) = 0; T_{DG} = 625 \text{ N}$$

$$\text{COEFF. OF } \mathbf{j}: -2400 \frac{T_{BE}}{52} + 7200 \frac{625}{25} = 0; T_{BE} = 975 \text{ N}$$

$$\text{COEFF. OF } \mathbf{k}: 140 T_{CF} + (24 \times 140) \frac{625}{25} - 168 \times 10 = 0; T_{CF} = 600 \text{ N}$$

$$\Sigma F = 0: \mathbf{A} + T_{BE} + T_{DG} + T_{CF} - 350\mathbf{j} = 0$$

$$\text{COEFF. OF } \mathbf{i}: A_x - \frac{12}{52} 975 - \frac{24}{25} 625 - 600 = 0; A_x = 2100 \text{ N}$$

$$\text{COEFF. OF } \mathbf{j}: A_y + \frac{7}{25} 625 - 350 = 0; A_y = 175 \text{ N}$$

$$\text{COEFF. OF } \mathbf{k}: A_z + \frac{7}{25} 975 = 0; A_z = -375 \text{ N}$$

$$\mathbf{A} = (2100 \text{ N})\mathbf{i} + (175 \text{ N})\mathbf{j} - (375 \text{ N})\mathbf{k}$$

## PROB. 4.124

$$a = 300 \text{ mm}$$

$$r_{HA} = 480\mathbf{i} - 300\mathbf{k}$$

$$\Sigma M_A = 0: r_{BA} \times T_{BE} + r_{FA} \times T_{CF} + r_{GA} \times T_{DG} + r_{HA} \times (-350\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 480 & 0 & 0 \\ T_{BE} & -12T_{BE}/52 & 5T_{BE}/52 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 140 & 0 \\ T_{CF} & -T_{CF} & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 140 & -300 \\ T_{DG} & -24T_{DG}/25 & 7T_{DG}/25 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 480 & 0 & -300 \\ 0 & -350 & 0 \end{vmatrix} = 0$$

$$\text{COEFF. OF } \mathbf{i}: 2100 \frac{T_{BE}}{52} - (300 \times 350) = 0; T_{DG} = 1250 \text{ N}$$

$$\text{COEFF. OF } \mathbf{j}: -2400 \frac{T_{BE}}{52} + 7200 \frac{1250}{25} = 0; T_{BE} = 1950 \text{ N}$$

$$\text{COEFF. OF } \mathbf{k}: 140 T_{CF} + (24 \times 140) \frac{1250}{25} - 168 \times 10 = 0; T_{CF} = 0$$

$$\Sigma F = 0: \mathbf{A} + T_{BE} + T_{DG} + T_{CF} - 350\mathbf{j} = 0$$

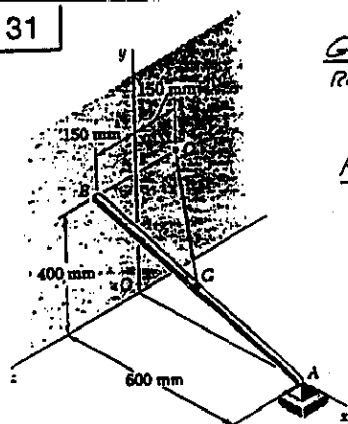
$$\text{COEFF. OF } \mathbf{i}: A_x - \frac{12}{52} 1950 - \frac{24}{25} 1250 + 0 = 0; A_x = 3000 \text{ N}$$

$$\text{COEFF. OF } \mathbf{j}: A_y + \frac{7}{25} 1250 - 350 = 0; A_y = 0$$

$$\text{COEFF. OF } \mathbf{k}: A_z + \frac{7}{25} 1950 = 0; A_z = -750 \text{ N}$$

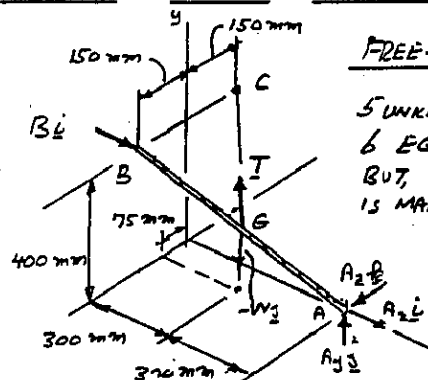
$$\mathbf{A} = (3000 \text{ N})\mathbf{i} - (750 \text{ N})\mathbf{k}$$

4.131



GIVEN: MASS OF  
ROD AB:  $m = 10 \text{ kg}$

FIND: (a) TENSION  
IN CORD CG,  
(b) REACTIONS  
AT A AND B.



FREE-BODY DIAGRAM

5 UNKNOWNS AND  
6 EQS. OF EQUIL.  
BUT, EQUILIBRIUM  
IS MAINTAINED ( $\sum M_A = 0$ )

$$W = mg = (10 \text{ kg})(9.81 \text{ m/s}^2) = 98.1 \text{ N}$$

$$\vec{GC} = -300\hat{i} + 200\hat{j} - 225\hat{k} \quad GC = 425 \text{ mm}$$

$$\vec{T} = T \frac{\vec{GC}}{GC} = \frac{T}{425} (-300\hat{i} + 200\hat{j} - 225\hat{k})$$

$$\vec{r}_{GA} = -100\hat{i} + 400\hat{j} + 150\hat{k} \text{ mm}$$

$$\vec{r}_{CA} = -300\hat{i} + 200\hat{j} + 75\hat{k} \text{ mm}$$

$$\sum \vec{M}_A = 0: \vec{r}_{BA} \times \vec{B} + \vec{r}_{CA} \times \vec{T} + \vec{r}_{GA} \times (-W\hat{j}) = 0$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -600 & 400 & 150 \\ B_x & B_y & B_z \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -300 & 200 & 75 \\ -\frac{T}{425}(-300 & 200 & -225) \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -100 & 400 & 150 \\ 0 & -98.1 & 0 \end{vmatrix} = 0$$

$$\text{COEFF. OF } \hat{i}: (-105.88 - 35.29)T + 7357.5 = 0$$

$$T = 52.12 \text{ N} \quad T = 52.1 \text{ N}$$

$$\text{COEFF. OF } \hat{j}: 150B_x - (300)(75) + (300)(225) \frac{52.12}{425} = 0$$

$$B_x = 73.58 \text{ N} \quad B_x = 73.6 \text{ N}$$

$$\sum F = 0: A_x + B_x + T - W = 0$$

$$\text{COEFF. OF } \hat{i}: A_x + 73.58 - 52.15 \frac{300}{425} = 0 \quad A_x = 37.8 \text{ N}$$

$$\text{COEFF. OF } \hat{j}: A_y + 52.15 \frac{200}{425} - 98.1 = 0 \quad A_y = 73.6 \text{ N}$$

$$\text{COEFF. OF } \hat{k}: A_z - 52.15 \frac{225}{425} = 0 \quad A_z = 27.6 \text{ N}$$

ALTERNATE COMPUTATION OF  $B_x$ :

$$\vec{AC} = -600\hat{i} + 400\hat{j} - 150\hat{k}; \quad \hat{r}_{AC} = \frac{\vec{AC}}{AC}$$

$$\sum \vec{M}_A = 0: \hat{r}_{AC} \cdot \vec{M}_A = \hat{r}_{AC} \cdot (\vec{r}_{BA} \times \vec{B}) + \hat{r}_{AC} \cdot (\vec{r}_{GA} \times (-W\hat{j})) = 0$$

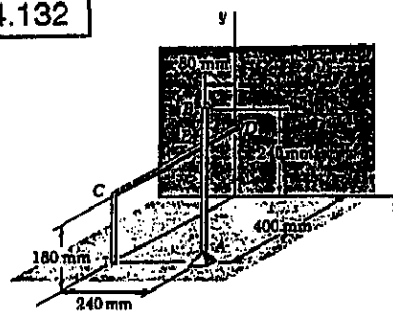
$$\begin{vmatrix} -600 & 400 & -150 \\ -600 & 400 & -150 \\ B_x & B_y & B_z \end{vmatrix} \frac{1}{AC} + \begin{vmatrix} -600 & 400 & -150 \\ -300 & 200 & 75 \\ -W \end{vmatrix} \frac{1}{AC} = 0$$

$$B(400 \times 150 + 400 \times 150) - W(300 \times 150 + 600 \times 75) = 0$$

$$120000B - 90000W = 0$$

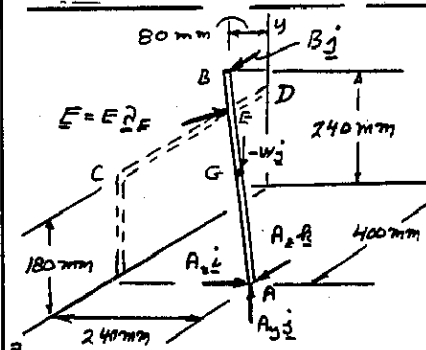
$$B = \frac{3}{4}W = \frac{3}{4}(98.1) = 73.6 \text{ N}; \quad B = 73.6 \text{ N}$$

4.132



GIVEN: MASS  
OF ROD AB  
 $m = 5 \text{ kg}$

FIND: (a) FORCE  
CD EXERTS  
ON AB  
(b) REACTIONS  
AT A AND B



FREE-BODY DIAGRAM

5 UNKNOWNS AND  
6 EQS. OF EQUIL.  
BUT, EQUILIBRIUM  
MAINTAINED  
( $\sum M_A = 0$ )

$$W = mg = (5 \text{ kg})(9.81 \text{ m/s}^2) = 49.05 \text{ N}$$

$\hat{r}_E$  = UNIT VECTOR  $\perp$  TO AB AND CD

$$\vec{AB} = -320\hat{i} + 240\hat{j} - 400\hat{k}$$

UNIT VECTOR ALONG ROD CD IS  $\hat{r}_E$

$$\hat{r}_E = \frac{\vec{AB} \times \vec{r}_E}{|\vec{AB} \times \vec{r}_E|} = \frac{(-320\hat{i} + 240\hat{j} - 400\hat{k}) \times \vec{r}_E}{|\vec{AB} \times \vec{r}_E|}$$

$$\hat{r}_E = \frac{320\hat{i} + 240\hat{j}}{\sqrt{320^2 + 240^2}} = \frac{320\hat{i} + 240\hat{j}}{400}; \quad \hat{r}_E = 0.8\hat{i} + 0.6\hat{j}$$

$$\vec{F} = F\hat{r}_E; \quad \vec{F} = F(0.8\hat{i} + 0.6\hat{j}) \quad (1)$$

$$\vec{r}_{BA} = \vec{AB} = -320\hat{i} + 240\hat{j} - 400\hat{k}$$

$$\vec{r}_{CA} = \frac{1}{2}\vec{AB} = -160\hat{i} + 120\hat{j} - 200\hat{k}$$

$$\vec{r}_{EA} = \frac{180}{240}\vec{AB} = -240\hat{i} + 180\hat{j} - 300\hat{k}$$

$$\sum \vec{M}_A = 0: \vec{r}_{BA} \times \vec{B} + \vec{r}_{CA} \times (-W\hat{j}) + \vec{r}_{EA} \times \vec{F} = 0$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -320 & 240 & -400 \\ B_x & B_y & B_z \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -160 & 120 & -200 \\ 0 & -W & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -240 & 180 & -300 \\ 0.6 & 0.8 & 0 \end{vmatrix} F = 0$$

Equate coefficients of unit vectors to zero

$$\text{COEFF. OF } \hat{i}: +160W + (-240)(0.8) - (180)(0.6)F = 0$$

$$160(49.05) - 300F = 0 \quad F = 26.16 \text{ N}$$

$$\text{COEFF. OF } \hat{j}: 240B_x - 200W + 300(0.8)F = 0$$

$$240B_x - 200(49.05) + 240(26.16) = 0; \quad B_x = 14.72 \text{ N}$$

$$\text{THUS: } \vec{F} = F(0.6\hat{i} + 0.8\hat{j}) = 26.16(0.6\hat{i} + 0.8\hat{j})$$

$$\vec{F} = (15.70\hat{i} + 20.9\hat{j}) \text{ N}$$

$$\vec{B} = B\hat{j}$$

$$\vec{B} = (14.72\hat{j}) \text{ N}$$

$$\sum \vec{F} = 0: \vec{A} + \vec{B} + \vec{F} - W\hat{j} = 0$$

$$\textcircled{1} A_x + 15.70 = 0$$

$$A_x = -15.70 \text{ N}$$

$$\textcircled{2} A_y + 20.9 - 49.05 = 0$$

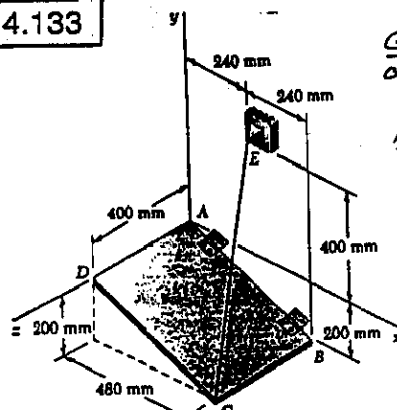
$$A_y = 28.1 \text{ N}$$

$$\textcircled{3} A_z + 14.72 = 0$$

$$A_z = -14.72 \text{ N}$$

$$\vec{A} = (-15.70\hat{i} + 28.1\hat{j} - 14.72\hat{k}) \text{ N}$$

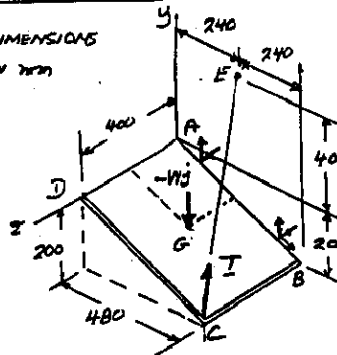
4.133



GIVEN: MASS  
OF PLATE  $m = 50 \text{ kg}$

FIND: TENSION  
IN WIRE CE

DIMENSIONS  
IN mm



FREE-BODY DIAGRAM

$$W = mg = (50 \text{ kg})(9.81 \text{ m/s}^2)$$

$$W = 490.5 \text{ N}$$

$$\vec{CE} = 240\hat{i} + 400\hat{j} - 400\hat{k}$$

$$CE = 760 \text{ mm}$$

$$T = T \frac{\vec{CE}}{CE}$$

$$T = \frac{T}{760} (-240\hat{i} + 400\hat{j} - 400\hat{k})$$

$$\vec{r}_{AB} = \frac{\vec{AB}}{AB} = \frac{480\hat{i} - 200\hat{j}}{520}$$

$$\vec{r}_{AB} = \frac{1}{13} (12\hat{i} - 5\hat{j})$$

$$\sum M_A = 0: \vec{r}_{AB} \cdot (\vec{r}_{CE} \times T) + \vec{r}_{AB} \cdot (\vec{r}_{GA} \times W) = 0$$

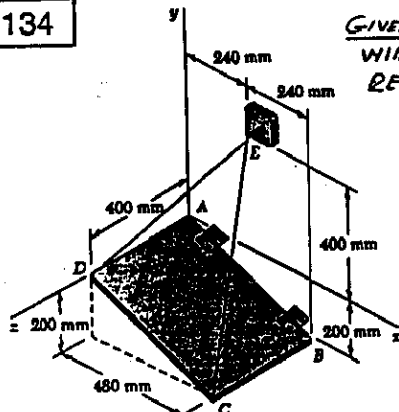
$$\vec{r}_{EA} = 240\hat{i} + 400\hat{j}; \vec{r}_{EA} = 240\hat{i} - 100\hat{j} + 200\hat{k}$$

$$\begin{vmatrix} 12 & -5 & 0 \\ 240 & 400 & 0 \\ 240 & 400 & -400 \end{vmatrix} \frac{T}{13 \times 760} + \begin{vmatrix} 12 & -5 & 0 \\ 240 & -100 & 200 \\ 0 & -W & 0 \end{vmatrix} \frac{1}{13} = 0$$

$$(-12 \times 400 \times 400 - 5 \times 240 \times 400) \frac{T}{760} + 12 \times 200 \times W = 0$$

$$T = 0.76W = 0.76(490.5 \text{ N}); \quad T = 373 \text{ N}$$

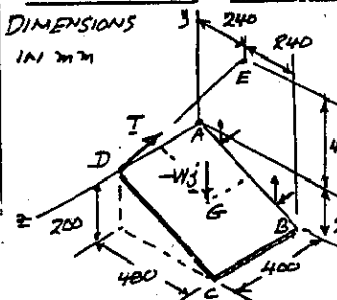
4.134



GIVEN: REMOVE  
WIRE CE AND  
REPLACE BY  
WIRE DE.  
MASS OF  
PLATE:  $m = 50 \text{ kg}$

FIND: TENSION  
IN WIRE DE

DIMENSIONS  
IN mm



FREE-BODY DIAGRAM

$$W = mg = (50 \text{ kg})(9.81 \text{ m/s}^2)$$

$$W = 490.5 \text{ N}$$

$$\vec{DE} = 240\hat{i} + 400\hat{j} - 400\hat{k}$$

$$DE = 614.5 \text{ mm}$$

$$T = T \frac{\vec{DE}}{DE}$$

$$T = \frac{T}{614.5} (240\hat{i} + 400\hat{j} - 400\hat{k})$$

(CONTINUED)

4.134 CONTINUED

$$\vec{r}_{AB} = \frac{\vec{AB}}{AB} = \frac{480\hat{i} - 200\hat{j}}{520}; \quad \vec{r}_{AB} = \frac{1}{13} (12\hat{i} - 5\hat{j})$$

$$\vec{r}_{EA} = 240\hat{i} + 400\hat{j}; \quad \vec{r}_{EA} = 240\hat{i} - 100\hat{j} + 200\hat{k}$$

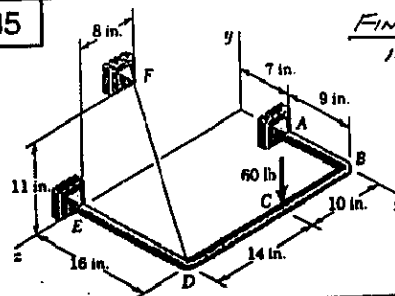
$$\sum M_A = 0: \vec{r}_{AB} \cdot (\vec{r}_{EA} \times T) + \vec{r}_{AB} \cdot (\vec{r}_{GA} \times W) = 0$$

$$\begin{vmatrix} 12 & -5 & 0 \\ 240 & 400 & 0 \\ 240 & 400 & -400 \end{vmatrix} \frac{T}{13 \times 614.5} + \begin{vmatrix} 12 & -5 & 0 \\ 240 & -100 & 200 \\ 0 & -W & 0 \end{vmatrix} \frac{1}{13} = 0$$

$$(-12 \times 400 \times 400 - 5 \times 240 \times 400) \frac{T}{614.5} + 12 \times 200 \times W = 0$$

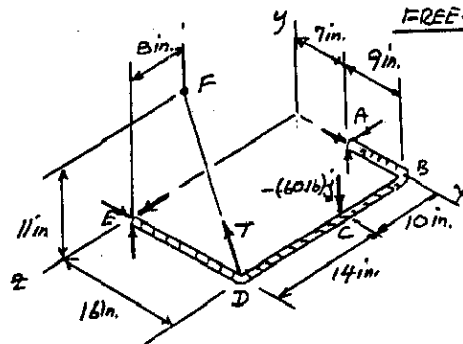
$$T = 0.6145W = 0.6145(490.5 \text{ N}); \quad T = 301 \text{ N}$$

4.135



FIND: TENSION  
IN CABLE DF

FREE-BODY DIAGRAM



$$\vec{DF} = -16\hat{i} + 11\hat{j} - 8\hat{k}; \quad DF = 21 \text{ in.}$$

$$T = T \frac{\vec{DF}}{DF} = \frac{T}{21} (-16\hat{i} + 11\hat{j} - 8\hat{k})$$

$$\vec{r}_{DE} = 16\hat{i}$$

$$\vec{r}_{DE} = 16\hat{i} - 14\hat{k}$$

$$\vec{r}_{EA} = \frac{\vec{EA}}{EA} = \frac{7\hat{i} - 24\hat{k}}{25}$$

$$\sum M_E = 0: \vec{r}_{EA} \cdot (\vec{r}_{DE} \times T) + \vec{r}_{EA} \cdot (\vec{r}_{GA} \times W) = 0$$

$$\begin{vmatrix} 7 & 0 & -24 \\ 16 & 0 & 0 \\ -16 & 11 & -8 \end{vmatrix} \frac{T}{21 \times 25} + \begin{vmatrix} 7 & 0 & -24 \\ 16 & 0 & -14 \\ 0 & -60 & 0 \end{vmatrix} \frac{1}{25} = 0$$

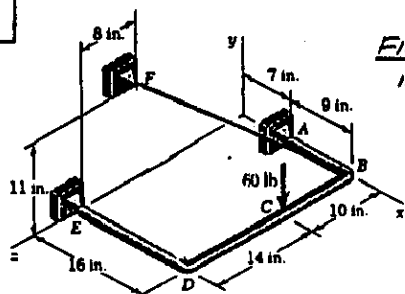
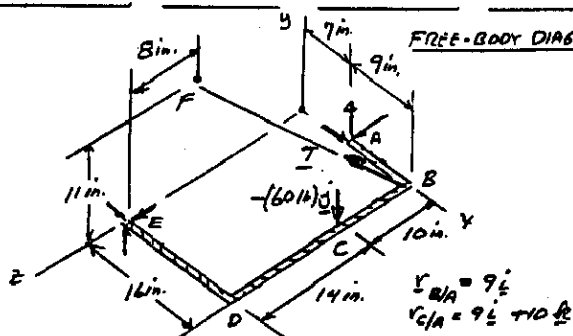
$$- \frac{24 \times 16 \times 11}{21 \times 25} T + \frac{7 \times 14 \times 60 + 24 \times 16 \times 60}{25} = 0$$

$$201.14T + 17,160 = 0$$

$$T = 85.31 \text{ lb}$$

$$T = 85.3 \text{ lb}$$

4.136

FIND: TENSION  
IN CABLE BF

FREE-BODY DIAGRAM

$$\vec{BF} = -16\hat{i} + 11\hat{j} + 16\hat{k} \quad BF = 25.16 \text{ in.}$$

$$T = T \frac{\vec{BF}}{BF} = \frac{T}{25.16} (-16\hat{i} + 11\hat{j} + 16\hat{k})$$

$$\vec{r}_{AE} = \frac{\vec{AE}}{AE} = \frac{7\hat{i} - 24\hat{k}}{25}$$

$$\sum M_{AE} = 0: \vec{r}_{AE} \cdot (\vec{r}_{BA} \times T) + \vec{r}_{AE} \cdot (\vec{r}_{CB} \times (-60\hat{j})) = 0$$

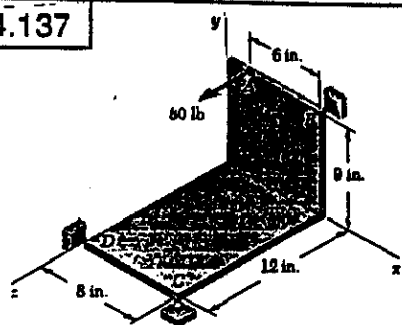
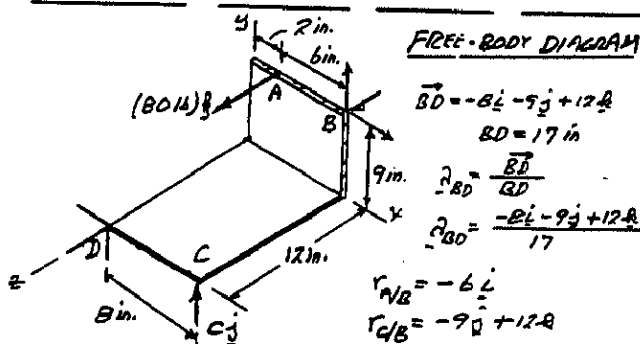
$$\begin{vmatrix} 7 & 0 & -24 \\ 9 & 0 & 0 \\ -16 & 11 & 16 \end{vmatrix} \frac{T}{25.16} + \begin{vmatrix} 7 & 0 & -24 \\ 9 & 0 & 10 \\ 0 & -60 & 0 \end{vmatrix} \frac{1}{25} = 0$$

$$-\frac{24 \times 9 \times 11}{25 \times 25.16} T + \frac{24 \times 9 \times 60 + 7 \times 10 \times 60}{25} = 0$$

$$94.4267 - 17.160 = 0$$

$$T = 181.716 \text{ lb}$$

4.137

FIND:  
REACTION AT C.

FREE-BODY DIAGRAM

$$\vec{BD} = -8\hat{i} - 9\hat{j} + 12\hat{k}$$

$$BD = 17 \text{ in.}$$

$$\vec{r}_{BD} = \frac{\vec{BD}}{BD}$$

$$\vec{r}_{BD} = \frac{-8\hat{i} - 9\hat{j} + 12\hat{k}}{17}$$

$$\vec{r}_{AB} = -6\hat{i}$$

$$\vec{r}_{CB} = -9\hat{j} + 12\hat{k}$$

(CONTINUED)

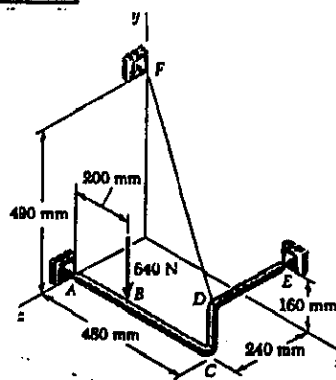
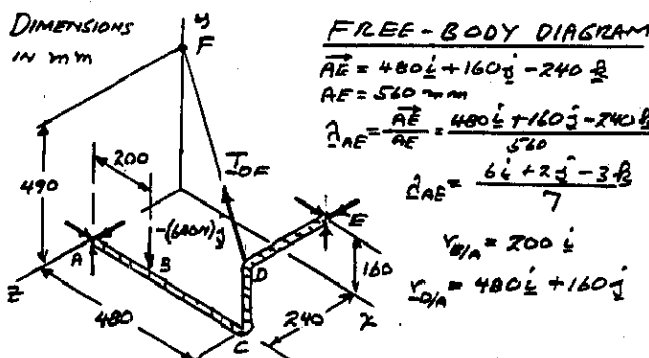
4.137 CONTINUED

$$\sum M_{BD} = 0: \vec{r}_{BD} \cdot (\vec{r}_{AB} \times (-60\hat{j})) + \vec{r}_{BD} \cdot (\vec{r}_{CB} \times (80\hat{i})) = 0$$

$$\begin{vmatrix} -8 & -9 & 12 \\ 0 & -9 & 12 \\ 0 & 0 & 0 \end{vmatrix} \frac{1}{17} + \begin{vmatrix} -8 & -9 & 12 \\ -6 & 0 & 0 \\ 0 & 0 & 80 \end{vmatrix} \frac{1}{17} = 0$$

$$\frac{B \times 12 \times C}{17} - \frac{9 \times 6 \times 80}{17} = 0; C = 45.16 \text{ lb} \quad C = (45.16)\hat{j}$$

4.138

FIND: TENSION  
IN WIRE DFDIMENSIONS  
IN mm

FREE-BODY DIAGRAM

$$\vec{AE} = 480\hat{i} + 160\hat{j} - 240\hat{k}$$

$$AE = 560 \text{ mm}$$

$$\vec{r}_{AE} = \frac{\vec{AE}}{AE} = \frac{480\hat{i} + 160\hat{j} - 240\hat{k}}{560}$$

$$\vec{r}_{AE} = \frac{6\hat{i} + 2\hat{j} - 3\hat{k}}{7}$$

$$\vec{r}_{BA} = 200\hat{i}$$

$$\vec{r}_{DA} = 480\hat{i} + 160\hat{j}$$

$$\vec{DF} = -480\hat{i} + 330\hat{j} - 240\hat{k}; \quad DF = 630 \text{ mm}$$

$$T_{DF} = T_{DF} \frac{\vec{DF}}{DF} = \frac{T_{DF}}{630} (-480\hat{i} + 330\hat{j} - 240\hat{k}) = \frac{T_{DF}}{630} (-16\hat{i} + 11\hat{j} - 8\hat{k})$$

$$\sum M_{AE} = \vec{r}_{AE} \cdot (\vec{r}_{BA} \times T_{DF}) + \vec{r}_{AE} \cdot (\vec{r}_{DA} \times (-600\hat{j})) = 0$$

$$\begin{vmatrix} 6 & 2 & -3 \\ 480 & 160 & 0 \\ -16 & 11 & -8 \end{vmatrix} \frac{T_{DF}}{21 \times 7} + \begin{vmatrix} 6 & 2 & -3 \\ 200 & 0 & 0 \\ 0 & -640 & 0 \end{vmatrix} \frac{1}{7} = 0$$

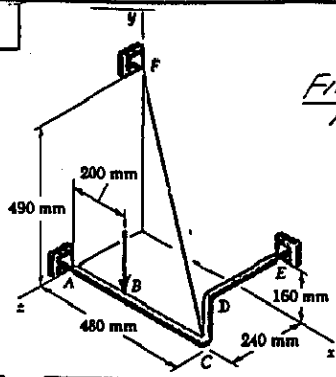
$$-6 \times 160 \times 8 + 2 \times 480 \times 8 - 3 \times 480 \times 11 - 3 \times 160 \times 16 \frac{T_{DF}}{7} + \frac{3 \times 200 \times 640}{7} = 0$$

$$-1120 T_{DF} + 384 \times 10^3 = 0$$

$$T_{DF} = 342.9 \text{ N}$$

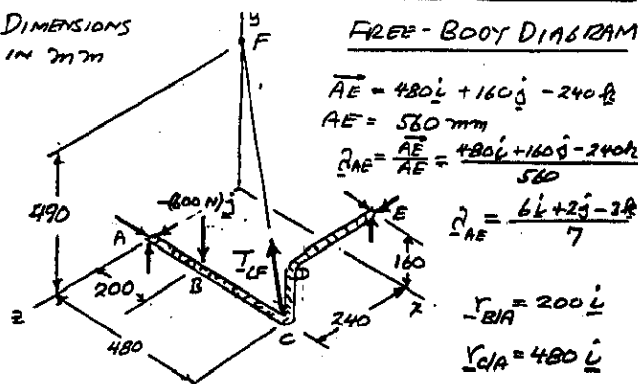
$$T_{DF} = 343 \text{ N}$$

4.139



FIND: TENSION  
IN WIRE CF

DIMENSIONS  
IN mm



FREE-BODY DIAGRAM

$$\vec{AE} = 480\hat{i} + 160\hat{j} - 240\hat{k}$$

$$AE = 560 \text{ mm}$$

$$\hat{e}_{AE} = \frac{\vec{AE}}{AE} = \frac{480\hat{i} + 160\hat{j} - 240\hat{k}}{560}$$

$$\hat{e}_{AE} = \frac{6\hat{i} + 2\hat{j} - 3\hat{k}}{7}$$

$$\vec{r}_{B/A} = 200\hat{i}$$

$$\vec{r}_{C/A} = 480\hat{i}$$

$$\vec{CF} = -480\hat{i} + 490\hat{j} - 240\hat{k} \quad CF = 726.7 \text{ mm}$$

$$\vec{T}_{CF} = T_{CF} \frac{\vec{CF}}{CF} = \frac{-480\hat{i} + 490\hat{j} - 240\hat{k}}{726.7}$$

$$\sum M_{AE} = 0: \hat{e}_{AE} \cdot (\vec{r}_{C/A} \times \vec{T}_{CF}) + \hat{e}_{AE} \cdot (\vec{r}_{B/A} \times (-800\hat{j})) = 0$$

$$\begin{vmatrix} 6 & 2 & -3 \\ 480 & 0 & 0 \\ -480 & 490 & -240 \end{vmatrix} \frac{T_{CF}}{726.7 \times 7} + \begin{vmatrix} 6 & 2 & -3 \\ 200 & 0 & 0 \\ 0 & -800 & 0 \end{vmatrix} \frac{1}{7} = 0$$

$$\frac{2 \times 480 \times 240 - 3 \times 480 \times 490}{726.7 \times 7} T_{CF} + \frac{3 \times 200 \times 640}{7} = 0$$

$$-653.91 T_{CF} + 384 \times 10^3 = 0; \quad T_{CF} = 587 \text{ N}$$

4.140 CONTINUED

$$\vec{AF} = 4\hat{i} - 2\hat{j} - 4\hat{k} \quad AF = 3 \text{ ft}$$

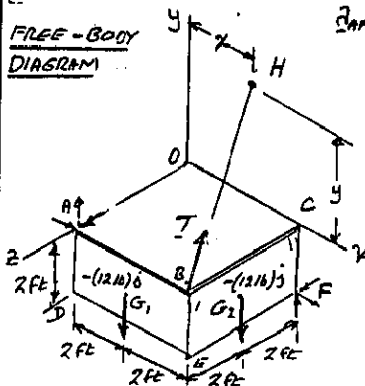
$$\hat{e}_{AF} = \frac{1}{3}(2\hat{i} - \hat{j} - 2\hat{k})$$

$$\vec{r}_{G_1/A} = 2\hat{i} - \hat{j}$$

$$\vec{r}_{G_2/A} = 4\hat{i} - \hat{j} - 2\hat{k}$$

$$\vec{r}_{B/A} = 4\hat{i}$$

FREE-BODY  
DIAGRAM



$$\sum M_{AF} = 0: \hat{e}_{AF} \cdot (\vec{r}_{G_1/A} \times (-12\hat{j})) + \hat{e}_{AF} \cdot (\vec{r}_{G_2/A} \times (-12\hat{j})) + \hat{e}_{AF} \cdot (\vec{r}_{B/A} \times T) = 0$$

$$\begin{vmatrix} 2 & -1 & -2 \\ 2 & -1 & 0 \\ 0 & -12 & 0 \end{vmatrix} \frac{1}{3} + \begin{vmatrix} 2 & -1 & -2 \\ 2 & -1 & -2 \\ 0 & -12 & 0 \end{vmatrix} \frac{1}{3} + \hat{e}_{AF} \cdot (\vec{r}_{B/A} \times T) = 0$$

$$(2 \times 2 \times 12) \frac{1}{3} + (2 \times 2 \times 12 + 2 \times 2 \times 12) \frac{1}{3} + \hat{e}_{AF} \cdot (\vec{r}_{B/A} \times T) = 0$$

$$\hat{e}_{AF} \cdot (\vec{r}_{B/A} \times T) = -32 \text{ OR } T \cdot (\hat{e}_{AF} \times \vec{r}_{B/A}) = -32 \quad (1)$$

PROJECTION OF T ON  $(\hat{e}_{AF} \times \vec{r}_{B/A})$  IS CONSTANT. THUS,  $T_{min}$  IS PARALLEL TO

$$\hat{e}_{AF} \times \vec{r}_{B/A} = \frac{1}{3}(2\hat{i} - \hat{j} - 2\hat{k}) \times 4\hat{i} = \frac{1}{3}(-8\hat{j} + 4\hat{k})$$

CORRESPONDING UNIT VECTOR IS  $\frac{1}{\sqrt{5}}(-2\hat{j} + \hat{k})$

$$T_{min} = T(-2\hat{j} + \hat{k}) \frac{1}{\sqrt{5}} \quad (2)$$

$$\text{EQ (1): } \frac{T}{\sqrt{5}}(-2\hat{j} + \hat{k}) \cdot \left[ \frac{1}{3}(2\hat{i} - \hat{j} - 2\hat{k}) \times 4\hat{i} \right] = -32$$

$$\frac{T}{\sqrt{5}}(-2\hat{j} + \hat{k}) \cdot \frac{1}{3}(-8\hat{j} + 4\hat{k}) = -32$$

$$\frac{T}{3\sqrt{5}}(16 + 4) = -32; \quad T = -\frac{3\sqrt{5}(32)}{20} = 4.8\sqrt{5}$$

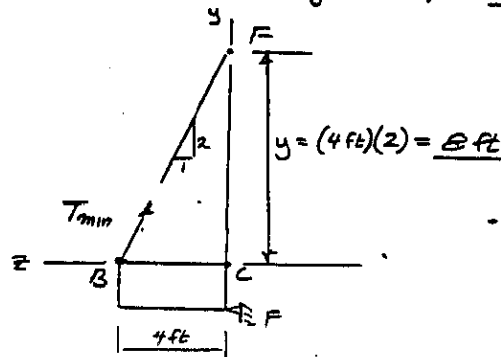
$$T = 10.733 \text{ lb}$$

$$\text{EQ (2): } T_{min} = T(-2\hat{j} + \hat{k}) \frac{1}{\sqrt{5}}$$

$$= 4.8\sqrt{5}(-2\hat{j} + \hat{k}) \frac{1}{\sqrt{5}}$$

$$T_{min} = -(9.6\hat{j}) + (4.8\hat{k})$$

SINCE  $T_{min}$  HAS NO  $\hat{i}$  COMPONENT, WIRE BH IS PARALLEL TO THE  $yz$  PLANE, AND  $x = 4 \text{ ft}$

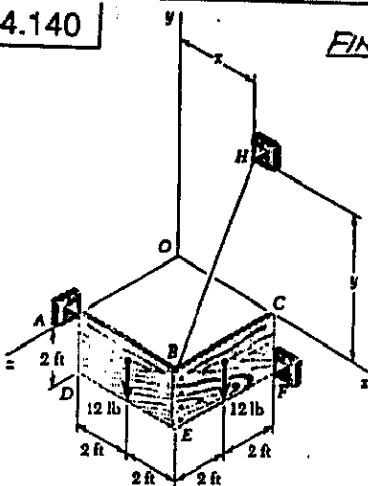


ANSWERS:

$$(a) x = 4 \text{ ft}, y = 8 \text{ ft}$$

$$(b) T_{min} = 10.73 \text{ lb}$$

4.140

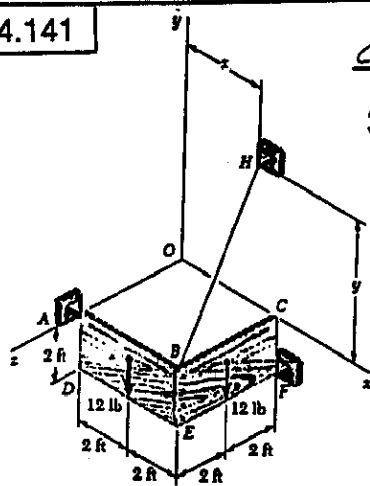


FIND: (a) LOCATION  
OF H IN  $xy$  PLANE  
FOR WHICH TENSION  
IN WIRE BH IS  
MINIMUM  
(b) CORRESPONDING  
MINIMUM TENSION

(CONTINUED)

$$\hat{e}_{CF} = \frac{\vec{CF}}{CF} = \frac{7\hat{i} - 24\hat{j}}{25}$$

4.141

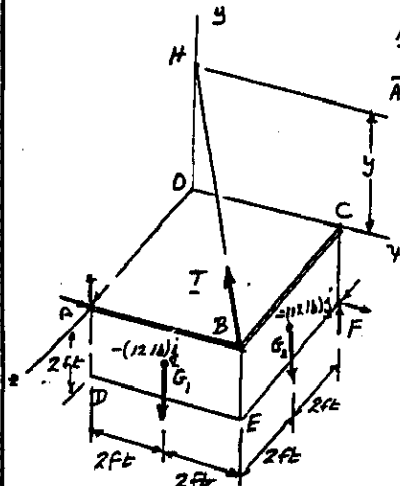


GIVEN: SUPPORT H  
MUST BE ON  
y AXIS (i.e.,  $x=0$ )

FIND:

- (a) DISTANCE y  
FOR WHICH  
TENSION IN WIRE  
BH IS MINIMUM.  
(b) CORRESPONDING  
MINIMUM TENSION.

FREE-BODY DIAGRAM



$$\vec{r}_{BH} = 4\hat{i} - 2\hat{j} - 4\hat{k}$$

$$\vec{r}_{BH} = \frac{1}{3}(2\hat{i} - \hat{j} - 2\hat{k})$$

$$y_{BH} = 2\hat{i} - \hat{j}$$

$$y_{BH} = 4\hat{i} - \hat{j} - 2\hat{k}$$

$$y_{BH} = 4\hat{i}$$

$$\sum M_A = 0: \vec{r}_{AH} \times (-12\hat{j}) + \vec{r}_{AD} \times (12\hat{j}) + \vec{r}_{AB} \times T = 0$$

$$\begin{vmatrix} 2 & -1 & 2 \\ 2 & -1 & 0 \\ 0 & -12 & 0 \end{vmatrix} + \begin{vmatrix} 2 & -1 & 2 \\ 4 & -1 & -2 \\ 0 & -12 & 0 \end{vmatrix} + \vec{r}_{AB} \times T = 0$$

$$(2 \times 2 \times 12) \frac{1}{3} + (-2 \times 2 \times 12 + 2 \times 4 \times 12) \frac{1}{3} + \vec{r}_{AB} \times T = 0$$

$$\vec{r}_{AB} \times T = -32 \quad (1)$$

$$\vec{r}_{BH} = -4\hat{i} + y\hat{j} - 4\hat{k} \quad BH = (32 + y^2)^{1/2}$$

$$T = T \frac{\vec{r}_{BH}}{BH} = T \frac{-4\hat{i} + y\hat{j} - 4\hat{k}}{(32 + y^2)^{1/2}}$$

$$\text{EQ. (1)} \quad \vec{r}_{AB} \times T = \begin{vmatrix} 2 & -1 & 2 \\ 4 & 0 & 0 \\ -4 & y & -4 \end{vmatrix} \frac{T}{3(32 + y^2)^{1/2}} = -32$$

$$(-16 - 8y)T = -3 \times 32(32 + y^2)^{1/2}; \quad T = \frac{96(32 + y^2)^{1/2}}{8y + 16} \quad (2)$$

$$\frac{dT}{dy} = 0: \frac{96(8y + 16) \frac{1}{2}(32 + y^2)^{-1/2}(2y) + (32 + y^2)^{1/2}(96)}{(8y + 16)^2}$$

$$\text{NUMERATOR} = 0: (8y + 16)y = (32 + y^2)8$$

$$8y^2 + 16y = 32 + 8y^2$$

$$y = 16 \text{ ft} \quad \leftarrow$$

$$\text{EQ. (2)}: T = \frac{96(32 + 16^2)^{1/2}}{8 \times 16 + 16} = 11.313 \text{ lb} \quad T = 11.31 \text{ lb} \quad \leftarrow$$

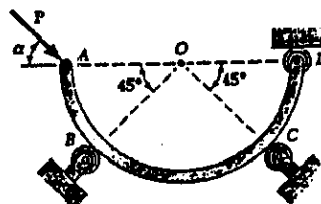
4.142 and 4.143

PROB. 4.141:

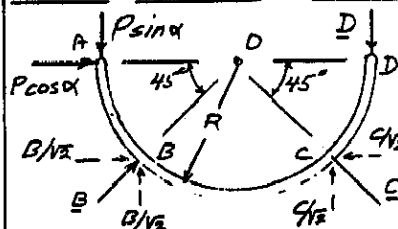
FOR  $\alpha = 45^\circ$ ,  
FIND REACTIONS  
AT B, C, AND D.

PROB. 4.142:

FIND RANGE OF  $\alpha$   
FOR EQUILIBRIUM.



FREE-BODY DIAGRAM



$$+\sum M_O = 0: (P \sin \alpha)R - D(R) = 0 \quad D = P \sin \alpha \quad (1)$$

$$+\sum F_x = 0: P \cos \alpha + B/\sqrt{2} - C/\sqrt{2} = 0 \quad (2)$$

$$+\sum F_y = 0: -P \sin \alpha + B/\sqrt{2} + C/\sqrt{2} - P \sin \alpha = 0$$

$$-2P \sin \alpha + B/\sqrt{2} + C/\sqrt{2} = 0 \quad (3)$$

$$(2) + (3): P(\cos \alpha - 2 \sin \alpha) + 2B/\sqrt{2} = 0$$

$$B = \frac{\sqrt{2}}{2}(2 \sin \alpha - \cos \alpha)P \quad (4)$$

$$(2) - (3): P(\cos \alpha + 2 \sin \alpha) - 2C/\sqrt{2} = 0$$

$$C = \frac{\sqrt{2}}{2}(2 \sin \alpha + \cos \alpha)P \quad (5)$$

PROB. 4.142 FOR  $\alpha = 45^\circ$ ;  $\sin \alpha = \cos \alpha = \frac{1}{\sqrt{2}}$

$$\text{EQ. (4)}: B = \frac{\sqrt{2}}{2}(\frac{2}{\sqrt{2}} - \frac{1}{\sqrt{2}})P = P; \quad B = P \angle 45^\circ \quad \leftarrow$$

$$\text{EQ. (5)}: C = \frac{\sqrt{2}}{2}(\frac{2}{\sqrt{2}} + \frac{1}{\sqrt{2}})P = \frac{3}{2}P; \quad C = \frac{3}{2}P \angle 45^\circ \quad \leftarrow$$

$$\text{EQ. (1)}: D = P/\sqrt{2} \quad D = P/\sqrt{2} \downarrow \quad \leftarrow$$

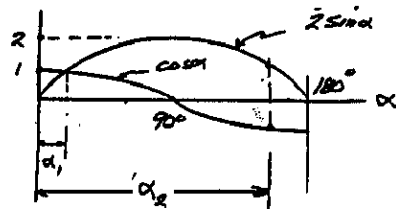
PROB. 4.143 RANGE OF  $\alpha$  FOR EQUILIBRIUM

FOR  $B \geq 0$ :

$$\text{FROM EQ. (4)}: 2 \sin \alpha - \cos \alpha \geq 0$$

FOR  $C \geq 0$ :

$$\text{FROM EQ. (5)}: 2 \sin \alpha + \cos \alpha \geq 0$$



$$2 \sin \alpha_1 \geq \cos \alpha_1$$

$$\tan \alpha_1 \geq 0.5$$

$$\alpha_1 \geq 26.6^\circ$$

$$2 \sin \alpha_2 \geq -\cos \alpha_2$$

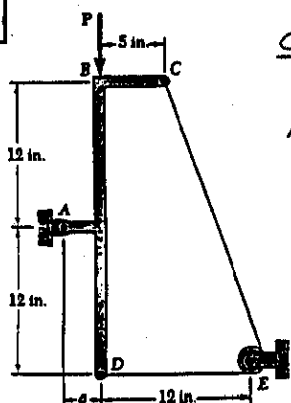
$$\tan \alpha_2 \geq -0.5$$

$$\alpha_2 \leq 153.4^\circ$$

$$26.6^\circ \leq \alpha \leq 153.4^\circ \quad \leftarrow$$

FOR THIS RANGE  $\sin \alpha \geq 0$ , THUS  
EQ. (1) YIELDS  $D \geq 0$ , O.K.

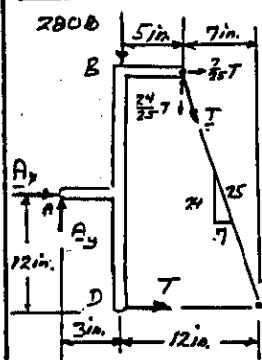
4.144



GIVEN:  $\alpha = 30^\circ$ ,  
 $P = 280 \text{ lb}$ .

FIND:

- (a) TENSION  
IN CABLE DEC.  
(b) REACTION  
AT A.



FREE-BODY DIAGRAM

$$+\sum M_A = 0: -(280 \text{ lb})(8 \text{ in}) - T(12 \text{ in}) - \frac{24}{25}T(12 \text{ in}) - \frac{24}{25}T(8 \text{ in}) = 0$$

$$(12 - 11.04)T = 840 \quad T = 875 \text{ lb}$$

$$+\sum F_x = 0: \frac{24}{25}(875 \text{ lb}) + 875 \text{ lb} + A_x = 0$$

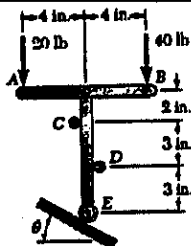
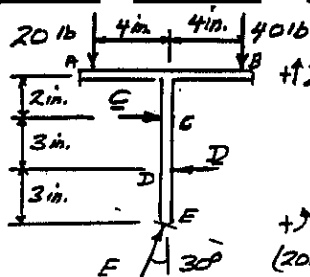
$$A_x = -1120 \quad A_x = 1120 \text{ lb} \leftarrow$$

$$+\sum F_y = 0: A_y - 280 \text{ lb} - \frac{24}{25}(875 \text{ lb}) = 0$$

$$A_y = 1120 \text{ lb} \quad A_y = 1120 \text{ lb} \uparrow$$

$$A = 1584 \text{ lb} \angle 45^\circ$$

4.145

GIVEN:  $\theta = 30^\circ$ .FIND: REACTIONS  
AT C, D, AND E.

FREE-BODY DIAGRAM

$$+\sum F_y = 0: E \cos 30^\circ - 20 - 40 = 0$$

$$E = \frac{60 \text{ lb}}{\cos 30^\circ} = 69.28 \text{ lb}$$

$$E = 69.3 \text{ lb} \angle 60^\circ$$

$$+\sum M_D = 0: (20 \text{ lb})(4 \text{ in}) - (40 \text{ lb})(4 \text{ in}) - C(3 \text{ in}) + E \sin 30^\circ(3 \text{ in}) = 0$$

$$-80 - 3C + 69.28(0.5)(3) = 0$$

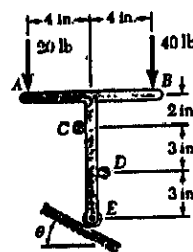
$$C = 7.974 \text{ lb} \quad C = 7.97 \text{ lb} \rightarrow$$

$$+\sum F_x = 0: E \sin 30^\circ + C - D = 0$$

$$(69.28 \text{ lb})(0.5) + 7.974 \text{ lb} - D = 0$$

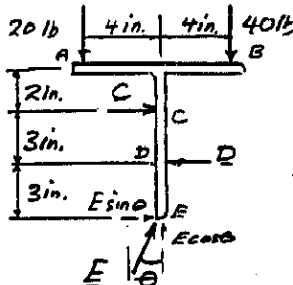
$$D = 42.61 \text{ lb} \quad D = 42.6 \text{ lb} \leftarrow$$

4.146



FIND:

- (a) SMALLEST  $\theta$   
FOR EQUILIBRIUM.  
(b) CORRESPONDING  
REACTIONS AT  
C, D, AND E



FREE-BODY DIAGRAM

$$+\sum F_y = 0: E \cos \theta - 20 - 40 = 0$$

$$E = \frac{60}{\cos \theta}$$

$$+\sum M_D = 0: (20 \text{ lb})(4 \text{ in}) - (40 \text{ lb})(4 \text{ in}) - C(3 \text{ in}) + \left(\frac{60}{\cos \theta} \sin \theta\right)(3 \text{ in}) = 0$$

$$C = \frac{1}{3}(180 \tan \theta - 80)$$

(a) For  $C = 0$ ,  $180 \tan \theta = 80$   
 $\tan \theta = \frac{4}{9}$ ;  $\theta = 23.9^\circ$   $\theta = 24.0^\circ$

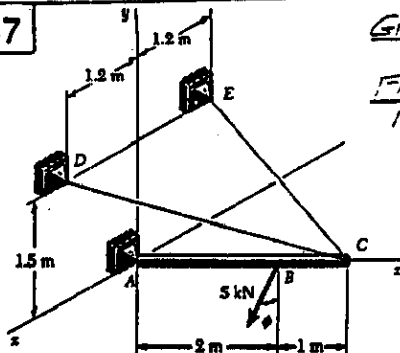
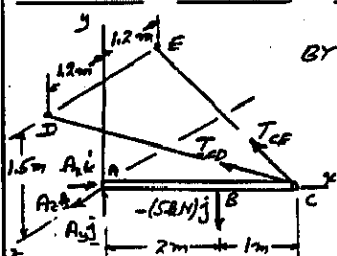
$$E = 60 / \cos 23.9^\circ = 65.66 \text{ lb}$$

$$+\sum F_x = 0: -D + C + E \sin \theta = 0$$

$$D = (65.66) \sin 23.9^\circ = 26.67 \text{ lb}$$

(b)  $C = 0$ ;  $D = 26.7 \text{ lb} \leftarrow$ ;  $E = 65.7 \text{ lb} \angle 24.0^\circ$

4.147

GIVEN:  $\phi = 0$ FIND: (a) TENSION  
IN CD AND CE  
(b) REACTIONS  
AT A.FREE-BODY DIAGRAM  
BY SYMMETRY WITH  $xy$  PLANE

$$T_{CD} = T_{CE} = T$$

$$\vec{CD} = -3\hat{i} + 1.5\hat{j} + 1.2\hat{k}$$

$$CD = 3.562 \text{ m}$$

$$T = T \frac{\vec{CD}}{CD} = T \frac{-3\hat{i} + 1.5\hat{j} + 1.2\hat{k}}{3.562}$$

$$T_{CE} = T \frac{-3\hat{i} + 1.5\hat{j} - 1.2\hat{k}}{3.562}$$

$$+\sum M_A = 0: -r_{DA} \times T_{CD} + r_{CA} \times T_{CE} + r_{BA} \times (-5\hat{k}) = 0$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 0 \\ 3.562 & 0 & 0 \end{vmatrix} T + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 0 \\ 3.562 & 0 & 0 \end{vmatrix} T + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ 0 & -5 & 0 \end{vmatrix} = 0$$

$$\text{COEFF. OF } T: 2[3 \times 1.5 \times \frac{1}{3.562}] - 10 = 0; T = 3.958 \text{ kN}$$

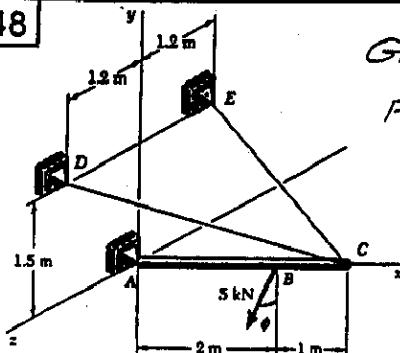
$$+\sum F_x = 0: A_x + T_{CD} + T_{CE} - 5 = 0; \text{COEFF. OF } T: A_x = 0$$

$$\text{COEFF. OF } T: A_x - 2[3.958 \times 3 / 3.562] = 0; A_x = 6.67 \text{ kN}$$

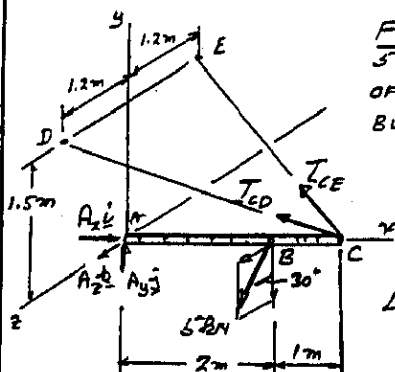
$$\text{COEFF. OF } T: A_y + 2[3.958 \times 1.5 / 3.562] - 5 = 0; A_y = 1.667 \text{ kN}$$

(a)  $T_{CD} = T_{CE} = 3.96 \text{ kN}$ . (b)  $A = (6.67 \text{ kN})\hat{i} + (1.667 \text{ kN})\hat{j}$

4.148

GIVEN:  $\phi = 30^\circ$ 

FIND:  
(a) TENSION  
IN CD AND CE.  
(b) REACTION  
AT A.



FREE-BODY DIAGRAM  
5 UNKNOWN AND 6 EQS.  
OF EQUILIBRIUM  
BUT, EQUIL. MAINTAINED  
( $\Sigma M_A = 0$ )

$$r_{B/A} = 2\hat{i}$$

$$r_{C/A} = 3\hat{i}$$

LOAD AT B,  
 $= -(5 \cos 30^\circ)\hat{j} + (5 \sin 30^\circ)\hat{k}$   
 $= -4.33\hat{j} + 2.5\hat{k}$

$$\vec{CD} = -3\hat{i} + 1.5\hat{j} + 1.2\hat{k} \quad CD = 3.562 \text{ m}$$

$$\vec{T}_{CD} = T_{CD} \frac{\vec{CD}}{CD} = \frac{T}{3.562} (-3\hat{i} + 1.5\hat{j} + 1.2\hat{k})$$

SIMILARLY,  $\vec{T}_{CE} = \frac{T}{2.562} (-3\hat{i} + 1.5\hat{j} - 1.2\hat{k})$

$$\Sigma M_A = 0: r_{C/A} \times T_{CD} + r_{C/A} \times T_{CE} + r_{B/A} \times (-4.33\hat{j} + 2.5\hat{k}) = 0$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 0 \\ -3 & 1.5 & 1.2 \end{vmatrix} \frac{T_{CD}}{3.562} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 0 \\ -3 & 1.5 & -1.2 \end{vmatrix} \frac{T_{CE}}{3.562} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ 0 & -4.33 & 2.5 \end{vmatrix} = 0$$

EQUATE COEFFICIENTS OF UNIT VECTORS TO ZERO

$$\textcircled{1} -3.6 \frac{T_{CD}}{3.562} + 3.6 \frac{T_{CE}}{3.562} - 5 = 0$$

$$-3.6 T_{CD} + 3.6 T_{CE} - 17.810 = 0 \quad (1)$$

$$\textcircled{2} 4.5 \frac{T_{CD}}{3.562} + 4.5 \frac{T_{CE}}{3.562} - 8.66 = 0$$

$$4.5 T_{CD} + 4.5 T_{CE} = 30.846 \quad (2)$$

$$(2) + 1.25(1): 9 T_{CE} - 53.11 = 0; T_{CE} = 5.901 \text{ kN}$$

$$\text{EQ (1): } -3.6 T_{CD} + 3.6(5.901) - 17.810 = 0$$

$$\Sigma F = 0: A + T_{CD} + T_{CE} - 4.33\hat{j} + 2.5\hat{k} = 0$$

$$\textcircled{3} A_x + \frac{0.954}{3.562}(-3) + \frac{5.901}{3.562}(-3) = 0; A_x = 5.77 \text{ kN}$$

$$\textcircled{4} A_y + \frac{0.954}{3.562}(1.5) + \frac{5.901}{3.562}(1.5) - 4.33 = 0$$

$$A_y = 1.443 \text{ kN}$$

$$\textcircled{5} A_z + \frac{0.954}{3.562}(1.2) + \frac{5.901}{3.562}(-1.2) + 2.5 = 0$$

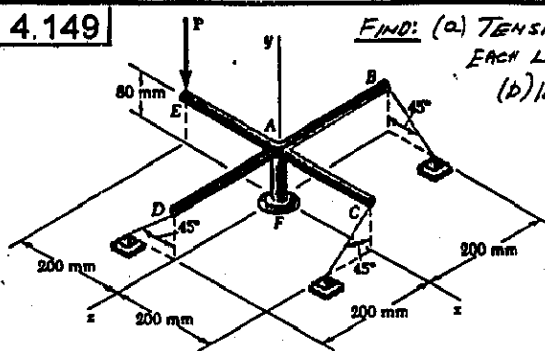
$$A_z = -0.833 \text{ kN}$$

ANSWERS:

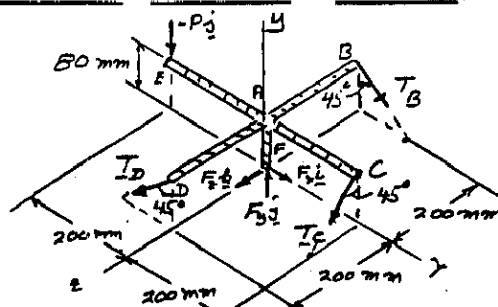
(a)  $T_{CD} = 0.954 \text{ kN}; T_{CE} = 5.90 \text{ kN}$

(b)  $A = (5.77 \text{ kN})\hat{i} + (1.443 \text{ kN})\hat{j} - (0.833 \text{ kN})\hat{k}$

4.149



FIND: (a) TENSION IN  
EACH LINK.  
(b) REACTION  
AT F



$$T_B = T_B (\hat{i} - \hat{j})/\sqrt{2}$$

$$T_C = T_C (-\hat{j} + \hat{k})/\sqrt{2}$$

$$T_D = T_D (-\hat{i} - \hat{j})/\sqrt{2}$$

$$r_{E/F} = -200\hat{i} + 80\hat{j}$$

$$r_{B/F} = 80\hat{j} - 200\hat{k}$$

$$r_{C/F} = 200\hat{i} + 80\hat{j}$$

$$r_{D/F} = 80\hat{j} + 200\hat{k}$$

$$\Sigma M_F = 0: r_{B/F} \times T_B + r_{C/F} \times T_C + r_{D/F} \times T_D + r_{E/F} \times (-P\hat{j}) = 0$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 80 & -200 & 0 \\ 0 & 1 & -1 \end{vmatrix} \frac{T_B}{\sqrt{2}} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -1 & 1 \\ 0 & 1 & 1 \end{vmatrix} \frac{T_C}{\sqrt{2}} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 80 & 200 \\ -1 & -1 & 0 \end{vmatrix} \frac{T_D}{\sqrt{2}} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -200 & 80 & 0 \\ 0 & -P & 0 \end{vmatrix} = 0$$

EQUATE COEFFICIENTS OF UNIT VECTORS TO ZERO

AND MULTIPLY EACH EQUATION BY  $\sqrt{2}$ .

$$\textcircled{1} -200 T_B + 80 T_C + 200 T_D = 0 \quad (1)$$

$$\textcircled{2} -200 T_B - 200 T_C - 200 T_D = 0 \quad (2)$$

$$\textcircled{3} -80 T_B - 200 T_C + 80 T_D + 200\sqrt{2} P = 0 \quad (3)$$

$$\frac{80}{200}(2): -80 T_B - 80 T_C - 80 T_D = 0 \quad (4)$$

$$(3) + (4): -160 T_B - 280 T_C + 200\sqrt{2} P = 0 \quad (5)$$

$$(1) + (2): -400 T_B - 120 T_C = 0$$

$$T_B = -\frac{120}{400} T_C = -0.3 T_C \quad (6)$$

$$(6) \rightarrow (5): -160(-0.3 T_C) - 280 T_C + 200\sqrt{2} P = 0$$

$$-232 T_C + 200\sqrt{2} P = 0$$

$$T_C = 1.2191 P$$

$$(6): T_B = -0.3(1.2191 P) = -0.36574 P$$

$$(2): -200(-0.36574 P) - 200(1.2191 P) - 200 T_D = 0$$

$$T_D = -0.8534 P$$

$$T_D = -0.853 P$$

$$\Sigma F = 0: F + T_B + T_C + T_D - P\hat{j} = 0$$

$$\textcircled{1} F_x + (-0.36574 P)/\sqrt{2} - (-0.8534 P)/\sqrt{2} = 0$$

$$F_x = -0.3448 P$$

$$F_x = -0.345 P$$

$$\textcircled{2} F_y - (-0.36574 P)/\sqrt{2} - (1.2191 P)/\sqrt{2} - (-0.8534 P)/\sqrt{2} - 200 = 0$$

$$F_y = P$$

$$F_y = P$$

$$\textcircled{3} F_z + (1.2191 P)/\sqrt{2} = 0$$

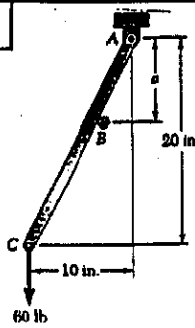
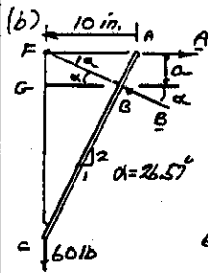
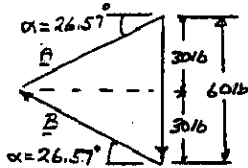
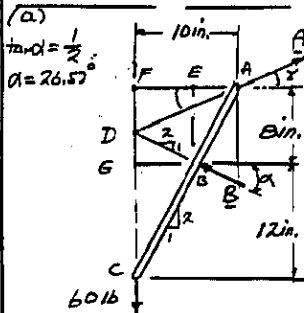
$$F_z = -0.8620 P$$

$$F_z = -0.862 P$$

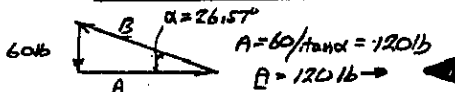
$$F = -0.345 P\hat{i} + P\hat{j} - 0.862 P\hat{k}$$



4.150

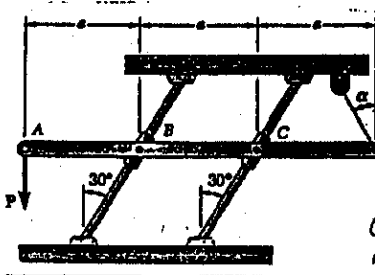
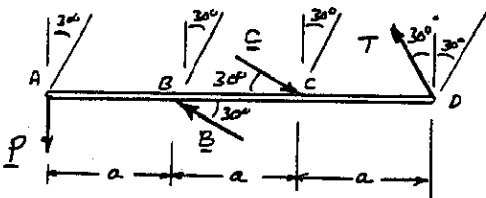
FIND:(a) REACTIONS AT A AND B WHEN  $\alpha = 81^\circ$ .(b) DISTANCE  $a$  FOR WHICH REACTION AT A IS HORIZONTAL AND CORRESPONDING REACTIONS AT A AND B.

FOR A HORIZONTAL,  
 $\Delta ABF: BF = AF \cos \alpha$   
 $\Delta BFG: FG = BF \sin \alpha$   
 $a = FG = AF \cos \alpha \sin \alpha$   
 $a = (10 \text{ in.}) \cos 26.57^\circ \sin 26.57^\circ; a = 4 \text{ in.}$

FORCE TRIANGLE

$$B = (60 \text{ lb}) \sin \alpha = 134.16 \text{ lb} \quad B = 134.16 \text{ lb} \angle 26.6^\circ$$

4.151

GIVEN:  
 $\alpha = 30^\circ$ FIND:(a) TENSION IN WIRE.  
(b) REACTIONS AT B AND C.

(CONTINUED)

4.151 CONTINUED

$$30^\circ \uparrow \Sigma F = 0: -P \cos 30^\circ + T \cos 60^\circ = 0$$

$$T = \frac{P \cos 30^\circ}{\cos 60^\circ} = P \frac{\sqrt{3}/2}{1/2} \quad T = \sqrt{3}P$$

$$+\circlearrowleft \Sigma M_B = 0: Pa - (C \sin 30^\circ)a + T \cos 30^\circ(2a) = 0$$

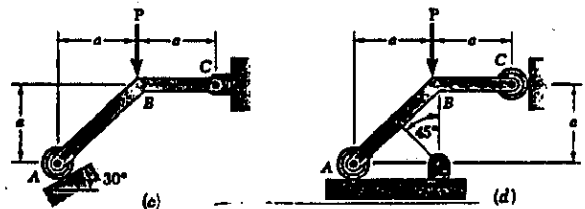
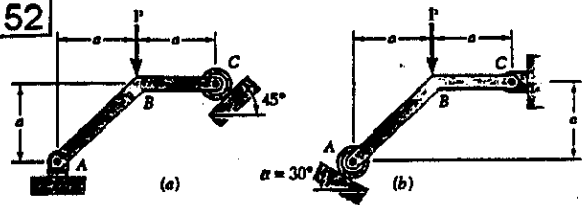
$$Pa - (\frac{1}{2}C)a + \sqrt{3}P(\frac{\sqrt{3}}{2})2a = 0$$

$$-\frac{1}{2}C + (1+\sqrt{3})P = 0; C = BP; C = BP \angle 30^\circ$$

$$\uparrow \Sigma F = 0: -B \cos 30^\circ + C \cos 30^\circ - T \sin 30^\circ = 0$$

$$-B \frac{\sqrt{3}}{2} + BP \frac{\sqrt{3}}{2} - \sqrt{3}P(\frac{1}{2}) = 0; D = 7P; B = 7P \angle 30^\circ$$

4.152

FIND: REACTIONS

$$(a) +\circlearrowleft \Sigma M = 0: -Pa + (C \sin 45^\circ)2a + (C \cos 45^\circ)a = 0$$

$$-Pa + C(\frac{\sqrt{2}}{2})2a + C(\frac{\sqrt{2}}{2})a = 0$$

$$3 \frac{C}{\sqrt{2}} = P; C = \frac{\sqrt{2}}{3}P$$

$$C = 0.471P \angle 45^\circ$$

$$+\circlearrowleft \Sigma F_x = 0: A_x - (\frac{\sqrt{2}}{3}P)\frac{1}{\sqrt{2}}; A_x = \frac{P}{3}$$

$$+\uparrow \Sigma F_y = 0: A_y - P + (\frac{\sqrt{2}}{3}P)\frac{1}{\sqrt{2}}; A_y = \frac{2P}{3}$$

$$\frac{2}{3}P \angle 26.6^\circ; A = 0.745P \angle 63.4^\circ$$

$$(b) +\circlearrowleft \Sigma M_C = 0: +Pa - (A \cos 30^\circ)2a + (A \sin 30^\circ)a = 0$$

$$+Pa - (1.732 - 0.5)P = 0; A = 1.232P$$

$$A = 1.232P \angle 60^\circ$$

$$+\circlearrowleft \Sigma F_x = 0: (1.232P) \sin 30^\circ + C_x = 0; C_x = 0.616P$$

$$+\uparrow \Sigma F_y = 0: (1.232P) \cos 30^\circ - P + C_y = 0; C_y = 0.067P$$

$$0.616P \angle 9.5^\circ; C = 0.620P \angle 6.2^\circ$$

$$(c) +\circlearrowleft \Sigma M_C = 0: +Pa - (A \cos 30^\circ)2a + (A \sin 30^\circ)a = 0$$

$$+Pa - (1.732 + 0.5)P = 0; A = 0.448P$$

$$A = 0.448P \angle 60^\circ$$

$$+\circlearrowleft \Sigma F_x = 0: -(0.448P) \sin 30^\circ + C_x = 0; C_x = 0.224P$$

$$+\uparrow \Sigma F_y = 0: (0.448P) \cos 30^\circ - P + C_y = 0; C_y = 0.112P$$

$$0.224P \angle 9.5^\circ; C = 0.234P \angle 6.2^\circ$$

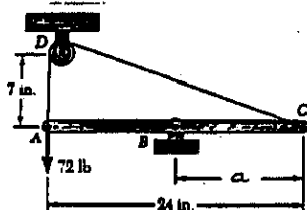
$$(d) \text{ FORCE } T \text{ EXERTED BY WIRE AND REACTIONS } A \text{ AND } C \text{ ALL INTERSECT AT POINT } D.$$

$$+\circlearrowleft \Sigma M_D = 0: Pa = 0$$

EQUILIBRIUM NOT MAINTAINED  
 ROD IS IMPROPERLY CONSTRAINED

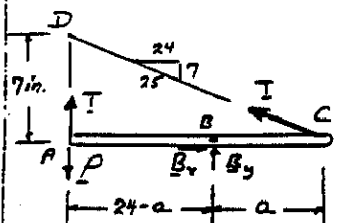
\*4.153

FOR THE RIGID BODIES OF THE FOLLOWING PROBLEMS, FIND THE VALUE OF  $a$  OR  $\alpha$  WHICH RESULTS IN IMPROPER CONSTRAINTS.



(a) PROB. 4.77

FREE-BODY DIAGRAM

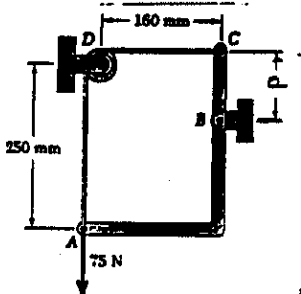


$$+\circlearrowleft \Sigma M_B = 0: P(24-a) - T(24-a) + \frac{7}{25}Ta = 0$$

$$T = \frac{P(24-a)}{24-a - \frac{7}{25}a}$$

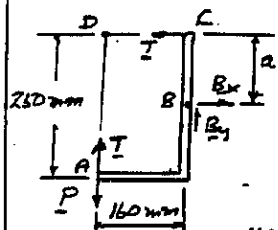
T BECOMES 0 WHEN  $24-a - \frac{7}{25}a = 0$

IMPROPER CONSTRAINT:  $a = 18.75 \text{ in.}$



(b) PROB. 4.78

FREE-BODY DIAGRAM

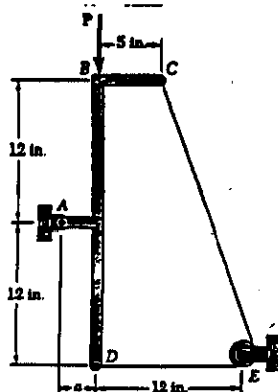


$$+\circlearrowleft \Sigma M_B = 0: P(160) - T(160) + T(a) = 0$$

$$T = \frac{160P}{160-a}$$

T BECOMES INFINITE WHEN  $160-a = 0$

IMPROPER CONSTRAINT:  $a = 160 \text{ mm}$

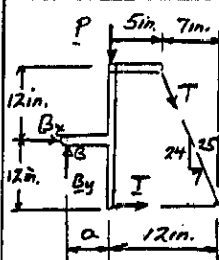


(c) PROB. 4.144

(CONTINUED)

\*4.153 CONTINUED

(c) PROB. 4.144 (CONTINUED)



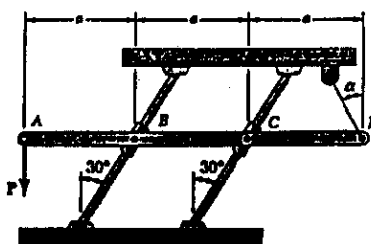
$$+\circlearrowleft \Sigma M_E = 0:$$

$$-Pa - \frac{7}{25}T(12) - \frac{24}{25}T(a+5) + T(12) = 0$$

$$T = \frac{Pa}{12 - \frac{24}{25}a - \frac{24}{25}a - \frac{120}{25}}$$

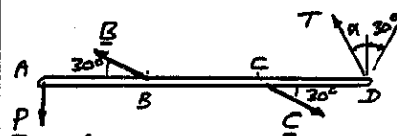
$$T = \frac{Pa}{3.84 - \frac{24}{25}a}$$

T BECOMES INFINITE WHEN  $3.84 - \frac{24}{25}a = 0$   
IMPROPER CONSTRAINT:  $a = 4 \text{ in.}$



(d) PROB. 4.151

FREE-BODY DIAGRAM

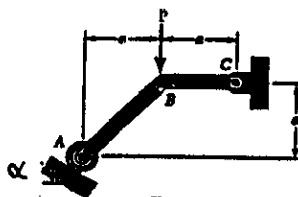


$$+\circlearrowleft \Sigma F = 0: -P \cos 30^\circ + T \cos(\alpha + 30^\circ) = 0$$

$$T = \frac{P \cos 30^\circ}{\cos(\alpha + 30^\circ)}$$

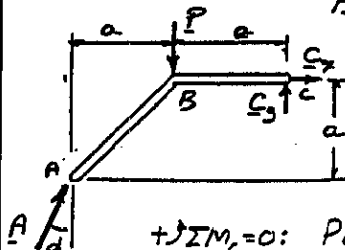
T BECOMES INFINITE WHEN  $\cos(\alpha + 30^\circ) = 0$

IMPROPER CONSTRAINT:  $\alpha + 30^\circ = 90^\circ$ ,  $\alpha = 60^\circ$



(e) PROB. 4.152

FREE-BODY DIAGRAM



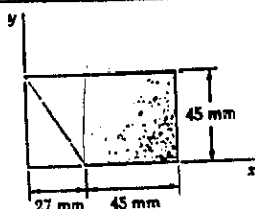
$$+\circlearrowleft \Sigma M_C = 0: Pa + (A \sin \alpha)a - (A \cos \alpha)2a = 0$$

$$A = \frac{Pa}{a(2 \cos \alpha - \sin \alpha)}$$

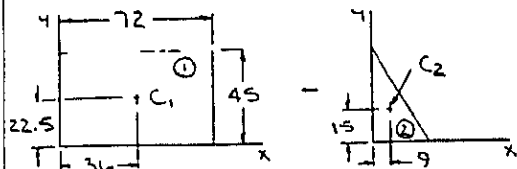
A BECOMES INFINITE WHEN  $2 \cos \alpha - \sin \alpha = 0$   
 $\tan \alpha = 2$   $\alpha = 63.43^\circ$

IMPROPER CONSTRAINT:  $\alpha = 63.4^\circ$

5.1



GIVEN: PLANE AREA  
SHOWN  
FIND:  $\bar{X}$  AND  $\bar{Y}$



DIMENSIONS IN mm				
	$A, \text{mm}^2$	$\bar{x}, \text{mm}$	$\bar{y}, \text{mm}$	$\bar{x}A, \text{mm}^3$
1	$72 \times 45 = 3240$	36	22.5	116 640
2	$-\frac{1}{2} \times 27 \times 45 = -607.5$	9	15	-5467.5
$\Sigma$	2632.5			111 172.5

THEN

$$\bar{X} \Sigma A = \Sigma \bar{x} A$$

$$\bar{X}(2632.5) = 111 172.5$$

$$\text{OR } \bar{X} = 42.2 \text{ mm}$$

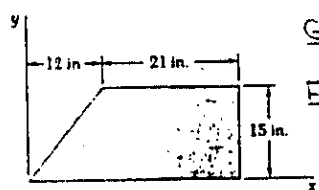
AND

$$\bar{Y} \Sigma A = \Sigma \bar{y} A$$

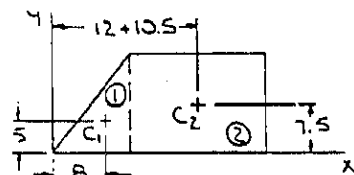
$$\bar{Y}(2632.5) = 63 787.5$$

$$\text{OR } \bar{Y} = 24.2 \text{ mm}$$

5.2



GIVEN: PLANE AREA  
SHOWN  
FIND:  $\bar{X}$  AND  $\bar{Y}$



DIMENSIONS IN IN.				
	$A, \text{in}^2$	$\bar{x}, \text{in.}$	$\bar{y}, \text{in.}$	$\bar{x}A, \text{in}^3$
1	$\frac{1}{2} \times 12 \times 15 = 90$	8	5	720
2	$21 \times 15 = 315$	22.5	7.5	7087.5
$\Sigma$	405			7807.5

THEN

$$\bar{X} \Sigma A = \Sigma \bar{x} A$$

$$\bar{X}(405) = 7807.5$$

$$\text{OR } \bar{X} = 19.28 \text{ in.}$$

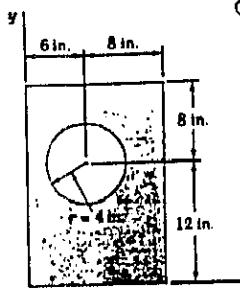
AND

$$\bar{Y} \Sigma A = \Sigma \bar{y} A$$

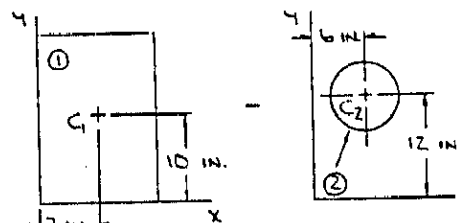
$$\bar{Y}(405) = 2812.5$$

$$\text{OR } \bar{Y} = 6.94 \text{ in.}$$

5.3



GIVEN: PLANE AREA  
SHOWN  
FIND:  $\bar{X}$  AND  $\bar{Y}$



	$A, \text{in}^2$	$\bar{x}, \text{in.}$	$\bar{y}, \text{in.}$	$\bar{x}A, \text{in}^3$	$\bar{y}A, \text{in}^3$
1	$14 \times 20 = 280$	7	10	1960	2800
2	$-\pi(4)^2 = -16\pi$	6	12	-301.59	-603.19
$\Sigma$	229.73			1658.41	2196.8

THEN

$$\bar{X} \Sigma A = \Sigma \bar{x} A$$

$$\bar{X}(229.73) = 1658.41$$

$$\text{OR } \bar{X} = 7.22 \text{ in.}$$

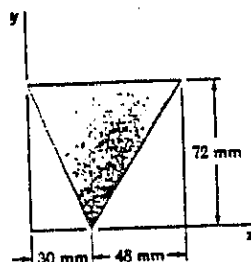
AND

$$\bar{Y} \Sigma A = \Sigma \bar{y} A$$

$$\bar{Y}(229.73) = 2196.8$$

$$\text{OR } \bar{Y} = 9.56 \text{ in.}$$

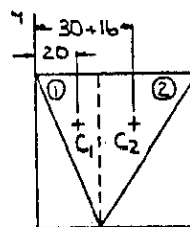
5.4



GIVEN: PLANE AREA  
SHOWN  
FIND:  $\bar{X}$  AND  $\bar{Y}$

FOR THE AREA AS A WHOLE, IT CAN BE  
CONCLUDED BY OBSERVATION THAT  
 $\bar{Y} = \frac{2}{3}(72 \text{ mm})$

$$\text{OR } \bar{Y} = 48.0 \text{ mm}$$



DIMENSIONS IN mm

	$A, \text{mm}^2$	$\bar{x}, \text{mm}$	$\bar{x}A, \text{mm}^3$
1	$\frac{1}{2} \times 30 \times 72 = 1080$	20	21600
2	$\frac{1}{2} \times 48 \times 72 = 1728$	46	79 488
$\Sigma$	2808		101 088

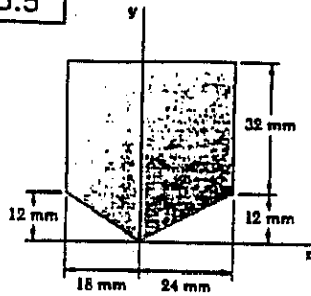
THEN

$$\bar{X} \Sigma A = \Sigma \bar{x} A$$

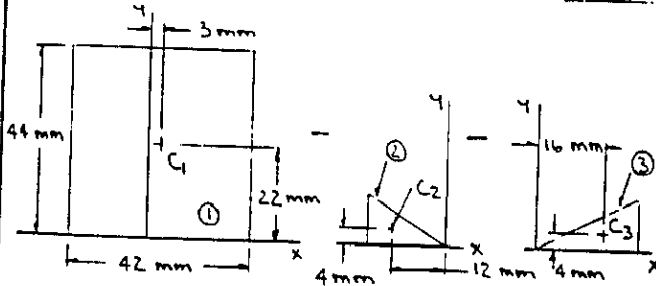
$$\bar{X}(2808) = 101 088$$

$$\text{OR } \bar{X} = 36.0 \text{ mm}$$

5.5



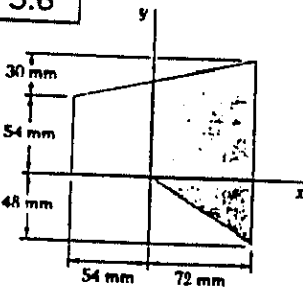
GIVEN: PLANE AREA  
SHOWN  
FIND:  $\bar{X}$  AND  $\bar{Y}$



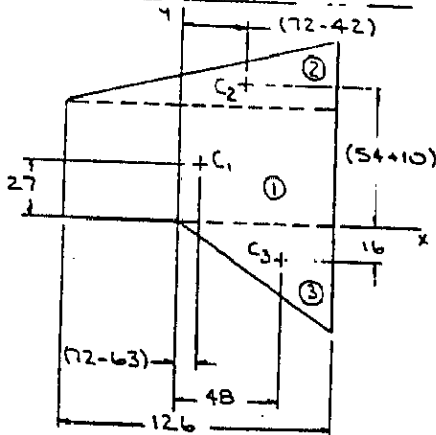
	A, mm <sup>2</sup>	$\bar{x}$ , mm	$\bar{y}$ , mm	$\bar{x}A$ , mm <sup>3</sup>	$\bar{y}A$ , mm <sup>3</sup>
1	$42 \times 44 = 1848$	3	22	5544	40656
2	$-\frac{1}{2} \times 12 \times 22 = -132$	-12	4	-1296	-432
3	$-\frac{1}{2} \times 12 \times 16 = -96$	16	4	-1248	-384
$\Sigma$	1596			4536	39648

THEN  $\bar{X}\Sigma A = \Sigma \bar{x}A$   
 $\bar{X}(1596) = 4536$   
 OR  $\bar{X} = 2.84$  mm  
 AND  $\bar{Y}\Sigma A = \Sigma \bar{y}A$   
 $\bar{Y}(1596) = 39648$   
 OR  $\bar{Y} = 24.8$  mm

5.6



GIVEN: PLANE AREA  
SHOWN  
FIND:  $\bar{X}$  AND  $\bar{Y}$



DIMENSIONS IN mm

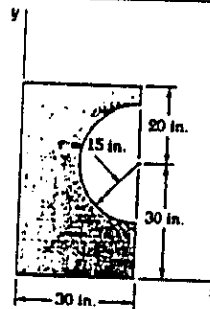
(CONTINUED)

5.6 CONTINUED

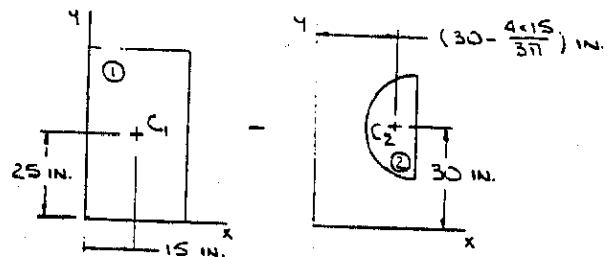
	A, mm <sup>2</sup>	$\bar{x}$ , mm	$\bar{y}$ , mm	$\bar{x}A$ , mm <sup>3</sup>	$\bar{y}A$ , mm <sup>3</sup>
1	$126 \times 54 = 6804$	9	27	18236	183708
2	$-\frac{1}{2} \times 126 \times 30 = -1890$	30	64	56700	120960
3	$-\frac{1}{2} \times 72 \times 48 = -1728$	48	-16	82944	-27648
$\Sigma$	10422			200880	277020

THEN  $\bar{X}\Sigma A = \Sigma \bar{x}A$   
 $\bar{X}(10422) = 200880$   
 OR  $\bar{X} = 19.27$  mm  
 AND  $\bar{Y}\Sigma A = \Sigma \bar{y}A$   
 $\bar{Y}(10422) = 277020$   
 OR  $\bar{Y} = 26.6$  mm

5.7



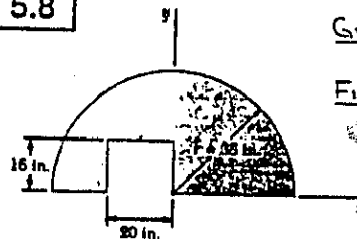
GIVEN: PLANE AREA  
SHOWN  
FIND:  $\bar{X}$  AND  $\bar{Y}$



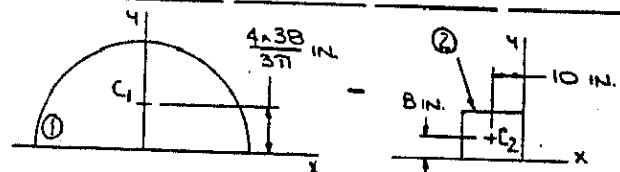
	A, in <sup>2</sup>	$\bar{x}$ , in.	$\bar{y}$ , in.	$\bar{x}A$ , in <sup>3</sup>	$\bar{y}A$ , in <sup>3</sup>
1	$30 \times 30 = 900$	15	25	22500	22500
2	$-\frac{\pi}{4}(15)^2 = -353.43$	23.634	30	-8353.0	-10602.9
$\Sigma$	1146.57			14147.0	26897

THEN  $\bar{X}\Sigma A = \Sigma \bar{x}A$   
 $\bar{X}(1146.57) = 14147.0$   
 OR  $\bar{X} = 12.34$  in.  
 AND  $\bar{Y}\Sigma A = \Sigma \bar{y}A$   
 $\bar{Y}(1146.57) = 26897$   
 OR  $\bar{Y} = 23.5$  in.

5.8



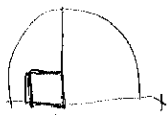
GIVEN: PLANE AREA  
SHOWN  
FIND:  $\bar{X}$  AND  $\bar{Y}$



(CONTINUED)

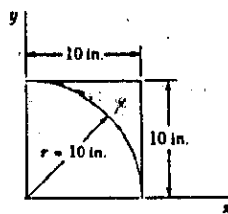
### 5.8 CONTINUED

	$A, \text{in}^2$	$\bar{x}, \text{in.}$	$\bar{y}, \text{in.}$	$\bar{x}A, \text{in}^3$	$\bar{y}A, \text{in}^3$
1	$\frac{\pi}{2}(38)^2 = 2268.2$	0	16.1277	0	36581
2	$-20 \times 16 = -320$	-10	8	3200	-2560
$\Sigma$	1948.23			3200	34021



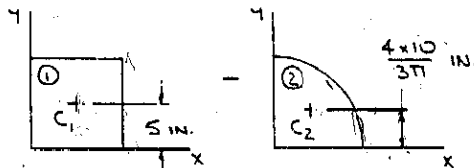
THEN  $\bar{x}\Sigma A = \Sigma \bar{x}A$   
 $\bar{x}(1948.23) = 3200$   
 OR  $\bar{x} = 1.643 \text{ in.}$   
 AND  $\bar{y}\Sigma A = \Sigma \bar{y}A$   
 $\bar{y}(1948.23) = 34021$   
 OR  $\bar{y} = 17.46 \text{ in.}$

### 5.9



GIVEN: PLANE AREA SHOWN  
 FIND:  $\bar{x}$  AND  $\bar{y}$

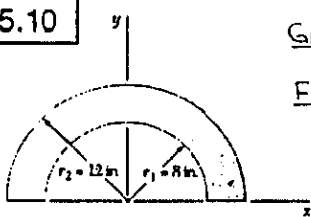
FIRST NOTE THAT SYMMETRY IMPLIES  $\bar{x} = \bar{y}$



	$A, \text{in}^2$	$\bar{y}, \text{in.}$	$\bar{y}A, \text{in}^3$
1	$10 \times 10 = 100$	5	500
2	$-\frac{\pi}{4}(10)^2 = -78.540$	4.2441	-333.33
$\Sigma$	21.460		166.67

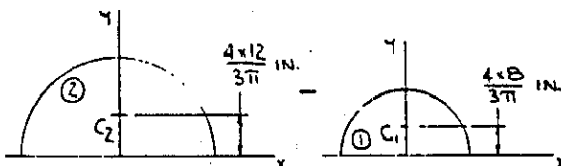
THEN  $\bar{y}\Sigma A = \Sigma \bar{y}A$   
 $\bar{y}(21.460) = 166.67$   
 OR  $\bar{x} = \bar{y} = 7.77 \text{ in.}$

### 5.10



GIVEN: PLANE AREA SHOWN  
 FIND:  $\bar{x}$  AND  $\bar{y}$

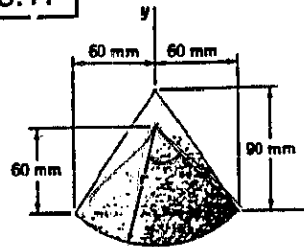
FIRST NOTE THAT SYMMETRY IMPLIES  $\bar{x} = 0$



	$A, \text{in}^2$	$\bar{y}, \text{in.}$	$\bar{y}A, \text{in}^3$
1	$-\frac{\pi}{2}(12)^2 = -100.531$	3.3953	-341.33
2	$\frac{\pi}{2}(12)^2 = 226.19$	5.0930	1151.99
$\Sigma$	125.659		810.66

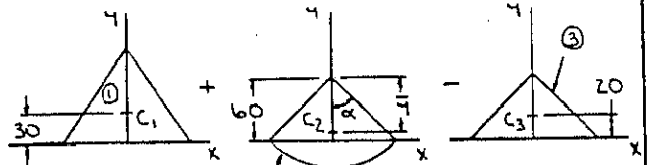
THEN  $\bar{y}\Sigma A = \Sigma \bar{y}A$   
 $\bar{y}(125.659) = 810.66$   
 OR  $\bar{y} = 6.45 \text{ in.}$

### 5.11



GIVEN: PLANE AREA SHOWN  
 FIND:  $\bar{x}$  AND  $\bar{y}$

FIRST NOTE THAT SYMMETRY IMPLIES  $\bar{x} = 0$

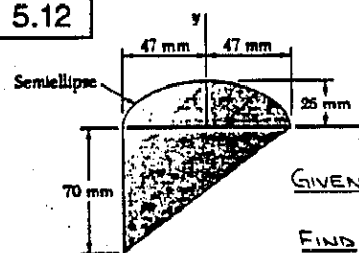


NOTE:  $r = 60\sqrt{2} \text{ mm}$   $\alpha = 45^\circ$   
 $\bar{y} = \frac{2r \sin \alpha}{3\alpha} = \frac{2(60\sqrt{2} \text{ mm}) \sin 45^\circ}{3 \times \frac{\pi}{4}}$  (FIG. 5.8A)

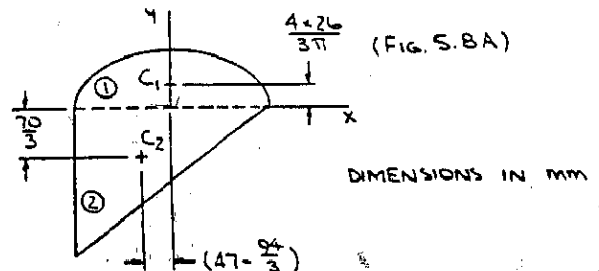
	$A, \text{mm}^2$	$\bar{y}, \text{mm}$	$\bar{y}A, \text{mm}^3$
1	$\frac{\pi}{2} \times 120 \times 90 = 5400$	30	162000
2	$\frac{\pi}{2}(60\sqrt{2})^2 = 5654.9$	$60 - 50.930 = 9.07$	51290
3	$-\frac{\pi}{2} \times 120 \times 60 = -3600$	20	-72000
$\Sigma$	7454.9		141290

THEN  $\bar{y}\Sigma A = \Sigma \bar{y}A$   
 $\bar{y}(7454.9) = 141290$   
 OR  $\bar{y} = 18.95 \text{ mm}$

### 5.12



GIVEN: PLANE AREA SHOWN  
 FIND:  $\bar{x}$  AND  $\bar{y}$



	$A, \text{mm}^2$	$\bar{x}, \text{mm}$	$\bar{y}, \text{mm}$	$\bar{x}A, \text{mm}^3$	$\bar{y}A, \text{mm}^3$
1	$\frac{\pi}{2} \times 47 \times 26 = 1919.51$	0	11.0347	0	21181
2	$\frac{1}{2} \times 47 \times 26 = 3290$	-15.6667	23.3333	-51543	-76766
$\Sigma$	5209.5			-51543	-55584

THEN  $\bar{x}\Sigma A = \Sigma \bar{x}A$   
 $\bar{x}(5209.5) = -51543$   
 OR  $\bar{x} = -9.89 \text{ mm}$   
 AND  $\bar{y}\Sigma A = \Sigma \bar{y}A$   
 $\bar{y}(5209.5) = -55584$   
 OR  $\bar{y} = -10.67 \text{ mm}$

# 5.16 CONTINUED

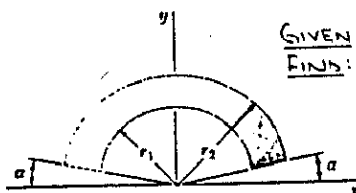
	A, mm <sup>2</sup>	$\bar{x}$ , mm	$\bar{y}$ , mm	$\bar{x}A$ , mm <sup>3</sup>	$\bar{y}A$ , mm <sup>3</sup>
1	$\frac{\pi}{4} \cdot 60 \cdot 150 = 2250$	48	42.857	108 000	96 429
2	$\frac{\pi}{4} \cdot 30 \cdot 18.75 = 140.625$	24	5.3571	-3 375	-153.35
$\Sigma$	2109.4			104 625	95 675

THEN  $\bar{x}\Sigma A = \Sigma \bar{x}A$   
 $\bar{x}(2109.4) = 104 625$   
 OR  $\bar{x} = 49.6$  mm

AND  $\bar{y}\Sigma A = \Sigma \bar{y}A$   
 $\bar{y}(2109.4) = 95 675$   
 OR  $\bar{y} = 45.4$  mm

# 5.17 and 5.18

5.17



GIVEN: PLANE AREA SHOWN  
 FIND:  $\bar{y}$

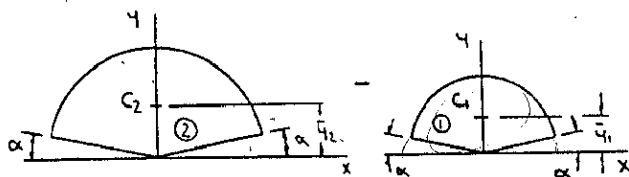


FIG. 5.BA:  $\bar{y}_2 = \frac{2}{3}r_2 \frac{\sin(\frac{\pi}{2}-\alpha)}{(\frac{\pi}{2}-\alpha)}$   $A_2 = (\frac{\pi}{2}-\alpha)r_2^2$   
 $= \frac{2}{3}r_2 \frac{\cos \alpha}{(\frac{\pi}{2}-\alpha)}$

SIMILARLY...  $\bar{y}_1 = \frac{2}{3}r_1 \frac{\cos \alpha}{(\frac{\pi}{2}-\alpha)}$   $A_1 = (\frac{\pi}{2}-\alpha)r_1^2$

THEN...  $\Sigma \bar{y}A = \frac{2}{3}r_2 \frac{\cos \alpha}{(\frac{\pi}{2}-\alpha)} [(\frac{\pi}{2}-\alpha)r_2^2] - \frac{2}{3}r_1 \frac{\cos \alpha}{(\frac{\pi}{2}-\alpha)} [(\frac{\pi}{2}-\alpha)r_1^2]$   
 $= \frac{2}{3}(\bar{r}_2^3 - \bar{r}_1^3) \cos \alpha$

AND  $\Sigma A = (\frac{\pi}{2}-\alpha)r_2^2 - (\frac{\pi}{2}-\alpha)r_1^2$   
 $= (\frac{\pi}{2}-\alpha)(r_2^2 - r_1^2)$

NOW  $\bar{y}\Sigma A = \Sigma \bar{y}A$   
 $\bar{y}[(\frac{\pi}{2}-\alpha)(r_2^2 - r_1^2)] = \frac{2}{3}(\bar{r}_2^3 - \bar{r}_1^3) \cos \alpha$   
 $\bar{y} = \frac{2}{3} \frac{\bar{r}_2^3 - \bar{r}_1^3}{\bar{r}_2^2 - \bar{r}_1^2} \frac{\cos \alpha}{\frac{\pi}{2} - \alpha}$

# 5.18

GIVEN: PLANE AREA SHOWN  
 SHOW:  $\bar{y}$  APPROACHES  $\bar{y}$  OF AN ARC OF RADIUS  $\frac{1}{2}(\bar{r}_1 + \bar{r}_2)$  AS  $\bar{r}_1 \rightarrow \bar{r}_2$

USING FIG. 5.BB,  $\bar{y}$  OF AN ARC OF RADIUS  $\frac{1}{2}(\bar{r}_1 + \bar{r}_2)$  IS...

$\bar{y} = \frac{1}{2}(\bar{r}_1 + \bar{r}_2) \frac{\sin(\frac{\pi}{2}-\alpha)}{(\frac{\pi}{2}-\alpha)}$   
 $= \frac{1}{2}(\bar{r}_1 + \bar{r}_2) \frac{\cos \alpha}{(\frac{\pi}{2}-\alpha)} \quad (1)$

(CONTINUED)

# 5.17 and 5.18 CONTINUED

FROM THE SOLUTION TO PROBLEM 5.17 HAVE

$\bar{y} = \frac{2}{3} \frac{\bar{r}_2^3 - \bar{r}_1^3}{\bar{r}_2^2 - \bar{r}_1^2} \frac{\cos \alpha}{\frac{\pi}{2} - \alpha}$

NOW...  $\frac{\bar{r}_2^3 - \bar{r}_1^3}{\bar{r}_2^2 - \bar{r}_1^2} = \frac{(\bar{r}_2 - \bar{r}_1)(\bar{r}_2^2 + \bar{r}_1\bar{r}_2 + \bar{r}_1^2)}{(\bar{r}_2 - \bar{r}_1)(\bar{r}_2 + \bar{r}_1)} = \frac{\bar{r}_2^2 + \bar{r}_1\bar{r}_2 + \bar{r}_1^2}{\bar{r}_2 + \bar{r}_1}$

LET  $\bar{r}_2 = \bar{r} + \Delta$

$\bar{r}_1 = \bar{r} - \Delta$

THEN  $\bar{r} = \frac{1}{2}(\bar{r}_1 + \bar{r}_2)$

AND  $\frac{\bar{r}_2^3 - \bar{r}_1^3}{\bar{r}_2^2 - \bar{r}_1^2} = \frac{(\bar{r} + \Delta)^2 + (\bar{r} + \Delta)(\bar{r} - \Delta) + (\bar{r} - \Delta)^2}{(\bar{r} + \Delta) + (\bar{r} - \Delta)} = \frac{3\bar{r}^2 + \Delta^2}{2\bar{r}}$

IN THE LIMIT AS  $\bar{r}_1 \rightarrow \bar{r}_2$ ,  $\Delta \rightarrow 0$ . THEN

$\frac{\bar{r}_2^3 - \bar{r}_1^3}{\bar{r}_2^2 - \bar{r}_1^2} = \frac{3}{2}\bar{r}$   
 $= \frac{3}{2} \times \frac{1}{2}(\bar{r}_1 + \bar{r}_2)$

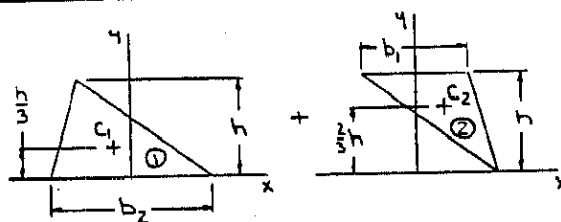
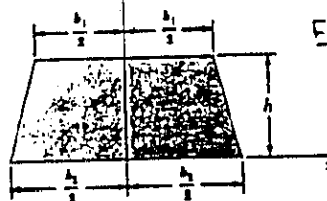
SO THAT  $\bar{y} = \frac{2}{3} \times \frac{3}{4}(\bar{r}_1 + \bar{r}_2) \frac{\cos \alpha}{\frac{\pi}{2} - \alpha}$   
 OR  $\bar{y} = \frac{1}{2}(\bar{r}_1 + \bar{r}_2) \frac{\cos \alpha}{\frac{\pi}{2} - \alpha}$

WHICH AGREES WITH EQ. (1).

# 5.19

GIVEN: PLANE AREA SHOWN

FIND:  $\bar{y}$

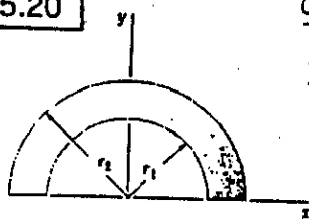


	A	$\bar{y}$	$\bar{y}A$
1	$\frac{1}{2}b_2h$	$\frac{1}{3}h$	$\frac{1}{6}b_2h^2$
2	$\frac{1}{2}b_1h$	$\frac{2}{3}h$	$\frac{1}{3}b_1h^2$
$\Sigma$	$\frac{1}{2}(b_1 + b_2)h$		$\frac{1}{6}(2b_1 + b_2)h$

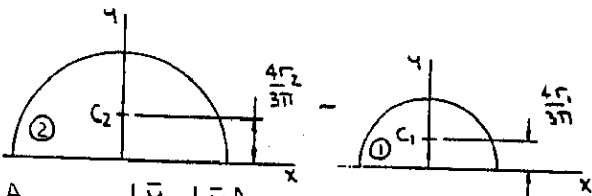
THEN  $\bar{y}\Sigma A = \Sigma \bar{y}A$

$\bar{y}[\frac{1}{2}(b_1 + b_2)h] = \frac{1}{6}(2b_1 + b_2)h^2$   
 OR  $\bar{y} = \frac{2b_1 + b_2}{b_1 + b_2} \frac{h}{3}$

5.20



GIVEN: PLANE AREA  
SHOWN,  $\bar{y} = \frac{3}{4} r_1$   
FIND:  $r_2/r_1$



	A	$\bar{y}$	$\bar{y}A$
1	$\frac{\pi}{2} r_2^2$	$\frac{4r_2}{3\pi}$	$-\frac{2}{3} r_2^3$
2	$\frac{\pi}{2} r_1^2$	$\frac{4r_1}{3\pi}$	$\frac{2}{3} r_1^3$
$\Sigma$	$\frac{\pi}{2} (r_2^2 - r_1^2)$		$\frac{2}{3} (r_2^3 - r_1^3)$

$$\text{THEN } \bar{y}\Sigma A = \Sigma \bar{y}A$$

$$\text{OR } \frac{3}{4} r_1 = \frac{\pi}{2} (r_2^2 - r_1^2) = \frac{2}{3} (r_2^3 - r_1^3)$$

$$\frac{9\pi}{16} \left[ \left( \frac{r_2}{r_1} \right)^2 - 1 \right] = \left( \frac{r_2}{r_1} \right)^3 - 1$$

$$\text{LET } p = \frac{r_2}{r_1}$$

$$\frac{9\pi}{16} [(p+1)(p-1)] = (p-1)(p^2+p+1)$$

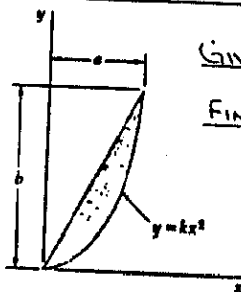
$$\text{OR } 16p^2 + (16-9\pi)p + (16-9\pi) = 0$$

$$\text{THEN } p = \frac{-(16-9\pi) \pm \sqrt{(16-9\pi)^2 - 4(16)(16-9\pi)}}{2(16)}$$

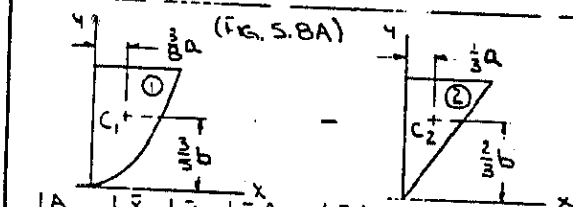
$$\text{OR } p = -0.5726 \quad p = 1.3397$$

TAKING THE POSITIVE ROOT...  $\frac{r_2}{r_1} = 1.340$

5.21



GIVEN: PLANE AREA  
SHOWN,  $\bar{x} = \bar{y}$   
FIND:  $a/b$



	A	$\bar{x}$	$\bar{y}$	$\bar{x}A$	$\bar{y}A$
1	$\frac{1}{6} ab$	$\frac{2}{3} a$	$\frac{2}{3} b$	$a^2 b/4$	$2ab^2/5$
2	$\frac{1}{6} ab$	$\frac{1}{3} a$	$\frac{1}{3} b$	$-a^2 b/6$	$-ab^2/3$
$\Sigma$	$\frac{1}{3} ab$			$a^2 b/12$	$ab^2/15$

(CONTINUED)

5.21 CONTINUED

$$\text{THEN } \bar{x}\Sigma A = \Sigma \bar{x}A$$

$$\bar{x} \left( \frac{1}{3} ab \right) = a^2 b/12$$

$$\text{OR } \bar{x} = \frac{1}{3} a$$

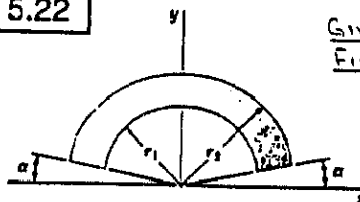
$$\bar{y}\Sigma A = \Sigma \bar{y}A$$

$$\bar{y} \left( \frac{1}{3} ab \right) = ab^2/15$$

$$\text{OR } \bar{y} = \frac{2}{5} b$$

$$\text{OR } \frac{\bar{x}}{\bar{y}} = \frac{4}{5}$$

5.22



GIVEN:  $\bar{y}$ ,  $\alpha = 60^\circ$   
FIND:  $r_2/r_1$  IF  $\bar{y} = r_1$

FROM THE SOLUTION TO PROBLEM 5.17 HAVE

$$\bar{y} = \frac{2}{3} \frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} \frac{\cos \alpha}{\frac{\pi}{2} - \alpha}$$

$$\text{WHEN } \bar{y} = r_1 \text{ AND } \alpha = 60^\circ \left( \frac{\pi}{3} \right)$$

$$r_1 = \frac{2}{3} \frac{r_2^3 [(\frac{r_2}{r_1})^3 - 1]}{r_1^2 [(\frac{r_2}{r_1})^2 - 1]} \frac{\cos \frac{\pi}{3}}{\frac{\pi}{2} - \frac{\pi}{3}}$$

$$1 = \frac{2}{\pi} \frac{p^3 - 1}{p^2 - 1} \quad \text{WHERE } p = \frac{r_2}{r_1}$$

$$\text{THEN } \frac{\pi}{2} = \frac{(p-1)(p^2+p+1)}{(p+1)(p-1)}$$

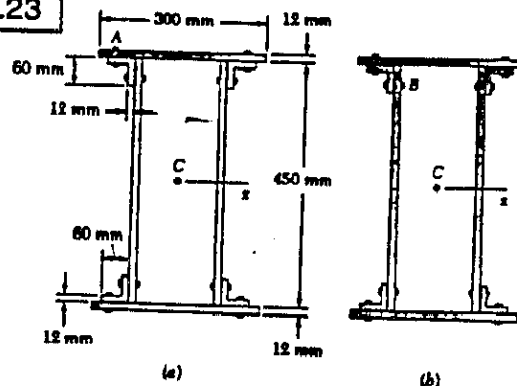
$$\text{OR } 2p^2 + (2-\pi)p + (2-\pi) = 0$$

$$\text{THEN } p = \frac{-(2-\pi) \pm \sqrt{(2-\pi)^2 - 4(2)(2-\pi)}}{2(2)}$$

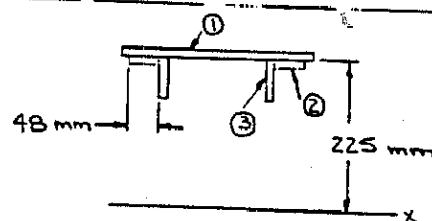
$$\text{OR } p = -0.522 \quad p = 1.093$$

$$\text{TAKING THE POSITIVE ROOT } \frac{r_2}{r_1} = 1.093$$

5.23



GIVEN:  $F_A \propto (Q_x)_A$ ,  $F_B \propto (Q_x)_B$ ,  $F_A = 280 \text{ N}$   
FIND:  $F_B$



FROM THE PROBLEM STATEMENT,  $F \propto Q_x$   
SO THAT  $\frac{F_A}{(Q_x)_A} = \frac{F_B}{(Q_x)_B}$  (CONTINUED)

# 5.23 CONTINUED

$$\text{OR } F_B = \frac{(Q_x)_B}{(Q_x)_A} F_A$$

$$\text{Now.. } Q_x = \sum \bar{y}A$$

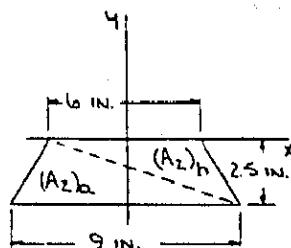
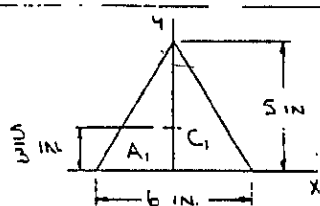
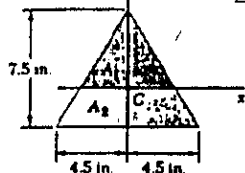
$$\text{Then } (Q_x)_A = [(225-6) \text{ mm}](300 \times 12) \text{ mm}^2 = 831.6 \times 10^3 \text{ mm}^3$$

$$\text{AND } (Q_x)_B = (Q_x)_A + 2[(225-6) \text{ mm}](48 \times 12) \text{ mm}^2 + 2[(225-30) \text{ mm}](60 \times 12) \text{ mm}^2 = 1364.688 \times 10^3 \text{ mm}^3$$

$$\text{FINALLY.. } F_B = \frac{1364.688 \times 10^3 \text{ mm}^3}{831.6 \times 10^3 \text{ mm}^3} \times 280 \text{ N} \text{ OR } F_B = 459 \text{ N}$$

# 5.24

GIVEN: PLANE AREA SHOWN  
FIND:  $(Q_x)_1, (Q_x)_2$   
EXPLAIN RESULTS



$$\text{HAVE.. } Q_x = \sum \bar{y}A$$

$$\text{Then } (Q_x)_1 = \left(\frac{3}{2} \text{ in.}\right) \left(\frac{1}{2} \times 6 \times 5\right) \text{ in}^2$$

$$(Q_x)_1 = 25 \text{ in}^3$$

$$\text{AND } (Q_x)_2 = \left(-\frac{3}{2} \times 2.5 \text{ in.}\right) \left(\frac{1}{2} \times 9 \times 2.5\right) \text{ in}^2 + \left(-\frac{1}{2} \times 2.5 \text{ in.}\right) \left(\frac{1}{2} \times 6 \times 2.5\right) \text{ in}^2$$

$$(Q_x)_2 = -25 \text{ in}^3$$

$$\text{Now.. } Q_x = (Q_x)_1 + (Q_x)_2 = 0$$

THIS RESULT IS EXPECTED SINCE  $\bar{y}$  IS A CENTROIDAL AXIS (THUS  $\bar{y} = 0$ )

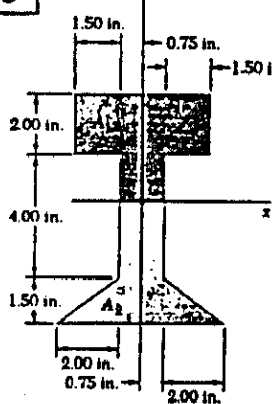
$$\text{AND } Q_x = \sum \bar{y}A = \bar{y} \sum A \quad (\bar{y} = 0 \Rightarrow Q_x = 0)$$

# 5.25

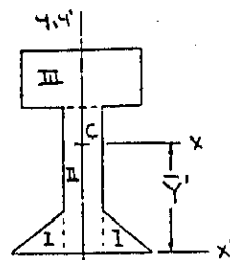
GIVEN: PLANE AREA SHOWN

FIND:  $(Q_x)_1, (Q_x)_2$

EXPLAIN RESULTS



# 5.25 CONTINUED



FIRST DETERMINE THE LOCATION OF THE CENTROIDS C. HAVE..

	A, in <sup>2</sup>	$\bar{y}$ , in.	$\bar{y}A$ , in <sup>3</sup>
I	$2\left(\frac{1}{2} \times 2 \times 1.5\right) = 3$	0.5	1.5
II	$1.5 \times 5.5 = 8.25$	2.75	22.6875
III	$4.5 \times 2 = 9$	6.5	58.5
$\Sigma$	20.25		82.6875

THEN

$$\bar{y} \Sigma A = \Sigma \bar{y}A$$

$$\bar{y}(20.25) = 82.6875$$

$$\text{OR } \bar{y} = 4.0833 \text{ in.}$$

$$\text{Now } Q_x = \sum \bar{y}A$$

$$\text{Then } (Q_x)_1 = \left[\frac{1}{2}(5.5 - 4.0833) \text{ in.}\right] \left[(1.5 \times 5.5 - 4.0833) \text{ in}^2\right] + [(6.5 - 4.0833) \text{ in.}](4.5)(2) \text{ in}^2$$

$$\text{OR } (Q_x)_1 = 23.3 \text{ in}^3$$

$$\text{AND } (Q_x)_2 = \left[-\frac{1}{2}(4.0833 \text{ in.})\right] \left[(1.5 \times 4.0833) \text{ in}^2\right] - [(4.0833 - 0.5) \text{ in.}]\left[2\left(\frac{1}{2} \times 2 \times 1.5\right) \text{ in}^2\right]$$

$$\text{OR } (Q_x)_2 = -23.3 \text{ in}^3$$

$$\text{Now.. } Q_x = (Q_x)_1 + (Q_x)_2 = 0$$

THIS RESULT IS EXPECTED SINCE  $\bar{y}$  IS A CENTROIDAL AXIS (THUS  $\bar{y} = 0$ )

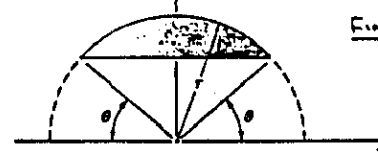
$$\text{AND } Q_x = \sum \bar{y}A = \bar{y} \Sigma A \quad (\bar{y} = 0 \Rightarrow Q_x = 0)$$

# 5.26

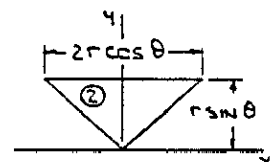
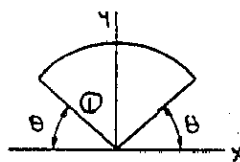
GIVEN: PLANE AREA SHOWN

FIND: (a)  $Q_x$

(b)  $\theta$  AND  $Q_x$  FOR THE MAXIMUM VALUE OF  $Q_x$



(a)



$$\text{HAVE } Q_x = \sum \bar{y}A \text{ AND USING FIG 5.8A...}$$

$$Q_x = \left(\frac{2}{3}r \frac{\sin(\frac{\pi}{2} - \theta)}{\frac{\pi}{2} - \theta}\right) \left[\left(\frac{\pi}{2} - \theta\right)r^2\right]$$

$$- \left(\frac{2}{3}r \sin \theta\right) \left(\frac{1}{2} \times 2r \cos \theta \times r \sin \theta\right)$$

$$= \frac{2}{3}r^3(\cos \theta - \cos \theta \sin^2 \theta)$$

$$\text{OR } Q_x = \frac{2}{3}r^3 \cos^3 \theta$$

(b) BY OBSERVATION,  $Q_x$  IS MAXIMUM FOR

$$\theta = 0$$

AND THEN

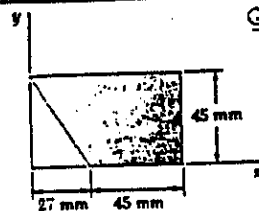
$$(Q_x)_{\text{MAX}} = \frac{2}{3}r^3$$

(CONTINUED)

6	111.50	114.50	0	24.14	0	24.14
$\Sigma$	227.3A			-370	344.1	

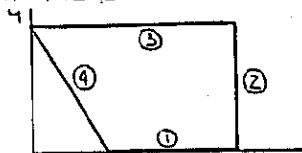


5.27



GIVEN: WIRE HAVING THE SHAPE OF THE PERIMETER OF THE PLANE AREA SHOWN  
FIND:  $\bar{X}$  AND  $\bar{Y}$

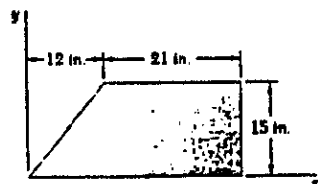
FIRST NOTE THAT BECAUSE THE WIRE IS HOMOGENEOUS, ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING LINE.



	L, mm	$\bar{x}$ , mm	$\bar{y}$ , mm	$\bar{x}L$ , mm <sup>2</sup>	$\bar{y}L$ , mm <sup>2</sup>
1	45	49.5	0	2227.5	0
2	45	72	22.5	3240	1012.5
3	72	36	45	2592	3240
4	$\sqrt{27^2 + 45^2} = 52.479$	13.5	22.5	708.47	1180.78
$\Sigma$	214.479			8768.0	5433.3

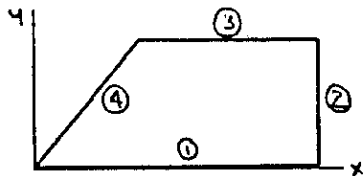
THEN  $\bar{X}\Sigma L = \Sigma \bar{x}L$   
 $\bar{X}(214.479) = 8768.0$   
 OR  $\bar{X} = 40.9$  mm  
 AND  $\bar{Y}\Sigma L = \Sigma \bar{y}L$   
 $\bar{Y}(214.479) = 5433.3$   
 OR  $\bar{Y} = 25.3$  mm

5.28



GIVEN: WIRE HAVING THE SHAPE OF THE PERIMETER OF THE PLANE AREA SHOWN  
FIND:  $\bar{X}$  AND  $\bar{Y}$

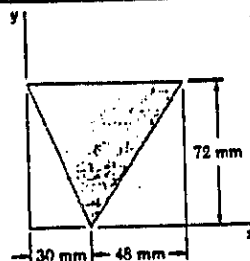
FIRST NOTE THAT BECAUSE THE WIRE IS HOMOGENEOUS, ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING LINE.



	L, in.	$\bar{x}$ , in.	$\bar{y}$ , in.	$\bar{x}L$ , in <sup>2</sup>	$\bar{y}L$ , in <sup>2</sup>
1	33	16.5	0	544.5	0
2	15	33	7.5	495	112.5
3	21	22.5	15	472.5	315
4	$\sqrt{12^2 + 15^2} = 19.2093$	6	7.5	115.256	144.070
$\Sigma$	88.209			1627.26	571.57

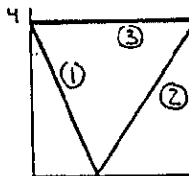
THEN  $\bar{X}\Sigma L = \Sigma \bar{x}L$   
 $\bar{X}(88.209) = 1627.26$   
 OR  $\bar{X} = 18.45$  in.  
 AND  $\bar{Y}\Sigma L = \Sigma \bar{y}L$   
 $\bar{Y}(88.209) = 571.57$   
 OR  $\bar{Y} = 6.48$  in.

5.29



GIVEN: WIRE HAVING THE SHAPE OF THE PERIMETER OF THE PLANE AREA SHOWN  
FIND:  $\bar{X}$  AND  $\bar{Y}$

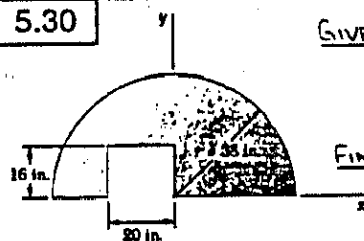
FIRST NOTE THAT BECAUSE THE WIRE IS HOMOGENEOUS, ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING LINE.



	L, mm	$\bar{x}$ , mm	$\bar{y}$ , mm	$\bar{x}L$ , mm <sup>2</sup>	$\bar{y}L$ , mm <sup>2</sup>
1	$\sqrt{30^2 + 72^2} = 78$	15	36	1170	2808
2	$\sqrt{48^2 + 72^2} = 86.533$	54	36	4672.8	3115.2
3	78	39	72	3042	5616
$\Sigma$	242.53			8884.8	11539.2

THEN  $\bar{X}\Sigma L = \Sigma \bar{x}L$   
 $\bar{X}(242.53) = 8884.8$   
 OR  $\bar{X} = 36.6$  mm  
 AND  $\bar{Y}\Sigma L = \Sigma \bar{y}L$   
 $\bar{Y}(242.53) = 11539.2$   
 OR  $\bar{Y} = 47.6$  mm

5.30



GIVEN: WIRE HAVING THE SHAPE OF THE PERIMETER OF THE PLANE AREA SHOWN  
FIND:  $\bar{X}$  AND  $\bar{Y}$

FIRST NOTE THAT BECAUSE THE WIRE IS HOMOGENEOUS, ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING LINE.

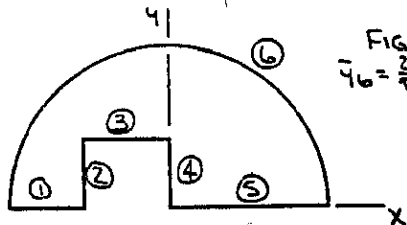


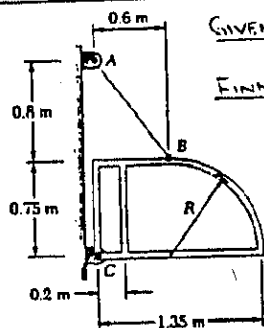
FIG. 5.8A  
 $\bar{y}_6 = \frac{2}{\pi}(38 \text{ in.})$

	L, in.	$\bar{x}$ , in.	$\bar{y}$ , in.	$\bar{x}L$ , in <sup>2</sup>	$\bar{y}L$ , in <sup>2</sup>
1	18	-29	0	-522	0
2	16	-20	8	-320	128
3	20	-10	16	-200	320
4	16	0	8	0	128
5	38	19	0	722	0
6	$\pi(38) = 119.381$	0	24.192	0	2888.1
$\Sigma$	227.38			-320	3464.1

### 5.30 CONTINUED

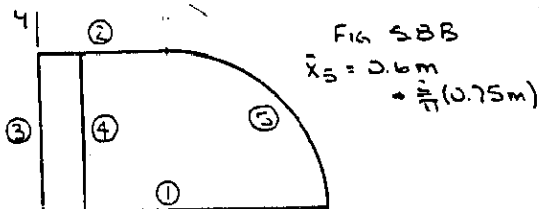
THEN  $\bar{X} \sum L = \sum \bar{x} L$   
 $\bar{X}(227.38) = -320$   
 OR  $\bar{X} = -1.407 \text{ IN.}$   
 AND  $\bar{Y} \sum L = \sum \bar{y} L$   
 $\bar{Y}(227.38) = 3464.1$   
 OR  $\bar{Y} = 15.23 \text{ IN.}$

### 5.31



GIVEN: MASS/LENGTH  $m' = 4.73 \text{ kg/m}$   
 FIND: (a)  $T_{BA}$   
 (b) REACTION  $C$  AT PIN  $C$

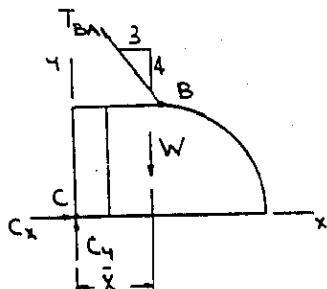
FIRST NOTE THAT BECAUSE THE FRAME IS FABRICATED FROM UNIFORM BAR STOCK, ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROIDS OF THE CORRESPONDING LINE.



	L, m	$\bar{x}, \text{m}$	$\bar{x}L, \text{m}^3$
1	1.35	0.675	0.91125
2	0.6	0.3	0.18
3	0.75	0	0
4	0.75	0.2	0.15
5	$\frac{\pi}{2}(0.75) = 1.17810$	1.07746	1.26736
$\Sigma$	4.62810		2.5106

THEN  $\bar{X} \sum L = \sum \bar{x} L$   
 $\bar{X}(4.62810) = 2.5106$   
 OR  $\bar{X} = 0.54247 \text{ m}$

THE FREE-BODY DIAGRAM OF THE FRAME IS THEN...



WHERE  $W = (m' \sum L)g$   
 $= 4.73 \text{ kg/m} \times 4.62810 \text{ m} \times 9.81 \frac{\text{m}}{\text{s}^2}$   
 $= 214.75 \text{ N}$

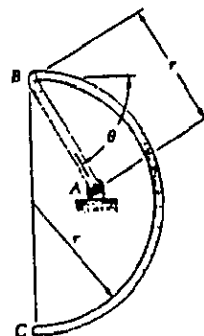
EQUILIBRIUM THEN REQUIRES...  
 (CONTINUED)

### 5.31 CONTINUED

(a)  $\Sigma M_C = 0: (1.55 \text{ m})(\frac{3}{5}T_{BA}) - (0.54247 \text{ m})(214.75 \text{ N}) = 0$   
 OR  $T_{BA} = 125.264 \text{ N}$   
 OR  $T_{BA} = 125.3 \text{ N}$

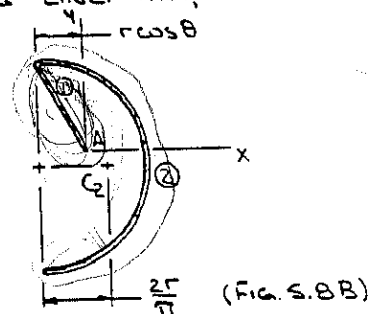
(b)  $\Sigma F_x = 0: C_x - \frac{3}{5}(125.264 \text{ N}) = 0$   
 OR  $C_x = 75.158 \text{ N}$   
 $\Sigma F_y = 0: C_y + \frac{4}{5}(125.264 \text{ N}) - (214.75 \text{ N}) = 0$   
 OR  $C_y = 114.539 \text{ N}$   
 THEN...  $C = 137.0 \text{ N} \angle 56.7^\circ$

### 5.32



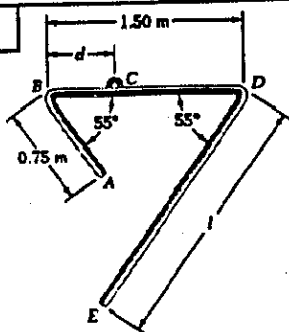
GIVEN: HOMOGENEOUS WIRE  
 FIND:  $\theta$  FOR EQUILIBRIUM

FIRST NOTE THAT FOR EQUILIBRIUM, THE CENTER OF GRAVITY OF THE WIRE MUST LIE ON A VERTICAL LINE THROUGH A. FURTHER, BECAUSE THE WIRE IS HOMOGENEOUS, ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROIDS OF THE CORRESPONDING LINE. THUS,



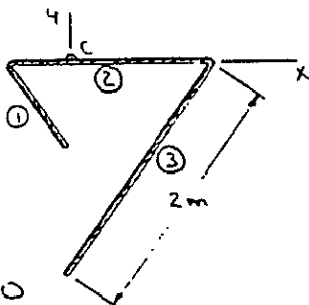
$\bar{X} = 0$   
 SO THAT  $\sum \bar{x} L = 0$   
 THEN...  $(-\frac{1}{2}r \cos \theta)(r) + (\frac{2r}{\pi} - r \cos \theta)(\pi r) = 0$   
 OR  $\cos \theta = \frac{4}{1+2\pi}$   
 $= 0.54921$   
 OR  $\theta = 56.7^\circ$

5.33



GIVEN: UNIFORM TUBING,  $l = 2$  m, BCD IS HORIZONTAL  
FIND:  $d$

FIRST NOTE THAT FOR EQUILIBRIUM, THE CENTER OF GRAVITY OF THE COMPONENT MUST LIE ON A VERTICAL LINE THROUGH C. FURTHER, BECAUSE THE TUBING IS UNIFORM, THE CENTER OF GRAVITY OF THE COMPONENT WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING LINE. THUS,



$$\bar{x} = 0$$

SO THAT  $\sum \bar{x}L = 0$

THEN  $-(d - \frac{0.75}{2} \cos 55^\circ)m = (0.75m)$

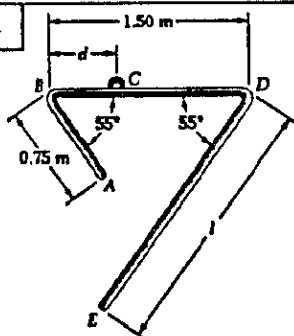
$$+ (0.75 - d)m = (1.5m)$$

$$+ [(1.5 - d)m - (\frac{1}{2} \cdot 2m \cdot \cos 55^\circ)] \cdot (2m) = 0$$

OR  $(0.75 + 1.5 + 2)d = [\frac{1}{2}(0.75)^2 - 2] \cos 55^\circ + (0.75)(1.5) + 3$

OR  $d = 0.739$  m

5.34

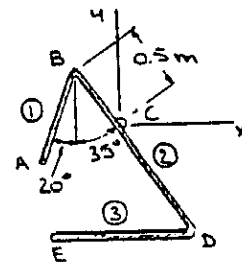


GIVEN: UNIFORM TUBING,  $d = 0.5$  m, DE IS HORIZONTAL  
FIND:  $l$

FIRST NOTE THAT FOR EQUILIBRIUM, THE CENTER OF GRAVITY OF THE COMPONENT MUST LIE ON A VERTICAL LINE THROUGH C. FURTHER, BECAUSE THE TUBING IS UNIFORM, THE CENTER OF GRAVITY OF THE COMPONENT WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING LINE. THUS,

(CONTINUED)

5.34 CONTINUED



$$\bar{x} = 0$$

SO THAT  $\sum \bar{x}L = 0$

$$\text{OR } -(\frac{0.75}{2} \sin 20^\circ + 0.5 \sin 35^\circ)m = (0.75m)$$

$$+ (0.25m \cdot \sin 35^\circ) + (1.5m)$$

$$+ (1.0 \cdot \sin 35^\circ - \frac{1}{2})m = (l m) = 0$$

OR  $-0.096193 + (\sin 35^\circ - \frac{1}{2})l = 0$

$$(\bar{x}L)_{AB} + (\bar{x}L)_{BC} + (\bar{x}L)_{DE} = 0$$

THIS EQUATION IMPLIES THAT THE CENTER OF GRAVITY OF DE MUST BE TO THE RIGHT OF C. THEN..

$$l^2 - 1.14715l + 0.192386 = 0$$

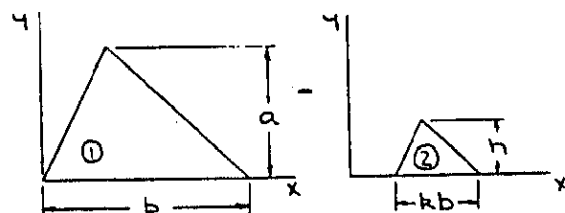
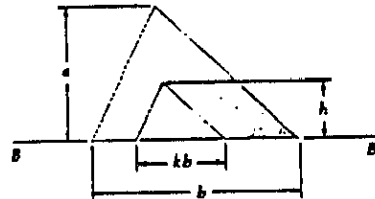
$$\text{OR } l = \frac{1.14715 \pm \sqrt{(-1.14715)^2 - 4(0.192386)}}{2}$$

OR  $l = 0.204$  m AND  $l = 0.943$  m

NOTE THAT  $\sin 35^\circ - \frac{1}{2} > 0$  FOR BOTH VALUES OF  $l$  SO BOTH VALUES ARE ACCEPTABLE.

5.35 and 5.36

GIVEN: PLANE AREA SHOWN



	A	$\bar{y}$	$\bar{y}A$
1	$\frac{1}{2}ba$	$\frac{2}{3}a$	$\frac{1}{3}a^2b$
2	$-\frac{1}{2}(kb)h$	$\frac{1}{2}h$	$-\frac{1}{4}krbh$
$\Sigma$	$\frac{1}{2}(a - kh)$	$\frac{1}{2}(a^2 - kh^2)$	

THEN  $\bar{y} \Sigma A = \Sigma \bar{y}A$

$$\bar{y} \left[ \frac{1}{2}(a - kh) \right] = \frac{1}{6}(a^2 - kh^2)$$

OR  $\bar{y} = \frac{a^2 - kh^2}{3(a - kh)} \quad (1)$

AND  $\frac{d\bar{y}}{dh} = \frac{1}{3} \frac{-2kh(a - kh) - (a^2 - kh^2)(-k)}{(a - kh)^2} = 0$

OR  $2h(a - kh) - a^2 + kh^2 = 0 \quad (2)$

5.35 FIND:  $h$  SO THAT  $\bar{y}$  IS MAXIMUM

(a)  $k = 0.10$

(b)  $k = 0.80$

(CONTINUED)

### 5.35 and 5.36 CONTINUED

SIMPLIFYING EQ. (2) YIELDS...

$$kh^2 - 2ah + a^2 = 0$$

THEN  $h = \frac{2a \pm \sqrt{(-2a)^2 - 4(k)(a^2)}}{2k}$

$$= \frac{a}{k} [1 \pm \sqrt{1-k}]$$

NOTE THAT ONLY THE NEGATIVE ROOT IS ACCEPTABLE SINCE  $h < a$ . THEN...

(a)  $k = 0.10$

$$h = \frac{a}{0.10} [1 - \sqrt{1-0.10}]$$

OR  $h = 0.513a$

(b)  $k = 0.80$

$$h = \frac{a}{0.80} [1 - \sqrt{1-0.80}]$$

OR  $h = 0.491a$

5.36 SHOW:  $\bar{Y} = \frac{2}{3}h$  FOR THE VALUE OF  $h$  WHICH MAXIMIZES  $\bar{Y}$

REARRANGING EQ. (2) (WHICH DEFINES THE VALUE OF  $h$  WHICH MAXIMIZES  $\bar{Y}$ ) YIELDS

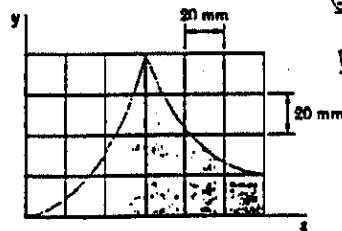
$$a^2 - kh^2 = 2h(a - kh)$$

THEN SUBSTITUTING INTO EQ. (1) (WHICH DEFINES  $\bar{Y}$ )...

$$\bar{Y} = \frac{1}{3(a - kh)} \cdot 2h(a - kh)$$

OR  $\bar{Y} = \frac{2}{3}h$

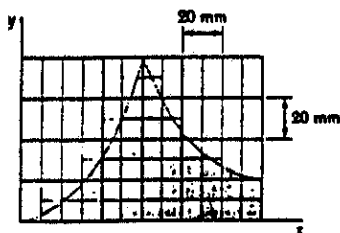
### 5.37 and 5.38



GIVEN: PLANE AREA

SHOWN  
FIND  $\bar{X}$  (5.37) AND  $\bar{Y}$  (5.38) USING APPROXIMATE MEANS

THE AREA IS FIRST DIVIDED INTO TWELVE VERTICAL STRIPS, EACH 10 MM WIDE, AND THEN THE AREA AND THE LOCATION OF THE CENTROID ARE APPROXIMATED FOR EACH STRIP. A 10x10-MM GRID IS USED TO FACILITATE APPROXIMATING THE VALUES.



(CONTINUED)

### 5.37 and 5.38 CONTINUED

STRIP	A, mm <sup>2</sup>	$\bar{x}$ , mm	$\bar{y}$ , mm	$\bar{x}A$ , mm <sup>3</sup>	$\bar{y}A$ , mm <sup>3</sup>
1	15	7	1	105	15
2	65	16	3	1040	195
3	150	26	7	3900	1050
4	250	36	14	9000	3500
5	400	47	21	18800	8400
6	650	57	33	37050	21450
7	700	63	36	44100	25200
8	520	74	27	38480	14040
9	390	83	18	32370	7020
10	295	94	15	27750	4425
11	240	104	12	24960	2880
12	210	113	11	23730	2310
$\Sigma$	3885			261265	90485

5.37

HAVE...

$$\bar{X}\Sigma A = \Sigma \bar{x}A$$

$$\bar{X}(3885) = 261265$$

OR  $\bar{X} = 67.2 \text{ mm}$

5.38

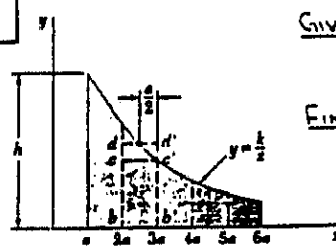
HAVE...

$$\bar{Y}\Sigma A = \Sigma \bar{y}A$$

$$\bar{Y}(3885) = 90485$$

OR  $\bar{Y} = 23.3 \text{ mm}$

5.39



GIVEN: PLANE AREA

SHOWN

$$\bar{x} = \frac{5a}{\ln 6}$$

FIND:  $\bar{x}$  USING

APPROXIMATE MEANS BASED ON RECTANGLES bcc'b'

HAVE  $y = \frac{h}{a}x$

THEN AT  $x=a$ ,  $y=h$ :  $h = \frac{h}{a}a$

SO THAT  $y = \frac{ah}{x}$

RECTANGLE	$x_c$	$y_c$	A	$\bar{x}$	$\bar{x}A$
1	2a	h/2	ah/2	1.5a	0.75a <sup>2</sup> h
2	3a	h/3	ah/3	2.5a	0.833a <sup>2</sup> h
3	4a	h/4	ah/4	3.5a	0.875a <sup>2</sup> h
4	5a	h/5	ah/5	4.5a	0.9a <sup>2</sup> h
5	6a	h/6	ah/6	5.5a	0.917a <sup>2</sup> h
$\Sigma$			1.45ah		4.275a <sup>2</sup> h

THEN  $\bar{X}\Sigma A = \Sigma \bar{x}A$

$$\bar{X}(1.45ah) = 4.275a^2h$$

OR  $\bar{X} = 2.9483a$

OR  $\bar{X} = 2.95a$

$$\% \text{ ERROR} = \frac{\left| \frac{3a}{\ln 6} - 2.9483a \right|}{\frac{3a}{\ln 6}} \cdot 100\%$$

OR  $\% \text{ ERROR} = 5.65\%$

5.40



GIVEN: PLANE AREA SHOWN,  
 $\bar{x} = 5a/\ln 6$

FIND:  $\bar{x}$  USING APPROXIMATE MEANS BASED ON RECTANGLES bdd' b'

HAVE  $y = \frac{h}{x}$   
 THEN AT  $x=a$ ,  $y=h$ :  $h = \frac{h}{a}$   
 SO THAT  $y = \frac{ah}{x}$  OR  $k = ah$

RECTANGLE	$x_{AV}$	$y_{AV}$	A	$\bar{x}$	$\bar{x}A$
1	1.5a	h/1.5	ah/1.5	1.5a	a <sup>2</sup> h
2	2.5a	h/2.5	ah/2.5	2.5a	a <sup>2</sup> h
3	3.5a	h/3.5	ah/3.5	3.5a	a <sup>2</sup> h
4	4.5a	h/4.5	ah/4.5	4.5a	a <sup>2</sup> h
5	5.5a	h/5.5	ah/5.5	5.5a	a <sup>2</sup> h
$\Sigma$			1.75642ah		5a <sup>2</sup> h

THEN:  $\bar{x}\Sigma A = \Sigma \bar{x}A$   
 $\bar{x}(1.75642ah) = 5a^2h$   
 OR  $\bar{x} = 2.8467a$   
 OR  $\bar{x} = 2.85a$

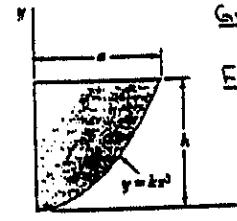
% ERROR =  $\frac{5a - 2.8467a}{5a} \times 100\%$   
 $\frac{2.1533a}{5a}$   
 OR % ERROR = 2.01%

5.41 CONTINUED

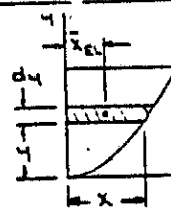
$$\bar{y}A = \int \bar{y}_{EL} dA: \bar{y}(\frac{1}{2}ah) = \frac{1}{3}ah^2$$

$$\bar{y} = \frac{2}{3}h$$

5.42



GIVEN: PLANE AREA SHOWN  
 FIND:  $\bar{x}$  AND  $\bar{y}$  USING DIRECT INTEGRATION



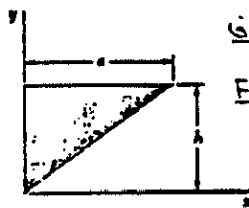
AT  $x=a$ ,  $y=h$ :  $h = ka^3$   
 OR  $k = \frac{h}{a^3}$

THEN  $x = \frac{a}{h^{1/3}} y^{1/3}$   
 NOW...  $dA = x dy$   
 $= \frac{a}{h^{1/3}} y^{1/3} dy$   
 $\bar{y}_{EL} = \frac{1}{2}y = \frac{1}{2} \frac{h^{2/3}}{a^{1/3}} y^{4/3}$

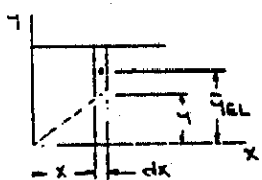
THEN...  $A = \int dA = \int_0^h \frac{a}{h^{1/3}} y^{1/3} dy = \frac{3}{4} \frac{a}{h^{1/3}} [y^{4/3}]_0^h = \frac{3}{4} ah$   
 AND...  $\int \bar{y}_{EL} dA = \int_0^h \frac{1}{2} \frac{a}{h^{1/3}} y^{1/3} [ \frac{h^{2/3}}{a^{1/3}} y^{4/3} ] dy = \frac{1}{2} \frac{a^2}{h^{2/3}} [ \frac{3}{5} y^{5/3} ]_0^h$   
 $= \frac{3}{10} a^2 h$   
 $\int \bar{y}_{EL} dA = \int_0^h y [ \frac{a}{h^{1/3}} y^{1/3} dy ] = \frac{a}{h^{1/3}} [ \frac{3}{4} y^{4/3} ]_0^h$   
 $= \frac{3}{4} ah^2$

$\bar{x}A = \int \bar{x}_{EL} dA: \bar{x}(\frac{3}{4}ah) = \frac{3}{10}a^2h$   
 $\bar{y}A = \int \bar{y}_{EL} dA: \bar{y}(\frac{3}{4}ah) = \frac{3}{10}ah^2$   
 $\bar{x} = \frac{2}{5}a$   
 $\bar{y} = \frac{2}{3}h$

5.41



GIVEN: PLANE AREA SHOWN  
 FIND:  $\bar{x}$  AND  $\bar{y}$  USING DIRECT INTEGRATION



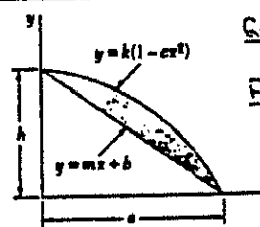
HAVE...  $y = \frac{h}{a}x$   
 AND  $dA = (h-y)dx$   
 $= h(1 - \frac{x}{a})dx$   
 $\bar{x}_{EL} = x$   
 $\bar{y}_{EL} = \frac{1}{2}(h+y)$   
 $= \frac{h}{2}(1 + \frac{x}{a})$

THEN...  $A = \int dA = \int_0^a h(1 - \frac{x}{a})dx = h[x - \frac{x^2}{2a}]_0^a$   
 $= \frac{1}{2}ah$

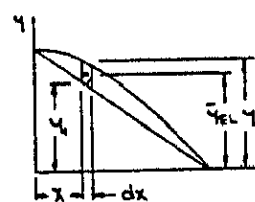
AND...  $\int \bar{x}_{EL} dA = \int_0^a x[h(1 - \frac{x}{a})dx] = h[\frac{x^2}{2} - \frac{x^3}{3a}]_0^a$   
 $= \frac{1}{6}a^2h$   
 $\int \bar{y}_{EL} dA = \int_0^a \frac{h}{2}(1 + \frac{x}{a})[h(1 - \frac{x}{a})dx]$   
 $= \frac{h^2}{2} \int_0^a (1 - \frac{x^2}{a^2})dx = \frac{h^2}{2} [x - \frac{x^3}{3a^2}]_0^a$   
 $= \frac{1}{3}ah^2$

$\bar{x}A = \int \bar{x}_{EL} dA: \bar{x}(\frac{1}{2}ah) = \frac{1}{6}a^2h$   
 $\bar{x} = \frac{1}{3}a$   
 (CONTINUED)

5.43



GIVEN: PLANE AREA SHOWN  
 FIND:  $\bar{x}$  AND  $\bar{y}$  USING DIRECT INTEGRATION



BY OBSERVATION...

$$y_1 = -\frac{h}{a}x + h$$

$$= h(1 - \frac{x}{a})$$

FOR  $y_2$ ...

AT  $x=0$ ,  $y=h$ :  $h = k(1-0)$   
 OR  $k = h$

AT  $x=a$ ,  $y=0$ :  $0 = h(1 - \frac{a}{a})$   
 OR  $C = \frac{h}{a^2}$

THEN...  $y_2 = h(1 - \frac{x^2}{a^2})$

NOW...  $dA = (y_2 - y_1)dx = h[(1 - \frac{x^2}{a^2}) - (1 - \frac{x}{a})]dx$   
 $= h(\frac{x}{a} - \frac{x^2}{a^2})dx$   
 $\bar{x}_{EL} = x$   
 $\bar{y}_{EL} = \frac{1}{2}(y_1 + y_2) = \frac{h}{2}[(1 - \frac{x}{a}) + (1 - \frac{x^2}{a^2})]$   
 $= \frac{h}{2}(2 - \frac{x}{a} - \frac{x^2}{a^2})$

THEN...  $A = \int dA = \int_0^a h(\frac{x}{a} - \frac{x^2}{a^2})dx = h[\frac{x^2}{2a} - \frac{x^3}{3a^2}]_0^a$   
 $= \frac{1}{6}ah$   
 (CONTINUED)

### 5.43 CONTINUED

$$\text{AND.. } \bar{x}_{EL} dA = \int_0^a x \left( h \left( \frac{x}{a} - \frac{x^2}{a^2} \right) dx \right) = h \left[ \frac{x^2}{2a} - \frac{x^3}{3a^2} \right]_0^a$$

$$= \frac{1}{12} a^2 h$$

$$\bar{y}_{EL} dA = \int_0^a \frac{h}{2} \left( 2 - \frac{x}{a} - \frac{x^2}{a^2} \right) \left( h \left( \frac{x}{a} - \frac{x^2}{a^2} \right) dx \right)$$

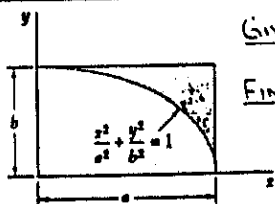
$$= \frac{h^2}{2} \int_0^a \left( 2 \frac{x}{a} - 3 \frac{x^2}{a^2} + \frac{x^4}{a^3} \right) dx$$

$$= \frac{h^2}{2} \left[ \frac{x^2}{a} - \frac{x^3}{a^2} + \frac{x^5}{5a^3} \right]_0^a = \frac{1}{10} a^2 h^2$$

$$\bar{x}A = \int \bar{x}_{EL} dA: \bar{x} \left( \frac{1}{6} a^2 h \right) = \frac{1}{12} a^2 h \quad \bar{x} = \frac{1}{2} a$$

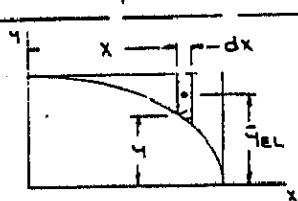
$$\bar{y}A = \int \bar{y}_{EL} dA: \bar{y} \left( \frac{1}{6} a^2 h \right) = \frac{1}{10} a^2 h^2 \quad \bar{y} = \frac{2}{5} h$$

### 5.44



GIVEN: PLANE AREA SHOWN

FIND:  $\bar{x}$  AND  $\bar{y}$  USING DIRECT INTEGRATION



HAVE..  $y = \frac{b}{a} \sqrt{a^2 - x^2}$

AND

$$dA = (b - y) dx$$

$$= \frac{b}{a} (a - \sqrt{a^2 - x^2}) dx$$

$$\bar{x}_{EL} = x$$

$$\bar{y}_{EL} = \frac{1}{2} (y + b)$$

$$= \frac{b}{2a} (a + \sqrt{a^2 - x^2})$$

$$\text{THEN.. } A = \int dA = \int_0^a \frac{b}{a} (a - \sqrt{a^2 - x^2}) dx$$

LET  $x = a \sin \theta: \sqrt{a^2 - x^2} = a \cos \theta$

$dx = a \cos \theta d\theta$

$$\text{THEN.. } A = \int_0^{\pi/2} \frac{b}{a} (a - a \cos \theta) (a \cos \theta d\theta)$$

$$= \frac{b}{a} \left[ a^2 \sin \theta - a^2 \left( \frac{\theta}{2} + \frac{\sin 2\theta}{4} \right) \right]_0^{\pi/2}$$

$$= ab \left( 1 - \frac{\pi}{4} \right)$$

$$\text{AND.. } \bar{x}_{EL} dA = \int_0^a x \left[ \frac{b}{a} (a - \sqrt{a^2 - x^2}) dx \right]$$

$$= \frac{b}{a} \left[ \frac{a}{2} x^2 - \frac{1}{3} (a^2 - x^2)^{3/2} \right]_0^a = \frac{1}{6} a^2 b$$

$$\bar{y}_{EL} dA = \int_0^a \frac{b}{2a} (a + \sqrt{a^2 - x^2}) \left[ \frac{b}{a} (a - \sqrt{a^2 - x^2}) dx \right]$$

$$= \frac{b^2}{2a^2} \int_0^a (x^2) dx = \frac{b^2}{2a^2} \left[ \frac{1}{3} x^3 \right]_0^a$$

$$= \frac{1}{6} a b^2$$

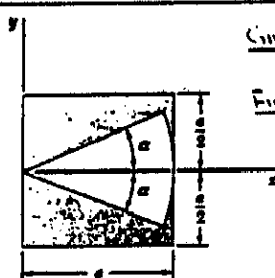
$$\bar{x}A = \int \bar{x}_{EL} dA: \bar{x} [ab(1 - \frac{\pi}{4})] = \frac{1}{6} a^2 b$$

$$\text{OR } \bar{x} = \frac{2a}{3(4 - \pi)}$$

$$\bar{y}A = \int \bar{y}_{EL} dA: \bar{y} [ab(1 - \frac{\pi}{4})] = \frac{1}{6} a b^2$$

$$\text{OR } \bar{y} = \frac{2b}{3(4 - \pi)}$$

### 5.45



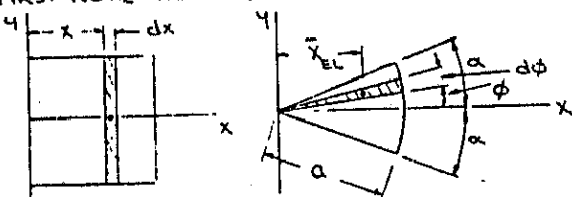
GIVEN: PLANE AREA SHOWN

FIND:  $\bar{x}$  AND  $\bar{y}$  USING DIRECT INTEGRATION

(CONTINUED)

### 5.45 CONTINUED

FIRST NOTE THAT SYMMETRY IMPLIES  $\bar{y} = 0$



$$dA = a dx$$

$$\bar{x}_{EL} = x$$

$$dA = \frac{1}{2} a (a d\phi)$$

$$\bar{x}_{EL} = \frac{2}{3} a \cos \phi$$

$$\text{THEN.. } A = \int dA = \int_0^a a dx = a^2$$

$$= a \left[ x \right]_0^a = \frac{a^2}{2} \left[ \phi \right]_0^{\pi/2} = a^2 (1 - \alpha)$$

$$\text{AND.. } \bar{x}_{EL} dA = \int_0^a x (a dx) = \frac{1}{2} a^2 \phi$$

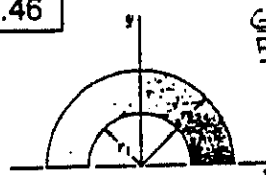
$$= a \left[ \frac{x^2}{2} \right]_0^a - \frac{1}{3} a^2 \left[ \sin \phi \right]_0^{\pi/2}$$

$$= a^3 \left( \frac{1}{2} - \frac{2}{3} \sin \alpha \right)$$

$$\bar{x}A = \int \bar{x}_{EL} dA: \bar{x} [a^2(1 - \alpha)] = a^3 \left( \frac{1}{2} - \frac{2}{3} \sin \alpha \right)$$

$$\text{OR } \bar{x} = \frac{3 - 4 \sin \alpha}{6(1 - \alpha)} a$$

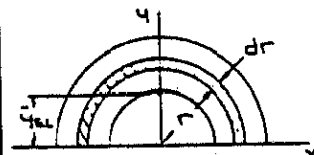
### 5.46



GIVEN: PLANE AREA SHOWN

FIND:  $\bar{x}$  AND  $\bar{y}$  USING DIRECT INTEGRATION

FIRST NOTE THAT SYMMETRY IMPLIES  $\bar{x} = 0$



HAVE..  $dA = \pi r dr$

AND  $\bar{y}_{EL} = \frac{2}{3} r$

(FIG. 5.8B)

$$\text{THEN.. } A = \int dA = \int_0^a \pi r dr = \frac{\pi}{2} [r^2]_0^a = \frac{\pi}{2} (a^2 - 0)$$

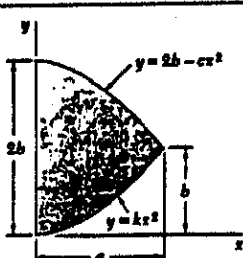
$$\text{AND.. } \bar{y}_{EL} dA = \int_0^a \frac{2}{3} r (\pi r dr) = \frac{2}{3} \pi \left[ \frac{1}{3} r^3 \right]_0^a$$

$$= \frac{2}{9} \pi (a^3 - 0)$$

$$\bar{y}A = \int \bar{y}_{EL} dA: \bar{y} \left[ \frac{\pi}{2} (a^2 - 0) \right] = \frac{2}{9} \pi (a^3 - 0)$$

$$\text{OR } \bar{y} = \frac{4}{3\pi} \frac{a^3 - 0}{a^2 - 0}$$

### 5.47



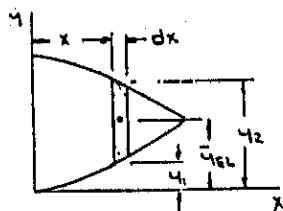
GIVEN: PLANE AREA SHOWN

FIND:  $\bar{x}$  AND  $\bar{y}$  USING DIRECT INTEGRATION

FIRST NOTE THAT SYMMETRY IMPLIES  $\bar{y} = b$

(CONTINUED)

# 5.47 CONTINUED



At  $x=a$ ,  $y=b$   
 $y_1: b = ka^2$  OR  $k = \frac{b}{a^2}$   
 THEN  $y_1 = \frac{b}{a^2} x^2$   
 $y_2: b = 2b - ca^2$   
 OR  $c = \frac{b}{a^2}$   
 THEN  $y_2 = b(2 - \frac{x^2}{a^2})$

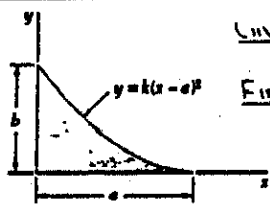
NOW..  $dA = (y_2 - y_1) dx = [b(2 - \frac{x^2}{a^2}) - \frac{b}{a^2} x^2] dx$   
 $= 2b(1 - \frac{x^2}{a^2}) dx$

AND  $\bar{x}_{EL} = x$   
 THEN..  $A = \int dA = \int_0^a 2b(1 - \frac{x^2}{a^2}) dx = 2b[x - \frac{x^3}{3a^2}]_0^a$   
 $= \frac{4}{3} ab$

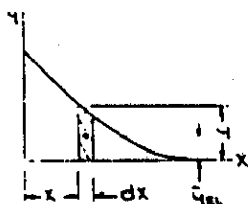
AND  $\int \bar{x}_{EL} dA = \int_0^a x(2b(1 - \frac{x^2}{a^2})) dx = 2b[\frac{x^2}{2} - \frac{x^4}{4a^2}]_0^a$   
 $= \frac{1}{2} a^2 b$

$\bar{x}A = \int \bar{x}_{EL} dA: \bar{x}(\frac{4}{3} ab) = \frac{1}{2} a^2 b \quad \bar{x} = \frac{3}{8} a$

# 5.48



(GIVEN: PLANE AREA SHOWN)  
 FIND:  $\bar{x}$  AND  $\bar{y}$   
 USING DIRECT INTEGRATION



At  $x=0$ ,  $y=b$   
 $b = k(0-a)^2$  OR  $k = \frac{b}{a^2}$   
 THEN  $y = \frac{b}{a^2} (x-a)^2$   
 NOW..  $\bar{x}_{EL} = x$   
 $\bar{y}_{EL} = \frac{1}{2} y = \frac{b}{2a^2} (x-a)^2$

AND  $dA = y dx = \frac{b}{a^2} (x-a)^2 dx$   
 THEN..  $A = \int dA = \int_0^a \frac{b}{a^2} (x-a)^2 dx = \frac{b}{3a^2} [(x-a)^3]_0^a$   
 $= \frac{1}{3} ab$

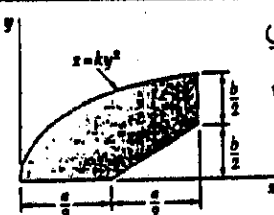
AND..  $\int \bar{x}_{EL} dA = \int_0^a x[\frac{b}{a^2} (x-a)^2] dx = \frac{b}{a^2} \int_0^a (x^3 - 2ax^2 + a^2x) dx$   
 $= \frac{b}{a^2} [\frac{x^4}{4} - \frac{2}{3} ax^3 + \frac{1}{2} a^2 x^2]_0^a = \frac{1}{12} a^2 b$

$\int \bar{y}_{EL} dA = \int_0^a \frac{b}{2a^2} (x-a)^2 [\frac{b}{a^2} (x-a)^2] dx$   
 $= \frac{b^2}{2a^4} [\frac{1}{5} (x-a)^5]_0^a = \frac{1}{10} ab^2$

$\bar{x}A = \int \bar{x}_{EL} dA: \bar{x}(\frac{1}{3} ab) = \frac{1}{12} a^2 b \quad \bar{x} = \frac{1}{4} a$

$\bar{y}A = \int \bar{y}_{EL} dA: \bar{y}(\frac{1}{3} ab) = \frac{1}{10} ab^2 \quad \bar{y} = \frac{3}{10} b$

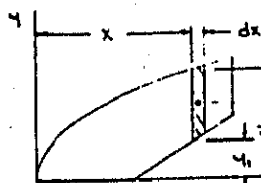
# 5.49



(GIVEN: PLANE AREA SHOWN)  
 FIND:  $\bar{x}$  AND  $\bar{y}$   
 USING DIRECT INTEGRATION

By OBSERVATION..  $y_1 = \frac{b}{a} x - \frac{b}{2} = b(\frac{x}{a} - \frac{1}{2})$   
 (CONTINUED)

# 5.49 CONTINUED



FOR  $y_2$  AT  $x=0$ ,  $y=b$   
 $a = kb^2$  OR  $k = \frac{a}{b^2}$   
 THEN  $y_2 = b\sqrt{\frac{x}{a}}$

NOW..  $\bar{x}_{EL} = x$   
 AND FOR  $0 \leq x \leq \frac{a}{2}$ :  
 $\bar{y}_{EL} = \frac{1}{2} y_2 \quad dA = y_2 dx$   
 $= \frac{1}{2} b\sqrt{\frac{x}{a}} = b\sqrt{\frac{x}{a}}$

FOR  $\frac{a}{2} \leq x \leq a$ :  $\bar{y}_{EL} = \frac{1}{2} (y_1 + y_2) = \frac{b}{2} (\frac{x}{a} - \frac{1}{2} + \sqrt{\frac{x}{a}})$   
 $dA = (y_2 - y_1) dx = b(\sqrt{\frac{x}{a}} - \frac{x}{a} + \frac{1}{2}) dx$

THEN..  $A = \int dA = \int_0^{a/2} b\sqrt{\frac{x}{a}} dx + \int_{a/2}^a b(\sqrt{\frac{x}{a}} - \frac{x}{a} + \frac{1}{2}) dx$   
 $= \frac{b}{\sqrt{a}} [\frac{2}{3} x^{3/2}]_0^{a/2} + b[\frac{2}{3} \sqrt{\frac{x}{a}} - \frac{x^2}{2a} + \frac{1}{2} x]_{a/2}^a$   
 $= \frac{2}{3} \frac{b}{\sqrt{a}} (\frac{a}{2})^{3/2} + b[\frac{2}{3} (\frac{a}{a}) - \frac{a^2}{2a} + \frac{1}{2} a - (\frac{2}{3} \sqrt{\frac{a}{a}} - \frac{a^2}{2a} + \frac{1}{2} \frac{a}{2})]$   
 $= \frac{13}{24} ab$

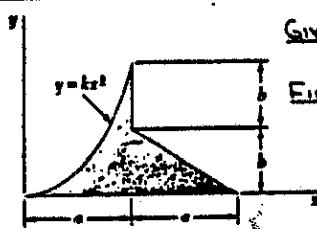
AND..  $\int \bar{x}_{EL} dA = \int_0^{a/2} x[b\sqrt{\frac{x}{a}}] dx + \int_{a/2}^a x[b(\sqrt{\frac{x}{a}} - \frac{x}{a} + \frac{1}{2})] dx$   
 $= \frac{b}{\sqrt{a}} [\frac{2}{5} x^{5/2}]_0^{a/2} + b[\frac{2}{5} \sqrt{\frac{x}{a}} - \frac{x^2}{2a} + \frac{1}{4} x]_{a/2}^a$   
 $= \frac{2}{5} \frac{b}{\sqrt{a}} (\frac{a}{2})^{5/2} + b[\frac{2}{5} (\frac{a}{a}) - \frac{a^2}{2a} + \frac{1}{4} a - (\frac{2}{5} \sqrt{\frac{a}{a}} - \frac{a^2}{2a} + \frac{1}{4} \frac{a}{2})]$   
 $= \frac{71}{240} a^2 b$

$\int \bar{y}_{EL} dA = \int_0^{a/2} \frac{1}{2} y_2 [b\sqrt{\frac{x}{a}}] dx + \int_{a/2}^a \frac{1}{2} (y_1 + y_2) [b(\sqrt{\frac{x}{a}} - \frac{x}{a} + \frac{1}{2})] dx$   
 $= \frac{b^2}{2a} [\frac{2}{5} x^{5/2}]_0^{a/2} + \frac{b^2}{2} [\frac{2}{5} (\frac{x}{a}) - \frac{1}{2a} (\frac{x^2}{2}) + \frac{1}{4} x]_{a/2}^a$   
 $= \frac{b^2}{40a} [(\frac{a}{2})^5 + (a)^5 - (\frac{a}{2})^5] - \frac{b^2}{4a} (\frac{a}{2} - \frac{1}{2})^3$   
 $= \frac{11}{40} ab^2$

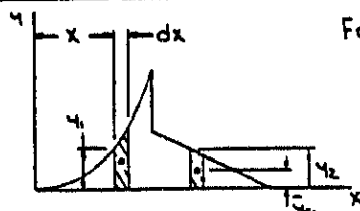
$\bar{x}A = \int \bar{x}_{EL} dA: \bar{x}(\frac{13}{24} ab) = \frac{71}{240} a^2 b \quad \bar{x} = \frac{71}{130} a$

$\bar{y}A = \int \bar{y}_{EL} dA: \bar{y}(\frac{13}{24} ab) = \frac{11}{40} ab^2 \quad \bar{y} = \frac{11}{26} b$   
 OR  $\bar{x} = 0.546a$   
 OR  $\bar{y} = 0.423b$

# 5.50



(GIVEN: PLANE AREA SHOWN)  
 FIND:  $\bar{x}$  AND  $\bar{y}$   
 USING DIRECT INTEGRATION



FOR  $y_1$  AT  $x=a$ ,  $y=2b$   
 $2b = ka^2$  OR  $k = \frac{2b}{a^2}$

THEN..  $y_1 = \frac{2b}{a^2} x^2$   
 BY OBSERVATION  
 $y_2 = -\frac{b}{a} x + 2b$   
 $= b(2 - \frac{x}{a})$

(CONTINUED)

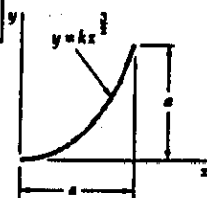




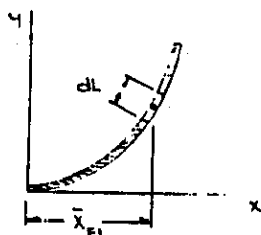
### 5.53 CONTINUED

Now..  $\bar{x}_{EL} = x$   
 AND  $dL = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \left\{1 + \left(a^{1/3} - x^{1/3}\right)^2 (-x^{-1/3})^2\right\}^{1/2} dx$   
 $= \frac{a^{1/3}}{x^{1/3}} dx$   
 THEN..  $L = \int dL = \int_0^a \frac{a^{1/3}}{x^{1/3}} dx = a^{1/3} \left[\frac{3}{2} x^{2/3}\right]_0^a = \frac{3}{2} a$   
 AND..  $\int \bar{x}_{EL} dL = \int_0^a x \left(\frac{a^{1/3}}{x^{1/3}} dx\right) = a^{1/3} \left[\frac{3}{5} x^{5/3}\right]_0^a = \frac{3}{5} a^2$   
 $\bar{x} L = \int \bar{x}_{EL} dL \quad \bar{x} \left(\frac{3}{2} a\right) = \frac{3}{5} a^2 \quad \bar{x} = \frac{2}{5} a \quad \blacktriangleleft$

### \* 5.54



GIVEN: HOMOGENEOUS WIRE SHOWN  
 FIND:  $\bar{x}$  USING DIRECT INTEGRATION



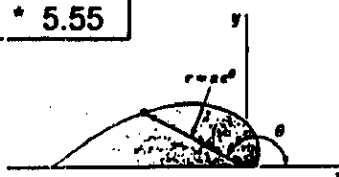
FIRST NOTE THAT BECAUSE THE WIRE IS HOMOGENEOUS, ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING LINE.

HAVE AT  $x=a, y=a$   
 $a = k a^2$  OR  $k = \frac{1}{a}$   
 THEN  $y = \frac{1}{a} x^2$   
 AND  $\frac{dy}{dx} = \frac{2}{a} x^{1/2}$

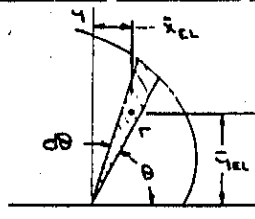
Now..  $\bar{x}_{EL} = x$   
 AND  $dL = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \left[1 + \left(\frac{2}{a} x^{1/2}\right)^2\right]^{1/2} dx$   
 $= \frac{1}{2a} \sqrt{4a + 9x} dx$   
 THEN..  $L = \int dL = \int_0^a \frac{1}{2a} \sqrt{4a + 9x} dx$   
 $= \frac{1}{2a} \left[\frac{2}{9} (4a + 9x)^{3/2}\right]_0^a = \frac{a}{27} [(13)^{3/2} - 8]$   
 $= 1.43971a$   
 AND..  $\int \bar{x}_{EL} dL = \int_0^a x \left[\frac{1}{2a} \sqrt{4a + 9x} dx\right]$

USE INTEGRATION BY PARTS WITH  
 $u = x \quad dv = \sqrt{4a + 9x} dx$   
 $du = dx \quad v = \frac{2}{27} (4a + 9x)^{3/2}$   
 THEN..  $\int \bar{x}_{EL} dL = \frac{1}{2a} \left\{ x \cdot \frac{2}{27} (4a + 9x)^{3/2} \right\}_0^a$   
 $= \frac{(13)^{3/2}}{27} a^2 - \frac{1}{27a} \left[ \frac{2}{45} (4a + 9x)^{5/2} \right]_0^a$   
 $= \frac{a^2}{27} \left\{ (13)^{3/2} - \frac{2}{45} [(13)^{5/2} - 32] \right\}$   
 $= 0.78566a^2$   
 $\bar{x} L = \int \bar{x}_{EL} dL \quad \bar{x} (1.43971a) = 0.78566a^2$   
 OR  $\bar{x} = 0.546a \quad \blacktriangleleft$

### \* 5.55



GIVEN: PLANE AREA SHOWN  
 FIND:  $\bar{x}$  AND  $\bar{y}$  USING DIRECT INTEGRATION



HAVE..  $\bar{x}_{EL} = \frac{2}{3} r \cos \theta$   
 $= \frac{2}{3} a r \cos \theta$   
 $\bar{y}_{EL} = \frac{2}{3} r \sin \theta$   
 $= \frac{2}{3} a r \sin \theta$

AND  $dA = \frac{1}{2} r \cdot r d\theta$   
 $= \frac{1}{2} a^2 d\theta$

THEN..  $A = \int dA = \int_0^{\pi/2} \frac{1}{2} a^2 d\theta = \frac{1}{2} a^2 \left[\theta\right]_0^{\pi/2} = \frac{1}{2} a^2 \left(\frac{\pi}{2} - 0\right) = \frac{\pi}{4} a^2$   
 $= 133.623 a^2$

AND  $\int \bar{x}_{EL} dA = \int_0^{\pi/2} \frac{2}{3} a r \cos \theta \left(\frac{1}{2} a^2 d\theta\right)$   
 $= \frac{1}{3} a^3 \int_0^{\pi/2} \cos \theta d\theta$

USE INTEGRATION BY PARTS WITH  
 $u = r \cos \theta \quad dv = \cos \theta d\theta$

$du = -r \sin \theta d\theta \quad v = \sin \theta$   
 THEN..  $\int r \cos \theta d\theta = r \sin \theta - \int \sin \theta (r \cos \theta d\theta)$

NOW LET  $u = r \cos \theta \quad dv = \sin \theta d\theta$   
 $du = -r \sin \theta d\theta \quad v = -\cos \theta$   
 THEN..  $\int r \cos \theta d\theta = r \sin \theta - 3 \int r \cos \theta d\theta$   
 $= r \sin \theta + 3 \cos \theta (r \cos \theta d\theta)$

SO THAT  $\int r \cos \theta d\theta = \frac{r}{10} (\sin \theta + 3 \cos \theta)$

$\therefore \int \bar{x}_{EL} dA = \frac{1}{3} a^3 \left[ \frac{r}{10} (\sin \theta + 3 \cos \theta) \right]_0^{\pi/2}$   
 $= \frac{a^3}{30} (3 - 3) = -1239.26 a^3$

ALSO..  $\int \bar{y}_{EL} dA = \int_0^{\pi/2} \frac{2}{3} a r \sin \theta \left(\frac{1}{2} a^2 d\theta\right)$   
 $= \frac{1}{3} a^3 \int_0^{\pi/2} \sin \theta d\theta$

USE INTEGRATION BY PARTS WITH  
 $u = r \sin \theta \quad dv = \sin \theta d\theta$

$du = r \cos \theta d\theta \quad v = -\cos \theta$   
 THEN..  $\int r \sin \theta d\theta = -r \cos \theta - \int (-\cos \theta) (r \cos \theta d\theta)$

NOW LET  $u = r \cos \theta \quad dv = \cos \theta d\theta$   
 $du = -r \sin \theta d\theta \quad v = \sin \theta$   
 THEN..  $\int r \sin \theta d\theta = -r \cos \theta + 3 \int r \sin \theta d\theta$   
 $= -r \cos \theta + 3 \sin \theta (r \cos \theta d\theta)$

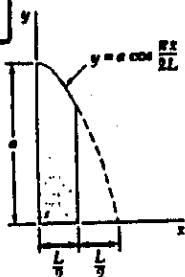
SO THAT  $\int r \sin \theta d\theta = \frac{r}{10} (-\cos \theta + 3 \sin \theta)$

$\therefore \int \bar{y}_{EL} dA = \frac{1}{3} a^3 \left[ \frac{r}{10} (-\cos \theta + 3 \sin \theta) \right]_0^{\pi/2}$   
 $= \frac{a^3}{30} (2 + 1) = 413.09 a^3$

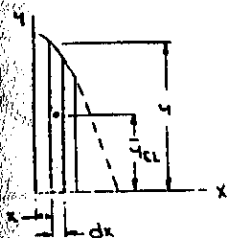
$\bar{x} A = \int \bar{x}_{EL} dA \quad \bar{x} (133.623 a^2) = -1239.26 a^3$   
 OR  $\bar{x} = -9.27a \quad \blacktriangleleft$

$\bar{y} A = \int \bar{y}_{EL} dA \quad \bar{y} (133.623 a^2) = 413.09 a^3$   
 OR  $\bar{y} = 3.09a \quad \blacktriangleleft$

5.56



GIVEN: PLANE AREA SHOWN  
FIND:  $\bar{x}$  AND  $\bar{y}$  USING DIRECT INTEGRATION



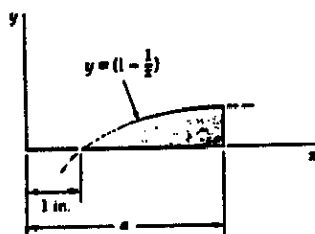
HAVE --  $\bar{x}_{EL} = x$   
 $\bar{y}_{EL} = \frac{1}{2}y = \frac{a}{2} \cos \frac{\pi x}{2L}$   
 AND --  $dA = y dx = a \cos \frac{\pi x}{2L} dx$   
 THEN --  $A = \int dA = \int_0^{L/2} a \cos \frac{\pi x}{2L} dx$   
 $= a \left[ \frac{2L}{\pi} \sin \frac{\pi x}{2L} \right]_0^{L/2}$   
 $= \frac{\sqrt{2}}{\pi} a L$

AND  $\int \bar{x}_{EL} dA = \int x (a \cos \frac{\pi x}{2L} dx)$   
 USE INTEGRATION BY PARTS WITH  
 $u = x$   $dv = \cos \frac{\pi x}{2L} dx$   
 $du = dx$   $v = \frac{2L}{\pi} \sin \frac{\pi x}{2L}$   
 THEN --  $\int x \cos \frac{\pi x}{2L} dx = \frac{2L}{\pi} x \sin \frac{\pi x}{2L} - \int \frac{2L}{\pi} \sin \frac{\pi x}{2L} dx$   
 $= \frac{2L}{\pi} \left( x \sin \frac{\pi x}{2L} + \frac{2L}{\pi} \cos \frac{\pi x}{2L} \right)$   
 $\therefore \int \bar{x}_{EL} dA = a \frac{2L}{\pi} \left[ x \sin \frac{\pi x}{2L} + \frac{2L}{\pi} \cos \frac{\pi x}{2L} \right]_0^{L/2}$   
 $= a \frac{2L}{\pi} \left[ \left( \frac{1}{\sqrt{2}} + \frac{\pi}{2} L \right) - \frac{2L}{\pi} \right]$   
 $= 0.106374 a L^2$

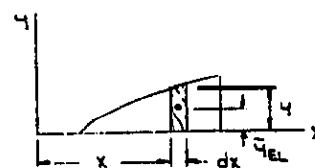
ALSO --  $\int \bar{y}_{EL} dA = \int_0^{L/2} \frac{a}{2} \cos \frac{\pi x}{2L} (a \cos \frac{\pi x}{2L} dx)$   
 $= \frac{a^2}{2} \left[ \frac{x}{2} + \frac{\sin \frac{\pi x}{L}}{2\pi} \right]_0^{L/2} = \frac{a^2}{2} \left( \frac{1}{4} + \frac{1}{2\pi} \right)$   
 $= 0.20458 a^2 L$

$\bar{x}A = \int \bar{x}_{EL} dA$   $\bar{x} \left( \frac{\sqrt{2}}{\pi} a L \right) = 0.106374 a L^2$   
 OR  $\bar{x} = 0.236 L$   
 $\bar{y}A = \int \bar{y}_{EL} dA$   $\bar{y} \left( \frac{\sqrt{2}}{\pi} a L \right) = 0.20458 a^2 L$   
 OR  $\bar{y} = 0.454 a$

5.57 and 5.58



GIVEN: PLANE AREA SHOWN  
FIND:  $\bar{x}$  AND  $\bar{y}$



HAVE --  $\bar{x}_{EL} = x$   
 $\bar{y}_{EL} = \frac{1}{2}y = \frac{1}{2} \left( 1 - \frac{x}{2} \right)$   
 AND  $dA = y dx = \left( 1 - \frac{x}{2} \right) dx$   
 (CONTINUED)

5.57 and 5.58 CONTINUED

THEN --  $A = \int dA = \int_0^2 \left( 1 - \frac{x}{2} \right) dx = \left[ x - \frac{x^2}{4} \right]_0^2$   
 $= (2 - \frac{4}{4}) = 1 \text{ in}^2$   
 AND  $\int \bar{x}_{EL} dA = \int_0^2 x \left( 1 - \frac{x}{2} \right) dx = \left[ \frac{x^2}{2} - \frac{x^3}{6} \right]_0^2$   
 $= \left( \frac{4}{2} - \frac{8}{6} \right) = \frac{2}{3} \text{ in}^3$   
 $\int \bar{y}_{EL} dA = \int_0^2 \frac{1}{2} \left( 1 - \frac{x}{2} \right) \left( 1 - \frac{x}{2} \right) dx = \frac{1}{2} \int_0^2 \left( 1 - \frac{x}{2} + \frac{x^2}{4} \right) dx$   
 $= \frac{1}{2} \left[ x - \frac{x^2}{4} + \frac{x^3}{12} \right]_0^2 = \frac{1}{2} \left( 2 - \frac{4}{4} + \frac{8}{12} \right) = \frac{1}{2} \left( 2 - 1 + \frac{2}{3} \right) = \frac{1}{2} \left( 1 + \frac{2}{3} \right) = \frac{5}{6} \text{ in}^3$   
 $\bar{x}A = \int \bar{x}_{EL} dA$   $\bar{x} = \frac{\frac{2}{3}}{1} = \frac{2}{3} \text{ in}$   
 $\bar{y}A = \int \bar{y}_{EL} dA$   $\bar{y} = \frac{\frac{5}{6}}{1} = \frac{5}{6} \text{ in}$

5.57 FIND:  $\bar{x}$  AND  $\bar{y}$  WHEN  $a = 2 \text{ in}$ .

HAVE --  $\bar{x} = \frac{\frac{2}{3}(2)^2 - 2 \cdot \frac{2}{3}}{2 - \ln 2 - 1}$  OR  $\bar{x} = 1.629 \text{ in}$

AND  $\bar{y} = \frac{2 - 2 \ln 2 - \frac{2}{3}}{2(2 - \ln 2 - 1)}$  OR  $\bar{y} = 0.1853 \text{ in}$

5.58 FIND:  $a$  SO THAT  $\frac{\bar{x}}{\bar{y}} = 9$

HAVE --  $\frac{\bar{x}}{\bar{y}} = \frac{\bar{x}A}{\bar{y}A} = \frac{\int \bar{x}_{EL} dA}{\int \bar{y}_{EL} dA}$

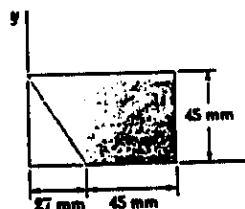
THEN --  $\frac{\frac{1}{2}a^2 - a + \frac{1}{2}}{\frac{1}{2}(a - 2 \ln a - \frac{a}{2})} = 9$

OR  $a^3 - 11a^2 + a + 18a \ln a + 9 = 0$

USING TRIAL AND ERROR OR NUMERICAL METHODS AND IGNORING THE TRIVIAL SOLUTION  $a = 1 \text{ in}$ , FIND --

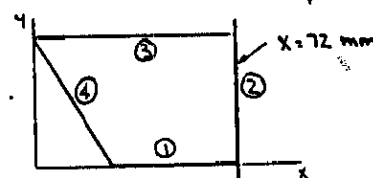
$a = 1.901 \text{ in}$  AND  $a = 3.74 \text{ in}$

5.59



GIVEN: PLANE AREA SHOWN  
FIND: VOLUME AND SURFACE AREA OF SOLID OBTAINED BY ROTATING THE AREA ABOUT  
(a) THE X AXIS  
(b) THE LINE  $x = 72 \text{ mm}$

FROM THE SOLUTION TO PROBLEM 5.1 HAVE  
 $A = 2632.5 \text{ mm}^2$   $\sum \bar{x}A = 111172.5 \text{ mm}^3$   
 $\sum \bar{y}A = 63787.5 \text{ mm}^3$



APPLYING THE THEOREMS OF PAPPUS-GULBINUS HAVE --

(a) ROTATION ABOUT THE X AXIS:

VOLUME =  $2\pi \bar{y} A = 2\pi (\sum \bar{y}A) = 2\pi (63787.5 \text{ mm}^3)$   
 OR VOLUME =  $401 \times 10^3 \text{ mm}^3$

AREA =  $2\pi \bar{y}_{LINE} L = 2\pi (\sum \bar{y}_{LINE} L)$   
 (CONTINUED)

# 5.59 CONTINUED

$$\begin{aligned} \text{AREA} &= 2\pi(\bar{q}_2 L_2 + \bar{q}_3 L_3 + \bar{q}_4 L_4) \\ &= 2\pi[(22.5)(45) + (45)(72) + (22.5)(\sqrt{27^2 + 45^2})] \\ &\quad \text{OR AREA} = 34.1 \times 10^3 \text{ mm}^2 \end{aligned}$$

(b) ROTATION ABOUT THE LINE  $x = 72 \text{ mm}$ :

$$\begin{aligned} \text{VOLUME} &= 2\pi(72 - \bar{x}_{\text{AREA}})A = 2\pi(72A - \sum \bar{x}A) \\ &= 2\pi[(72 \text{ mm})(2632.5 \text{ mm}^2) - (111)(172.5 \text{ mm}^3)] \\ &\quad \text{OR VOLUME} = 492 \times 10^3 \text{ mm}^3 \end{aligned}$$

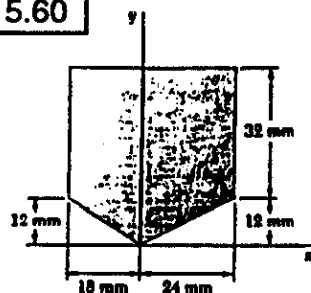
$$\begin{aligned} \text{AREA} &= 2\pi \bar{x}_{\text{LINE}} L = 2\pi(\sum \bar{x}_{\text{LINE}})L \\ &= 2\pi(\bar{x}_1 L_1 + \bar{x}_3 L_3 + \bar{x}_4 L_4) \end{aligned}$$

WHERE  $\bar{x}_1$ ,  $\bar{x}_3$ , AND  $\bar{x}_4$  ARE MEASURED WITH RESPECT TO THE LINE  $x = 72 \text{ mm}$ . THEN

$$\begin{aligned} \text{AREA} &= 2\pi[(22.5)(45) + (36)(72) \\ &\quad + (\frac{45+72}{2})(\sqrt{27^2 + 45^2})] \end{aligned}$$

$$\text{OR AREA} = 41.9 \times 10^3 \text{ mm}^2$$

# 5.60



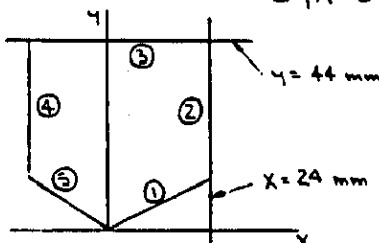
GIVEN: PLANE AREA SHOWN

FIND: VOLUME AND SURFACE AREA OF SOLID OBTAINED BY ROTATING THE AREA ABOUT

- (a) THE LINE  $y = 44 \text{ mm}$   
(b) THE LINE  $x = 24 \text{ mm}$

FROM THE SOLUTION TO PROBLEM 5.5 HAVE  
 $A = 1596 \text{ mm}^2$

$$\begin{aligned} \sum \bar{x}A &= 4536 \text{ mm}^3 \\ \sum \bar{y}A &= 39648 \text{ mm}^3 \end{aligned}$$



APPLYING THE THEOREMS OF PAPPUS-GULBINUS HAVE-

(a) ROTATION ABOUT THE LINE  $y = 44 \text{ mm}$ :

$$\begin{aligned} \text{VOLUME} &= 2\pi(44 - \bar{y}_{\text{AREA}})A = 2\pi(44A - \sum \bar{y}A) \\ &= 2\pi[(44 \text{ mm})(1596 \text{ mm}^2) - (39648 \text{ mm}^3)] \\ &\quad \text{OR VOLUME} = 192.1 \times 10^3 \text{ mm}^3 \end{aligned}$$

$$\begin{aligned} \text{AREA} &= 2\pi \bar{y}_{\text{LINE}} L = 2\pi(\sum \bar{y}_{\text{LINE}})L \\ &= 2\pi(\bar{y}_1 L_1 + \bar{y}_2 L_2 + \bar{y}_4 L_4 + \bar{y}_5 L_5) \end{aligned}$$

WHERE  $\bar{y}_1, \dots, \bar{y}_5$  ARE MEASURED WITH RESPECT TO THE LINE  $y = 44 \text{ mm}$ . THEN--

$$\begin{aligned} \text{AREA} &= 2\pi[(38)(\sqrt{24^2 + 12^2}) + (16)(32) + (16)(32) \\ &\quad + (38)(\sqrt{18^2 + 12^2})] \\ &\quad \text{OR AREA} = 18.01 \times 10^3 \text{ mm}^2 \end{aligned}$$

(b) ROTATION ABOUT THE LINE  $x = 24 \text{ mm}$ :

$$\begin{aligned} \text{VOLUME} &= 2\pi(24 - \bar{x}_{\text{AREA}})A = 2\pi(24A - \sum \bar{x}A) \\ &= 2\pi[(24 \text{ mm})(1596 \text{ mm}^2) - (4536 \text{ mm}^3)] \\ &\quad \text{OR VOLUME} = 212 \times 10^3 \text{ mm}^3 \end{aligned}$$

$$\begin{aligned} \text{AREA} &= 2\pi \bar{x}_{\text{LINE}} L = 2\pi(\sum \bar{x}_{\text{LINE}})L \\ &= 2\pi(\bar{x}_1 L_1 + \bar{x}_3 L_3 + \bar{x}_4 L_4 + \bar{x}_5 L_5) \end{aligned}$$

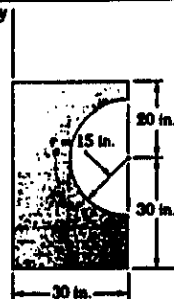
WHERE  $\bar{x}_1, \dots, \bar{x}_5$  ARE MEASURED WITH RESPECT (CONTINUED)

# 5.60 CONTINUED

TO THE LINE  $x = 24 \text{ mm}$ . THEN--

$$\begin{aligned} \text{AREA} &= 2\pi[(12)(\sqrt{24^2 + 12^2}) + (21)(42) + (42)(32) \\ &\quad + (33)(\sqrt{18^2 + 12^2})] \\ &\quad \text{OR AREA} = 20.5 \times 10^3 \text{ mm}^2 \end{aligned}$$

# 5.61



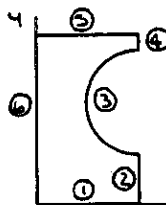
GIVEN: PLANE AREA SHOWN

FIND: VOLUME AND SURFACE AREA OF SOLID OBTAINED BY ROTATING THE AREA ABOUT

- (a) THE  $x$  AXIS  
(b) THE  $y$  AXIS

FROM THE SOLUTION TO PROBLEM 5.7 HAVE

$$\begin{aligned} A &= 1146.57 \text{ in}^2 \\ \sum \bar{x}A &= 14147.0 \text{ in}^3 \\ \sum \bar{y}A &= 26897 \text{ in}^3 \end{aligned}$$



APPLYING THE THEOREMS OF PAPPUS-GULBINUS HAVE--

(a) ROTATION ABOUT THE  $x$  AXIS:

$$\begin{aligned} \text{VOLUME} &= 2\pi \bar{y}_{\text{AREA}} A = 2\pi \sum \bar{y}A = 2\pi(26897 \text{ in}^3) \\ &\quad \text{OR VOLUME} = 169.0 \times 10^3 \text{ in}^3 \end{aligned}$$

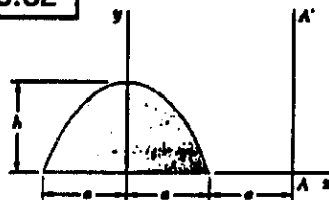
$$\begin{aligned} \text{AREA} &= 2\pi \bar{y}_{\text{LINE}} L = 2\pi(\sum \bar{y}_{\text{LINE}})L \\ &= 2\pi(\bar{y}_2 L_2 + \bar{y}_3 L_3 + \bar{y}_4 L_4 + \bar{y}_5 L_5 + \bar{y}_6 L_6) \\ &= 2\pi[(7.5)(15) + (30)(\pi(15)) + (47.5)(5) \\ &\quad + (50)(30) + (25)(50)] \\ &\quad \text{OR AREA} = 22.4 \times 10^3 \text{ in}^2 \end{aligned}$$

(b) ROTATION ABOUT THE  $y$  AXIS:

$$\begin{aligned} \text{VOLUME} &= 2\pi \bar{x}_{\text{AREA}} A = 2\pi \sum \bar{x}A = 2\pi(14147.0 \text{ in}^3) \\ &\quad \text{OR VOLUME} = 88.9 \times 10^3 \text{ in}^3 \end{aligned}$$

$$\begin{aligned} \text{AREA} &= 2\pi \bar{x}_{\text{LINE}} L = 2\pi(\sum \bar{x}_{\text{LINE}})L \\ &= 2\pi(\bar{x}_1 L_1 + \bar{x}_2 L_2 + \bar{x}_3 L_3 + \bar{x}_4 L_4 + \bar{x}_5 L_5) \\ &= 2\pi[(15)(30) + (30)(15) + (30 - \frac{2(15)}{\pi})(\pi(15)) \\ &\quad + (30)(5) + (15)(30)] \\ &\quad \text{OR AREA} = 15.48 \times 10^3 \text{ in}^2 \end{aligned}$$

# 5.62



GIVEN: PLANE PARABOLIC AREA SHOWN

FIND: VOLUME OF SOLID OBTAINED BY ROTATING THE AREA ABOUT

- (a) THE  $x$  AXIS  
(b) THE LINE  $AA'$

FIRST, FROM FIG. 5.8A HAVE..  $A = \frac{4}{3}ah$   
 $\bar{y} = \frac{3}{5}h$

APPLYING THE SECOND THEOREM OF PAPPUS-GULBINUS HAVE..

(a) ROTATION ABOUT THE  $x$  AXIS:

(CONTINUED)

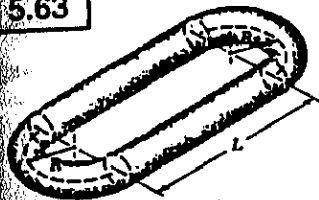
## 5.62 CONTINUED

$$\text{VOLUME} = 2\pi \bar{y} A = 2\pi \left(\frac{2}{3}h\right) \left(\frac{4}{3}ah\right) \\ \text{OR VOLUME} = \frac{16}{15}\pi a h^2$$

(b) ROTATION ABOUT THE LINE AA':

$$\text{VOLUME} = 2\pi (2a) A = 2\pi (2a) \left(\frac{4}{3}ah\right) \\ \text{OR VOLUME} = \frac{16}{3}\pi a^2 h$$

## 5.63



(GIVEN:  $d = 6 \text{ mm}$ ,  $R = 10 \text{ mm}$ ,  $L = 30 \text{ mm}$ )  
 FIND: VOLUME  $V$  AND SURFACE AREA  $A_s$  OF THE LINK

FIRST NOTE THAT THE AREA  $A$  AND THE CIRCUMFERENCE  $C$  OF THE CROSS SECTION OF THE BAR ARE

$$A = \frac{\pi}{4} d^2 \quad C = \pi d$$

OBSERVING THAT THE SEMICIRCULAR ENDS OF THE LINK CAN BE OBTAINED BY ROTATING THE CROSS SECTION THROUGH A HORIZONTAL SEMICIRCULAR ARC OF RADIUS  $R$ . THEN, APPLYING THE THEOREMS OF PAPPUS-GULBINUS HAVE..

$$\text{VOLUME: } V = 2(V_{\text{SIDE}}) + 2(V_{\text{END}}) \\ = 2(AL) + 2(\pi RA) = 2(L + \pi R)A \\ = 2[(30 \text{ mm}) + \pi(10 \text{ mm})] \left[\frac{\pi}{4}(6 \text{ mm})^2\right] \\ \text{OR } V = 3470 \text{ mm}^3$$

$$\text{AREA: } A_s = 2(A_{\text{SIDE}}) + 2(A_{\text{END}}) \\ = 2(CL) + 2(\pi RC) = 2(L + \pi R)C \\ = 2[(30 \text{ mm}) + \pi(10 \text{ mm})] [\pi(6 \text{ mm})] \\ \text{OR } A_s = 2320 \text{ mm}^2$$

## 5.64

(GIVEN: FIRST FOUR SHAPES OF FIG. 5.21)  
 FIND: VOLUME OF EACH SHAPE

FOLLOWING THE SECOND THEOREM OF PAPPUS-GULBINUS, IN EACH CASE A SPECIFIC GENERATING AREA  $A$  WILL BE ROTATED ABOUT THE  $x$  AXIS TO PRODUCE THE GIVEN SHAPE. VALUES OF  $\bar{y}$  ARE FROM FIG. 5.8A.

(1) HEMISPHERE: THE GENERATING AREA IS A QUARTER CIRCLE

$$\bar{y} = \frac{4a}{3\pi} \quad \text{HAVE.. } V = 2\pi \bar{y} A = 2\pi \left(\frac{4a}{3\pi}\right) \left(\frac{\pi}{4} a^2\right) \\ \text{OR } V = \frac{2}{3}\pi a^3$$

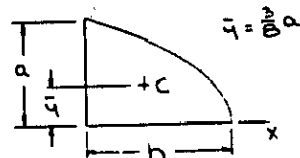
(2) SEMIELLIPSOID OF REVOLUTION: THE GENERATING AREA IS A QUARTER ELLIPSE

$$\bar{y} = \frac{4a}{3\pi} \quad \text{HAVE.. } V = 2\pi \bar{y} A \\ = 2\pi \left(\frac{4a}{3\pi}\right) \left(\frac{\pi}{4} ha\right) \\ \text{OR } V = \frac{2}{3}\pi a^2 h$$

(CONTINUED)

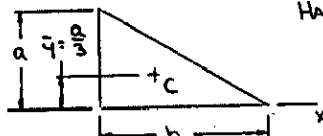
## 5.64 CONTINUED

(3) PARABOLOID OF REVOLUTION: THE GENERATING AREA IS A QUARTER PARABOLA



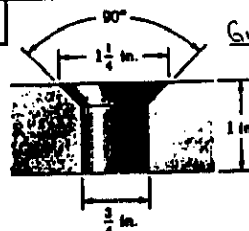
$$\text{HAVE.. } V = 2\pi \bar{y} A \\ = 2\pi \left(\frac{3}{8}a\right) \left(\frac{2}{3}ah\right) \\ \text{OR } V = \frac{1}{2}\pi a^2 h$$

(4) CONE: THE GENERATING AREA IS A TRIANGLE



$$\text{HAVE.. } V = 2\pi \bar{y} A \\ = 2\pi \left(\frac{a}{3}\right) \left(\frac{1}{2}ha\right) \\ \text{OR } V = \frac{1}{3}\pi a^2 h$$

## 5.65



(GIVEN: COUNTERSUNK HOLE SHOWN)

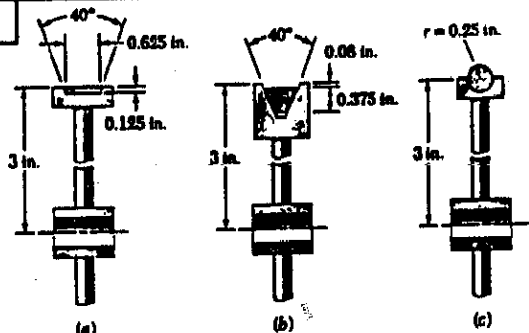
FIND: VOLUME OF STEEL REMOVED DURING COUNTERSINKING PROCESS

THE REQUIRED VOLUME CAN BE GENERATED BY ROTATING THE AREA SHOWN ABOUT THE  $y$  AXIS. APPLYING THE SECOND THEOREM OF PAPPUS-GULBINUS HAVE..

$$V = 2\pi \bar{x} A \\ = 2\pi \left[\frac{3}{8} + \frac{1}{4}\left(\frac{1}{4}\right)\right] \text{IN.} \cdot \left[\frac{1}{2} \times \frac{1}{4} \text{IN.} \times \frac{1}{4} \text{IN.}\right] \\ \text{OR } V = 0.0900 \text{ IN}^3$$

ALL DIMENSIONS ARE IN INCHES

## 5.66



(GIVEN: THREE DRIVE BELT PROFILES, EACH BELT CONTACTS ONE-HALF OF THE CIRCUMFERENCE OF ITS PULLEY)

FIND: CONTACT AREA BETWEEN EACH BELT AND ITS PULLEY

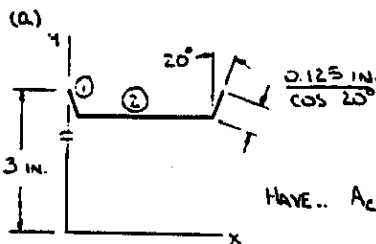
APPLYING THE FIRST THEOREM OF PAPPUS-GULBINUS, THE CONTACT AREA  $A_c$  OF A BELT (CONTINUED)

# 5.66 CONTINUED

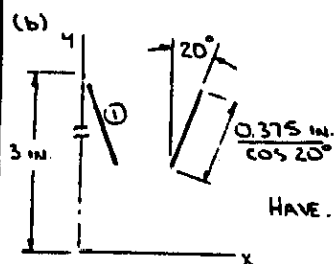
IS GIVEN BY

$$A_c = \pi \bar{Y} L + \pi \sum \bar{q} L$$

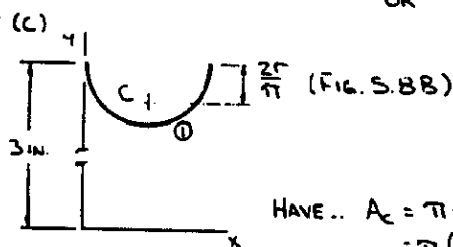
WHERE THE INDIVIDUAL LENGTHS ARE THE LENGTHS OF THE BELT CROSS SECTION WHICH ARE IN CONTACT WITH THE PULLEY.



HAVE..  $A_c = \pi [2(\bar{q}_1 L_1) + \bar{q}_2 L_2]$   
 $= \pi [2(3 - \frac{0.125}{2})(\frac{0.125}{\cos 20^\circ}) + (3 - 0.125)(0.625)]$   
 OR  $A_c = 8.10 \text{ in}^2$

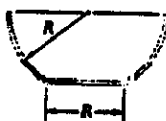


HAVE..  $A_c = \pi [2(\bar{q}_1 L_1)]$   
 $= 2\pi (3 - 0.08 - \frac{0.375}{2}) (\frac{0.375}{\cos 20^\circ})$   
 OR  $A_c = 6.85 \text{ in}^2$



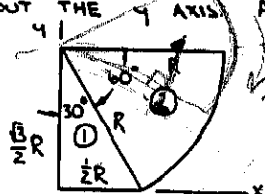
HAVE..  $A_c = \pi (\bar{q}_1 L_1)$   
 $= \pi (3 - \frac{0.25}{2}) (\pi \cdot 0.25)$   
 OR  $A_c = 7.01 \text{ in}^2$

# 5.67



GIVEN: BOWL SHOWN,  $R = 250 \text{ mm}$   
 FIND: VOLUME  $V$  IN LITERS

THE VOLUME CAN BE GENERATED BY ROTATING THE TRIANGLE AND CIRCULAR SECTOR SHOWN ABOUT THE  $Y$  AXIS. APPLYING THE SECOND THEOREM OF PAPPUS-GULBINUS AND USING FIG. S.8A, HAVE..



(CONTINUED)

# 5.67 CONTINUED

$$V = 2\pi \bar{X} A = 2\pi \sum \bar{X} A = 2\pi (\bar{X}_1 A_1 + \bar{X}_2 A_2)$$

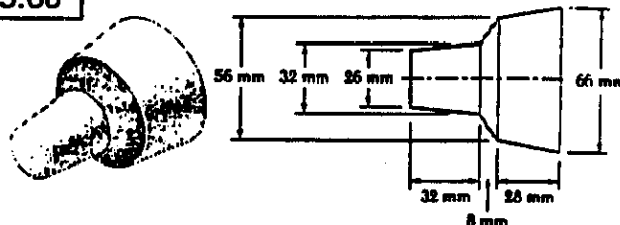
$$= 2\pi [(\frac{1}{3} \times \frac{1}{2} R)(\frac{1}{2} \times \frac{1}{2} R)(\frac{\sqrt{3}}{2} R) + (\frac{2R \sin 30^\circ}{3} \cos 30^\circ)(\frac{\pi}{6} R^2)]$$

$$= 2\pi (\frac{R^3}{16\sqrt{3}} + \frac{R^3}{2\sqrt{3}}) = \frac{3\sqrt{3}}{8} \pi R^3$$

$$= \frac{3\sqrt{3}}{8} \pi (0.25 \text{ m})^3 = 0.031883 \text{ m}^3 = \frac{10^3}{1 \text{ m}^3}$$

OR  $V = 31.9 \text{ l}$

# 5.68



GIVEN: LAMP SHADE SHOWN, DENSITY  $\rho = 2800 \frac{\text{kg}}{\text{m}^3}$ ,  
 THICKNESS  $t = 1 \text{ mm}$   
 FIND: MASS  $m$

THE MASS OF THE SHADE IS GIVEN BY  
 $m = \rho V = \rho A t$   
 WHERE  $A$  IS THE SURFACE AREA OF THE SHADE. THIS AREA CAN BE GENERATED BY ROTATING THE LINE SHOWN ABOUT THE  $X$  AXIS. APPLYING THE FIRST THEOREM OF PAPPUS-GULBINUS HAVE..

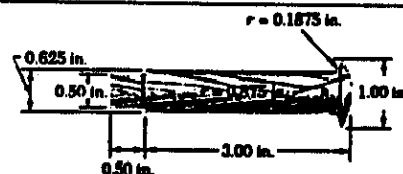
$$A = 2\pi \bar{Y} L = 2\pi \sum \bar{q} L = 2\pi (\bar{q}_1 L_1 + \bar{q}_2 L_2 + \bar{q}_3 L_3 + \bar{q}_4 L_4)$$

$$= 2\pi [(\frac{13}{2})(13) + (\frac{13+16}{2})(\sqrt{32^2+3^2}) + (\frac{16+28}{2})(\sqrt{8^2+12^2}) + (\frac{28+33}{2})(\sqrt{28^2+5^2})]$$

$$= 2\pi (1735.33 \text{ mm}^2)$$

THEN..  $m = 2800 \frac{\text{kg}}{\text{m}^3} \times [2\pi (1735.33 \text{ mm}^2)] \times 1 \text{ mm} = \frac{1 \text{ m}^3}{10^9 \text{ mm}^3}$   
 OR  $m = 0.0305 \text{ kg}$

# 5.69



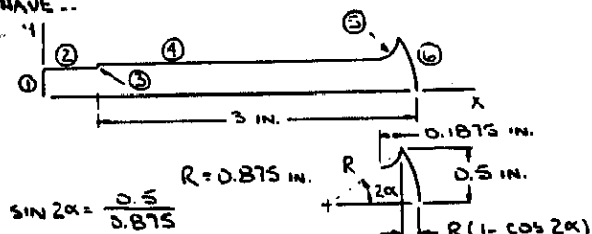
GIVEN: 20,000 PEGS HAVING SHAPE SHOWN, 2 COATS OF PAINT, 1 GALLON PAINT / 100  $\text{ft}^2$   
 FIND: NUMBER OF GALLONS NEEDED

THE NUMBER OF GALLONS OF PAINT NEEDED IS GIVEN BY  
 NUMBER OF GALLONS = (NUMBER OF PEGS) / (SURFACE AREA OF 1 PEG) (2 COATS)  
 (1 GALLON / 100  $\text{ft}^2$ )

(CONTINUED)

## 5.69 CONTINUED

OR NUMBER OF GALLONS =  $400 A_5$  ( $A_5 = ft^2$ )  
 WHERE  $A_5$  IS THE SURFACE AREA OF ONE PEG.  
 $A_5$  CAN BE GENERATED BY ROTATING THE LINE  
 SHOWN ABOUT THE X AXIS. USING THE FIRST  
 THEOREM OF PAPPUS-GULDINUS AND FIG. 5.8B,  
 HAVE..



$$A_5 = 2\pi \bar{Y}L = 2\pi \sum \bar{y}L$$

L, IN	$\bar{y}$ , IN	$\bar{y}L$ , IN <sup>2</sup>
1 0.25	0.125	0.03125
2 0.5	0.25	0.125
3 0.0625	$0.25 + 0.3125 = 0.5625$	0.035156
4 $3 - 0.875(1 - \cos 34.850^\circ)$ $= 0.1975 = 2.6556$	0.3125	0.061719
5 $\frac{1}{2} \cdot 0.1875 = 0.09375$	$0.5 - \frac{2 \cdot 0.1875}{\pi} = 0.38063$	0.035613
6 $2\alpha(0.875)$	$\frac{0.875 \sin 17.425^\circ}{\alpha} = 0.137314$	0.0127314

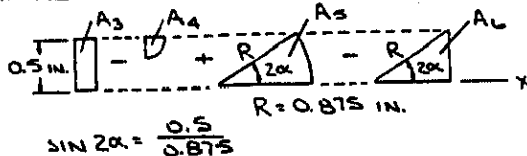
$$\sum \bar{y}L = 1.25312 \text{ IN}^2$$

THEN..  $A_5 = 2\pi(1.25312 \text{ IN}^2) = \frac{144}{144 \text{ IN}^2} = 0.054678 \text{ IN}^2$

FINALLY.. NUMBER OF GALLONS =  $400 \times 0.054678$   
 $= 21.87$  GALLONS  
 $\therefore$  ORDER 22 GALLONS

## 5.70 CONTINUED

COMPONENTS AS INDICATED.



APPLYING THE SECOND THEOREM OF PAPPUS-GULDINUS AND THEN USING FIG. 5.8A, HAVE..

$$V_{\text{PEG}} = 2\pi \bar{Y}A = 2\pi \sum \bar{y}A$$

A, IN <sup>2</sup>	$\bar{y}$ , IN	$\bar{y}A$ , IN <sup>3</sup>
1 $0.5 \times 0.25 = 0.125$	0.125	0.015625
2 $[3 - 0.875(1 - \cos 34.850^\circ) - 0.1875]$ $= (0.3125) = 0.082987$	0.15625	0.012967
3 $0.1875 \times 0.5 = 0.09375$	0.25	0.023438
4 $\frac{\pi}{4}(0.1875)^2 = 0.027612$	$0.5 - \frac{2 \times 0.1875}{\pi} = 0.42042$	0.011609
5 $\alpha(0.875)^2$	$\frac{2 \times 0.875 \sin 17.425^\circ}{2\alpha} = 0.04005$	0.00005
6 $\frac{1}{2}(0.875 \cos 34.850^\circ)(0.5)$ $= -0.179517$	$\frac{1}{2}(0.5) = 0.166667$	-0.029920

$$\sum \bar{y}A = 0.167252 \text{ IN}^3$$

$$\text{THEN.. } V_{\text{PEG}} = 2\pi(0.167252 \text{ IN}^3) = 1.05088 \text{ IN}^3$$

$$\text{NOW.. } V_{\text{DOWEL}} = \frac{\pi}{4}(\text{DIAMETER})^2(\text{LENGTH}) = \frac{\pi}{4}(1 \text{ IN.})^2(4 \text{ IN.})$$

$$= 3.14159 \text{ IN}^3$$

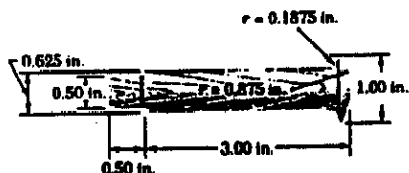
$$\text{THEN.. } \% \text{ WASTE} = \frac{V_{\text{WASTE}}}{V_{\text{DOWEL}}} \times 100\%$$

$$= \frac{V_{\text{DOWEL}} - V_{\text{PEG}}}{V_{\text{DOWEL}}} \times 100\%$$

$$= \left(1 - \frac{1.05088}{3.14159}\right) \times 100\%$$

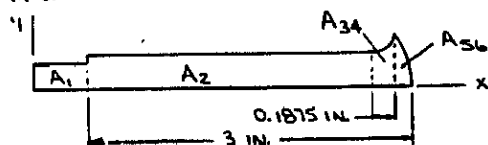
$$\text{OR } \% \text{ WASTE} = 66.5\%$$

## 5.70



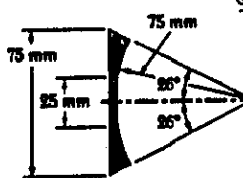
GIVEN: PEG HAVING THE SHAPE SHOWN, INITIAL  
 SIZE OF DOWEL.. 1 IN. DIA.  $\times$  4 IN. LONG  
 FIND: PERCENT (VOLUME) OF DOWEL THAT  
 BECOMES WASTE

TO OBTAIN THE SOLUTION IT IS FIRST  
 NECESSARY TO DETERMINE THE VOLUME OF  
 THE PEG. THAT VOLUME CAN BE GENERATED  
 BY ROTATING THE AREA SHOWN ABOUT THE  
 X AXIS.



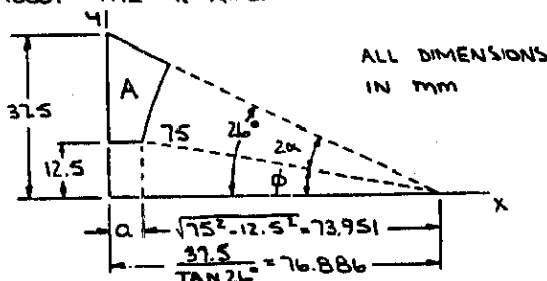
THE GENERATING AREA IS NEXT DIVIDED INTO SIX  
 (CONTINUED)

## 5.71



GIVEN: BRASS PLATE,  
 DENSITY  $\rho = 8470 \text{ kg/m}^3$   
 FIND: MASS  $m$

THE MASS OF THE ESCUTCHEON IS GIVEN BY  
 $m = \rho V$   
 WHERE  $V$  IS THE VOLUME OF THE PLATE.  $V$  CAN  
 BE GENERATED BY ROTATING THE AREA  $A$   
 ABOUT THE X AXIS.



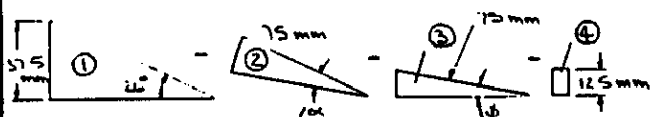
(CONTINUED)

### 5.71 CONTINUED

HAVE...  $u = 76.886 - 73.951 = 2.935 \text{ mm}$   
 AND...  $\sin \phi = \frac{12.5}{75} \Rightarrow \phi = 9.5941^\circ$

THEN  $2\alpha = 26^\circ - 9.5941^\circ = 16.4059^\circ$   
 AND  $\alpha = 8.2030^\circ$

THE AREA A CAN BE OBTAINED BY COMBINING THE FOLLOWING FOUR AREAS AS INDICATED.



APPLYING THE SECOND THEOREM OF PAPPUS-GULDINUS AND THEN USING FIG. 5.8A, HAVE...

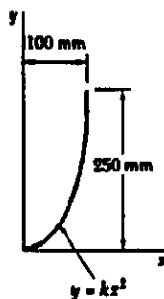
$$V = 2\pi \bar{Y}A = 2\pi \sum \bar{Y}A$$

A, mm <sup>2</sup>	$\bar{Y}$ , mm	$\bar{Y}A$ , mm <sup>3</sup>
1 $\frac{1}{2} \times 76.886 \times 37.5$ = 1441.61	$\frac{1}{3}(37.5) = 12.5$	18 020.13
2 $-\frac{1}{2}(75)^2$	$\frac{2(75)\sin(8.203^\circ)}{3\alpha} \times \sin(8.203^\circ + 9.5941^\circ)$	-12 265.30
3 $-\frac{1}{2} \times 73.951 \times 12.5$ = -462.19	$\frac{1}{3}(12.5) = 4.1667$	-1925.81
4 $-2.935 \times 12.5$ = -36.688	$\frac{1}{2}(12.5) = 6.25$	-229.30

$$\sum \bar{Y}A = 3599.72 \text{ mm}^3$$

THEN  $V = 2\pi(3599.72 \text{ mm}^3) = 22 617.7 \text{ mm}^3$   
 SO THAT  $m = 8470 \frac{\text{kg}}{\text{m}^3} \times 22 617.7 \times 10^{-9} \text{ m}^3$   
 $= 0.1916 \text{ kg}$   
 OR  $m = 191.6 \text{ g}$

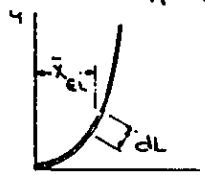
### 5.72



GIVEN: SHADE SHOWN  
 FIND: OUTER SURFACE AREA

FIRST NOTE THAT THE REQUIRED SURFACE AREA A CAN BE GENERATED BY ROTATING THE PARABOLIC CROSS SECTION THROUGH  $\pi$  RADIANS ABOUT THE Y AXIS. APPLYING THE FIRST THEOREM OF PAPPUS-GULDINUS HAVE

$$A = \pi \bar{X}L$$



NOW... AT  $x = 100 \text{ mm}$ ,  $y = 250 \text{ mm}$   
 $250 = k(100)^2$   
 OR  $k = 0.025 \text{ mm}^{-1}$

AND  $\bar{x}_{EL} = x$   
 $dL = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$   
 WHERE  $\frac{dy}{dx} = 2kx$

THEN...  $dL = \sqrt{1 + 4k^2x^2} dx$   
 HAVE...  $\bar{x}L = \int \bar{x}_{EL} dL = \int_0^{100} x(\sqrt{1 + 4k^2x^2}) dx$   
 (CONTINUED)

### 5.72 CONTINUED

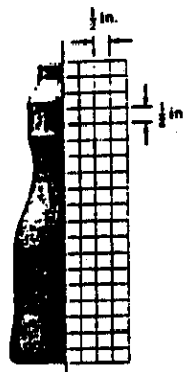
$$\bar{x}L = \left[ \frac{1}{3} \frac{1}{4k^2} (1 + 4k^2x^2)^{3/2} \right]_0^{100}$$

$$= \frac{1}{12} \frac{1}{(0.025)^2} \left\{ [1 + 4(0.025)^2(100)^2]^{3/2} - (1)^{3/2} \right\}$$

$$= 17 543.3 \text{ mm}^2$$

FINALLY...  $A = \pi(17 543.3 \text{ mm}^2)$   
 OR  $A = 55.1 \times 10^3 \text{ mm}^2$

### 5.73



GIVEN: BOTTLE OF CROSS SECTION SHOWN,  
 $W = 0.131 \text{ lb}$ ,  
 SPECIFIC WEIGHT  
 $\gamma = 59.0 \text{ lb/ft}^3$

FIND: AVERAGE WALL THICKNESS  $t$

THE WEIGHT OF THE BOTTLE IS GIVEN BY  
 $W = \gamma V = \gamma A_s t$   
 WHERE  $A_s$  IS THE SURFACE AREA OF THE BOTTLE.  $A_s$  CAN BE GENERATED BY ROTATING THE CURVE BOUNDING THE CROSS SECTION ABOUT THE VERTICAL AXIS OF SYMMETRY. APPROXIMATING THE PORTION OF THIS CURVE TO THE RIGHT OF THE VERTICAL AXIS WITH A SERIES OF SHORT, STRAIGHT LINE SEGMENTS AND THEN APPROXIMATING THE LENGTH AND THE VALUE OF  $\bar{x}$  FOR EACH SEGMENT USING THE GIVEN GRID,  $A_s$  IS THEN DETERMINED USING THE FIRST THEOREM OF PAPPUS-GULDINUS.

$$A_s = 2\pi \bar{x}L = 2\pi \sum \bar{x}L$$

WITH THE ELEVEN SEGMENTS NUMBERED STARTING AT THE TOP, HAVE...

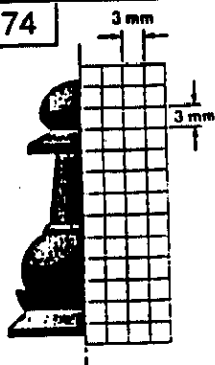


	L, IN.	$\bar{x}$ , IN.	$\bar{x}L$ , IN <sup>2</sup>
1	0.76	0.38	0.2888
2	0.48	0.76	0.3648
3	0.88	0.98	0.8624
4	1.06	1.20	1.272
5	0.36	1.08	0.3888
6	1.12	0.98	1.0976
7	1.78	1.32	2.3496
8	2.50	1.66	4.15
9	1.12	1.74	1.9488
10	0.48	1.68	0.8064
11	1.56	0.78	1.2168
$\Sigma$			14.7460

THEN...  $A_s = 2\pi(14.7460 \text{ IN}^2) = 92.652 \text{ IN}^2$

FINALLY...  $0.131 \text{ lb} = 59.0 \frac{\text{lb}}{\text{ft}^3} \times 92.652 \text{ IN}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ IN}}\right)^3 t$   
 OR  $t = 0.0414 \text{ IN.}$

5.74



GIVEN: PAWN OF CROSS SECTION SHOWN, DENSITY  $\rho = 7310 \text{ kg/m}^3$   
 FIND: MASS  $m$

THE MASS OF THE PAWN  $m$ , GIVEN BY  
 $m = \rho V$   
 WHERE  $V$  IS THE VOLUME OF THE PAWN.  $V$  CAN BE GENERATED BY ROTATING THE CROSS-SECTIONAL AREA OF THE PAWN ABOUT THE VERTICAL AXIS OF SYMMETRY. APPROXIMATING THIS AREA WITH A TRIANGLE AND A SERIES OF RECTANGLES AND TRAPEZOIDS AND APPROXIMATING THE DIMENSIONS OF THESE ELEMENTS USING THE GIVEN GRID,  $V$  IS THEN DETERMINED USING THE SECOND THEOREM OF PAPPUS-GULBINUS.  
 $V = 2\pi \bar{x} A = 2\pi \sum \bar{x} A$

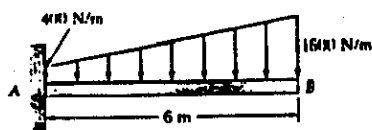
WITH THE AREAS TAKEN STARTING AT THE TOP, HAVE..

	A, mm <sup>2</sup>	$\bar{x}$ , mm	$\bar{x}A$ , mm <sup>3</sup>
1	$\frac{1}{2} \cdot 3 \cdot 1.5 = 2.25$	1.0	2.25
2	$\frac{1}{2} \cdot 3 \cdot (3.9) = 5.85$	1.75	10.24
3	$\frac{1}{2} \cdot 3 \cdot (3.9 + 3.6) = 6.75$	3.4	22.95
4	$\frac{1}{2} \cdot 3 \cdot (2.1) = 3.15$	3.3	10.26
5	$\frac{1}{2} \cdot 3 \cdot (3.6 - 2.55) = 1.485$	3.0	4.455
6	$\frac{1}{2} \cdot 3 \cdot (2.25 + 5.15) = 10.44$	3.5	36.54
7	$\frac{1}{2} \cdot 3 \cdot (2.25 + 1.2) = 2.025$	4.1	8.3025
8	$\frac{1}{2} \cdot 3 \cdot (1.55 + 2.55) = 6.075$	2.5	15.1875
9	$\frac{1}{2} \cdot 3 \cdot (0.9) = 1.35$	3.1	4.185
10	$\frac{1}{2} \cdot 3 \cdot (3.15 + 5.85) = 14.175$	3.8	53.865
11	$\frac{1}{2} \cdot 3 \cdot (1.65 + 2.15) = 4.65$	6.7	31.155
12	$\frac{1}{2} \cdot 3 \cdot (2.25 + 2.85) = 7.425$	6.8	50.49
13	$\frac{1}{2} \cdot 3 \cdot (2.15 + 1.65) = 3.075$	6.7	20.6025
14	$\frac{1}{2} \cdot 3 \cdot (1.5) = 2.25$	7.2	16.2
15	$\frac{1}{2} \cdot 3 \cdot (3 + 1.95) = 6.975$	6.95	48.375
16	$\frac{1}{2} \cdot 3 \cdot (1.95 + 4.35) = 9.525$	7.35	69.975
17	$\frac{1}{2} \cdot 3 \cdot (4.35 + 1.2) = 8.25$	7.9	65.175
$\Sigma$	104.33		453.12

THEN..  $V = 2\pi (453.12 \text{ mm}^3) = 2847.0 \text{ mm}^3$

FINALLY..  $m = 7310 \frac{\text{kg}}{\text{m}^3} \cdot 2847.0 \cdot 10^{-9} \text{ m}^3$   
 OR  $m = 0.0208 \text{ kg}$

5.75

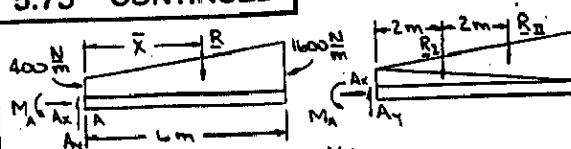


GIVEN: BEAM AND LOADING SHOWN

FIND: (a) RESULTANT  $R$   
 (b) REACTIONS AT A

(CONTINUED)

5.75 CONTINUED



(a) HAVE..  $R_1 = \frac{1}{2} (6 \text{ m}) (400 \frac{\text{N}}{\text{m}}) = 1200 \text{ N}$

$R_2 = \frac{1}{2} (6 \text{ m}) (1600 \frac{\text{N}}{\text{m}}) = 4800 \text{ N}$

THEN..  $\Sigma F_y = 0: -R = -R_1 - R_2$

OR  $R = 1200 + 4800 = 6000 \text{ N}$

AND  $\Sigma M_A = 0: -\bar{x} (6000) = -2(1200) - 4(4800)$

OR  $\bar{x} = 3.6 \text{ m}$

$\therefore R = 6000 \text{ N} \uparrow, \bar{x} = 3.6 \text{ m}$

(b) REACTIONS

$\Sigma F_x = 0: A_x = 0$

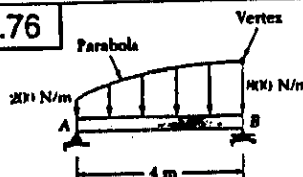
$\uparrow \Sigma F_y = 0: A_y - 6000 \text{ N} = 0 \quad A_y = 6000 \text{ N}$

$\therefore A = 6000 \text{ N} \uparrow$

$\Sigma M_A = 0: M_A - (3.6 \text{ m})(6000 \text{ N}) = 0$

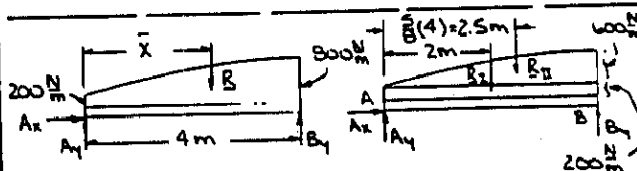
OR  $M_A = 21.6 \text{ kN}\cdot\text{m}$

5.76



GIVEN: BEAM AND LOADING SHOWN

FIND: (a) RESULTANT  $R$   
 (b) REACTIONS AT SUPPORTS



(a) HAVE..  $R_1 = (4 \text{ m}) (200 \frac{\text{N}}{\text{m}}) = 800 \text{ N}$

$R_2 = \frac{2}{3} (4 \text{ m}) (1600 \frac{\text{N}}{\text{m}}) = 1600 \text{ N}$

THEN..  $\Sigma F_y = 0: -R = -R_1 - R_2$

OR  $R = 800 + 1600 = 2400 \text{ N}$

AND  $\Sigma M_A = 0: -\bar{x} (2400) = -2(800) - 2.5(1600)$

OR  $\bar{x} = \frac{7}{3} \text{ m}$

$\therefore R = 2400 \text{ N} \uparrow, \bar{x} = 2.33 \text{ m}$

(b) REACTIONS

$\Sigma F_x = 0: A_x = 0$

$\Sigma M_A = 0: (4 \text{ m}) B_y - (\frac{7}{3} \text{ m}) (2400 \text{ N}) = 0$

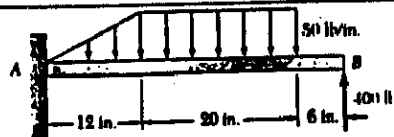
OR  $B_y = 1400 \text{ N}$

$\uparrow \Sigma F_y = 0: A_y + 1400 \text{ N} - 2400 \text{ N} = 0$

OR  $A_y = 1000 \text{ N}$

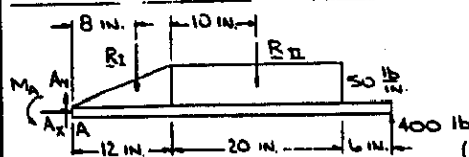
$\therefore A = 1000 \text{ N} \uparrow, B = 1400 \text{ N} \uparrow$

5.77



GIVEN: BEAM AND LOADING SHOWN

FIND: REACTIONS AT A



(CONTINUED)

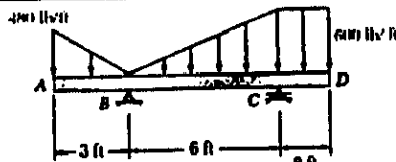


### 5.77 CONTINUED

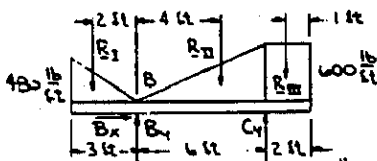
HAVE...  $R_I = \frac{1}{2}(12 \text{ in.})(50 \frac{\text{lb}}{\text{in.}}) = 300 \text{ lb}$   
 $R_{II} = (20 \text{ in.})(50 \frac{\text{lb}}{\text{in.}}) = 1000 \text{ lb}$

THEN...  $\sum F_x = 0: A_x = 0$   
 $\uparrow \sum F_y = 0: A_y - 300 \text{ lb} - 1000 \text{ lb} - 400 \text{ lb} = 0$   
 OR  $A_y = 900 \text{ lb}$   $A = 900 \text{ lb} \downarrow$   
 $\rightarrow \sum M_A = 0: M_A - (8 \text{ in.})(300 \text{ lb}) - (22 \text{ in.})(1000 \text{ lb}) - (2 \text{ in.})(400 \text{ lb}) = 0$   
 OR  $M_A = 9200 \text{ lb} \cdot \text{in.}$   
 $M_A = 9200 \text{ lb} \cdot \text{in.}$

### 5.78



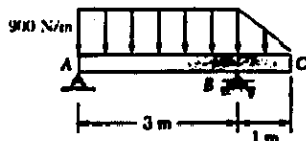
GIVEN: BEAM AND LOADING SHOWN  
 FIND: REACTIONS AT SUPPORTS



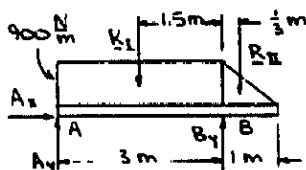
HAVE...  $R_I = \frac{1}{2}(3 \text{ ft})(400 \frac{\text{lb}}{\text{ft}}) = 720 \text{ lb}$   
 $R_{II} = \frac{1}{2}(6 \text{ ft})(400 \frac{\text{lb}}{\text{ft}}) = 1800 \text{ lb}$   
 $R_{III} = (2 \text{ ft})(400 \frac{\text{lb}}{\text{ft}}) = 1200 \text{ lb}$

THEN...  $\sum F_x = 0: B_x = 0$   
 $\rightarrow \sum M_B = 0: (2 \text{ ft})(720 \text{ lb}) - (4 \text{ ft})(1800 \text{ lb}) + (6 \text{ ft})(C_y) - (7 \text{ ft})(1200 \text{ lb}) = 0$   
 OR  $C_y = 2360 \text{ lb}$   $C = 2360 \text{ lb} \uparrow$   
 $\uparrow \sum F_y = 0: -720 \text{ lb} + B_y - 1800 \text{ lb} + 2360 \text{ lb} - 1200 \text{ lb} = 0$   
 OR  $B_y = 1360 \text{ lb}$   $B = 1360 \text{ lb} \uparrow$

### 5.79



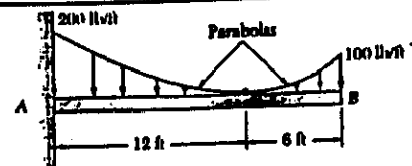
GIVEN: BEAM AND LOADING SHOWN  
 FIND: REACTIONS AT SUPPORTS



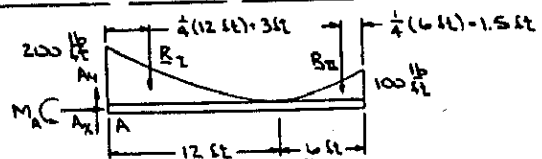
HAVE...  $R_I = (3 \text{ m})(900 \frac{\text{N}}{\text{m}}) = 2700 \text{ N}$   
 $R_{II} = \frac{1}{2}(1 \text{ m})(900 \frac{\text{N}}{\text{m}}) = 450 \text{ N}$

NOW...  $\sum F_x = 0: A_x = 0$   
 $\rightarrow \sum M_B = 0: -(3 \text{ m})A_y + (1.5 \text{ m})(2700 \text{ N}) - (\frac{1}{3} \text{ m})(450 \text{ N}) = 0$   
 OR  $A_y = 1300 \text{ N}$   $A = 1300 \text{ N} \uparrow$   
 $\uparrow \sum F_y = 0: 1300 \text{ N} - 2700 \text{ N} + B_y - 450 \text{ N} = 0$   
 OR  $B_y = 1850 \text{ N}$   $B = 1850 \text{ N} \uparrow$

### 5.80



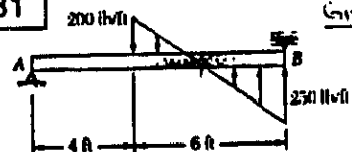
GIVEN: BEAM AND LOADING SHOWN  
 FIND: REACTIONS AT A



HAVE...  $R_I = \frac{1}{3}(12 \text{ ft})(200 \frac{\text{lb}}{\text{ft}}) = 800 \text{ lb}$   
 $R_{II} = \frac{1}{3}(6 \text{ ft})(100 \frac{\text{lb}}{\text{ft}}) = 200 \text{ lb}$

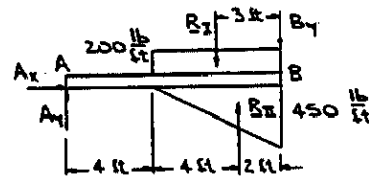
THEN...  $\sum F_x = 0: A_x = 0$   
 $\uparrow \sum F_y = 0: A_y - 800 \text{ lb} - 200 \text{ lb} = 0$   
 OR  $A_y = 1000 \text{ lb}$   $A = 1000 \text{ lb} \uparrow$   
 $\rightarrow \sum M_A = 0: M_A - (3 \text{ ft})(800 \text{ lb}) - (16.5 \text{ ft})(200 \text{ lb}) = 0$   
 OR  $M_A = 5700 \text{ lb} \cdot \text{ft}$   
 $M_A = 5700 \text{ lb} \cdot \text{ft}$

### 5.81



GIVEN: BEAM AND LOADING SHOWN  
 FIND: REACTIONS AT SUPPORTS

FIRST REPLACE THE GIVEN LOADING WITH THE LOADING SHOWN BELOW. THE TWO LOADINGS ARE EQUIVALENT BECAUSE BOTH ARE DEFINED BY A LINEAR RELATION BETWEEN LOAD AND DISTANCE AND THE VALUES AT THE END POINTS ARE THE SAME.



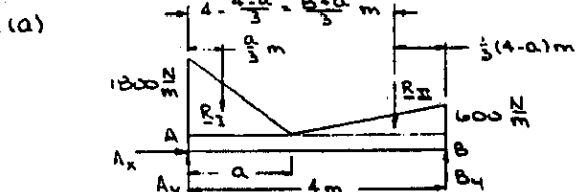
HAVE...  $R_I = (6 \text{ ft})(200 \frac{\text{lb}}{\text{ft}}) = 1200 \text{ lb}$   
 $R_{II} = \frac{1}{2}(6 \text{ ft})(450 \frac{\text{lb}}{\text{ft}}) = 1350 \text{ lb}$

THEN...  $\sum F_x = 0: A_x = 0$   
 $\rightarrow \sum M_B = 0: -(10 \text{ ft})A_y + (3 \text{ ft})(1200 \text{ lb}) - (2 \text{ ft})(1350 \text{ lb}) = 0$   
 OR  $A_y = 90 \text{ lb}$   $A = 90 \text{ lb} \uparrow$   
 $\uparrow \sum F_y = 0: 90 \text{ lb} - 1200 \text{ lb} + 1350 \text{ lb} - B_y = 0$   
 OR  $B_y = 240 \text{ lb}$   $B = 240 \text{ lb} \uparrow$

5.86

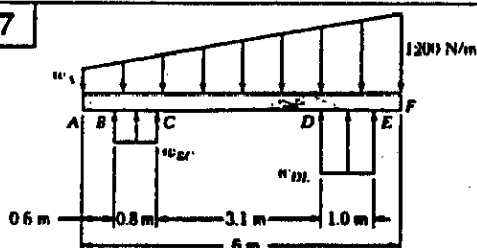


GIVEN: BEAM AND LOADING SHOWN  
 FIND: (a)  $a$  so that  $B_y$  is minimum  
 (b) REACTIONS AT SUPPORTS

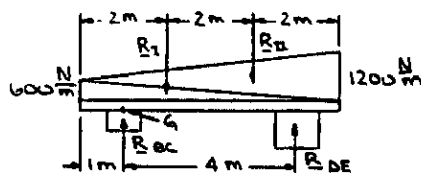


HAVE..  $R_1 = \frac{1}{2}(a \text{ m})(1800 \frac{\text{N}}{\text{m}}) = 900a \text{ N}$   
 $R_2 = \frac{1}{2}[(4-a) \text{ m}](600 \frac{\text{N}}{\text{m}}) = 300(4-a) \text{ N}$   
 THEN..  $\sum M_A = 0: -(\frac{a}{3} \text{ m})(900a \text{ N}) - (\frac{4-a}{3} \text{ m})(300(4-a) \text{ N}) + (4 \text{ m})B_y = 0$   
 OR  $B_y = 50a^2 - 100a + 800$  (1)  
 THEN..  $\frac{dB_y}{da} = 100a - 100 = 0$  OR  $a = 1.00 \text{ m}$   
 (b) Eq. (1)..  $B_y = 50(1)^2 - 100(1) + 800 = 750 \text{ N}$   
 AND..  $\sum F_x = 0: A_x = 0$   
 $\sum F_y = 0: A_y - 900(1) \text{ N} - 300(4-1) \text{ N} + 750 \text{ N} = 0$   
 OR  $A_y = 1050 \text{ N}$   $A = 1050 \text{ N} \uparrow$

5.87

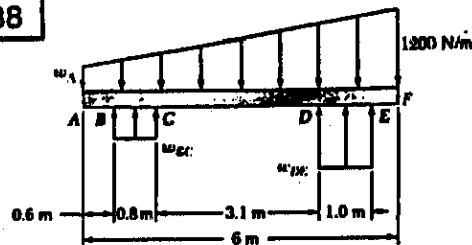


GIVEN: BEAM AND LOADING SHOWN,  $W_A = 600 \frac{\text{N}}{\text{m}}$   
 FIND:  $W_{BC}$  AND  $W_{DE}$

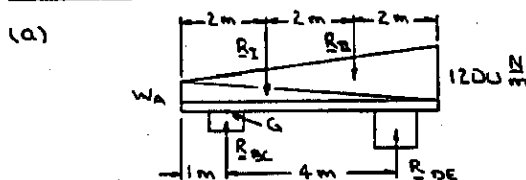


HAVE..  $R_1 = \frac{1}{2}(6 \text{ m})(600 \frac{\text{N}}{\text{m}}) + 1800 \text{ N}$   
 $R_2 = \frac{1}{2}(6 \text{ m})(1200 \frac{\text{N}}{\text{m}}) = 3600 \text{ N}$   
 $R_{BC} = (0.8 \text{ m})(W_{BC} \frac{\text{N}}{\text{m}}) = (0.8 W_{BC}) \text{ N}$   
 $R_{DE} = (1.0 \text{ m})(W_{DE} \frac{\text{N}}{\text{m}}) = (W_{DE}) \text{ N}$   
 THEN..  $\sum M_G = 0: -(1 \text{ m})(1800 \text{ N}) - (3 \text{ m})(3600 \text{ N}) + (4 \text{ m})(W_{DE} \text{ N}) = 0$   
 OR  $W_{DE} = 3150 \frac{\text{N}}{\text{m}}$   
 AND..  $\sum F_y = 0: (0.8 W_{BC}) \text{ N} - 1800 \text{ N} - 3600 \text{ N} + 3150 \text{ N} = 0$   
 OR  $W_{BC} = 2812.5 \frac{\text{N}}{\text{m}}$   
 $W_{BC} = 2810 \frac{\text{N}}{\text{m}}$

5.88

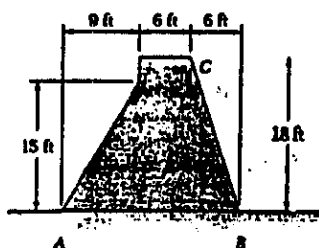


GIVEN: BEAM AND LOADING SHOWN  
 FIND: (a)  $W_A$  so that  $W_{BC} = W_{DE}$   
 (b)  $W_{BC}$  AND  $W_{DE}$

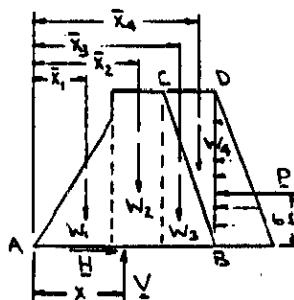


HAVE..  $R_1 = \frac{1}{2}(6 \text{ m})(W_A \frac{\text{N}}{\text{m}}) = (3 W_A) \text{ N}$   
 $R_2 = \frac{1}{2}(6 \text{ m})(1200 \frac{\text{N}}{\text{m}}) = 3600 \text{ N}$   
 $R_{BC} = (0.8 \text{ m})(W_{BC} \frac{\text{N}}{\text{m}}) = (0.8 W_{BC}) \text{ N}$   
 $R_{DE} = (1 \text{ m})(W_{DE} \frac{\text{N}}{\text{m}}) = (W_{DE}) \text{ N}$   
 THEN..  $\sum F_y = 0: (0.8 W_{BC}) \text{ N} - (3 W_A) \text{ N} - 3600 \text{ N} + (W_{DE}) \text{ N} = 0$   
 OR  $0.8 W_{BC} + W_{DE} = 3600 + 3 W_A$   
 NOW..  $W_{BC} = W_{DE} \Rightarrow W_{BC} = W_{DE} = 2000 + \frac{3}{2} W_A$  (1)  
 ALSO..  $\sum M_G = 0: -(1 \text{ m})(3 W_A \text{ N}) - (3 \text{ m})(3600 \text{ N}) + (4 \text{ m})(W_{DE} \text{ N}) = 0$   
 OR  $W_{DE} = 2700 + \frac{3}{2} W_A$  (2)  
 EQUATING EQS. (1) AND (2)..  
 $2000 + \frac{3}{2} W_A = 2700 + \frac{3}{2} W_A$   
 OR  $W_A = \frac{700 \text{ N}}{\text{m}}$   $W_A = 764 \frac{\text{N}}{\text{m}}$   
 (b) Eq. (1)  $\Rightarrow W_{BC} = W_{DE} = 2000 + \frac{3}{2}(\frac{700 \text{ N}}{\text{m}})$   
 OR  $W_{BC} = W_{DE} = 3270 \frac{\text{N}}{\text{m}}$

5.89



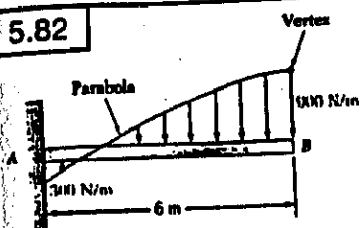
GIVEN: DAM CROSS SECTION SHOWN, WIDTH = 1 ft  
 FIND: (a) REACTION FORCES EXERTED ON BASE OF DAM  
 (b) POINT OF APPLICATION OF REACTION FORCES  
 (c) RESULTANT FORCE ON FACE OF DAM



THE FREE BODY SHOWN CONSISTS OF A 1-ft THICK SECTION OF THE DAM AND THE TRIANGULAR SECTION BCD OF WATER ABOVE THE DAM.

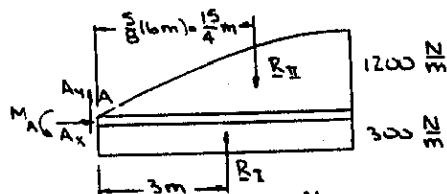
NOTE:  $\bar{x}_1 = 6 \text{ ft}$   
 $\bar{x}_2 = (9+3) \text{ ft} = 12 \text{ ft}$   
 $\bar{x}_3 = (15+2) \text{ ft} = 17 \text{ ft}$   
 $\bar{x}_4 = (15+4) \text{ ft} = 19 \text{ ft}$   
 (CONTINUED)

5.82



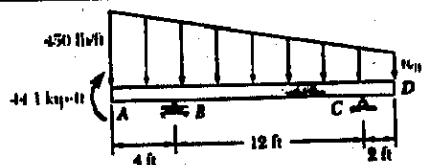
GIVEN: BEAM AND LOADING SHOWN  
FIND: REACTIONS AT A

FIRST REPLACE THE GIVEN LOADING WITH THE LOADING SHOWN BELOW. THE TWO LOADINGS ARE EQUIVALENT BECAUSE BOTH ARE DEFINED BY A PARABOLIC RELATION BETWEEN LOAD AND DISTANCE AND THE VALUES AT THE END POINTS ARE THE SAME.

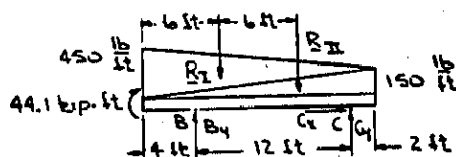


HAVE..  $R_I = (6\text{ m})(300 \frac{\text{N}}{\text{m}}) = 1800 \text{ N}$   
 $R_{II} = \frac{2}{3}(6\text{ m})(1200 \frac{\text{N}}{\text{m}}) = 4800 \text{ N}$   
 THEN..  $\sum F_x = 0: A_x = 0$   
 $\sum F_y = 0: A_y + 1800\text{ N} - 4800\text{ N} = 0$   
 OR  $A_y = 3000 \text{ N}$   $A = 3000 \text{ N}$   
 $\sum M_A = 0: M_A + (3\text{ m})(1800\text{ N}) - (\frac{15}{4}\text{ m})(4800\text{ N}) = 0$   
 OR  $M_A = 12.6 \text{ kN}\cdot\text{m}$   
 $M_A = 12.6 \text{ kN}\cdot\text{m}$

5.83



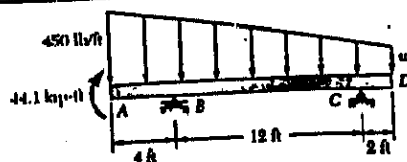
GIVEN: BEAM AND LOADING SHOWN,  $W_0 = 150 \frac{\text{lb}}{\text{ft}}$   
FIND: REACTIONS AT SUPPORTS



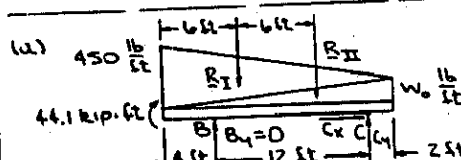
HAVE..  $R_I = \frac{1}{2}(18\text{ ft})(450 \frac{\text{lb}}{\text{ft}}) = 4050 \text{ lb}$   
 $R_{II} = \frac{1}{2}(18\text{ ft})(150 \frac{\text{lb}}{\text{ft}}) = 1350 \text{ lb}$

THEN..  $\sum F_x = 0: C_x = 0$   
 $\sum M_B = 0: -(44,100 \text{ kip}\cdot\text{ft}) - (2\text{ ft})(4050\text{ lb}) - (8\text{ ft})(1350\text{ lb}) + (12\text{ ft})C_y = 0$   
 OR  $C_y = 5250 \text{ lb}$   $C = 5250 \text{ lb}$   
 $\sum F_y = 0: B_y - 4050\text{ lb} - 1350\text{ lb} + 5250\text{ lb} = 0$   
 OR  $B_y = 150 \text{ lb}$   $B = 150 \text{ lb}$

5.84



GIVEN: BEAM AND LOADING SHOWN  
FIND: (a)  $W_0$  SO THAT  $B = 0$   
(b) REACTION AT C



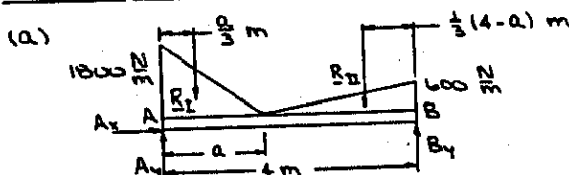
HAVE..  $R_I = \frac{1}{2}(18\text{ ft})(450 \frac{\text{lb}}{\text{ft}}) = 4050 \text{ lb}$   
 $R_{II} = \frac{1}{2}(18\text{ ft})(W_0 \frac{\text{lb}}{\text{ft}}) = 9W_0 \text{ lb}$   
 THEN..  $\sum M_C = 0: -(44,100 \text{ lb}\cdot\text{ft}) + (10\text{ ft})(4050\text{ lb}) + (4\text{ ft})(9W_0\text{ lb}) = 0$   
 OR  $W_0 = 100 \frac{\text{lb}}{\text{ft}}$

(b)  $\sum F_x = 0: C_x = 0$   
 $\sum F_y = 0: -4050\text{ lb} - (9 \times 100)\text{ lb} + C_y = 0$   
 OR  $C_y = 4950 \text{ lb}$   $C = 4950 \text{ lb}$

5.85



GIVEN: BEAM AND LOADING SHOWN  
FIND: (a)  $a$  SO THAT  $A_y = B_y$   
(b) REACTIONS AT SUPPORTS



HAVE..  $R_I = \frac{1}{2}(a\text{ m})(1800 \frac{\text{N}}{\text{m}}) = 900a \text{ N}$   
 $R_{II} = \frac{1}{2}[(4-a)\text{ m}](600 \frac{\text{N}}{\text{m}}) = 300(4-a) \text{ N}$

THEN..  $\sum F_y = 0: A_y - 900a - 300(4-a) + B_y = 0$   
 OR  $A_y + B_y = 1200 + 600a$

NOW  $A_y = B_y \Rightarrow A_y = B_y = 600 + 300a \text{ (N)}$  (1)

ALSO..  $\sum M_B = 0: -(4\text{ m})A_y + [(4-\frac{a}{2})\text{ m}][900a\text{ N}] + [\frac{1}{2}(4-a)\text{ m}][300(4-a)\text{ N}] = 0$   
 OR  $A_y = 400 + 700a - 50a^2$  (2)

EQUATING Eqs. (1) AND (2)

$600 + 300a = 400 + 700a - 50a^2$

OR  $a^2 - 8a + 4 = 0$

THEN..  $a = \frac{8 \pm \sqrt{64 - 4(1)(4)}}{2}$

OR  $a = 0.53590\text{ m}$   $a = 7.4641\text{ m}$

NOW  $a \leq 4\text{ m} \Rightarrow a = 0.536\text{ m}$

(b) HAVE..  $\sum F_x = 0: A_x = 0$   
 Eq. (1)..  $A_y = B_y = 600 + 300(0.53590)$   
 $= 761 \text{ N}$   
 $\therefore A = B = 761 \text{ N}$

# 5.89 CONTINUED

(a) Now...  $W = \gamma V$  SO THAT  
 $W_1 = (150 \frac{\text{lb}}{\text{ft}^3}) [\frac{1}{2} (9 \text{ ft}) (15 \text{ ft}) (1 \text{ ft})] = 10,125 \text{ lb}$   
 $W_2 = (150 \frac{\text{lb}}{\text{ft}^3}) [(6 \text{ ft}) (18 \text{ ft}) (1 \text{ ft})] = 16,200 \text{ lb}$   
 $W_3 = (150 \frac{\text{lb}}{\text{ft}^3}) [\frac{1}{2} (6 \text{ ft}) (18 \text{ ft}) (1 \text{ ft})] = 8100 \text{ lb}$   
 $W_4 = (62.4 \frac{\text{lb}}{\text{ft}^3}) [\frac{1}{2} (6 \text{ ft}) (18 \text{ ft}) (1 \text{ ft})] = 3369.6 \text{ lb}$   
 Also...  $P = \frac{1}{2} A p = \frac{1}{2} [(18 \text{ ft}) (1 \text{ ft})] [(62.4 \frac{\text{lb}}{\text{ft}^3}) (18 \text{ ft})]$   
 $= 10,108.8 \text{ lb}$

THEN...  $\sum F_x = 0: H - 10,108.8 \text{ lb} = 0$   
 OR  $H = 10.11 \text{ kips}$   
 $\sum F_y = 0: V - 10,125 \text{ lb} - 16,200 \text{ lb} - 8100 \text{ lb}$   
 $- 3369.6 \text{ lb} = 0$   
 OR  $V = 37,794.6 \text{ lb}$   $V = 37.8 \text{ kips}$

(b) HAVE...  $\sum M_A = 0: x (37,794.6 \text{ lb}) - (6 \text{ ft}) (10,125 \text{ lb})$   
 $- (12 \text{ ft}) (16,200 \text{ lb}) - (17 \text{ ft}) (8100 \text{ lb})$   
 $- (9 \text{ ft}) (3369.6 \text{ lb})$   
 $+ (6 \text{ ft}) (10,108.8 \text{ lb}) = 0$   
 OR...  $37,794.6 x - 60,750 - 194,400 - 137,700$   
 $- 64,022.4 + 60,652.8 = 0$   
 OR  $x = 10.48 \text{ ft}$

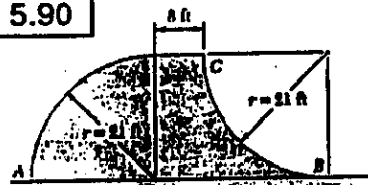
(c) CONSIDER WATER SECTION BCD AS THE FREE BODY

HAVE...  $\sum F = 0$   
 $P = 10,108.8 \text{ lb}$   
 $W_4 = 3369.6 \text{ lb}$   
 $\theta = 18.43^\circ$   
 THEN...  $-R = 10.66 \text{ kips}$   $\angle 18.43^\circ$   
 OR  $R = 10.66 \text{ kips}$   $\angle 18.43^\circ$

ALTERNATIVE SOLUTION

CONSIDER THE FACE BC OF THE DAM.  
 HAVE...  $BC = \sqrt{6^2 + 18^2} = 18.9737 \text{ ft}$   
 $\tan \theta = \frac{6}{18}$   $\theta = 18.43^\circ$   
 AND...  $P = \gamma h = 62.4 \frac{\text{lb}}{\text{ft}^3} \times 18 \text{ ft}$   
 $= 1123.2 \frac{\text{lb}}{\text{ft}^2}$   
 THEN...  $R = \frac{1}{2} A p$   
 $= \frac{1}{2} [(18.9737 \text{ ft}) (1 \text{ ft})]$   
 $\times (1123.2 \frac{\text{lb}}{\text{ft}^2})$   
 $= 10,655.6 \text{ lb}$   
 $\therefore R = 10.66 \text{ kips}$   $\angle 18.43^\circ$

# 5.90

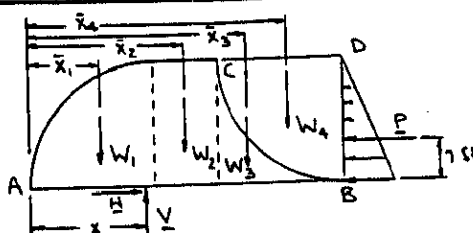


GIVEN: DAM CROSS SECTION SHOWN. WIDTH = 1 ft  
 FIND: (a) REACTION FORCES EXERTED ON BASE OF DAM

(b) POINT OF APPLICATION OF REACTION FORCES  
 (c) RESULTANT FORCE ON FACE OF DAM

THE FREE BODY SHOWN (TOP OF NEXT COLUMN) CONSISTS OF A 1-ft THICK SECTION OF THE DAM AND THE QUARTER CIRCULAR SECTION OF WATER ABOVE THE DAM. (CONTINUED)

# 5.90 CONTINUED



NOTE:  $\bar{x}_1 = (21 - \frac{4 \times 21}{3\pi}) \text{ ft} = 12.0873 \text{ ft}$   
 $\bar{x}_2 = (21 - 4) \text{ ft} = 25 \text{ ft}$   
 $\bar{x}_4 = (50 - \frac{4 \times 21}{3\pi}) \text{ ft} = 41.087 \text{ ft}$

FOR AREA 3 FIRST NOTE...



	A	$\bar{x}$
I	$r^2$	$\frac{1}{2} r$
II	$-\frac{\pi}{4} r^2$	$r - \frac{4r}{3\pi}$

THEN...  $\bar{x}_3 = 29 \text{ ft} + \left[ \frac{\frac{1}{2} (21)^2 + (21 - \frac{4 \times 21}{3\pi}) (-\frac{\pi}{4} \times 21^2)}{(21)^2 - \frac{\pi}{4} (21)^2} \right] \text{ ft}$   
 $= (29 + 4.6907) \text{ ft} = 33.691 \text{ ft}$

(a) NOW...  $W = \gamma V$  SO THAT  
 $W_1 = (150 \frac{\text{lb}}{\text{ft}^3}) [\frac{\pi}{4} (21 \text{ ft})^2 (1 \text{ ft})] = 51,954 \text{ lb}$   
 $W_2 = (150 \frac{\text{lb}}{\text{ft}^3}) [(8 \text{ ft}) (21 \text{ ft}) (1 \text{ ft})] = 25,200 \text{ lb}$   
 $W_3 = (150 \frac{\text{lb}}{\text{ft}^3}) [\frac{\pi}{4} (21^2 - 21^2) \text{ ft}^2 (1 \text{ ft})] = 14,196 \text{ lb}$   
 $W_4 = (62.4 \frac{\text{lb}}{\text{ft}^3}) [\frac{\pi}{4} (21 \text{ ft})^2 (1 \text{ ft})] = 21,613 \text{ lb}$   
 ALSO  $P = \frac{1}{2} A p = \frac{1}{2} [(21 \text{ ft}) (1 \text{ ft})] [(62.4 \frac{\text{lb}}{\text{ft}^3}) (21 \text{ ft})]$   
 $= 13,759 \text{ lb}$

THEN...  $\sum F_x = 0: H - 13,759 \text{ lb} = 0$   
 OR  $H = 13.76 \text{ kips}$   
 $\sum F_y = 0: V - 51,954 \text{ lb} - 25,200 \text{ lb} - 14,196 \text{ lb}$   
 $- 21,613 \text{ lb} = 0$   
 OR  $V = 112,963 \text{ lb}$   $V = 113.0 \text{ kips}$

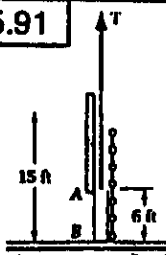
(b) HAVE...  $\sum M_A = 0: x (112,963 \text{ lb})$   
 $- (12.0873 \text{ ft}) (51,954 \text{ lb})$   
 $- (25 \text{ ft}) (25,200 \text{ lb})$   
 $- (33.691 \text{ ft}) (14,196 \text{ lb})$   
 $- (41.087 \text{ ft}) (21,613 \text{ lb})$   
 $+ (7 \text{ ft}) (13,759 \text{ lb}) = 0$   
 OR  $112,963 x - 627,980 - 630,000 - 478,280$   
 $- 888,010 + 96,313 = 0$   
 OR  $x = 22.4 \text{ ft}$

(c) CONSIDER WATER SECTION BCD AS THE FREE BODY

HAVE...  $\sum F = 0$   
 $P = 13,759 \text{ lb}$   
 $W_4 = 21,613 \text{ lb}$   
 $\theta = 57.5^\circ$   
 $-R$

THEN...  $-R = 25.6 \text{ kips}$   $\angle 57.5^\circ$   
 OR  $R = 25.6 \text{ kips}$   $\angle 57.5^\circ$

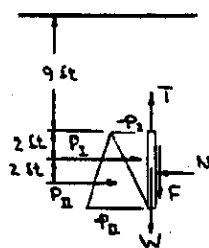
5.91



GIVEN: 6-ft GATE,  $W = 1000 \text{ lb}$   
 FRICTION FORCE  $F = 0.1$   
 = RESULTANT PRESSURE  
 FORCE  $P$ ,  $x = 6 + \frac{15}{3}$

FIND:  $T$

CONSIDER THE FREE-BODY DIAGRAM OF THE GATE. NOW..



$$P_1 = \frac{1}{2} A p_1 = \frac{1}{2} [(6 \text{ ft})(15 \text{ ft})] \left( (2.4 \frac{\text{lb}}{\text{ft}^3}) (9 \text{ ft}) \right)$$

$$= 10,108.8 \text{ lb}$$

$$P_2 = \frac{1}{2} A p_2 = \frac{1}{2} [(6 \text{ ft})(15 \text{ ft})] \left( (2.4 \frac{\text{lb}}{\text{ft}^3}) (15 \text{ ft}) \right)$$

$$= 16,848 \text{ lb}$$

$$\text{THEN.. } F = 0.1 P = 0.1 (P_1 + P_2)$$

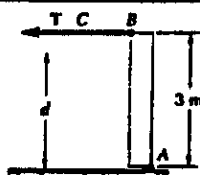
$$= 0.1 (10,108.8 + 16,848) \text{ lb}$$

$$= 2695.7 \text{ lb}$$

$$\text{FINALLY.. } +\sum F_y = 0: T - 2695.7 \text{ lb} - 1000 \text{ lb} = 0$$

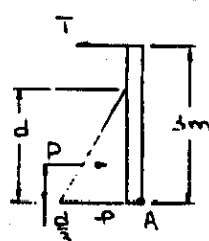
$$\text{OR } T = 3700 \text{ lb}$$

5.92



GIVEN: 3-4-m SIDE,  
 $T_{\text{max}} = 0.2 (200 \text{ kN})$   
 $p = 10^3 \frac{\text{kg}}{\text{m}^3}$   
 FIND:  $d_{\text{max}}$

CONSIDER THE FREE-BODY DIAGRAM OF THE SIDE.



$$\text{HAVE.. } P = \frac{1}{2} A p = \frac{1}{2} A (p g d)$$

$$\text{NOW.. } \sum M_A = 0: h T - \frac{d}{3} P = 0$$

$$\text{WHERE } h = 3 \text{ m}$$

$$\text{THEN FOR } d_{\text{max}}:$$

$$(3 \text{ m}) (0.2 \times 200 \times 10^3 \text{ N})$$

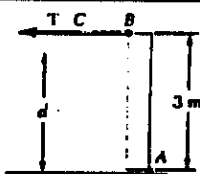
$$= \frac{d_{\text{max}}}{3} \left[ \frac{1}{2} (4 \text{ m}) d_{\text{max}} \right] (10^3 \frac{\text{kg}}{\text{m}^3}) (9.81 \frac{\text{m}}{\text{s}^2})$$

$$= 0$$

$$\text{OR } 120 \text{ N} \cdot \text{m} - 6.54 d_{\text{max}}^2 \frac{\text{N}}{\text{m}} = 0$$

$$\text{OR } d_{\text{max}} = 2.64 \text{ m}$$

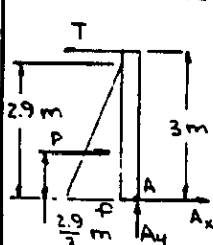
5.93



GIVEN: 3-4-m SIDE,  
 $p = 1263 \frac{\text{kg}}{\text{m}^3}$ ,  
 $d = 2.9 \text{ m}$

FIND:  $T$ , REACTION AT A

CONSIDER THE FREE-BODY DIAGRAM OF THE SIDE.



$$\text{HAVE.. } P = \frac{1}{2} A p = \frac{1}{2} A (p g d)$$

$$= \frac{1}{2} [(2.9 \text{ m})(4 \text{ m})]$$

$$= 208.40 \text{ kN}$$

$$\text{THEN.. } +\sum F_y = 0: A_y = 0$$

$$+\sum M_A = 0: (3 \text{ m}) T - (\frac{2.9}{3} \text{ m}) (208.40 \text{ kN}) = 0$$

$$\text{OR } T = 67.151 \text{ kN} \quad T = 67.2 \text{ kN}$$

$$+\sum F_x = 0: A_x + 208.40 \text{ kN} - 67.151 \text{ kN} = 0$$

$$\text{OR } A_x = -141.249 \text{ kN} \quad A_x = 141.2 \text{ kN}$$

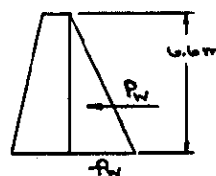
5.94



GIVEN:  $p_s = 1.76 \times 10^3 \frac{\text{kg}}{\text{m}^3}$ ,  
 WIDTH = 1 m,  
 $d_s = 2 \text{ m}$

FIND: PERCENTAGE  
 INCREASE OF FORCE  
 ON DAM FACE  
 BECAUSE OF SILT

FIRST DETERMINE THE FORCE ON THE DAM FACE WITHOUT THE SILT. HAVE..



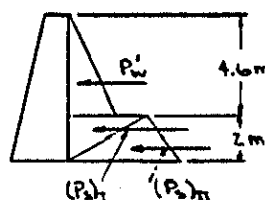
$$P_w = \frac{1}{2} A p_w = \frac{1}{2} A (p g h)$$

$$= \frac{1}{2} [(6.6 \text{ m})(1 \text{ m})]$$

$$= 103.79 \text{ kN}$$

$$= 213.66 \text{ kN}$$

NEXT DETERMINE THE FORCE ON THE DAM FACE WITH THE SILT. HAVE..



$$P'_w = \frac{1}{2} [(4.6 \text{ m})(1 \text{ m})]$$

$$= 103.79 \text{ kN}$$

$$(P_s)_1 = \frac{1}{2} [(2 \text{ m})(1 \text{ m})] [(1.76 \times 10^3 \frac{\text{kg}}{\text{m}^3}) (9.81 \frac{\text{m}}{\text{s}^2}) (4.6 \text{ m})]$$

$$= 79.42 \text{ kN}$$

$$(P_s)_2 = \frac{1}{2} [(2 \text{ m})(1 \text{ m})] [(1.76 \times 10^3 \frac{\text{kg}}{\text{m}^3}) (9.81 \frac{\text{m}}{\text{s}^2}) (6.6 \text{ m})]$$

$$= 113.95 \text{ kN}$$

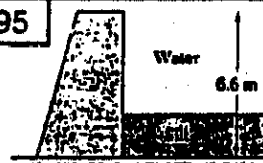
$$\text{THEN.. } P' = P'_w + (P_s)_1 + (P_s)_2 = 297.16 \text{ kN}$$

THE PERCENTAGE INCREASE % INC. IS THEN

$$\text{GIVEN BY.. } \% \text{ INC.} = \frac{P' - P_w}{P_w} \times 100\% = \frac{(297.16 - 213.66)}{213.66} \times 100\%$$

$$\text{OR } \% \text{ INC.} = 39.1\%$$

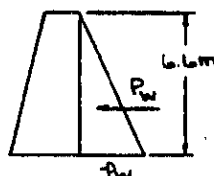
5.95



GIVEN:  $(F_{\text{BASE}})_{\text{MAX}} = 1.2 \times$   
 FORCE OF WATER,  
 $p_s = 1.76 \times 10^3 \frac{\text{kg}}{\text{m}^3}$ ,  
 WIDTH = 1 m,  
 RATE  $r_s$  AT WHICH  
 SILT IS DEPOSITED  
 $= 12 \text{ mm/YEAR}$

FIND: NUMBER OF YEARS  $N$  UNTIL DAM  
 BECOMES UNSAFE

FIRST DETERMINE THE FORCE ON THE DAM FACE BEFORE ANY SILT IS DEPOSITED. HAVE..



$$P_w = \frac{1}{2} A p_w = \frac{1}{2} A (p g h)$$

$$= \frac{1}{2} [(6.6 \text{ m})(1 \text{ m})]$$

$$= 213.66 \text{ kN}$$

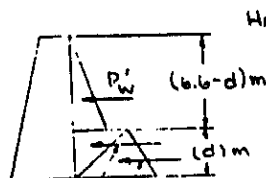
THE MAXIMUM ALLOWED FORCE  
 $P_{\text{ALLOW}}$  ON THE DAM IS THEN..

$$P_{\text{ALLOW}} = 1.2 P_w = 1.2 (213.66 \text{ kN}) = 256.39 \text{ kN}$$

NEXT DETERMINE THE FORCE  $P'$  ON THE DAM  
 FACE AFTER A DEPTH  $d$  OF SILT HAS SETTLED.

(CONTINUED)

# 5.95 CONTINUED



$(P_1)_1$   $(P_1)_2$

$$\begin{aligned} \text{HAVE...} \\ P_w &= \frac{1}{2}[(6.6-d)m \cdot (1m)] \cdot (10^3 \frac{\text{kg}}{\text{m}^3}) (9.81 \frac{\text{m}}{\text{s}^2}) (6.6-d)m \\ &= 4.905(6.6-d)^2 \text{ kN} \\ (P_1)_1 &= \frac{1}{2}[(d)m \cdot (1m)] \cdot (1.76 \cdot 10^3 \frac{\text{kg}}{\text{m}^3}) (9.81 \frac{\text{m}}{\text{s}^2}) \cdot (6.6-d)m \\ &= 8.6328(6.6-d)^2 \text{ kN} \\ (P_1)_2 &= \frac{1}{2}[(d)m \cdot (1m)] \cdot (1.76 \cdot 10^3 \frac{\text{kg}}{\text{m}^3}) (9.81 \frac{\text{m}}{\text{s}^2}) \cdot (6.6m) \\ &= 56.976d \text{ kN} \end{aligned}$$

$$\begin{aligned} \text{THEN } P' &= P_w + (P_1)_1 + (P_1)_2 \\ &= [4.905(6.6-d)^2 + 8.6328(6.6-d)^2 + 56.976d] \text{ kN} \\ &= (213.66 + 49.206d - 3.7278d^2) \text{ kN} \end{aligned}$$

NOW REQUIRE THAT  $P' = P_{\text{allow}}$  TO DETERMINE THE MAXIMUM VALUE OF  $d$ .

$$\begin{aligned} \therefore (213.66 + 49.206d - 3.7278d^2) \text{ kN} &= 256.39 \text{ kN} \\ \text{OR } 3.7278d^2 - 49.206d + 42.73 &= 0 \end{aligned}$$

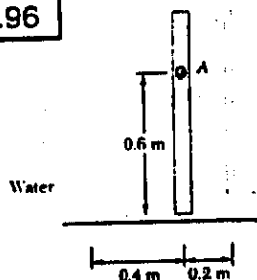
$$\text{THEN... } d = \frac{49.206 \pm \sqrt{(-49.206)^2 - 4(3.7278)(42.73)}}{2(3.7278)}$$

$$\text{OR } d = 0.93456 \text{ m AND } d = 12.2652 \text{ m}$$

$$\text{NOW, } d \leq 6.6 \text{ m AND } d = 0.93456 \text{ m} = 12 \times 10^3 \frac{\text{m}}{\text{YEAR}} \cdot N$$

$$\text{OR } N = 71.9 \text{ YEARS}$$

# 5.96



GIVEN: 1-m GATE,  $M_R = 490 \text{ N}\cdot\text{m}$ ,  $\Gamma_w = 0.1 \frac{\text{m}}{\text{min}}$ ,  $\Gamma_{MA} = 0.2 \frac{\text{m}}{\text{min}}$ ,  $\rho_{MA} = 789 \frac{\text{kg}}{\text{m}^3}$   
FIND: TIME  $t_R$  WHEN GATE ROTATES, DIRECTION OF ROTATION

CONSIDER THE FREE-BODY DIAGRAM OF THE GATE.

FIRST NOTE...  $V = A_{\text{BASE}} d$  AND  $V = \Gamma t$

$$\text{THEN... } d_w = \frac{0.1 \frac{\text{m}}{\text{min}} \cdot t(\text{min})}{(0.4\text{m})(1\text{m})}$$

$$= 0.25t \text{ (m)}$$

$$d_{MA} = \frac{0.2 \frac{\text{m}}{\text{min}} \cdot t(\text{min})}{(0.2\text{m})(1\text{m})} = t \text{ (m)}$$

$$\begin{aligned} \text{NOW... } P &= \frac{1}{2} A \cdot p = \frac{1}{2} A (\rho g h) \text{ SO THAT} \\ P_w &= \frac{1}{2} [(0.25t)m \cdot (1m)] (10^3 \frac{\text{kg}}{\text{m}^3}) (9.81 \frac{\text{m}}{\text{s}^2}) (0.25t)m \\ &= 306.56t^2 \text{ N} \\ P_{MA} &= \frac{1}{2} [(t)m \cdot (1m)] (789 \frac{\text{kg}}{\text{m}^3}) (9.81 \frac{\text{m}}{\text{s}^2}) (t)m \\ &= 3870t^2 \text{ N} \end{aligned}$$

NOW ASSUME THAT THE GATE WILL ROTATE CLOCKWISE AND WHEN  $d_{MA} \leq 0.6 \text{ m}$ . WHEN (CONTINUED)

# 5.96 CONTINUED

ROTATION OF THE GATE IS IMPENDING, REQUIRE  $\Sigma M_A: M_R = (0.6m - \frac{1}{3}d_{MA})P_{MA} - (0.6m - \frac{1}{3}d_w)P_w$

$$\begin{aligned} \text{SUBSTITUTING...} \\ 490 \text{ N}\cdot\text{m} &= (0.6 - \frac{1}{3}t)m \cdot (3870t^2) \text{ N} \\ &\quad - (0.6 - \frac{1}{3} \cdot 0.25t)m \cdot (306.56t^2) \text{ N} \end{aligned}$$

$$\text{SIMPLIFYING... } 1264.45t^3 - 2138.1t^2 + 490 = 0$$

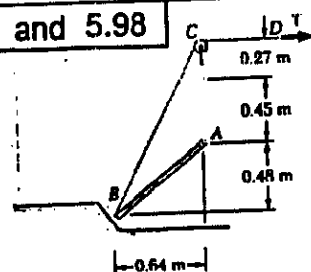
SOLVING (POSITIVE ROOTS ONLY)...

$$t = 0.59451 \text{ min AND } t = 1.52411 \text{ min}$$

NOW CHECK ASSUMPTION USING THE SMALLER ROOT. HAVE...

$$\begin{aligned} d_{MA} &= (t)m = 0.59451 \text{ m} < 0.6 \text{ m} \\ \therefore t &= 0.59451 \text{ min} = 35.7 \text{ s} \\ \text{AND THE GATE ROTATES COUNTERCLOCKWISE} \end{aligned}$$

# 5.97 and 5.98



GIVEN: 0.5-0.8-m GATE, WATER FRICTIONLESS STOP AT B

FIRST CONSIDER THE FORCE OF THE WATER ON THE GATE. HAVE  $P = \frac{1}{2} A \cdot p = \frac{1}{2} A (\rho g h)$  SO THAT...

$$\begin{aligned} P_1 &= \frac{1}{2} [(0.5\text{m})(0.8\text{m})] \cdot (10^3 \frac{\text{kg}}{\text{m}^3}) (9.81 \frac{\text{m}}{\text{s}^2}) (0.45\text{m}) \\ &= 882.9 \text{ N} \\ P_2 &= \frac{1}{2} [(0.5\text{m})(0.8\text{m})] \cdot (10^3 \frac{\text{kg}}{\text{m}^3}) (9.81 \frac{\text{m}}{\text{s}^2}) (0.93\text{m}) \\ &= 1824.66 \text{ N} \end{aligned}$$

# 5.97 FIND: REACTIONS AT A AND B WHEN $T = 0$

HAVE...

$$\begin{aligned} \Sigma M_A &= 0: \frac{1}{2} (0.8\text{m})(882.9 \text{ N}) \\ &\quad + \frac{1}{2} (0.8\text{m})(1824.66 \text{ N}) \\ &\quad - (0.8\text{m})B = 0 \end{aligned}$$

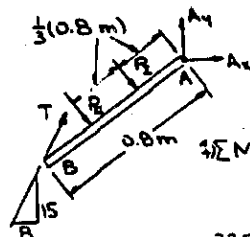
$$\text{OR } B = 1510.74 \text{ N}$$

$$\text{OR } B = 1511 \text{ N } \Delta 53.1^\circ$$

$$\Sigma F = 0: A + 1510.74 \text{ N} - 882.9 \text{ N} - 1824.66 \text{ N} = 0$$

$$\text{OR } A = 1197 \text{ N } \Delta 53.1^\circ$$

# 5.98 FIND: T TO OPEN GATE



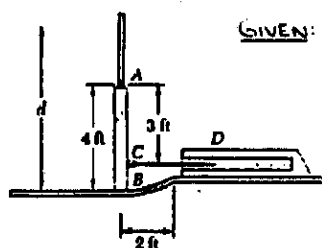
FIRST NOTE THAT WHEN THE GATE BEGINS TO OPEN, THE REACTION AT B = 0. THEN...

$$\begin{aligned} \Sigma M_A &= 0: \frac{1}{2} (0.8\text{m})(882.9 \text{ N}) \\ &\quad + \frac{1}{2} (0.8\text{m})(1824.66 \text{ N}) \\ &\quad - (0.45 + 0.27)m \cdot (T) = 0 \end{aligned}$$

$$\text{OR } 235.44 + 973.152 - 0.33882T = 0$$

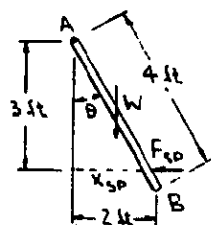
$$\text{OR } T = 3570 \text{ N}$$

### 5.99 and 5.100



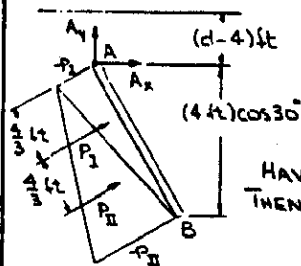
GIVEN: 4=2-ft GATE,  $k=828 \frac{\text{lb}}{\text{ft}}$ ,  
SPRING IS UNDEFORMED  
WHEN GATE IS VERTICAL  
WATER

FIRST DETERMINE THE FORCES EXERTED  
ON THE GATE BY THE SPRING AND THE  
WATER WHEN B IS AT THE END OF THE  
CYLINDRICAL PORTION OF THE FLOOR.



$$\begin{aligned} \text{HAVE.. } \sin \theta &= \frac{2}{4} \quad \theta = 30^\circ \\ \text{THEN } x_{sp} &= (3 \text{ ft}) \tan 30^\circ \\ \text{AND } F_{sp} &= k x_{sp} \\ &= 828 \frac{\text{lb}}{\text{ft}} \cdot 3 \text{ ft} \cdot \tan 30^\circ \\ &= 1434.14 \text{ lb} \end{aligned}$$

Assume  $d \geq 4 \text{ ft}$



$$\begin{aligned} \text{HAVE.. } P &= \frac{1}{2} A p = \frac{1}{2} A (\gamma h) \\ \text{THEN.. } P_1 &= \frac{1}{2} [(4 \text{ ft})(2 \text{ ft})] \\ &\quad \cdot [(62.4 \frac{\text{lb}}{\text{ft}^3})(d-4) \text{ ft}] \\ &= 249.6(d-4) \text{ lb} \\ P_2 &= \frac{1}{2} [(4 \text{ ft})(2 \text{ ft})] \\ &\quad \cdot [(62.4 \frac{\text{lb}}{\text{ft}^3})(d-4+4 \cos 30^\circ)] \\ &= 249.6(d-0.53590) \text{ lb} \end{aligned}$$

5.99 FIND:  $d, W=0$

USING THE ABOVE FREE-BODY DIAGRAMS  
OF THE GATE, HAVE..

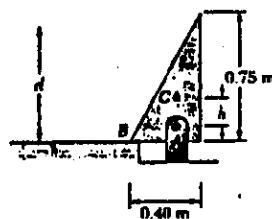
$$\begin{aligned} \sum M_A = 0: & \left(\frac{4}{3} \text{ ft}\right) [249.6(d-4) \text{ lb}] \\ & + \left(\frac{8}{3} \text{ ft}\right) [249.6(d-0.53590) \text{ lb}] \\ & - (3 \text{ ft}) (1434.14 \text{ lb}) = 0 \\ \text{OR } (332.8 d - 1331.2) & + (665.6 d - 356.70) \\ & - 4302.4 = 0 \\ \text{OR } d &= 6.00 \text{ ft} \\ d \geq 4 \text{ ft} \Rightarrow \text{ASSUMPTION CORRECT} \quad \therefore d &= 6.00 \text{ ft} \end{aligned}$$

5.100 FIND:  $d, W=1000 \text{ lb}$

USING THE ABOVE FREE-BODY DIAGRAMS OF  
THE GATE, HAVE..

$$\begin{aligned} \sum M_A = 0: & \left(\frac{4}{3} \text{ ft}\right) [249.6(d-4) \text{ lb}] \\ & + \left(\frac{8}{3} \text{ ft}\right) [249.6(d-0.53590) \text{ lb}] \\ & - (3 \text{ ft}) (1434.14 \text{ lb}) - (1 \text{ ft}) (1000 \text{ lb}) = 0 \\ \text{OR } (332.8 d - 1331.2) & + (665.6 d - 356.70) - 4302.4 \\ & - 1000 = 0 \\ \text{OR } d &= 7.00 \text{ ft} \\ d \geq 4 \text{ ft} \Rightarrow \text{ASSUMPTION CORRECT} \quad \therefore d &= 7.00 \text{ ft} \end{aligned}$$

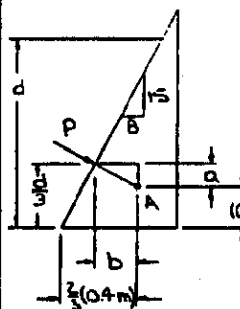
### 5.101 and 5.102



GIVEN: PRISMATICALLY  
SHAPED GATE,  
WATER

FIRST NOTE THAT WHEN THE  
GATE IS ABOUT TO OPEN  
(CLOCKWISE ROTATION IS  
IMPENDING),  $B_y \rightarrow 0$  AND THE  
LINE OF ACTION OF THE  
RESULTANT  $P$  OF THE PRESSURE  
FORCES PASSES THROUGH THE  
PIN AT A. IN ADDITION, IF IT  
 $\frac{1}{3}(0.4 \text{ m})$   $\frac{1}{3}(0.75 \text{ m}) = 0.25 \text{ m}$   
THE GATE IS

HOMOGENEOUS, THEN ITS  
CENTER OF GRAVITY  $C$  COINCIDES WITH THE  
CENTROID OF THE TRIANGULAR AREA. THEN...



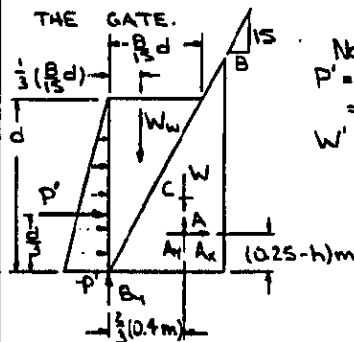
$$\begin{aligned} a &= \frac{d}{3} - (0.25 - h) \\ \text{AND } b &= \frac{2}{3}(0.4) - \frac{8}{15} \left(\frac{d}{3}\right) \end{aligned}$$

NOW  $\frac{a}{b} = \frac{8}{15}$   
SO THAT

$$\begin{aligned} \frac{\frac{d}{3} - (0.25 - h)}{\frac{2}{3}(0.4) - \frac{8}{15} \left(\frac{d}{3}\right)} &= \frac{8}{15} \\ \text{SIMPLIFYING YIELDS..} \\ \frac{289}{45} d + 15h &= \frac{70.6}{12} \quad (1) \end{aligned}$$

ALTERNATIVE SOLUTION

CONSIDER A FREE BODY CONSISTING OF A  
1-m THICK SECTION OF THE GATE AND THE  
TRIANGULAR SECTION BDE OF WATER ABOVE  
THE GATE.



$$\begin{aligned} \text{Now..} \\ P' &= \frac{1}{2} A p' = \frac{1}{2} (d \times 1 \text{ m}) (\rho g d) \\ &= \frac{1}{2} \rho g d^2 \quad (\text{N}) \\ W' &= \rho g V = \rho g \left(\frac{1}{2} \times \frac{8}{15} d \times d \times 1 \text{ m}\right) \\ &= \frac{4}{15} \rho g d^2 \quad (\text{N}) \end{aligned}$$

THEN WITH  $B_y = 0$  (AS EXPLAINED ABOVE), HAVE..

$$\begin{aligned} \sum M_A = 0: & \left[\frac{1}{3}(0.4) - \frac{1}{3} \left(\frac{8}{15} d\right)\right] \left(\frac{4}{15} \rho g d^2\right) \\ & - \left[\frac{d}{3} - (0.25 - h)\right] \left(\frac{1}{2} \rho g d^2\right) = 0 \end{aligned}$$

$$\begin{aligned} \text{SIMPLIFYING YIELDS..} \\ \frac{289}{45} d + 15h &= \frac{70.6}{12} \\ \text{AS ABOVE.} \end{aligned}$$

(CONTINUED)

# 5.101 and 5.102 CONTINUED

5.101 FIND:  $d$ ,  $h = 0.10$  m

SUBSTITUTING INTO EQ. (1)...

$$\frac{289}{45}d + 15(0.10) = \frac{70.6}{12}$$

$$\text{OR } d = 0.683 \text{ m}$$

5.102 FIND:  $h$ ,  $d = 0.75$  m

SUBSTITUTING INTO EQ. (1)...

$$\frac{289}{45}(0.75) + 15h = \frac{70.6}{12}$$

$$\text{OR } h = 0.0711 \text{ m}$$

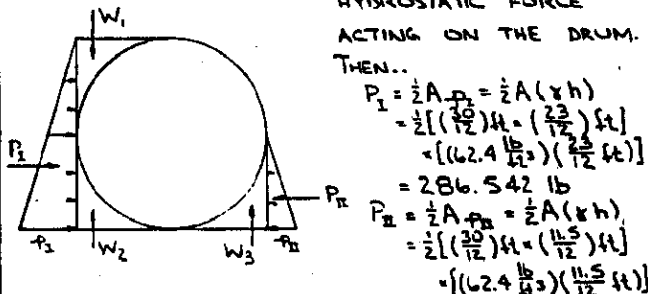
5.103

GIVEN: WIDTH = 30 IN., WATER

FIND: RESULTANT  $R$  OF PRESSURE FORCES ACTING ON DRUM



CONSIDER THE ELEMENTS OF WATER SHOWN. THE RESULTANT OF THE WEIGHTS OF WATER ABOVE EACH SECTION OF THE DRUM AND THE RESULTANTS OF THE PRESSURE FORCES ACTING ON THE VERTICAL SURFACES OF THE ELEMENTS IS EQUAL TO THE RESULTANT HYDROSTATIC FORCE ACTING ON THE DRUM.



THEN...

$$P_1 = \frac{1}{2}A_1p_1 = \frac{1}{2}A_1(\gamma h)$$

$$= \frac{1}{2}\left[\left(\frac{30}{12}\right)\text{ft}\right]\left(\frac{23}{12}\right)\text{ft}\left[\left(\frac{62.4}{12}\right)\left(\frac{23}{12}\right)\text{ft}\right]$$

$$= 286.542 \text{ lb}$$

$$P_2 = \frac{1}{2}A_2p_2 = \frac{1}{2}A_2(\gamma h)$$

$$= \frac{1}{2}\left[\left(\frac{30}{12}\right)\text{ft}\right]\left(\frac{11.5}{12}\right)\text{ft}\left[\left(\frac{62.4}{12}\right)\left(\frac{11.5}{12}\right)\text{ft}\right]$$

$$= 71.635 \text{ lb}$$

$$W_1 = \gamma V_1 = \left(\frac{62.4}{12}\right)\left[\left(\frac{11.5}{12}\right)^2\text{ft}^2\right]\left(\frac{30}{12}\right)\text{ft}$$

$$= 30.746 \text{ lb}$$

$$W_2 = \gamma V_2 = \left(\frac{62.4}{12}\right)\left[\left(\frac{11.5}{12}\right)^2\text{ft}^2 + \frac{\pi}{4}\left(\frac{11.5}{12}\right)^2\text{ft}^2\right]\left(\frac{30}{12}\right)\text{ft}$$

$$= 255.80 \text{ lb}$$

$$W_3 = \gamma V_3 = \left(\frac{62.4}{12}\right)\left[\frac{\pi}{4}\left(\frac{11.5}{12}\right)^2\text{ft}^2\right]\left(\frac{30}{12}\right)\text{ft}$$

$$= 112.525 \text{ lb}$$

$$\text{THEN... } \sum F_x: R_x = (286.542 - 71.635)\text{lb} = 214.91 \text{ lb}$$

$$+ \sum F_y: R_y = (-30.746 + 255.80 + 112.525) \text{ lb}$$

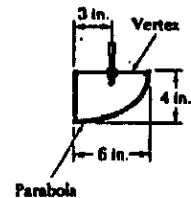
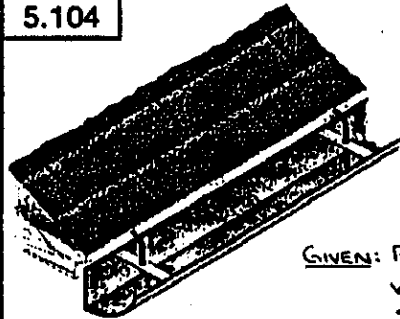
$$= 337.58 \text{ lb}$$

$$\text{FINALLY... } R = \sqrt{R_x^2 + R_y^2} \quad \tan \theta = \frac{R_y}{R_x}$$

$$= 400.18 \text{ lb} \quad \theta = 57.5^\circ$$

$$\therefore R = 400 \text{ lb } \angle 57.5^\circ$$

5.104

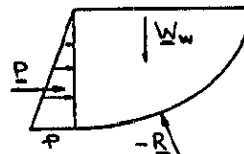


GIVEN: PARABOLIC GUTTER, WATER, HANGERS SPACED 2 ft APART

FIND: (a) THE RESULTANT  $R$  OF THE PRESSURE FORCES EXERTED ON A 2-ft SECTION OF GUTTER

(b) THE FORCE-COUPLE SYSTEM EXERTED ON A HANGER AT THE GUTTER

(a) CONSIDER A 2-ft-LONG PARABOLIC SECTION OF WATER. THEN...



$$P = \frac{1}{2}A_1p_1 = \frac{1}{2}A_1(\gamma h)$$

$$= \frac{1}{2}\left[\left(\frac{3}{12}\right)\text{ft}\right]\left(2\text{ft}\right)\left[\left(\frac{62.4}{12}\right)\left(\frac{4}{12}\right)\text{ft}\right]$$

$$= 6.9333 \text{ lb}$$

$$W_w = \gamma V$$

$$= \left(\frac{62.4}{12}\right)\left[\left(\frac{3}{12}\right)\text{ft}\right]\left(\frac{4}{12}\right)\text{ft}\left(2\text{ft}\right)$$

$$= 13.8667 \text{ lb}$$

$$\text{NOW... } \sum F = 0: (-R) - P + W_w = 0$$

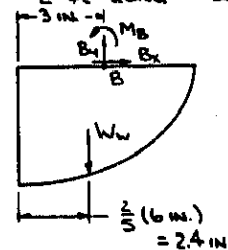
SO THAT

$$R = \sqrt{P^2 + W_w^2} \quad \tan \theta = \frac{W_w}{P}$$

$$= 15.5034 \text{ lb} \quad \theta = 63.4^\circ$$

$$\therefore R = 15.50 \text{ lb } \angle 63.4^\circ$$

(b) CONSIDER THE FREE-BODY DIAGRAM OF A 2-ft-LONG SECTION OF WATER AND GUTTER.



THEN...

$$\sum F_x = 0: B_x = 0$$

$$+ \sum F_y = 0: B_y - 13.8667 \text{ lb} = 0$$

$$\text{OR } B_y = 13.8667 \text{ lb}$$

$$\sum M_B = 0: M_B + (3 - 2.4)\text{in.}$$

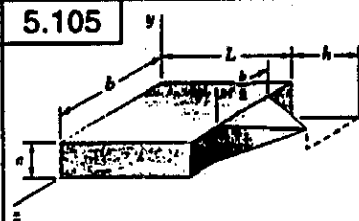
$$= (13.8667 \text{ lb})(2.4 \text{ in.})$$

$$\text{OR } M_B = -8.320 \text{ lb-in.}$$

THE FORCE-COUPLE SYSTEM EXERTED ON THE HANGER IS THEN...

$$13.87 \text{ lb } \uparrow, 8.32 \text{ lb-in.}$$

5.105



GIVEN: COMPOSITE BODY SHOWN

FIND: (a)  $\bar{x}$ ,  $h = \frac{1}{2}L$

(b)  $\frac{h}{L}$ ,  $\bar{x} = L$

	V	$\bar{x}$	$\bar{x}V$
RECTANGULAR PRISM	$Lab$	$\frac{1}{2}L$	$\frac{1}{2}L^2ab$
PYRAMID	$\frac{1}{3}a(\frac{1}{2}h)$	$L + \frac{1}{4}h$	$\frac{1}{6}abh(L + \frac{1}{4}h)$

$$\text{THEN... } \sum V = ab(L + \frac{1}{6}h) \quad \sum \bar{x}V = \frac{1}{6}ab[3L^2 + h(L + \frac{1}{4}h)]$$

(CONTINUED)



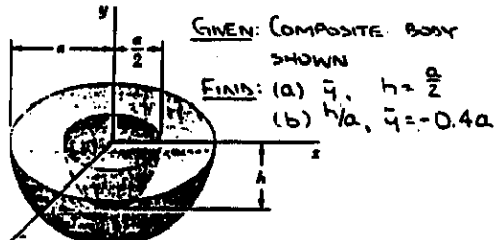
# 5.105 CONTINUED

Now..  $\bar{X} \Sigma V = \Sigma \bar{x} V$  SO THAT  
 $\bar{X} [ab(L + \frac{1}{6}h)] = \frac{1}{6}ab(3L^2 + hL + \frac{1}{4}h^2)$   
 OR  $\bar{X}(1 + \frac{1}{6}\frac{h}{L}) = \frac{1}{6}L(3 + \frac{h}{L} + \frac{1}{4}(\frac{h}{L})^2)$  (1)

(a)  $\bar{X} = ?$  WHEN  $h = \frac{1}{2}L$   
 SUBSTITUTING  $\frac{h}{L} = \frac{1}{2}$  INTO EQ. (1)..  
 $\bar{X}(1 + \frac{1}{6}(\frac{1}{2})) = \frac{1}{6}L[3 + (\frac{1}{2}) + \frac{1}{4}(\frac{1}{2})^2]$   
 OR  $\bar{X} = \frac{57}{104}L$   $\bar{X} = 0.548L$

(b)  $\frac{h}{L} = ?$  WHEN  $\bar{X} = L$   
 SUBSTITUTING INTO EQ. (1)..  
 $L(1 + \frac{1}{6}\frac{h}{L}) = \frac{1}{6}L(3 + \frac{h}{L} + \frac{1}{4}(\frac{h}{L})^2)$   
 OR..  $1 + \frac{1}{6}\frac{h}{L} = \frac{1}{2} + \frac{1}{6}\frac{h}{L} + \frac{1}{24}(\frac{h}{L})^2$   
 OR  $\frac{h}{L} = 12$   $\therefore \frac{h}{L} = 2.8$

# 5.106



	V	$\bar{y}$	$\bar{y}V$
HEMISPHERE	$\frac{2}{3}\pi a^3$	$-\frac{3}{8}a$	$-\frac{1}{4}\pi a^4$
SEMI-ELLIPOID	$-\frac{2}{3}\pi(\frac{a}{2})^2h = -\frac{1}{6}\pi a^2h$	$-\frac{3}{8}h$	$+\frac{1}{6}\pi a^2h^2$

THEN..  $\Sigma V = \frac{2}{3}\pi a^2(4a - h)$   $\Sigma \bar{y}V = -\frac{\pi}{16}a^2(4a^2 - h^2)$

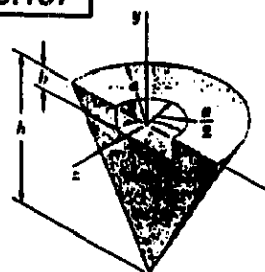
Now..  $\bar{Y} \Sigma V = \Sigma \bar{y}V$  SO THAT  
 $\bar{Y}[\frac{2}{3}\pi a^2(4a - h)] = -\frac{\pi}{16}a^2(4a^2 - h^2)$   
 OR  $\bar{Y}(4 - \frac{h}{a}) = -\frac{1}{8}a[4 - (\frac{h}{a})^2]$  (1)

(a)  $\bar{Y} = ?$  WHEN  $h = \frac{a}{2}$   
 SUBSTITUTING  $\frac{h}{a} = \frac{1}{2}$  INTO EQ. (1)..  
 $\bar{Y}(4 - \frac{1}{2}) = -\frac{1}{8}a[4 - (\frac{1}{2})^2]$   
 OR  $\bar{Y} = -\frac{43}{112}a$   $\bar{Y} = -0.402a$

(b)  $\frac{h}{a} = ?$  WHEN  $\bar{Y} = -0.4a$   
 SUBSTITUTING INTO EQ. (1)..  
 $(-0.4a)(4 - \frac{h}{a}) = -\frac{1}{8}a[4 - (\frac{h}{a})^2]$   
 OR  $3(\frac{h}{a})^2 - 3.2(\frac{h}{a}) + 0.8 = 0$   
 THEN..  $\frac{h}{a} = \frac{3.2 \pm \sqrt{(3.2)^2 - 4(3)(0.8)}}{2(3)}$   
 $= \frac{3.2 \pm 0.8}{6}$

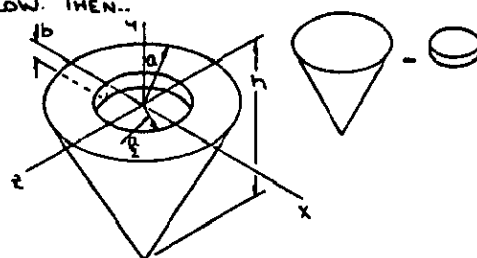
OR  $\frac{h}{a} = \frac{2}{3}$  AND  $\frac{h}{a} = \frac{2}{3}$

# 5.107



GIVEN: COMPOSITE BODY SHOWN  
 FIND:  $\bar{Y}$

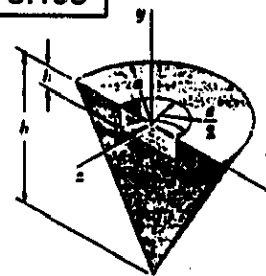
FIRST NOTE THAT THE VALUES OF  $\bar{Y}$  WILL BE THE SAME FOR THE GIVEN BODY AND THE BODY SHOWN BELOW. THEN..



	V	$\bar{y}$	$\bar{y}V$
CONE	$\frac{1}{3}\pi a^2h$	$-\frac{3}{8}h$	$-\frac{1}{24}\pi a^2h^2$
CYLINDER	$-\pi(\frac{a}{2})^2b = -\frac{1}{4}\pi a^2b$	$-\frac{1}{2}b$	$-\frac{1}{8}\pi a^2b^2$
$\Sigma$	$\frac{\pi}{12}a^2(4h - 3b)$		$-\frac{\pi}{24}a^2(2h^2 - 3b^2)$

HAVE..  $\bar{Y} \Sigma V = \Sigma \bar{y}V$   
 THEN..  $\bar{Y}[\frac{\pi}{12}a^2(4h - 3b)] = -\frac{\pi}{24}a^2(2h^2 - 3b^2)$   
 OR  $\bar{Y} = -\frac{2h^2 - 3b^2}{2(4h - 3b)}$

# 5.108



GIVEN: COMPOSITE BODY SHOWN  
 FIND:  $\bar{Z}$

FIRST NOTE THAT THE BODY CAN BE FORMED BY REMOVING A 'HALF-CYLINDER' FROM A 'HALF-CONE'.

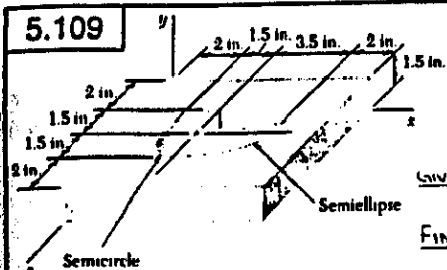


	V	$\bar{z}$	$\bar{z}V$
HALF-CONE	$\frac{1}{6}\pi a^2h$	$-\frac{3}{8}h$	$-\frac{1}{48}\pi a^2h^2$
HALF-CYLINDER	$-\frac{1}{2}(\frac{a}{2})^2b = -\frac{1}{8}\pi a^2b$	$-\frac{3}{8}b$	$-\frac{3}{64}\pi a^2b^2$
$\Sigma$	$\frac{\pi}{24}a^2(4h - 3b)$		$-\frac{1}{12}\pi a^2(2h - b)$

\*FROM SAMPLE PROBLEM 5.13

HAVE..  $\bar{Z} \Sigma V = \Sigma \bar{z}V$   
 THEN..  $\bar{Z}[\frac{\pi}{24}a^2(4h - 3b)] = -\frac{1}{12}\pi a^2(2h - b)$   
 OR  $\bar{Z} = -\frac{2a}{\pi} \frac{2h - b}{4h - 3b}$

5.109

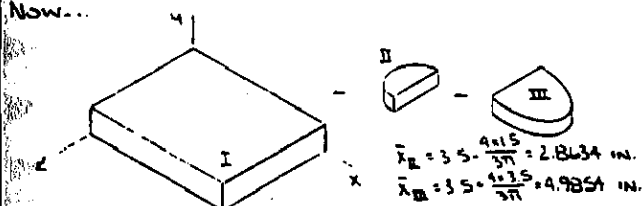


GIVEN: SAND MOLD SHOWN

FIND: LOCATION OF CENTER OF GRAVITY

FIRST ASSUME THAT THE MOLD IS HOMOGENEOUS SO THAT ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING VOLUME. SYMMETRY THEN IMPLIES  $\bar{z} = 3.5$  IN.

NOW...

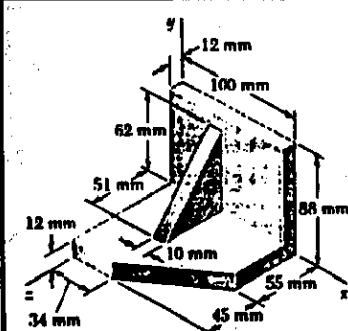


	$V, \text{in}^3$	$\bar{x}, \text{in}$	$\bar{y}, \text{in}$	$\bar{x}V, \text{in}^4$	$\bar{y}V, \text{in}^4$
I	$(9)(1.5)(1) = 13.5$	4.5	0.75	425.25	10.875
II	$-\frac{\pi}{2}(1.5)^2(0.75) = -2.6507$	2.8634	1.125	-7.5900	-2.9820
III	$-\frac{\pi}{2}(1.5)^2(0.75) = -2.6507$	2.8634	1.125	-7.5900	-2.9820
$\Sigma$	8.1986			340.66	5.911

HAVE...  $\bar{x}\Sigma V = \Sigma \bar{x}V$ :  $\bar{x}(8.1986 \text{ in}^3) = 340.66 \text{ in}^4$   
OR  $\bar{x} = 4.15 \text{ in}$

AND  $\bar{y}\Sigma V = \Sigma \bar{y}V$ :  $\bar{y}(8.1986 \text{ in}^3) = 5.911 \text{ in}^4$   
OR  $\bar{y} = 0.72 \text{ in}$

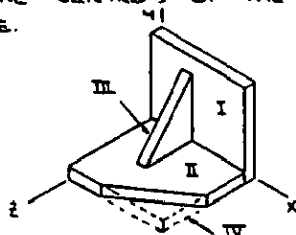
5.110 and 5.111



GIVEN: STOP BRACKET SHOWN

FIND:  $\bar{x}$  (5.110)  
 $\bar{z}$  (5.111)

FIRST ASSUME THAT THE BRACKET IS HOMOGENEOUS SO THAT ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING VOLUME.



$$\bar{x}_{III} = 34 + \frac{1}{2}(10) = 39 \text{ mm}$$

$$\bar{z}_{III} = 12 + \frac{1}{2}(88) = 56 \text{ mm}$$

$$\bar{x}_{II} = 34 + \frac{1}{2}(66) = 78 \text{ mm}$$

$$\bar{z}_{II} = 55 + \frac{1}{2}(45) = 85 \text{ mm}$$

(CONTINUED)

5.110 and 5.111 CONTINUED

	$V, \text{mm}^3$	$\bar{x}, \text{mm}$	$\bar{z}, \text{mm}$	$\bar{x}V, \text{mm}^4$	$\bar{z}V, \text{mm}^4$
I	$(100)(88)(12) = 105600$	50	6	5280000	633600
II	$(100)(12)(88) = 105600$	50	56	5280000	5913600
III	$\frac{1}{2}(100)(12)(51) = 31500$	39	29	1246500	913500
IV	$-\frac{1}{2}(66)(12)(45) = -17820$	78	85	-1389960	-1514700
$\Sigma$	209190			9786630	5490990

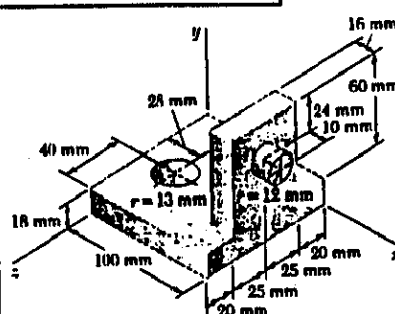
5.110

HAVE...  $\bar{x}\Sigma V = \Sigma \bar{x}V$   
 $\bar{x}(209190 \text{ mm}^3) = 9786630 \text{ mm}^4$   
OR  $\bar{x} = 46.8 \text{ mm}$

5.111

HAVE...  $\bar{z}\Sigma V = \Sigma \bar{z}V$   
 $\bar{z}(209190 \text{ mm}^3) = 5490990 \text{ mm}^4$   
OR  $\bar{z} = 26.2 \text{ mm}$

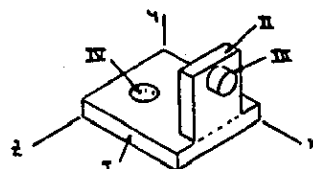
5.112 and 5.115



GIVEN: MACHINE ELEMENT SHOWN

FIND:  $\bar{x}$  (5.112)  
 $\bar{y}$  (5.115)

FIRST ASSUME THAT THE MACHINE ELEMENT IS HOMOGENEOUS SO THAT ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING VOLUME.



	$V, \text{mm}^3$	$\bar{x}, \text{mm}$	$\bar{y}, \text{mm}$	$\bar{x}V, \text{mm}^4$	$\bar{y}V, \text{mm}^4$
I	$(100)(18)(90) = 162000$	50	9	8100000	1458000
II	$(16)(60)(50) = 48000$	92	48	4416000	2304000
III	$\pi(12)^2(10) = 4523.9$	105	54	475010	244290
IV	$-\pi(13)^2(18) = -9556.7$	28	9	-267590	-86010
$\Sigma$	204967.2			12723420	3920280

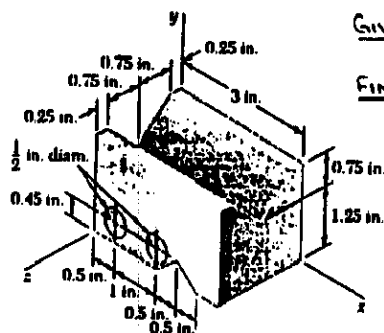
5.112

HAVE...  $\bar{x}\Sigma V = \Sigma \bar{x}V$   
 $\bar{x}(204967.2 \text{ mm}^3) = 12723420 \text{ mm}^4$   
OR  $\bar{x} = 62.1 \text{ mm}$

5.115

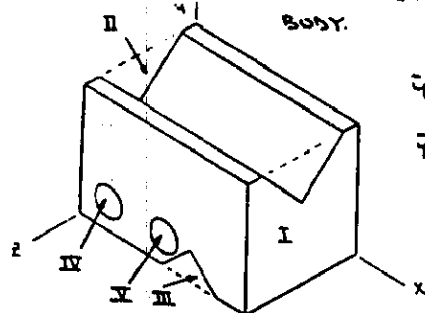
HAVE...  $\bar{y}\Sigma V = \Sigma \bar{y}V$   
 $\bar{y}(204967.2 \text{ mm}^3) = 3920280 \text{ mm}^4$   
OR  $\bar{y} = 19.13 \text{ mm}$

# 5.113 and 5.114



GIVEN: MACHINE ELEMENT SHOWN  
FIND:  $\bar{X}$  (5.113)  
 $\bar{Y}$  (5.114)

FIRST ASSUME THAT THE MACHINE ELEMENT IS HOMOGENEOUS SO THAT ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROIDS OF THE CORRESPONDING VOLUME. ALSO NOTE THAT THE TWO HOLES AND THE V-NOTCH EXTEND THROUGH THE BODY.



$$\bar{y}_{II} = 1.25 + \frac{2}{3}(0.75) = 1.75 \text{ in.}$$

$$\bar{y}_{III} = \frac{2}{3}(0.45) = 0.15 \text{ in.}$$

	V, in <sup>3</sup>	$\bar{x}$ , in.	$\bar{y}$ , in.	$\bar{x}V$ , in <sup>4</sup>	$\bar{y}V$ , in <sup>4</sup>
I	$(3 \times 2)(2) = 12$	1.5	1	18	12
II	$-\frac{1}{2}(1.5)(0.75)(3) = -1.6875$	1.5	1.75	-2.53125	-2.9531
III	$-\frac{1}{2}(1)(0.45)(2) = -0.45$	2	0.15	-0.90	-0.0675
IV	$-\pi(\frac{1}{4})^2(2) = -0.39270$	0.5	0.45	-0.19635	-0.17672
V	$-\pi(\frac{1}{4})^2(2) = -0.39270$	1.5	0.45	-0.58905	-0.17672
$\Sigma$	9.0771			13.7834	8.6260

5.113

HAVE..  $\bar{X}\Sigma V = \Sigma \bar{x}V$   
 $\bar{X}(9.0771 \text{ in}^3) = 13.7834 \text{ in}^4$   
OR  $\bar{X} = 1.518 \text{ in.}$

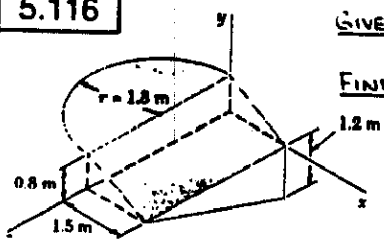
5.114

HAVE..  $\bar{Y}\Sigma V = \Sigma \bar{y}V$   
 $\bar{Y}(9.0771 \text{ in}^3) = 8.6260 \text{ in}^4$   
OR  $\bar{Y} = 0.950 \text{ in.}$

5.115

SEE SOLUTION TO PROBLEM 5.112

5.116

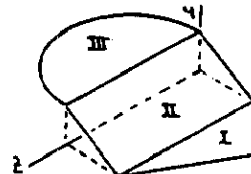


GIVEN: SHEET-METAL FORM SHOWN  
FIND: LOCATION OF CENTER OF GRAVITY

FIRST ASSUME THAT THE SHEET METAL IS (CONTINUED)

# 5.116 CONTINUED

HOMOGENEOUS SO THAT THE CENTER OF GRAVITY OF THE FORM WILL COINCIDE WITH THE CENTROIDS OF THE CORRESPONDING AREA.



$$\bar{y}_I = -\frac{1}{3}(1.2) = -0.4 \text{ m}$$

$$\bar{y}_2 = \frac{1}{3}(3.6) = 1.2 \text{ m}$$

$$\bar{x}_{III} = -\frac{4(1.8)}{3\pi} = -\frac{2.4}{\pi} \text{ m}$$

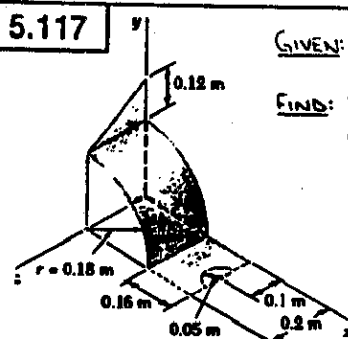
	A, m <sup>2</sup>	$\bar{x}$ , m	$\bar{y}$ , m	$\bar{x}A$ , m <sup>3</sup>	$\bar{y}A$ , m <sup>3</sup>	$\bar{z}A$ , m <sup>3</sup>
I	$\frac{1}{2}(3.6)(1.2) = 2.16$	1.5	-0.4	1.2	3.24	-0.864
II	$(3.6)(1.1) = 6.12$	0.75	0.4	1.8	4.59	2.448
III	$\frac{1}{2}(1.8)^2 = 5.0894$	$-\frac{2.4}{\pi}$	0.8	1.8	-3.888	4.0715
$\Sigma$	13.3694			3.942	5.6555	22.769

HAVE..  $\bar{X}\Sigma V = \Sigma \bar{x}V$ :  $\bar{X}(13.3694 \text{ m}^2) = 3.942 \text{ m}^3$   
OR  $\bar{X} = 0.295 \text{ m}$

$\bar{Y}\Sigma V = \Sigma \bar{y}V$ :  $\bar{Y}(13.3694 \text{ m}^2) = 5.6555 \text{ m}^3$   
OR  $\bar{Y} = 0.423 \text{ m}$

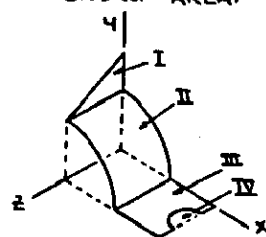
$\bar{Z}\Sigma V = \Sigma \bar{z}V$ :  $\bar{Z}(13.3694 \text{ m}^2) = 22.769 \text{ m}^3$   
OR  $\bar{Z} = 1.703 \text{ m}$

5.117



GIVEN: SHEET-METAL FORM SHOWN  
FIND: LOCATION OF CENTER OF GRAVITY

FIRST ASSUME THAT THE SHEET METAL IS HOMOGENEOUS SO THAT THE CENTER OF GRAVITY OF THE FORM WILL COINCIDE WITH THE CENTROIDS OF THE CORRESPONDING AREA.



$$\bar{y}_I = 0.18 + \frac{1}{3}(0.12) = 0.22 \text{ m}$$

$$\bar{y}_2 = \frac{1}{3}(0.2 \text{ m})$$

$$\bar{x}_{II} = \bar{y}_{II} = \frac{2(0.18)}{\pi} = \frac{0.36}{\pi} \text{ m}$$

$$\bar{x}_{IV} = 0.34 - \frac{4(0.05)}{3\pi} = 0.31878 \text{ m}$$

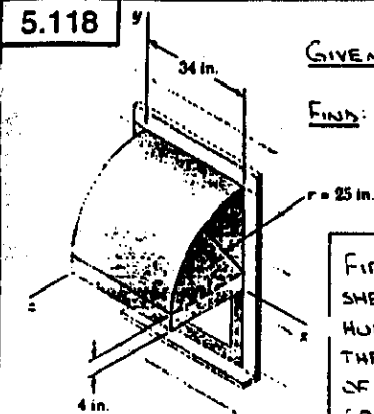
	A, m <sup>2</sup>	$\bar{x}$ , m	$\bar{y}$ , m	$\bar{x}A$ , m <sup>3</sup>	$\bar{y}A$ , m <sup>3</sup>	$\bar{z}A$ , m <sup>3</sup>
I	$\frac{1}{2}(0.2)(0.12) = 0.012$	0	0.22	0	0.00264	0.0008
II	$\frac{\pi}{2}(0.18)(0.2) = 0.018\pi$	$\frac{0.36}{\pi}$	$\frac{0.36}{\pi}$	0.1	0.00648	0.005655
III	$(0.16)(0.2) = 0.032$	0.26	0	0.1	0.00832	0
IV	$-\frac{1}{2}(0.05)^2 = -0.00125\pi$	0.31878	0	0.1	-0.001258	0
$\Sigma$	0.096622			0.03548	0.00912	0.009262

HAVE..  $\bar{X}\Sigma V = \Sigma \bar{x}V$ :  $\bar{X}(0.096622 \text{ m}^2) = 0.013548 \text{ m}^3$   
OR  $\bar{X} = 0.1402 \text{ m}$

$\bar{Y}\Sigma V = \Sigma \bar{y}V$ :  $\bar{Y}(0.096622 \text{ m}^2) = 0.00912 \text{ m}^3$   
OR  $\bar{Y} = 0.0944 \text{ m}$

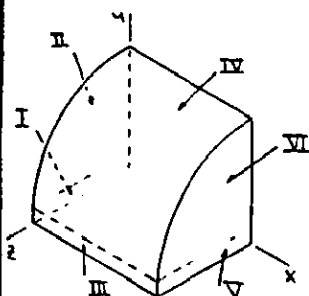
$\bar{Z}\Sigma V = \Sigma \bar{z}V$ :  $\bar{Z}(0.096622 \text{ m}^2) = 0.009262 \text{ m}^3$   
OR  $\bar{Z} = 0.0959 \text{ m}$

5.118



**GIVEN:** SHEET-METAL  
AWNING SHOWN  
**FIND:** LOCATION OF CENTER  
OF GRAVITY

FIRST ASSUME THAT THE  
SHEET METAL IS  
HOMOGENEOUS SO THAT  
THE CENTER OF GRAVITY  
OF THE AWNING WILL  
COINCIDE WITH THE  
CENTROID OF THE  
CORRESPONDING AREA.



$$\bar{y}_{II} = \bar{y}_{III} = 4 + \frac{4 \cdot 25}{3\pi} = 14.6103 \text{ in.}$$

$$\bar{z}_{II} = \bar{z}_{III} = \frac{4 \cdot 25}{3\pi} = \frac{100}{3\pi} \text{ in.}$$

$$\bar{y}_{IV} = 4 + \frac{2 \cdot 25}{\pi} = 19.9155 \text{ in.}$$

$$\bar{z}_{IV} = \frac{2 \cdot 25}{\pi} = \frac{50}{\pi} \text{ in.}$$

$$A_{II} = A_{III} = \frac{\pi}{4} (25)^2 = 156.25\pi \text{ in}^2$$

$$A_{IV} = \frac{\pi}{2} (25)(34) = 425\pi \text{ in}^2$$

	A, in <sup>2</sup>	$\bar{y}$ , in.	$\bar{z}$ , in.	$\bar{y}A$ , in <sup>3</sup>	$\bar{z}A$ , in <sup>3</sup>
I	(4)(25) = 100	2	12.5	200	1250
II	156.25π = 490.87	14.6103	$\frac{100}{3\pi}$	7171.8	5208.3
III	(4)(34) = 136	2	25	272	3400
IV	425π = 1335.18	19.9155	$\frac{50}{\pi}$	26,591	21,250
V	(4)(25) = 100	2	12.5	200	1250
VI	156.25π = 490.87	14.6103	$\frac{100}{3\pi}$	7171.8	5208.3
Σ	2652.9			41,606.6	37,566.6

NOW... SYMMETRY IMPLIES  $\bar{x} = 17.00 \text{ in.}$

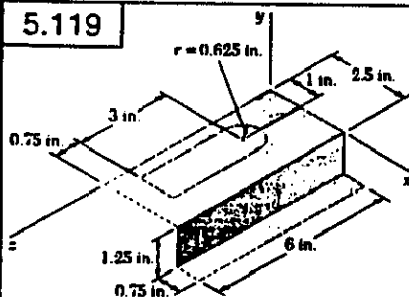
AND  $\bar{y}\Sigma A = \Sigma \bar{y}A$ :  $\bar{y}(2652.9 \text{ in}^2) = 41,606.6 \text{ in}^3$

OR  $\bar{y} = 15.68 \text{ in.}$

$\bar{z}\Sigma A = \Sigma \bar{z}A$ :  $\bar{z}(2652.9 \text{ in}^2) = 37,566.6 \text{ in}^3$

OR  $\bar{z} = 14.16 \text{ in.}$

5.119



**GIVEN:** SHEET-METAL  
BRACKET SHOWN  
**FIND:** LOCATION OF  
CENTER OF  
GRAVITY

FIRST ASSUME THAT THE SHEET METAL IS  
HOMOGENEOUS SO THAT THE CENTER OF GRAVITY  
OF THE BRACKET WILL COINCIDE WITH THE  
CENTROID OF THE CORRESPONDING AREA. THEN  
(SEE DIAGRAM AT THE TOP OF NEXT COLUMN)

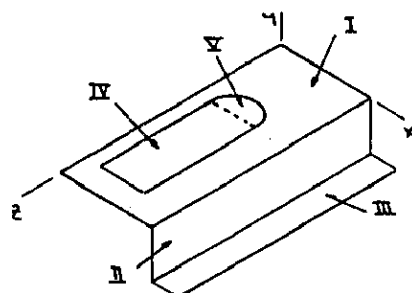
$$\bar{z}_{II} = 2.25 - \frac{4 \cdot 0.625}{3\pi}$$

$$= 1.98474 \text{ in.}$$

$$A_{II} = -\frac{\pi}{2} (0.625)^2$$

$$= -0.61359 \text{ in}^2$$

5.119 CONTINUED



	A, in <sup>2</sup>	$\bar{x}$ , in.	$\bar{y}$ , in.	$\bar{z}$ , in.	$\bar{x}A$ , in <sup>3</sup>	$\bar{y}A$ , in <sup>3</sup>	$\bar{z}A$ , in <sup>3</sup>
I	(12.5)(6) = 75	1.25	0	3	18.75	0	45
II	(1.25)(6) = 7.5	2.5	-0.625	3	18.75	-4.6875	22.5
III	(0.75)(6) = 4.5	2.875	-1.25	3	12.9375	-5.425	13.5
IV	(2)(3) = 6	1	0	3.75	-3.75	0	-14.0625
V	-0.61359	1	0	1.98474	-0.61359	0	-1.21782
Σ	22.6364				46.0739	-10.3125	65.7197

HAVE...  $\bar{x}\Sigma A = \Sigma \bar{x}A$ :  $\bar{x}(22.6364 \text{ in}^2) = 46.0739 \text{ in}^3$

OR  $\bar{x} = 2.04 \text{ in.}$

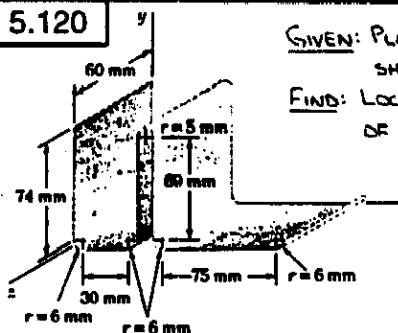
$\bar{y}\Sigma A = \Sigma \bar{y}A$ :  $\bar{y}(22.6364 \text{ in}^2) = -10.3125 \text{ in}^3$

OR  $\bar{y} = -0.456 \text{ in.}$

$\bar{z}\Sigma A = \Sigma \bar{z}A$ :  $\bar{z}(22.6364 \text{ in}^2) = 65.7197 \text{ in}^3$

OR  $\bar{z} = 2.90 \text{ in.}$

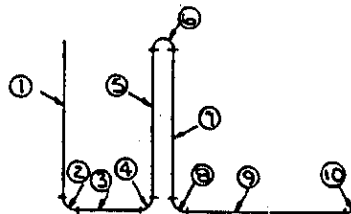
5.120



**GIVEN:** PLASTIC ORGANIZER  
SHOWN  
**FIND:** LOCATION OF CENTER  
OF GRAVITY

FIRST ASSUME THAT THE PLASTIC IS HOMOGENEOUS  
SO THAT THE CENTER OF GRAVITY OF THE  
ORGANIZER WILL COINCIDE WITH THE CENTROID OF  
THE CORRESPONDING AREA. NOW NOTE THAT  
SYMMETRY IMPLIES

$\bar{z} = 30 \text{ mm}$



$$\bar{x}_2 = 6 - \frac{2 \cdot 6}{\pi} = 2.1803 \text{ mm}$$

$$\bar{x}_4 = 36 + \frac{2 \cdot 6}{\pi} = 39.820 \text{ mm}$$

$$\bar{x}_8 = 58 + \frac{2 \cdot 6}{\pi} = 54.180 \text{ mm}$$

$$\bar{x}_{10} = 133 + \frac{2 \cdot 6}{\pi} = 136.820 \text{ mm}$$

$$\bar{y}_2 = \bar{y}_4 = \bar{y}_8 = \bar{y}_{10} = 6 - \frac{2 \cdot 6}{\pi} = 2.1803 \text{ mm}$$

$$\bar{y}_6 = 75 + \frac{2 \cdot 6}{\pi} = 78.183 \text{ mm}$$

(CONTINUED)

### 5.120 CONTINUED

$$A_2 = A_4 = A_6 = A_{10} = \frac{\pi}{2} \cdot 6 \cdot 60 = 565.49 \text{ mm}^2$$

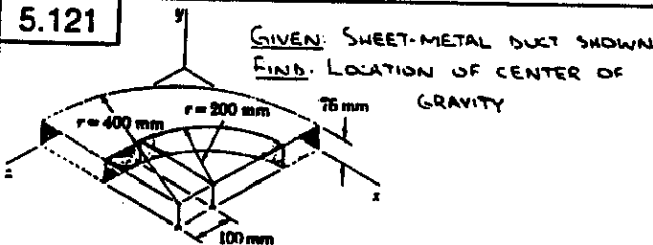
$$A_6 = \pi \cdot 5 \cdot 60 = 942.48 \text{ mm}^2$$

	A, mm <sup>2</sup>	$\bar{x}$ , mm	$\bar{y}$ , mm	$\bar{x}A$ , mm <sup>3</sup>	$\bar{y}A$ , mm <sup>3</sup>
1	(74)(60) = 4440	0	43	0	190 920
2	565.49	2.1803	2.1803	1233	1233
3	(30)(60) = 1800	21	0	37 800	0
4	565.49	39.820	2.1803	22 518	1233
5	(69)(60) = 4140	42	40.5	173 880	167 670
6	942.48	47	78.183	44 297	73 686
7	(69)(60) = 4140	52	40.5	215 280	167 670
8	565.49	54.180	2.1803	30 638	1233
9	(75)(60) = 4500	95.5	0	429 750	0
10	565.49	136.820	2.1803	77 370	1233
$\Sigma$	22 224.44			1 032 766	604 878

HAVE...  $\bar{x}\Sigma A = \Sigma \bar{x}A$ :  $\bar{x}(22 224.44 \text{ mm}^2) = 1 032 766 \text{ mm}^3$   
OR  $\bar{x} = 46.5 \text{ mm}$   $\blacktriangleleft$

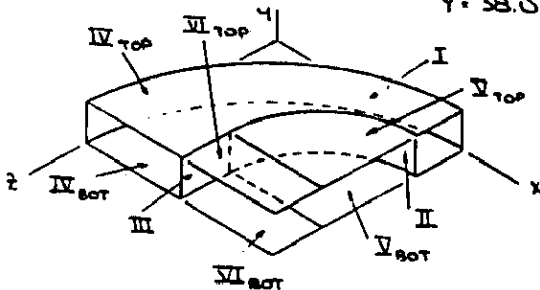
$\bar{y}\Sigma A = \Sigma \bar{y}A$ :  $\bar{y}(22 224.44 \text{ mm}^2) = 604 878 \text{ mm}^3$   
OR  $\bar{y} = 27.2 \text{ mm}$   $\blacktriangleleft$

### 5.121



FIRST ASSUME THAT THE SHEET METAL IS HOMOGENEOUS SO THAT THE CENTER OF GRAVITY OF THE DUCT WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING AREA. NOW NOTE THAT SYMMETRY IMPLIES

$$\bar{y} = 38.0 \text{ mm} \quad \blacktriangleleft$$



$$\bar{x}_I = \bar{z}_I = 400 - \frac{2 \cdot 400}{\pi} = 145.352 \text{ mm}$$

$$\bar{x}_{II} = 400 - \frac{2 \cdot 200}{\pi} = 272.68 \text{ mm} \quad \bar{z}_{II} = 300 - \frac{2 \cdot 200}{\pi} = 172.676 \text{ mm}$$

$$\bar{x}_{IV} = \bar{z}_{IV} = 400 - \frac{4 \cdot 400}{3\pi} = 230.23 \text{ mm}$$

$$\bar{x}_8 = 400 - \frac{4 \cdot 200}{3\pi} = 315.12 \text{ mm} \quad \bar{z}_8 = 300 - \frac{4 \cdot 200}{3\pi} = 215.12 \text{ mm}$$

ALSO NOTE THAT THE CORRESPONDING TOP AND BOTTOM AREAS WILL CONTRIBUTE EQUALLY WHEN DETERMINING  $\bar{x}$  AND  $\bar{z}$ . THUS...

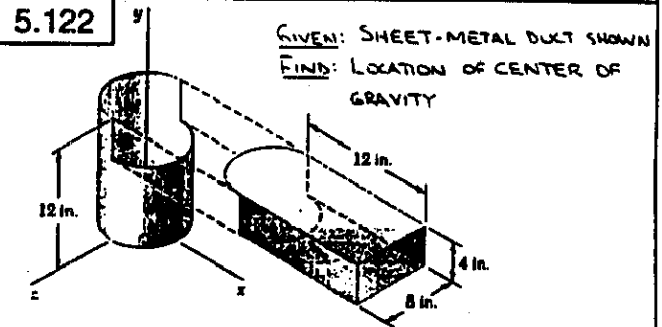
(CONTINUED)

### 5.121 CONTINUED

	A, mm <sup>2</sup>	$\bar{x}$ , mm	$\bar{z}$ , mm	$\bar{x}A$ , mm <sup>3</sup>	$\bar{z}A$ , mm <sup>3</sup>
I	$\frac{\pi}{2}(400)(76) = 47 752$	145.352	145.352	6 940 850	6 940 850
II	$\frac{\pi}{2}(200)(76) = 23 876$	272.68	172.676	6 510 510	4 122 810
III	$(100)(76) = 7600$	200	350	1 520 000	2 660 000
IV	$\frac{\pi}{2}(400)^2 = 251 321$	230.23	230.23	57 843 020	57 843 020
V	$-2 \cdot \frac{\pi}{2}(200)^2 = -62 832$	315.12	215.12	-19 799 620	-13 516 420
VI	$-2(100 \cdot 200) = -40 000$	300	350	-12 000 000	-14 000 000
$\Sigma$	227 723			41 034 760	44 070 260

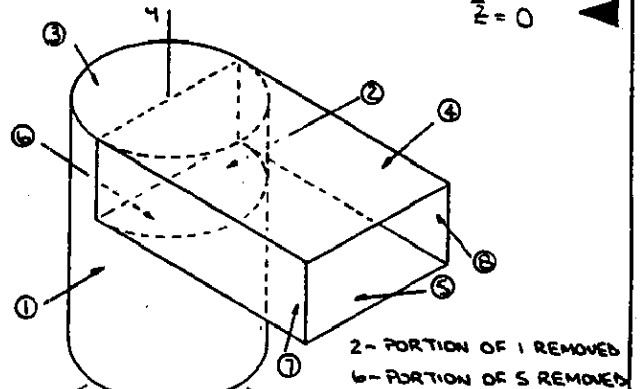
HAVE...  $\bar{x}\Sigma A = \Sigma \bar{x}A$ :  $\bar{x}(227 723 \text{ mm}^2) = 41 034 760 \text{ mm}^3$   
OR  $\bar{x} = 180.2 \text{ mm} \quad \blacktriangleleft$   
 $\bar{z}\Sigma A = \Sigma \bar{z}A$ :  $\bar{z}(227 723 \text{ mm}^2) = 44 070 260 \text{ mm}^3$   
OR  $\bar{z} = 193.5 \text{ mm} \quad \blacktriangleleft$

### 5.122



FIRST ASSUME THAT THE SHEET METAL IS HOMOGENEOUS SO THAT THE CENTER OF GRAVITY OF THE DUCT ASSEMBLY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING AREA. NOW NOTE THAT SYMMETRY IMPLIES

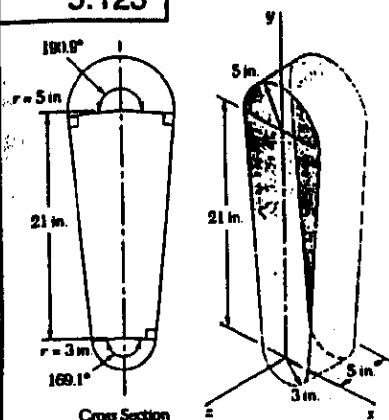
$$\bar{z} = 0 \quad \blacktriangleleft$$



	A, in <sup>2</sup>	$\bar{x}$ , in.	$\bar{y}$ , in.	$\bar{x}A$ , in <sup>3</sup>	$\bar{y}A$ , in <sup>3</sup>
1	$\pi(8)(12) = 301.59$	0	6	0	1809.54
2	$-\frac{\pi}{2}(8)(4) = -50.27$	$\frac{4}{\pi} = 1.2732$	10	-128	-502.7
3	$\frac{\pi}{2}(4)^2 = 25.13$	$-\frac{4}{\pi} = -1.2732$	12	-42.667	301.56
4	$(12)(8) = 96$	6	12	576	1152
5	$(12)(8) = 96$	6	8	576	768
6	$-\frac{\pi}{2}(4)^2 = -25.13$	$\frac{4}{\pi} = 1.2732$	8	-42.667	-201.04
7	$(12)(4) = 48$	6	10	288	480
8	$(12)(4) = 48$	6	10	288	480
$\Sigma$	539.32			1514.666	4287.36

HAVE...  $\bar{x}\Sigma A = \Sigma \bar{x}A$ :  $\bar{x}(539.32 \text{ in}^2) = 1514.666 \text{ in}^3$   
OR  $\bar{x} = 2.81 \text{ in.} \quad \blacktriangleleft$   
 $\bar{y}\Sigma A = \Sigma \bar{y}A$ :  $\bar{y}(539.32 \text{ in}^2) = 4287.36 \text{ in}^3$   
OR  $\bar{y} = 7.95 \text{ in.} \quad \blacktriangleleft$

# 5.123

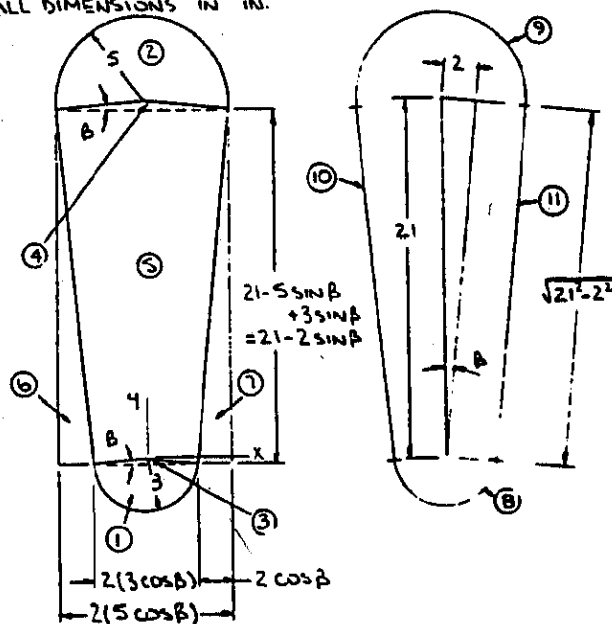


GIVEN: SHEET-METAL COVER SHOWN  
FIND: LOCATION OF CENTER OF GRAVITY

FIRST ASSUME THAT THE SHEET METAL IS HOMOGENEOUS SO THAT THE CENTER OF GRAVITY OF THE COVER WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING AREA. NOW NOTE THAT SYMMETRY IMPLIES

$$\bar{X} = 0$$

ALL DIMENSIONS IN IN.



FIRST NOTE...  $\beta = 90^\circ - \frac{169.1^\circ}{2} = 5.45^\circ$

$$\bar{y}_1 = -\frac{2(3) \sin\left(\frac{169.1^\circ}{2}\right)}{3\left(\frac{169.1^\circ}{2} \cdot \frac{\pi}{180^\circ}\right)} \quad A_1 = \left(\frac{169.1^\circ}{2} \cdot \frac{\pi}{180^\circ}\right)(3)^2$$

$$= -1.3492 \text{ IN.}$$

$$\bar{y}_2 = 21 + \frac{2(5) \sin\left(\frac{190.9^\circ}{2}\right)}{3\left(\frac{190.9^\circ}{2} \cdot \frac{\pi}{180^\circ}\right)} \quad A_2 = \left(\frac{190.9^\circ}{2} \cdot \frac{\pi}{180^\circ}\right)(5)^2$$

$$= 22.99 \text{ IN.}$$

$$\bar{y}_3 = -\frac{1}{3}(3 \sin 5.45^\circ) \quad A_3 = -\frac{1}{2}[2(3 \cos 5.45^\circ)] \cdot (3 \sin 5.45^\circ)$$

$$= -0.18995 \text{ IN.}$$

$$= -0.8509 \text{ IN}^2$$

$$\bar{y}_4 = 21 - \frac{1}{3}(5 \sin 5.45^\circ) \quad A_4 = \frac{1}{2}[2(5 \cos 5.45^\circ)] \cdot (5 \sin 5.45^\circ)$$

$$= 20.68 \text{ IN.}$$

$$A_4 = 2.364 \text{ IN}^2 \quad (\text{CONTINUED})$$

# 5.123 CONTINUED

$$\bar{y}_5 = \frac{1}{2}(21 - 2 \sin 5.45^\circ) - 3 \sin 5.45^\circ$$

$$= 10.120 \text{ IN}$$

$$\bar{y}_6 = \bar{y}_7 = \frac{1}{3}(21 - 2 \sin 5.45^\circ) - 3 \sin 5.45^\circ$$

$$= 6.652 \text{ IN.}$$

$$\bar{y}_8 = -\frac{3 \sin\left(\frac{169.1^\circ}{2}\right)}{\left(\frac{169.1^\circ}{2} \cdot \frac{\pi}{180^\circ}\right)}$$

$$= -2.024 \text{ IN.}$$

$$\bar{y}_9 = 21 + \frac{5 \sin\left(\frac{190.9^\circ}{2}\right)}{\left(\frac{190.9^\circ}{2} \cdot \frac{\pi}{180^\circ}\right)}$$

$$= 23.99 \text{ IN.}$$

$$\bar{y}_{10} = \bar{y}_{11} = \bar{y}_5 = 10.120 \text{ IN.}$$

$$A_5 = (21 - 2 \sin 5.45^\circ) \cdot 2(5 \cos 5.45^\circ)$$

$$= 207.2 \text{ IN}^2$$

$$A_6 = A_7 = -\frac{1}{2}(2 \cos 5.45^\circ) \cdot (21 - 2 \sin 5.45^\circ)$$

$$= -20.72 \text{ IN}^2$$

$$A_8 = [(169.1^\circ \cdot \frac{\pi}{180^\circ})(3)](5)$$

$$= 44.27 \text{ IN}^2$$

$$A_9 = [(190.9^\circ \cdot \frac{\pi}{180^\circ})(5)](5)$$

$$= 83.30 \text{ IN}^2$$

$$A_{10} = A_{11} = (\sqrt{21^2 + 2^2})(5)$$

$$= 104.52 \text{ IN}^2$$

	A, IN <sup>2</sup>	$\bar{y}$ , IN.	$\bar{z}$ , IN.	$\bar{y}A$ , IN <sup>3</sup>	$\bar{z}A$ , IN <sup>3</sup>
1	13.281	-1.3492	-5	-17.919	-66.41
2	41.65	22.99	-5	957.5	-208.3
3	-0.8509	-0.18995	-5	0.16162	4.255
4	2.364	20.68	-5	48.89	-11.820
5	207.2	10.120	-5	2097	-1036.0
6	-20.72	6.652	-5	-137.83	103.60
7	-20.72	6.652	-5	-137.83	103.60
8	44.27	-2.024	-2.5	-89.60	-110.68
9	83.30	23.99	-2.5	1998.4	-208.3
10	104.52	10.120	-2.5	1057.7	-261.3
11	104.52	10.120	-2.5	1057.7	-261.3
$\Sigma$	558.8			6834	-1952.7

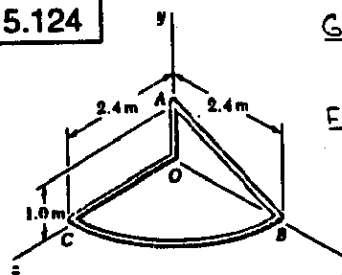
$$\text{HAVE... } \bar{Y} \Sigma A = \Sigma \bar{y}A: \bar{Y}(558.8 \text{ IN}^2) = 6834 \text{ IN}^3$$

$$\text{OR } \bar{Y} = 12.23 \text{ IN.}$$

$$\bar{Z} \Sigma A = \Sigma \bar{z}A: \bar{Z}(558.8 \text{ IN}^2) = -1952.7 \text{ IN}^3$$

$$\text{OR } \bar{Z} = -3.49 \text{ IN.}$$

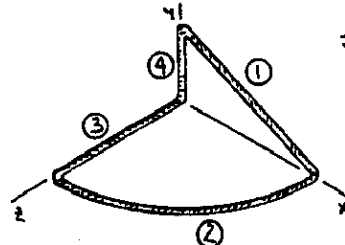
# 5.124



GIVEN: UNIFORM WIRE BENT INTO THE SHAPE SHOWN

FIND: LOCATION OF CENTER OF GRAVITY

FIRST ASSUME THAT THE WIRE IS HOMOGENEOUS SO THAT ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING LINE.



$$\bar{x}_2 = \bar{x}_3 = \frac{2 \cdot 2.4}{\pi} = \frac{4.8}{\pi} \text{ m}$$

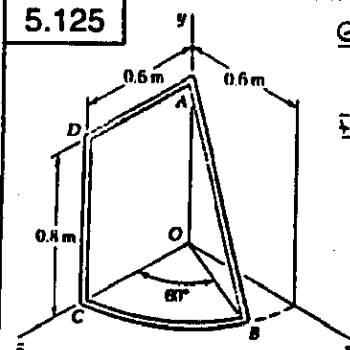
(CONTINUED)

# 5.124 CONTINUED

	L, m	$\bar{x}$ , m	$\bar{y}$ , m	$\bar{z}$ , m	$\bar{x}L$ , m <sup>2</sup>	$\bar{y}L$ , m <sup>2</sup>	$\bar{z}L$ , m <sup>2</sup>
1	2.6	1.2	0.5	0	3.12	1.3	0
2	$\frac{2}{3} \times 2.4 = 1.28$	$\frac{4.8}{\pi}$	0	$\frac{2.4}{\pi}$	5.76	0	5.76
3	2.4	0	0	1.2	0	0	2.88
4	1.0	0	0.5	0	0	0.5	0
$\Sigma$	9.7699				8.88	1.8	8.64

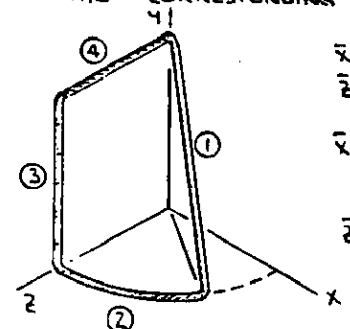
HAVE..  $\bar{X}\Sigma L = \Sigma \bar{x}L$ :  $\bar{X}(9.7699 \text{ m}) = 8.88 \text{ m}^2$   
OR  $\bar{X} = 0.909 \text{ m}$   
 $\bar{Y}\Sigma L = \Sigma \bar{y}L$ :  $\bar{Y}(9.7699 \text{ m}) = 1.8 \text{ m}^2$   
OR  $\bar{Y} = 0.1842 \text{ m}$   
 $\bar{Z}\Sigma L = \Sigma \bar{z}L$ :  $\bar{Z}(9.7699 \text{ m}) = 8.64 \text{ m}^2$   
OR  $\bar{Z} = 0.884 \text{ m}$

# 5.125



GIVEN: UNIFORM WIRE BENT INTO THE SHAPE SHOWN  
FIND: LOCATION OF CENTER OF GRAVITY

FIRST ASSUME THAT THE WIRE IS HOMOGENEOUS SO THAT ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING LINE.

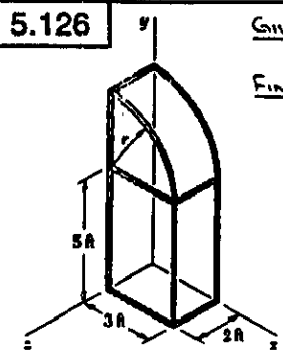


$\bar{x}_1 = 0.3 \sin 60^\circ = 0.15\sqrt{3} \text{ m}$   
 $\bar{z}_1 = 0.3 \cos 60^\circ = 0.15 \text{ m}$   
 $\bar{x}_2 = \left( \frac{0.6 \sin 30^\circ}{\frac{\pi}{6}} \right) \sin 30^\circ = \frac{0.9}{\pi} \text{ m}$   
 $\bar{z}_2 = \left( \frac{0.6 \sin 30^\circ}{\frac{\pi}{6}} \right) \cos 30^\circ = \frac{0.9}{\pi} \sqrt{3} \text{ m}$   
 $L_2 = \left( \frac{\pi}{3} \right) (0.6) = 0.2\pi \text{ m}$

	L, m	$\bar{x}$ , m	$\bar{y}$ , m	$\bar{z}$ , m	$\bar{x}L$ , m <sup>2</sup>	$\bar{y}L$ , m <sup>2</sup>	$\bar{z}L$ , m <sup>2</sup>
1	1.0	0.15 $\sqrt{3}$	0.4	0.15	0.25981	0.4	0.15
2	0.2 $\pi$	$\frac{0.9}{\pi}$	0	$\frac{0.9\sqrt{3}}{\pi}$	0.18	0	0.31177
3	0.8	0	0.4	0.6	0	0.32	0.48
4	0.6	0	0.8	0.3	0	0.48	0.18
$\Sigma$	3.0283				0.43981	1.20	1.12177

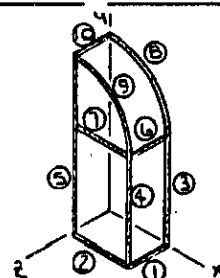
HAVE..  $\bar{X}\Sigma L = \Sigma \bar{x}L$ :  $\bar{X}(3.0283 \text{ m}) = 0.43981 \text{ m}^2$   
OR  $\bar{X} = 0.1452 \text{ m}$   
 $\bar{Y}\Sigma L = \Sigma \bar{y}L$ :  $\bar{Y}(3.0283 \text{ m}) = 1.20 \text{ m}^2$   
OR  $\bar{Y} = 0.396 \text{ m}$   
 $\bar{Z}\Sigma L = \Sigma \bar{z}L$ :  $\bar{Z}(3.0283 \text{ m}) = 1.12177 \text{ m}^2$   
OR  $\bar{Z} = 0.370 \text{ m}$

# 5.126



GIVEN: PORTION OF GREENHOUSE FRAME SHOWN  
FIND: LOCATION OF CENTER OF GRAVITY

FIRST ASSUME THAT THE CHANNELS ARE HOMOGENEOUS SO THAT THE CENTER OF GRAVITY OF THE FRAME WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING LINE.

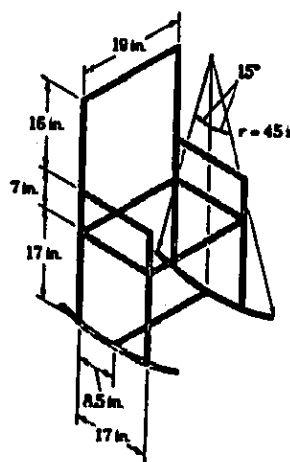


$\bar{x}_B = \bar{x}_A = \frac{2 \times 3}{\pi} = \frac{6}{\pi} \text{ ft}$   
 $\bar{y}_B = \bar{y}_A = 5 + \frac{2 \times 3}{\pi} = 6.9099 \text{ ft}$

	L, ft	$\bar{x}$ , ft	$\bar{y}$ , ft	$\bar{z}$ , ft	$\bar{x}L$ , ft <sup>2</sup>	$\bar{y}L$ , ft <sup>2</sup>	$\bar{z}L$ , ft <sup>2</sup>
1	2	3	0	1	6	0	2
2	3	1.5	0	2	4.5	0	6
3	5	3	2.5	0	15	12.5	0
4	5	3	2.5	2	15	12.5	10
5	8	0	4	2	0	32	16
6	2	3	5	1	6	10	2
7	3	1.5	5	2	4.5	15	6
8	$\frac{2}{3} \times 3 = 4.7124$	$\frac{6}{\pi}$	6.9099	0	9	32.562	0
9	$\frac{2}{3} \times 3 = 4.7124$	$\frac{6}{\pi}$	6.9099	2	9	32.562	9.4248
10	2	0	8	1	0	16	2
$\Sigma$	39.4248				69	163.124	53.4248

HAVE..  $\bar{X}\Sigma L = \Sigma \bar{x}L$ :  $\bar{X}(39.4248 \text{ ft}) = 69 \text{ ft}^2$   
OR  $\bar{X} = 1.750 \text{ ft}$   
 $\bar{Y}\Sigma L = \Sigma \bar{y}L$ :  $\bar{Y}(39.4248 \text{ ft}) = 163.124 \text{ ft}^2$   
OR  $\bar{Y} = 4.14 \text{ ft}$   
 $\bar{Z}\Sigma L = \Sigma \bar{z}L$ :  $\bar{Z}(39.4248 \text{ ft}) = 53.4248 \text{ ft}^2$   
OR  $\bar{Z} = 1.355 \text{ ft}$

# \* 5.127



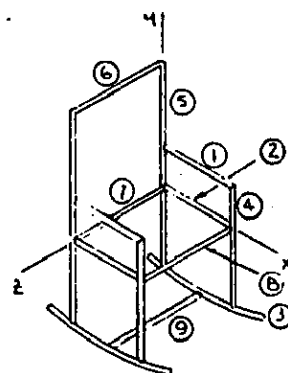
GIVEN: ROCKING CHAIR FRAME SHOWN  
FIND: ANGLE BETWEEN CHAIR BACK AND VERTICAL

FIRST ASSUME THAT THE TUBING IS HOMOGENEOUS SO THAT THE CENTER OF GRAVITY OF THE FRAME WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING LINE. ALSO, NOTE THAT THE CENTER OF GRAVITY MUST LIE ON A VERTICAL LINE THAT PASSES THROUGH THE POINT OF CONTACT OF A

ROCKER AND THE GROUND.

(CONTINUED)

# 5.127 CONTINUED



$$a = \sqrt{45^2 - 8.5^2} \text{ IN.}$$

$$b = 45 - (a - 17)$$

$$= 17.8101 \text{ IN.}$$

$$\bar{y}_3 = -[17 - (\frac{45 \sin 15^\circ}{\pi/12} - a)] \quad L_3 = \frac{\pi}{6} (45)$$

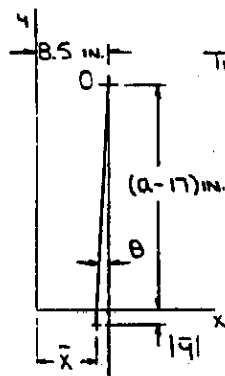
$$= -17.2978 \text{ IN.} \quad = 23.562 \text{ IN.}$$

NOTE: TO ACCOUNT FOR THE TWO SIDES OF THE CHAIR, THE LENGTHS OF MEMBERS 1-5 WILL BE COUNTED TWICE

L, IN.	$\bar{x}$ , IN.	$\bar{y}$ , IN.	$\bar{x}L$ , IN <sup>2</sup>	$\bar{y}L$ , IN <sup>2</sup>
1 2(17)	8.5	7	289	238
2 2(17)	8.5	0	289	0
3 2(23.562)	8.5	-17.2978	400.55	-815.14
4 2(24)	17	-5	816	-240
5 2(40)	0	3	0	240
6 19	0	23	0	437
7 19	0	0	0	0
8 19	17	0	323	0
9 19	8.5	-17.8101	161.5	-338.39
$\Sigma$ 319.124			2279.1	-478.53

HAVE...  $\bar{x}\Sigma L = \Sigma \bar{x}L: \bar{x}(319.124 \text{ IN.}) = 2279.1 \text{ IN}^2$   
OR  $\bar{x} = 7.1417 \text{ IN.}$

$\bar{y}\Sigma L = \Sigma \bar{y}L: \bar{y}(319.124 \text{ IN.}) = -478.53 \text{ IN}^2$   
OR  $\bar{y} = -1.49951 \text{ IN.}$



THEN...  $\tan \theta = \frac{8.5 - 7.1417}{(45^2 - 8.5^2 - 17) \cdot 1.49951}$

$$= 0.047345$$

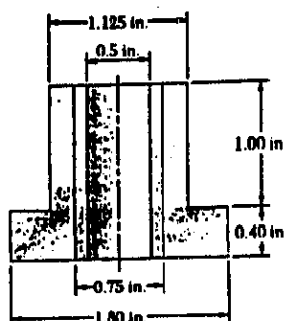
OR  $\theta = 2.7106^\circ$

∴ THE ANGLE FORMED BY THE BACK OF THE CHAIR AND THE VERTICAL IS

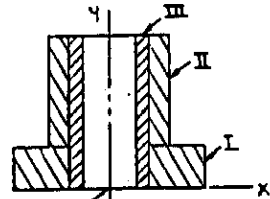
2.71°

# 5.128

GIVEN: BRONZE BUSHING AND STEEL SLEEVE SHOWN  
 $\gamma_{BR} = 0.318 \text{ lb/IN}^3$   
 $\gamma_{ST} = 0.284 \text{ lb/IN}^3$   
FIND: LOCATION OF CENTER OF GRAVITY



FIRST NOTE THAT SYMMETRY IMPLIES  $\bar{x} = \bar{z} = 0$



NOW...  $W = \gamma V$

$$\bar{y}_I = 0.20 \text{ IN.} \quad W_I = 0.284 \frac{\text{lb}}{\text{IN}^3} \cdot \frac{\pi}{4} (1.0^2 - 0.75^2) \text{ IN}^2 = 0.4 \text{ IN.}$$

$$= 0.23889 \text{ lb}$$

$$\bar{y}_{II} = 0.90 \text{ IN.} \quad W_{II} = 0.284 \frac{\text{lb}}{\text{IN}^3} \cdot \frac{\pi}{4} (1.125^2 - 0.75^2) \text{ IN}^2 = 1 \text{ IN.}$$

$$= 0.156834 \text{ lb}$$

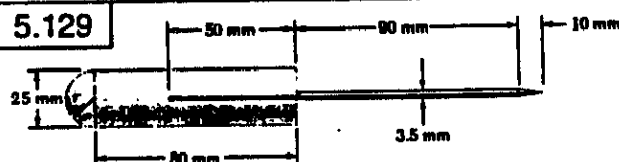
$$\bar{y}_{III} = 0.70 \text{ IN.} \quad W_{III} = 0.318 \frac{\text{lb}}{\text{IN}^3} \cdot \frac{\pi}{4} (0.75^2 - 0.5^2) \text{ IN}^2 = 1.4 \text{ IN.}$$

$$= 0.109269 \text{ lb}$$

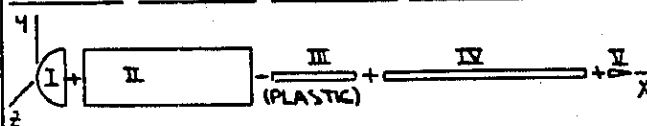
	W, lb	$\bar{y}$ , IN.	$\bar{y}W$ , IN·lb
I	0.23889	0.20	0.047778
II	0.156834	0.90	0.141151
III	0.109269	0.70	0.076488
$\Sigma$	0.50499		0.26542

HAVE...  $\bar{y}\Sigma W = \Sigma \bar{y}W: \bar{y}(0.50499 \text{ lb}) = 0.26542 \text{ IN} \cdot \text{lb}$   
OR  $\bar{y} = 0.526 \text{ IN. (ABOVE BASE)}$

# 5.129



GIVEN: AWW HAVING PLASTIC HANDLE AND STEEL BLADE,  $\rho_R = 1030 \text{ kg/m}^3$ ,  $\rho_{ST} = 7860 \text{ kg/m}^3$   
FIND: LOCATION OF CENTER OF GRAVITY



FIRST NOTE THAT SYMMETRY IMPLIES  $\bar{y} = \bar{z} = 0$

$$\bar{x}_I = \frac{\pi}{8} (12.5) = 7.8125 \text{ mm} \quad m_I = \rho_R V_I = 1030 \frac{\text{kg}}{\text{m}^3} \cdot \frac{\pi}{8} (0.0125 \text{ m})^3$$

$$= 4.2133 \times 10^{-3} \text{ kg}$$

$$\bar{x}_{II} = 52.5 \text{ mm} \quad m_{II} = \rho_R V_{II} = 1030 \frac{\text{kg}}{\text{m}^3} \cdot \frac{\pi}{4} (0.025 \text{ m})^2 (0.08 \text{ m})$$

$$= 40.448 \times 10^{-3} \text{ kg}$$

$$\bar{x}_{III} = 92.5 - 25 = 67.5 \text{ mm} \quad m_{III} = \rho_R V_{III} = -1030 \frac{\text{kg}}{\text{m}^3} \cdot \frac{\pi}{4} (0.0035 \text{ m})^2 (0.05 \text{ m})$$

$$= -0.49549 \times 10^{-3} \text{ kg}$$

(CONTINUED)



# 5.129 CONTINUED

$$\bar{x}_{II} = 182.5 - 70 = 112.5 \text{ mm}$$

$$m_{II} = \rho_{st} V_{II} = 7860 \frac{\text{kg}}{\text{m}^3} \cdot \frac{\pi}{4} (0.0035 \text{ m})^2 (0.14 \text{ m}) = 10.5871 \times 10^{-3} \text{ kg}$$

$$\bar{x}_{III} = 182.5 + \frac{1}{4}(10) = 185 \text{ mm}$$

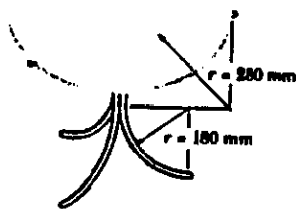
$$m_{III} = \rho_{st} V_{III} = 7860 \frac{\text{kg}}{\text{m}^3} \cdot \frac{\pi}{4} (0.0075 \text{ m})^2 (0.01 \text{ m}) = 0.25207 \times 10^{-3} \text{ kg}$$

m, kg	$\bar{x}$ , mm	$\bar{x}m$ , kg·mm
I 4.2133 × 10 <sup>-3</sup>	7.8125	32.916 × 10 <sup>-3</sup>
II 10.448 × 10 <sup>-3</sup>	52.5	2123.5 × 10 <sup>-3</sup>
III -0.49549 × 10 <sup>-3</sup>	67.5	-33.447 × 10 <sup>-3</sup>
IV 10.5871 × 10 <sup>-3</sup>	112.5	1191.05 × 10 <sup>-3</sup>
V 0.25207 × 10 <sup>-3</sup>	185	46.633 × 10 <sup>-3</sup>
Σ 55.005 × 10 <sup>-3</sup>		3360.7 × 10 <sup>-3</sup>

HAVE...  $\bar{X} \Sigma m = \Sigma \bar{x}m$ :  $\bar{X}(55.005 \times 10^{-3} \text{ kg}) = 3360.7 \times 10^{-3} \text{ kg} \cdot \text{mm}$   
OR  $\bar{X} = 61.098 \text{ mm}$

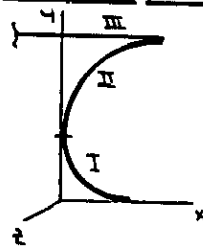
∴ THE CENTER OF GRAVITY IS 61.1 mm FROM THE END OF THE HANDLE.

# 5.130



GIVEN: TABLE WITH GLASS TOP ( $\rho_L = 2190 \text{ kg/m}^3$ ) AND STEEL TUBING LEGS ( $\rho_{st} = 7860 \text{ kg/m}^3$ ),  $d_{\text{TOP}} = 600 \text{ mm}$ ,  $t_{\text{TOP}} = 10 \text{ mm}$ ,  $A_{\text{TUBING}} = 150 \text{ mm}^2$  ( $d_{\text{O TUBING}} = 24 \text{ mm}$ )

FINN: LOCATION OF CENTER OF GRAVITY



FIRST NOTE THAT SYMMETRY IMPLIES  $\bar{x} = \bar{z} = 0$

ALSO, TO ACCOUNT FOR THE THREE LEGS, THE MASSES OF COMPONENTS I AND II WILL EACH BE MULTIPLIED BY THREE.

$$\bar{y}_I = 12 + 180 - \frac{2 \cdot 180}{\pi} = 77.408 \text{ mm}$$

$$\bar{y}_{II} = 12 + 180 + \frac{2 \cdot 280}{\pi} = 370.25 \text{ mm}$$

$$\bar{y}_{III} = 24 + 180 + 280 + 5 = 489 \text{ mm}$$

$$m_I = \rho_{st} V_I = 7860 \frac{\text{kg}}{\text{m}^3} \cdot \frac{\pi}{2} (0.180 \text{ m})^2 \cdot \frac{1}{2} (0.180 \text{ m}) = 0.33335 \text{ kg}$$

$$m_{II} = \rho_{st} V_{II} = 7860 \frac{\text{kg}}{\text{m}^3} \cdot \frac{\pi}{2} (0.280 \text{ m})^2 \cdot \frac{1}{2} (0.280 \text{ m}) = 0.51855 \text{ kg}$$

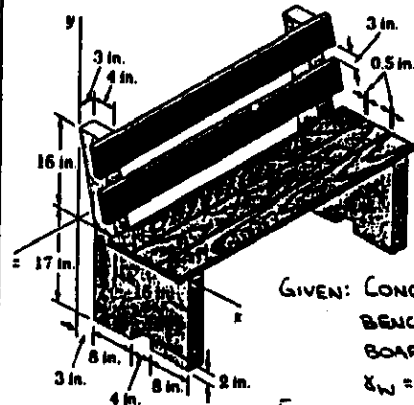
$$m_{III} = \rho_L V_{III} = 2190 \frac{\text{kg}}{\text{m}^3} \cdot \frac{\pi}{4} (0.6 \text{ m})^2 \cdot 0.010 \text{ m} = 6.1921 \text{ kg}$$

m, kg	$\bar{y}$ , mm	$\bar{y}m$ , kg·mm
I 3(0.33335)	77.408	77.412
II 3(0.51855)	370.25	575.98
III 6.1921	489	3027.9
Σ 8.7478		3681.3

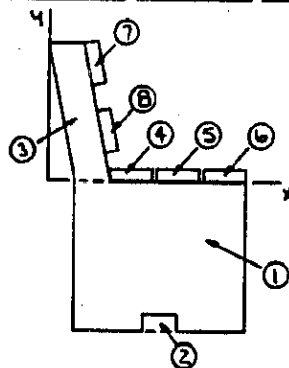
HAVE...  $\bar{Y} \Sigma m = \Sigma \bar{y}m$ :  $\bar{Y}(8.7478 \text{ kg}) = 3681.3 \text{ kg} \cdot \text{mm}$   
OR  $\bar{Y} = 420.8 \text{ mm}$

∴ THE CENTER OF GRAVITY IS 421 mm ABOVE THE FLOOR.

# \* 5.131



GIVEN: CONCRETE AND WOOD BENCH,  $1\frac{1}{2} \times 5 \times 48$ -IN. BOARDS,  $\gamma_c = 0.084 \text{ lb/in}^3$ ,  $\gamma_w = 0.017 \text{ lb/in}^3$   
FIND:  $\bar{x}$  AND  $\bar{y}$  COORDINATES OF CENTER OF GRAVITY



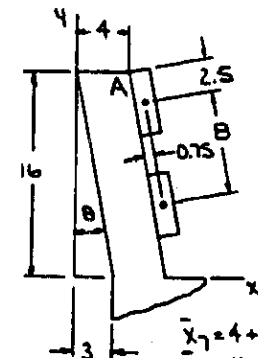
FIRST NOTE TO ACCOUNT FOR THE TWO CONCRETE ENDS, THE WEIGHTS OF COMPONENTS 1-3 WILL BE COUNTED TWICE.

$$W_1 = \gamma_c V_1 = 0.084 \frac{\text{lb}}{\text{in}^3} \cdot (20 \times 17 \times 3) \text{ in}^3 = 85.68 \text{ lb}$$

$$W_2 = \gamma_c V_2 = -0.084 \frac{\text{lb}}{\text{in}^3} \cdot (4 \times 2 \times 3) \text{ in}^3 = -2.016 \text{ lb}$$

$$W_3 = \gamma_c V_3 = 0.084 \frac{\text{lb}}{\text{in}^3} \cdot (4 \times 16 \times 3) \text{ in}^3 = 16.128 \text{ lb}$$

$$W_4 = W_5 = W_6 = W_7 = W_8 = \gamma_w V_{\text{boards}} = 0.017 \frac{\text{lb}}{\text{in}^3} \cdot (5 \times 1\frac{1}{2} \times 48) \text{ in}^3 = 6.12 \text{ lb}$$



ALL DIMENSIONS IN IN.

$$\tan \theta = \frac{3}{16} \Rightarrow \theta = 10.619^\circ$$

$$\bar{x}_7 = 4 + 2.5 \sin \theta + 0.75 \cos \theta = 5.1979 \text{ in.}$$

$$\bar{y}_7 = 16 - 2.5 \cos \theta + 0.75 \sin \theta = 13.6810 \text{ in.}$$

$$\bar{x}_8 = \bar{x}_7 + 8 \sin \theta = 6.6722 \text{ in.} \quad \bar{y}_8 = \bar{y}_7 - 8 \cos \theta = 5.8180 \text{ in.}$$

	W, lb	$\bar{x}$ , in.	$\bar{y}$ , in.	$\bar{x}W$ , in·lb	$\bar{y}W$ , in·lb
1	2(85.68)	13	-8.5	2227.7	-1456.56
2	2(-2.016)	13	-16	-52.416	64.512
3	2(16.128)	3.5	8	112.896	258.05
4	6.12	9.5	0.75	58.14	4.59
5	6.12	15	0.75	91.8	4.59
6	6.12	20.5	0.75	125.46	4.59
7	6.12	5.1979	13.6810	31.811	83.728
8	6.12	6.6722	5.8180	40.834	35.606

(CONTINUED)

# 5.131 CONTINUED

THEN..  $\Sigma W = 230.18 \text{ lb}$

$$\Sigma \bar{r}W = 2636.2 \text{ in}\cdot\text{lb} \quad \Sigma \bar{y}W = -1000.89 \text{ in}\cdot\text{lb}$$

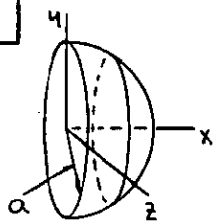
NOW..  $\bar{X}\Sigma W = \Sigma \bar{r}W$ :  $\bar{X}(230.18 \text{ lb}) = 2636.2 \text{ in}\cdot\text{lb}$

$$\text{OR } \bar{X} = 11.45 \text{ in.} \quad \blacktriangleleft$$

$\bar{Y}\Sigma W = \Sigma \bar{y}W$ :  $\bar{Y}(230.18 \text{ lb}) = -1000.89 \text{ in}\cdot\text{lb}$

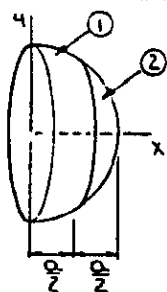
$$\text{OR } \bar{Y} = -4.35 \text{ in.} \quad \blacktriangleleft$$

# 5.132



GIVEN: A HEMISPHERE WHICH IS CUT INTO TWO COMPONENTS OF EQUAL WIDTH AS SHOWN

FIND:  $\bar{X}$  OF EACH COMPONENT



CHOOSE AS THE ELEMENT OF VOLUME A DISK OF RADIUS  $r$  AND THICKNESS  $dx$ . THEN

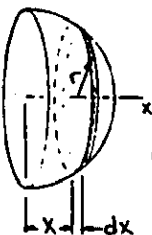
$$dV = \pi r^2 dx, \quad \bar{x}_{EL} = x$$

THE EQUATION OF THE GENERATING CURVE IS

$$x^2 + y^2 = a^2 \text{ SO THAT}$$

$$r^2 = a^2 - x^2 \text{ AND THEN}$$

$$dV = \pi(a^2 - x^2) dx$$



COMPONENT 1

$$V_1 = \int_0^{a/2} \pi(a^2 - x^2) dx = \pi[a^2x - \frac{x^3}{3}]_0^{a/2}$$

$$= \frac{11}{24} \pi a^3$$

$$\text{AND.. } \int \bar{x}_{EL} dV = \int_0^{a/2} x[\pi(a^2 - x^2) dx]$$

$$= \pi[a^2 \frac{x^2}{2} - \frac{x^4}{4}]_0^{a/2}$$

$$= \frac{7}{64} \pi a^4$$

$$\text{NOW.. } \bar{x}_1 V_1 = \int \bar{x}_{EL} dV: \bar{x}_1 (\frac{11}{24} \pi a^3) = \frac{7}{64} \pi a^4$$

$$\text{OR } \bar{x}_1 = \frac{21}{88} a \quad \blacktriangleleft$$

COMPONENT 2

$$V_2 = \int_{a/2}^a \pi(a^2 - x^2) dx = \pi[a^2x - \frac{x^3}{3}]_{a/2}^a$$

$$= \pi[a^2(a) - \frac{a^3}{3}] - [a^2(\frac{a}{2}) - \frac{(\frac{a}{2})^3}{3}]$$

$$= \frac{5}{24} \pi a^3$$

$$\text{AND } \int \bar{x}_{EL} dV = \int_{a/2}^a x[\pi(a^2 - x^2) dx] = \pi[a^2 \frac{x^2}{2} - \frac{x^4}{4}]_{a/2}^a$$

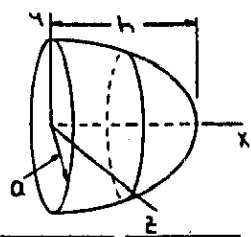
$$= \pi[a^2(\frac{a^2}{2}) - \frac{a^4}{4}] - [a^2(\frac{(\frac{a}{2})^2}{2}) - \frac{(\frac{a}{2})^4}{4}]$$

$$= \frac{9}{64} \pi a^4$$

$$\text{NOW.. } \bar{x}_2 V_2 = \int \bar{x}_{EL} dV: \bar{x}_2 (\frac{5}{24} \pi a^3) = \frac{9}{64} \pi a^4$$

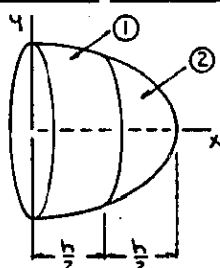
$$\text{OR } \bar{x}_2 = \frac{27}{40} a \quad \blacktriangleleft$$

# 5.133



GIVEN: A SEMIELLIPSOID OF REVOLUTION WHICH IS CUT INTO TWO COMPONENTS OF EQUAL WIDTH AS SHOWN

FIND:  $\bar{X}$  OF EACH COMPONENT



CHOOSE AS THE ELEMENT OF VOLUME A DISK OF RADIUS  $r$  AND THICKNESS  $dx$ . THEN

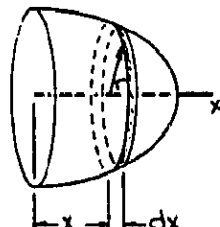
$$dV = \pi r^2 dx, \quad \bar{x}_{EL} = x$$

THE EQUATION OF THE GENERATING CURVE IS

$$\frac{x^2}{h^2} + \frac{y^2}{a^2} = 1 \text{ SO THAT}$$

$$r^2 = \frac{a^2}{h^2}(h^2 - x^2) \text{ AND THEN}$$

$$dV = \pi \frac{a^2}{h^2}(h^2 - x^2) dx$$



COMPONENT 1

$$V_1 = \int_0^{h/2} \pi \frac{a^2}{h^2}(h^2 - x^2) dx$$

$$= \pi \frac{a^2}{h^2} [h^2x - \frac{x^3}{3}]_0^{h/2}$$

$$= \frac{11}{24} \pi a^2 h$$

$$\text{AND } \int \bar{x}_{EL} dV = \int_0^{h/2} x[\pi \frac{a^2}{h^2}(h^2 - x^2) dx]$$

$$= \pi \frac{a^2}{h^2} [h^2 \frac{x^2}{2} - \frac{x^4}{4}]_0^{h/2}$$

$$= \frac{7}{64} \pi a^2 h^2$$

$$\text{NOW.. } \bar{x}_1 V_1 = \int \bar{x}_{EL} dV: \bar{x}_1 (\frac{11}{24} \pi a^2 h) = \frac{7}{64} \pi a^2 h^2$$

$$\text{OR } \bar{x}_1 = \frac{21}{88} h \quad \blacktriangleleft$$

COMPONENT 2

$$V_2 = \int_{h/2}^h \pi \frac{a^2}{h^2}(h^2 - x^2) dx = \pi \frac{a^2}{h^2} [h^2x - \frac{x^3}{3}]_{h/2}^h$$

$$= \pi \frac{a^2}{h^2} \{ [h^2(h) - \frac{(h)^3}{3}] - [h^2(\frac{h}{2}) - \frac{(\frac{h}{2})^3}{3}] \}$$

$$= \frac{5}{24} \pi a^2 h$$

$$\text{AND } \int \bar{x}_{EL} dV = \int_{h/2}^h x[\pi \frac{a^2}{h^2}(h^2 - x^2) dx]$$

$$= \pi \frac{a^2}{h^2} [h^2 \frac{x^2}{2} - \frac{x^4}{4}]_{h/2}^h$$

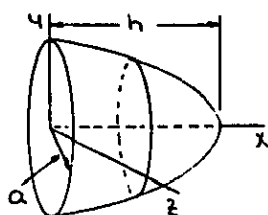
$$= \pi \frac{a^2}{h^2} \{ [h^2(\frac{h^2}{2}) - \frac{(h)^4}{4}] - [h^2(\frac{(h/2)^2}{2}) - \frac{(\frac{h}{2})^4}{4}] \}$$

$$= \frac{9}{64} \pi a^2 h^2$$

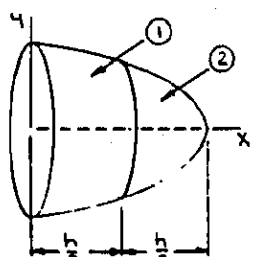
$$\text{NOW.. } \bar{x}_2 V_2 = \int \bar{x}_{EL} dV: \bar{x}_2 (\frac{5}{24} \pi a^2 h) = \frac{9}{64} \pi a^2 h^2$$

$$\text{OR } \bar{x}_2 = \frac{27}{40} h \quad \blacktriangleleft$$

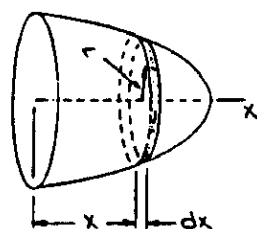
5.134



GIVEN: A PARABOLOID OF REVOLUTION WHICH IS CUT INTO TWO COMPONENTS OF EQUAL WIDTH AS SHOWN  
FIND:  $\bar{x}$  OF EACH COMPONENT



CHOOSE AS THE ELEMENT OF VOLUME A DISK OF RADIUS  $r$  AND THICKNESS  $dx$ . THEN  
 $dV = \pi r^2 dx$ ,  $\bar{x}_{EL} = x$   
 THE EQUATION OF THE GENERATING CURVE IS  $x = h - \frac{h}{a^2} z^2$  SO THAT  
 $r^2 = \frac{a^2}{h} (h - x)$  AND THEN  
 $dV = \pi \frac{a^2}{h} (h - x) dx$



COMPONENT 1

$$V_1 = \int_0^{h/2} \pi \frac{a^2}{h} (h - x) dx$$

$$= \pi \frac{a^2}{h} \left[ hx - \frac{x^2}{2} \right]_0^{h/2}$$

$$= \frac{3}{8} \pi a^2 h$$

$$\text{AND... } \int \bar{x}_{EL} dV = \int_0^{h/2} x \left( \pi \frac{a^2}{h} (h - x) \right) dx = \pi \frac{a^2}{h} \left[ h \frac{x^2}{2} - \frac{x^3}{3} \right]_0^{h/2}$$

$$= \frac{1}{12} \pi a^2 h^2$$

$$\text{NOW... } \bar{x}_1 V_1 = \int \bar{x}_{EL} dV: \bar{x}_1 \left( \frac{3}{8} \pi a^2 h \right) = \frac{1}{12} \pi a^2 h^2$$

$$\text{OR } \bar{x}_1 = \frac{2}{5} h$$

COMPONENT 2

$$V_2 = \int_{h/2}^h \pi \frac{a^2}{h} (h - x) dx = \pi \frac{a^2}{h} \left[ hx - \frac{x^2}{2} \right]_{h/2}^h$$

$$= \pi \frac{a^2}{h} \left[ h(h) - \frac{(h)^2}{2} \right] - \left[ h \left( \frac{h}{2} \right) - \frac{(h/2)^2}{2} \right]$$

$$= \frac{5}{8} \pi a^2 h$$

$$\text{AND... } \int \bar{x}_{EL} dV = \int_{h/2}^h x \left( \pi \frac{a^2}{h} (h - x) \right) dx = \pi \frac{a^2}{h} \left[ h \frac{x^2}{2} - \frac{x^3}{3} \right]_{h/2}^h$$

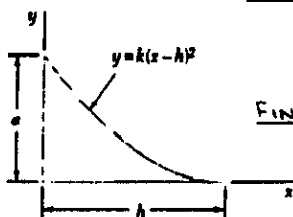
$$= \pi \frac{a^2}{h} \left[ h \left( \frac{h^2}{2} \right) - \frac{(h)^3}{3} \right] - \left[ h \left( \frac{h^2}{2} \right) - \frac{(h/2)^3}{3} \right]$$

$$= \frac{1}{12} \pi a^2 h^2$$

$$\text{NOW... } \bar{x}_2 V_2 = \int \bar{x}_{EL} dV: \bar{x}_2 \left( \frac{5}{8} \pi a^2 h \right) = \frac{1}{12} \pi a^2 h^2$$

$$\text{OR } \bar{x}_2 = \frac{2}{5} h$$

5.135

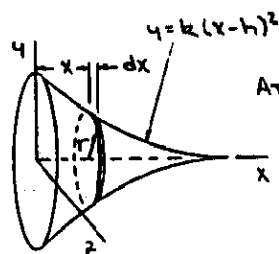


GIVEN: VOLUME GENERATED BY ROTATING THE AREA SHOWN ABOUT THE X AXIS

FIND: LOCATION OF THE CENTROID OF THE VOLUME

FIRST NOTE THAT SYMMETRY IMPLIES  
 (CONTINUES)

5.135 CONTINUED



$\bar{y} = 0$   
 $\bar{z} = 0$

At  $x=0$ ,  $y=a$ :  $a = k(-h)^2$   
 OR  $k = a/h^2$

CHOOSE AS THE ELEMENT OF VOLUME A DISK OF RADIUS  $r$  AND THICKNESS  $dx$ . THEN

$$dV = \pi r^2 dx, \bar{x}_{EL} = x$$

$$\text{NOW } r = \frac{a}{h^2} (x-h)^2 \text{ SO THAT}$$

$$dV = \pi \frac{a^2}{h^4} (x-h)^4 dx$$

$$\text{THEN... } V = \int_0^h \pi \frac{a^2}{h^4} (x-h)^4 dx = \frac{\pi}{5} \frac{a^2}{h^4} [x-h]^5 \Big|_0^h$$

$$= \frac{1}{5} \pi a^2 h$$

$$\text{AND } \int \bar{x}_{EL} dV = \int_0^h x \left( \pi \frac{a^2}{h^4} (x-h)^4 \right) dx$$

$$= \pi \frac{a^2}{h^4} \int_0^h (x^5 - 4hx^4 + 6h^2x^3 - 4h^3x^2 + h^4x) dx$$

$$= \pi \frac{a^2}{h^4} \left[ \frac{x^6}{6} - \frac{4}{5}hx^5 + \frac{3}{2}h^2x^4 - \frac{4}{3}h^3x^3 + \frac{1}{2}h^4x^2 \right]_0^h$$

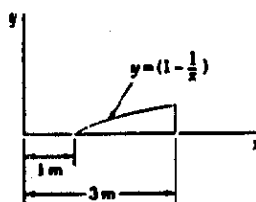
$$= \frac{1}{30} \pi a^2 h^2$$

$$\text{NOW... } \bar{x} V = \int \bar{x}_{EL} dV: \bar{x} \left( \frac{1}{5} \pi a^2 h \right) = \frac{1}{30} \pi a^2 h^2$$

$$\text{OR } \bar{x} = \frac{1}{6} h$$

5.136

GIVEN: VOLUME GENERATED BY ROTATING THE AREA SHOWN ABOUT THE X AXIS  
FIND: LOCATION OF THE CENTROID OF THE VOLUME



FIRST NOTE THAT SYMMETRY IMPLIES

$\bar{y} = 0$   
 $\bar{z} = 0$

CHOOSE AS THE ELEMENT OF VOLUME A DISK OF RADIUS  $r$  AND THICKNESS  $dx$ . THEN

$$dV = \pi r^2 dx, \bar{x}_{EL} = x$$

$$\text{NOW } r = 1 - \frac{x}{2} \text{ SO THAT}$$

$$dV = \pi \left( 1 - \frac{x}{2} \right)^2 dx$$

$$= \pi \left( 1 - \frac{x}{2} + \frac{x^2}{4} \right) dx$$

$$\text{THEN... } V = \int_0^2 \pi \left( 1 - \frac{x}{2} + \frac{x^2}{4} \right) dx = \pi \left[ x - \frac{x^2}{2} + \frac{x^3}{12} \right]_0^2$$

$$= \pi \left[ (2 - 2 + \frac{8}{12}) - (0 - 0 + 0) \right]$$

$$= 0.46944 \pi \text{ m}^3$$

$$\text{AND } \int \bar{x}_{EL} dV = \int_0^2 x \left( \pi \left( 1 - \frac{x}{2} + \frac{x^2}{4} \right) \right) dx = \pi \left[ \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{16} \right]_0^2$$

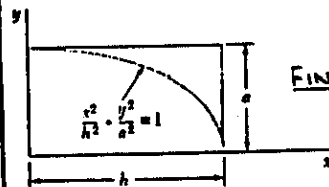
$$= \pi \left[ \left( \frac{4}{2} - \frac{8}{6} + \frac{16}{16} \right) - (0 - 0 + 0) \right]$$

$$= 1.09861 \pi \text{ m}^4$$

$$\text{NOW... } \bar{x} V = \int \bar{x}_{EL} dV: \bar{x} (0.46944 \pi \text{ m}^3) = 1.09861 \pi \text{ m}^4$$

$$\text{OR } \bar{x} = 2.34 \text{ m}$$

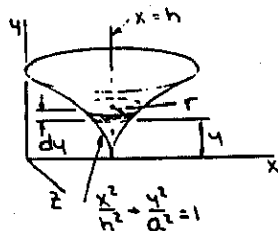
5.137



GIVEN: VOLUME GENERATED BY ROTATING THE AREA SHOWN ABOUT THE LINE  $x=h$

FIND: LOCATION OF THE CENTROID OF THE VOLUME

FIRST NOTE THAT SYMMETRY IMPLIES  $\bar{x}=h$   
 $\bar{z}=0$



CHOOSE AS THE ELEMENT OF VOLUME A DISK OF RADIUS  $r$  AND THICKNESS  $dy$ . THEN  
 $dV = \pi r^2 dy$ .  $\bar{y}_{EL} = y$   
NOW  $x^2 = a^2 - y^2$

SO THAT  
 $r = h - \frac{1}{a} \sqrt{a^2 - y^2}$

THEN  $dV = \pi \frac{h^2}{a^2} (a - \sqrt{a^2 - y^2})^2 dy$

AND  $V = \int_0^a \pi \frac{h^2}{a^2} (a - \sqrt{a^2 - y^2})^2 dy$

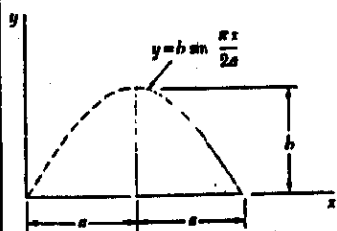
LET  $y = a \sin \theta \Rightarrow dy = a \cos \theta d\theta$   
THEN  $V = \pi \frac{h^2}{a^2} \int_0^{\pi/2} (a - \sqrt{a^2 - a^2 \sin^2 \theta})^2 a \cos \theta d\theta$   
 $= \pi \frac{h^2}{a^2} \int_0^{\pi/2} (a^2 - 2a(a \cos \theta) + (a^2 - a^2 \sin^2 \theta)) a \cos \theta d\theta$   
 $= \pi a h^2 \int_0^{\pi/2} (2 \cos \theta - 2 \cos \theta - \sin^2 \theta \cos \theta) d\theta$   
 $= \pi a h^2 [2 \sin \theta - 2(\frac{\theta}{2} + \frac{\sin 2\theta}{4}) - \frac{1}{3} \sin^3 \theta]_0^{\pi/2}$   
 $= \pi a h^2 [2 - 2(\frac{\pi/2}{2}) - \frac{1}{3}]$   
 $= 0.095870 \pi a h^2$

AND  $\bar{y}_{EL} dV = \int_0^a y [\pi \frac{h^2}{a^2} (a - \sqrt{a^2 - y^2})^2 dy]$   
 $= \pi \frac{h^2}{a^2} \int_0^a (2a^2 y - 2ay \sqrt{a^2 - y^2} - y^3) dy$   
 $= \pi \frac{h^2}{a^2} [a^2 y^2 + \frac{2}{3} a (a^2 - y^2)^{3/2} - \frac{1}{4} y^4]_0^a$   
 $= \pi \frac{h^2}{a^2} [a^3 - \frac{2}{3} a^3] - [\frac{1}{4} a^4]$   
 $= \frac{1}{12} \pi a^2 h^2$

NOW  $\bar{y} V = \bar{y}_{EL} dV: y(0.095870 \pi a h^2) = \frac{1}{12} \pi a^2 h^2$

OR  $\bar{y} = 0.869a$

5.138



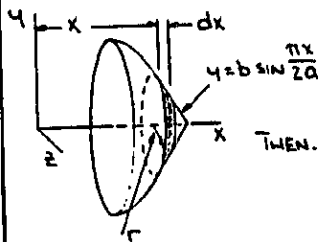
GIVEN: VOLUME GENERATED BY ROTATING THE AREA SHOWN ABOUT THE X AXIS

FIND: LOCATION OF THE CENTROID OF THE VOLUME

FIRST NOTE THAT SYMMETRY IMPLIES  $\bar{y}=0$   
 $\bar{z}=0$

CHOOSE AS THE ELEMENT OF VOLUME A DISK OF RADIUS  $r$  AND THICKNESS  $dx$ . THEN  
(CONTINUED)

5.138 CONTINUED



$dV = \pi r^2 dx$ ,  $\bar{y}_{EL} = x$   
NOW  $r = b \sin \frac{\pi x}{2a}$

SO THAT  
 $dV = \pi b^2 \sin^2 \frac{\pi x}{2a} dx$   
THEN  $V = \int_0^{2a} \pi b^2 \sin^2 \frac{\pi x}{2a} dx$   
 $= \pi b^2 [\frac{x}{2} - \frac{\sin \frac{\pi x}{a}}{2 \frac{\pi}{a}}]_0^{2a}$   
 $= \pi b^2 [(\frac{2a}{2}) - (\frac{a}{2})]$   
 $= \frac{1}{2} \pi a b^2$

AND  $\bar{x} dV = \int_0^{2a} x [\pi b^2 \sin^2 \frac{\pi x}{2a} dx]$

USE INTEGRATION BY PARTS WITH

$u = x$   $dv = \sin^2 \frac{\pi x}{2a}$   
 $du = dx$   $v = \frac{x}{2} - \frac{\sin \frac{\pi x}{a}}{2 \frac{\pi}{a}}$

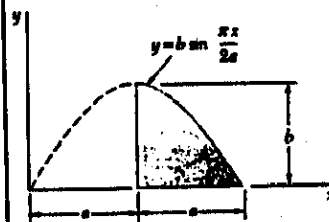
THEN  $\bar{x} dV = \pi b^2 \{ [x(\frac{x}{2} - \frac{\sin \pi x/a}{2 \pi/a})]_0^{2a} - [\frac{2a}{2} (\frac{x}{2} - \frac{\sin \pi x/a}{2 \pi/a})] dx \}$   
 $= \pi b^2 \{ [2a(\frac{2a}{2}) - a(\frac{a}{2})] - [\frac{1}{2} x^2 + \frac{a^2}{2 \pi^2} \cos \frac{\pi x}{a}]_0^{2a} \}$   
 $= \pi b^2 \{ (\frac{2}{2} a^2) - [\frac{1}{2} (2a)^2 + \frac{a^2}{2 \pi^2} - \frac{1}{2} (a)^2 + \frac{a^2}{2 \pi^2}] \}$   
 $= \pi a^2 b^2 (\frac{3}{4} - \frac{\pi^2}{2})$   
 $= 0.64868 \pi a^2 b^2$

NOW  $\bar{x} V = \bar{x}_{EL} dV: \bar{x} (\frac{1}{2} \pi a b^2) = 0.64868 \pi a^2 b^2$   
OR  $\bar{x} = 1.297a$

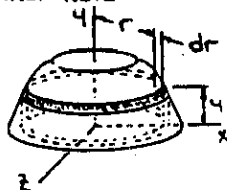
5.139

GIVEN: VOLUME GENERATED BY ROTATING THE AREA SHOWN ABOUT THE Y AXIS

FIND: LOCATION OF THE CENTROID OF THE VOLUME



FIRST NOTE THAT SYMMETRY IMPLIES  $\bar{x}=0$   
 $\bar{z}=0$



CHOOSE AS THE ELEMENT OF VOLUME A CYLINDRICAL SHELL OF RADIUS  $r$ , THICKNESS  $dr$ , AND HEIGHT  $y$ . THEN  
 $dV = (2\pi r)(y)dr$ ,  $\bar{y}_{EL} = \frac{1}{2} y$

NOW  $y = b \sin \frac{\pi x}{2a}$  SO THAT  
 $dV = 2\pi b r \sin \frac{\pi x}{2a} dr$

THEN  $V = \int_0^{2a} 2\pi b r \sin \frac{\pi x}{2a} dr$

USE INTEGRATION BY PARTS WITH

$u = r$   $dv = \sin \frac{\pi x}{2a}$   
 $du = dr$   $v = -\frac{2a}{\pi} \cos \frac{\pi x}{2a}$

THEN  $V = 2\pi b [ (rX - \frac{2a}{\pi} \cos \frac{\pi x}{2a}) ]_0^{2a}$   
 $= 2\pi b [ -\frac{2a}{\pi} ((2a)(-1)) + [\frac{4a^2}{\pi} \sin \frac{\pi x}{2a}]_0^{2a} ]$   
(CONTINUED)

# 5.139 CONTINUED

$$V = 2\pi b \left( \frac{4a^2}{\pi} - \frac{4a^2}{\pi} \right) = 8a^2b \left( 1 - \frac{1}{\pi} \right) = 5.4535 a^2b$$

$$\text{ALSO } \int \bar{y}_{\text{el}} dV = \int_a^{2a} \left( \frac{1}{2} b \sin \frac{\pi r}{2a} \right) (2\pi b r \sin \frac{\pi r}{2a} dr) = \pi b^2 \int_a^{2a} r \sin^2 \frac{\pi r}{2a} dr$$

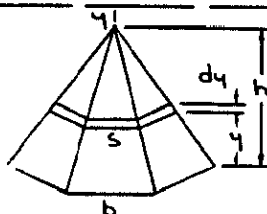
USE INTEGRATION BY PARTS WITH  
 $u = r$   $dv = \sin^2 \frac{\pi r}{2a} dr$   
 $du = dr$   $v = \frac{r}{2} - \frac{\sin \frac{\pi r}{a}}{2\pi/a}$

$$\begin{aligned} \text{THEN.. } \int \bar{y}_{\text{el}} dV &= \pi b^2 \left\{ \left( r \right) \left( \frac{r}{2} - \frac{\sin \frac{\pi r}{a}}{2\pi/a} \right) \right\}_a^{2a} \\ &\quad - \int_a^{2a} \left( \frac{r}{2} - \frac{\sin \frac{\pi r}{a}}{2\pi/a} \right) dr \} \\ &= \pi b^2 \left\{ \left( (2a) \left( \frac{2a}{2} \right) - (a) \left( \frac{a}{2} \right) \right) \right. \\ &\quad \left. - \left[ \frac{r^2}{4} + \frac{a^2}{2\pi^2} \cos \frac{\pi r}{a} \right]_a^{2a} \right\} \\ &= \pi b^2 \left\{ \frac{3}{2} a^2 - \left( \frac{(2a)^2}{4} + \frac{a^2}{2\pi^2} - \left( \frac{a^2}{4} + \frac{a^2}{2\pi^2} \right) \right) \right\} \\ &= \pi a^2 b^2 \left( \frac{3}{4} - \frac{1}{\pi} \right) = 2.0379 a^2 b^2 \end{aligned}$$

$$\text{Now.. } \bar{y} V = \int \bar{y}_{\text{el}} dV: \bar{y} (5.4535 a^2 b) = 2.0379 a^2 b^2 \quad \text{OR } \bar{y} = 0.374 b$$

# 5.140

GIVEN: A REGULAR PYRAMID OF HEIGHT  $h$  AND  $N$  SIDES  
SHOW:  $\bar{y} = \frac{h}{4}$  ABOVE THE BASE



CHOOSE AS THE ELEMENT OF VOLUME A HORIZONTAL SLICE OF THICKNESS  $dy$ . FOR ANY NUMBER  $N$  OF SIDES, THE AREA OF THE BASE OF THE

PYRAMID IS GIVEN BY  
 $A_{\text{BASE}} = kb^2$

WHERE  $k = k(N)$ ; SEE NOTE BELOW. USING SIMILAR TRIANGLES HAVE

$$\frac{s}{b} = \frac{h-y}{h}$$

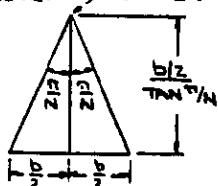
$$\text{OR } s = \frac{b}{h} (h-y)$$

$$\begin{aligned} \text{THEN.. } dV &= A_{\text{SLICE}} dy = ks^2 dy = k \frac{b^2}{h^2} (h-y)^2 dy \\ \text{AND } V &= \int_0^h k \frac{b^2}{h^2} (h-y)^2 dy = k \frac{b^2}{h^2} \left[ -\frac{1}{3} (h-y)^3 \right]_0^h \\ &= \frac{1}{3} kb^2 h \end{aligned}$$

$$\begin{aligned} \text{ALSO.. } \bar{y}_{\text{el}} &= y \quad \text{SO THEN } \int \bar{y}_{\text{el}} dV = \int_0^h y \left( k \frac{b^2}{h^2} (h-y)^2 dy \right) = k \frac{b^2}{h^2} \int_0^h (hy - 2hy^2 + y^3) dy \\ &= k \frac{b^2}{h^2} \left[ \frac{1}{2} hy^2 - \frac{2}{3} hy^3 + \frac{1}{4} y^4 \right]_0^h = \frac{1}{12} kb^2 h^2 \end{aligned}$$

$$\text{Now.. } \bar{y} V = \int \bar{y}_{\text{el}} dV: \bar{y} \left( \frac{1}{3} kb^2 h \right) = \frac{1}{12} kb^2 h^2 \quad \text{OR } \bar{y} = \frac{1}{4} h \quad \text{Q.E.D.}$$

NOTE: CENTER OF BASE

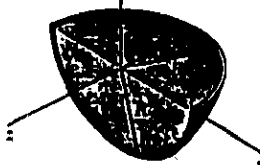


$$\begin{aligned} A_{\text{BASE}} &= N \left( \frac{1}{2} b \cdot \frac{b/2 \tan \frac{\pi}{N}}{\tan \frac{\pi}{N}} \right) \\ &= \frac{N}{4 \tan \frac{\pi}{N}} b^2 \\ &= k(N) b^2 \end{aligned}$$

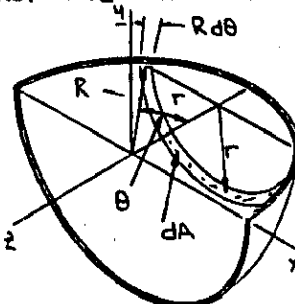
# 5.141

GIVEN: ONE-HALF OF A THIN, UNIFORM HEMISPHERICAL SHELL

FIND: LOCATION OF CENTROID USING DIRECT INTEGRATION



FIRST NOTE THAT SYMMETRY IMPLIES  $\bar{x} = 0$



THE ELEMENT OF AREA  $dA$  OF THE SHELL SHOWN IS OBTAINED BY CUTTING THE SHELL WITH TWO PLANES PARALLEL TO THE  $xy$  PLANE. NOW  $dA = (\pi r)(R d\theta)$ ,  $\bar{y}_{\text{el}} = -\frac{2r}{\pi}$  WHERE  $r = R \sin \theta$

$$\text{SO THAT } dA = \pi R^2 \sin \theta d\theta, \quad \bar{y}_{\text{el}} = -\frac{2R}{\pi} \sin \theta$$

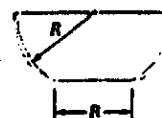
$$\text{THEN } A = \int_0^{\pi/2} \pi R^2 \sin \theta d\theta = \pi R^2 [-\cos \theta]_0^{\pi/2} = \pi R^2$$

$$\begin{aligned} \text{AND } \int \bar{y}_{\text{el}} dA &= \int_0^{\pi/2} \left( -\frac{2R}{\pi} \sin \theta \right) (\pi R^2 \sin \theta d\theta) \\ &= -2R^3 \left[ \frac{\theta}{2} - \frac{\sin 2\theta}{4} \right]_0^{\pi/2} \\ &= -\frac{\pi}{2} R^3 \end{aligned}$$

$$\text{Now.. } \bar{y} A = \int \bar{y}_{\text{el}} dA: \bar{y} (\pi R^2) = -\frac{\pi}{2} R^3 \quad \text{OR } \bar{y} = -\frac{1}{2} R$$

SYMMETRY IMPLIES  $\bar{z} = \bar{y} \therefore \bar{z} = -\frac{1}{2} R$

# 5.142



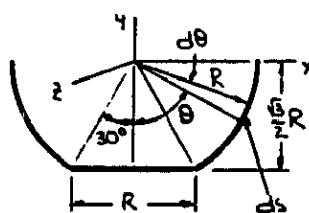
GIVEN: PUNCH BOWL OF UNIFORM WALL THICKNESS  $t$ ,  $R = 250$  mm,  $t \ll R$

FIND: LOCATION OF THE CENTER OF GRAVITY OF  
 (a) THE BOWL  
 (b) THE PUNCH

(a) BOWL

FIRST NOTE THAT SYMMETRY IMPLIES  $\bar{x} = 0$   
 $\bar{z} = 0$

FOR THE COORDINATE AXES SHOWN BELOW. NOW ASSUME THAT THE BOWL MAY BE TREATED AS A SHELL; THE CENTER OF GRAVITY OF THE BOWL WILL COINCIDE WITH THE CENTROIDS OF THE SHELL.



FOR THE WALLS OF THE BOWL, AN ELEMENT OF AREA IS OBTAINED BY ROTATING THE ARC  $ds$  ABOUT THE  $y$  AXIS. THEN  $dA_{\text{wall}} = (2\pi R \sin \theta)(R d\theta)$  (CONTINUES)

# 5.142 CONTINUED

AND  $(\bar{q}_{el})_{wall} = -R \cos \theta$   
 THEN  $A_{wall} = \int_{\pi/6}^{\pi/2} 2\pi R^2 \sin \theta d\theta = 2\pi R^2 [-\cos \theta]_{\pi/6}^{\pi/2}$   
 $= \pi \sqrt{3} R^2$

AND  $\bar{q}_{wall} A_{wall} = \int_{\pi/6}^{\pi/2} (\bar{q}_{el})_{wall} dA$   
 $= \int_{\pi/6}^{\pi/2} (-R \cos \theta) (2\pi R^2 \sin \theta d\theta)$   
 $= \pi R^3 [\cos^2 \theta]_{\pi/6}^{\pi/2}$   
 $= -\frac{3}{4} \pi R^3$

BY OBSERVATION...  $A_{base} = \frac{\pi}{4} R^2$ ,  $\bar{q}_{base} = -\frac{\sqrt{3}}{2} R$

NOW...  $\bar{q} \Sigma A = \Sigma \bar{q} A$

OR...  $\bar{q} (\pi \sqrt{3} R^2 - \frac{\pi}{4} R^2) = -\frac{3}{4} \pi R^3 + \frac{\pi}{4} R^2 (-\frac{\sqrt{3}}{2} R)$

OR  $\bar{q} = -0.48763 R$   $R = 250 \text{ mm}$

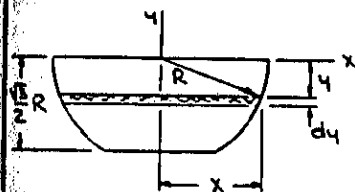
$\therefore \bar{q} = -121.9 \text{ mm}$

(b) PUNCH

FIRST NOTE THAT SYMMETRY IMPLIES  $\bar{y} = 0$

$\bar{z} = 0$

AND THAT BECAUSE THE PUNCH IS HOMOGENEOUS, ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROIDS OF THE CORRESPONDING VOLUME.



CHOOSE AS THE ELEMENT OF VOLUME A DISK OF RADIUS  $x$  AND THICKNESS  $dy$ . THEN

$dV = \pi x^2 dy$ ,  $\bar{q}_{el} = y$   
 NOW...  $x^2 + y^2 = R^2$

SO THAT  $dV = \pi(R^2 - y^2) dy$   
 THEN  $V = \int_{-\frac{\sqrt{3}}{2}R}^0 \pi(R^2 - y^2) dy = \pi [R^2 y - \frac{1}{3} y^3]_{-\frac{\sqrt{3}}{2}R}^0$   
 $= -\pi [R^2 (-\frac{\sqrt{3}}{2}R) - \frac{1}{3} (-\frac{\sqrt{3}}{2}R)^3] = \frac{3}{8} \pi \sqrt{3} R^3$

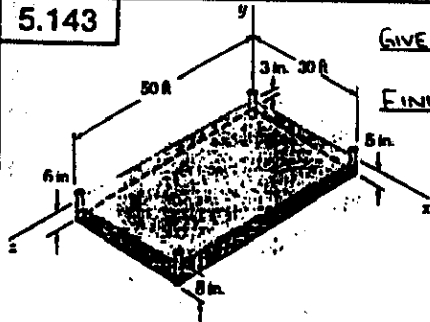
AND  $(\bar{q}_{el} dV) = \int_{-\frac{\sqrt{3}}{2}R}^0 y [\pi(R^2 - y^2) dy] = \pi [\frac{1}{2} R^2 y^2 - \frac{1}{4} y^4]_{-\frac{\sqrt{3}}{2}R}^0$   
 $= -\pi [\frac{1}{2} R^2 (-\frac{\sqrt{3}}{2}R)^2 - \frac{1}{4} (-\frac{\sqrt{3}}{2}R)^4] = -\frac{15}{64} \pi R^4$

NOW...  $\bar{q} V = (\bar{q}_{el} dV)$ :  $\bar{q} (\frac{3}{8} \pi \sqrt{3} R^3) = -\frac{15}{64} \pi R^4$

OR  $\bar{q} = -\frac{5}{8\sqrt{3}} R$   $R = 250 \text{ mm}$

$\therefore \bar{q} = -92.2 \text{ mm}$

# 5.143



GIVEN: GRAVEL BASE,  
 $y = a + bx + cz$

FIND: VOLUME OF GRAVEL,  $\bar{x}$

FIRST DETERMINE THE CONSTANTS  $a$ ,  $b$ , AND  $c$

AT  $x=0, z=0$ :  $y=3 \text{ in.}$ :  $-\frac{3}{12} \text{ ft} = a$   $a = -\frac{1}{4} \text{ ft}$

$x=30 \text{ ft}, z=0$ :  $y=5 \text{ in.}$ :  $-\frac{5}{12} \text{ ft} = -\frac{1}{4} \text{ ft} + b(30 \text{ ft})$

$b = -\frac{1}{180}$

$x=0, z=50 \text{ ft}$ :  $y=6 \text{ in.}$ :  $-\frac{6}{12} \text{ ft} = -\frac{1}{4} \text{ ft} + c(50 \text{ ft})$

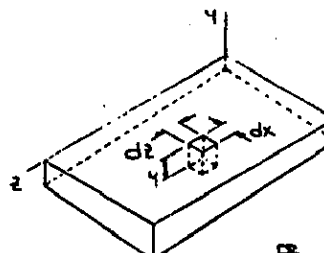
(CONTINUED)

# 5.143 CONTINUED

OR  $C = -\frac{1}{200}$

$\therefore y = -\frac{1}{4} - \frac{1}{180}x - \frac{1}{200}z$

$= -\frac{1}{4} (1 + \frac{1}{45}x + \frac{1}{50}z)$  WHERE ALL DIMENSIONS ARE IN FEET



CHOOSE AS THE ELEMENT OF VOLUME A FILAMENT OF BASE  $dx \cdot dz$  AND HEIGHT  $y$ . THEN  $dV = y dx dz$ ,  $\bar{x}_{el} = x$

OR  $dV = \frac{1}{4} (1 + \frac{1}{45}x + \frac{1}{50}z) dx dz$

THEN  $V = \int_0^{30} \int_0^{50} \frac{1}{4} (1 + \frac{1}{45}x + \frac{1}{50}z) dx dz$

$= \frac{1}{4} \int_0^{50} [x + \frac{1}{90}x^2 + \frac{1}{50}xz]_0^{30} dz$

$= \frac{1}{4} \int_0^{50} [30 + \frac{(30)^2}{90} + \frac{1}{50}(30)z] dz$

$= \frac{1}{4} [40z + \frac{1}{10}z^2]_0^{50} = \frac{1}{4} [40(50) + \frac{1}{10}(50)^2]$

$= 687.5 \text{ ft}^3$   $V = 688 \text{ ft}^3$

AND  $\int \bar{x}_{el} dV = \int_0^{50} \int_0^{30} x [\frac{1}{4} (1 + \frac{1}{45}x + \frac{1}{50}z)] dx dz$

$= \frac{1}{4} \int_0^{50} [\frac{x^2}{2} + \frac{1}{135}x^3 + \frac{1}{50}xz^2]_0^{30} dz$

$= \frac{1}{4} \int_0^{50} [\frac{(30)^2}{2} + \frac{(30)^3}{135} + \frac{1}{50}(30)z^2] dz$

$= \frac{1}{4} [450z + 200z + \frac{1}{10}z^2]_0^{50}$

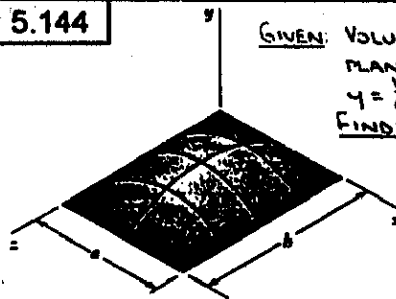
$= \frac{1}{4} [650(50) + \frac{1}{10}(50)^2]$

$= 10,937.5 \text{ ft}^4$

NOW...  $\bar{x} V = \int \bar{x}_{el} dV$ :  $\bar{x} (687.5 \text{ ft}^3) = 10,937.5 \text{ ft}^4$

OR  $\bar{x} = 15.91 \text{ ft}$

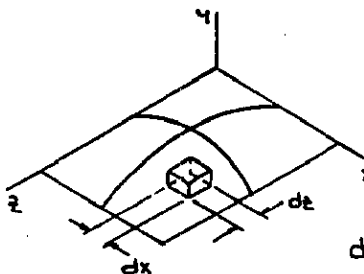
# 5.144



GIVEN: VOLUME BETWEEN THE  $xz$  PLANE AND THE SURFACE  $y = \frac{16h}{a^2b} (ax-x^2)(bz-z^2)$

FIND: LOCATION OF THE CENTROID USING DIRECT INTEGRATION

FIRST NOTE THAT SYMMETRY IMPLIES  $\bar{x} = \frac{a}{2}$ ,  $\bar{z} = \frac{b}{2}$



CHOOSE AS THE ELEMENT OF VOLUME A FILAMENT OF BASE  $dx \cdot dz$  AND HEIGHT  $y$ . THEN  $dV = y dx dz$ ,  $\bar{q}_{el} = \frac{1}{2} y$

OR  $dV = \frac{16h}{a^2b} (ax-x^2)(bz-z^2) dx dz$

THEN  $V = \int_0^b \int_0^a \frac{16h}{a^2b} (ax-x^2)(bz-z^2) dx dz$

(CONTINUED)

# 5.144 CONTINUED

$$V = \frac{16h}{a^2b^2} \int_0^b (bz - z^2) \left[ \frac{a}{2}x^2 - \frac{1}{3}x^3 \right]_0^a dz$$

$$= \frac{16h}{a^2b^2} \left[ \frac{a}{2}(a^2) - \frac{1}{3}(a^3) \right] \left[ \frac{b}{2}z^2 - \frac{1}{3}z^3 \right]_0^b$$

$$= \frac{8ah}{3b^2} \left[ \frac{b}{2}(b^2) - \frac{1}{3}(b^3) \right] = \frac{4}{9}abh$$

$$\text{AND } \bar{y}_{EL} dV = \int_0^b \int_0^a \frac{16h}{a^2b^2} (ax - x^2)(bz - z^2) \times \left[ \frac{16h}{a^2b^2} (ax - x^2)(bz - z^2) dx dz \right]$$

$$= \frac{128h^2}{a^4b^4} \int_0^b \int_0^a (a^2x^2 - 2ax^3 + x^4) \times (b^2z^2 - 2bz^3 + z^4) dx dz$$

$$= \frac{128h^2}{a^4b^4} \int_0^b (b^2z^2 - 2bz^3 + z^4) \left[ \frac{a^2}{3}x^3 - \frac{a}{2}x^4 + \frac{1}{5}x^5 \right]_0^a dz$$

$$= \frac{128h^2}{a^4b^4} \left[ \frac{a^2}{3}(a)^3 - \frac{a}{2}(a)^4 + \frac{1}{5}(a)^5 \right] \times \left[ \frac{b^2}{3}z^3 - \frac{b}{2}z^4 + \frac{1}{5}z^5 \right]_0^b$$

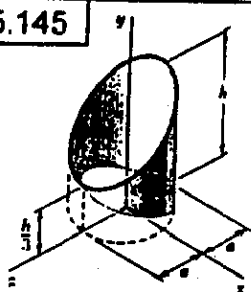
$$= \frac{64ah^2}{15b^4} \left[ \frac{b^2}{3}(b)^3 - \frac{b}{2}(b)^4 + \frac{1}{5}(b)^5 \right]$$

$$= \frac{32}{225} abh^2$$

$$\text{Now.. } \bar{y}V = \int \bar{y}_{EL} dV: \quad \bar{y} \left( \frac{4}{9}abh \right) = \frac{32}{225} abh^2$$

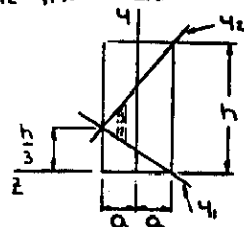
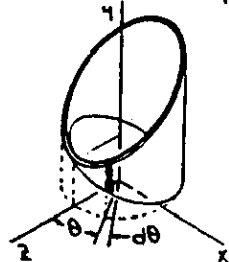
$$\text{OR } \bar{y} = \frac{8}{25}h$$

# 5.145



GIVEN: THE PORTION OF A CIRCULAR PIPE SHOWN  
FIND: LOCATION OF THE CENTROIDS

FIRST NOTE THAT SYMMETRY IMPLIES  $\bar{x} = 0$   
 ASSUME THAT THE PIPE HAS A UNIFORM WALL THICKNESS  $t$  AND CHOOSE AS THE ELEMENT OF VOLUME A VERTICAL STRIP OF WIDTH  $a d\theta$  AND HEIGHT  $(y_2 - y_1)$ . THEN



$$dV = (y_2 - y_1)t a d\theta, \quad \bar{y}_{EL} = \frac{1}{2}(y_1 + y_2), \quad \bar{z}_{EL} = z$$

$$\text{Now.. } y_1 = \frac{h}{2a}z + \frac{h}{2}, \quad y_2 = -\frac{h}{2a}z + \frac{h}{2}$$

$$= \frac{h}{6a}(z + a), \quad = \frac{h}{3a}(-z + 2a)$$

$$\text{AND } z = a \cos \theta$$

$$\text{THEN } (y_2 - y_1) = \frac{h}{3a}(-a \cos \theta + 2a) - \frac{h}{6a}(a \cos \theta + a)$$

$$= \frac{h}{2}(1 - \cos \theta)$$

$$\text{AND } (y_1 + y_2) = \frac{h}{6a}(a \cos \theta + a) + \frac{h}{3a}(-a \cos \theta + 2a)$$

$$= \frac{h}{6}(5 - \cos \theta)$$

(CONTINUED)

# 5.145 CONTINUED

$$\therefore dV = \frac{ah^2}{2}(1 - \cos \theta) d\theta, \quad \bar{y}_{EL} = \frac{h}{12}(5 - \cos \theta), \quad \bar{z}_{EL} = a \cos \theta$$

$$\text{THEN } V = 2 \int_0^{\pi} \frac{ah^2}{2}(1 - \cos \theta) d\theta = ah^2 [\theta - \sin \theta]_0^{\pi}$$

$$= \pi ah^2$$

$$\text{AND } \int \bar{y}_{EL} dV = 2 \int_0^{\pi} \frac{h}{12}(5 - \cos \theta) \left[ \frac{ah^2}{2}(1 - \cos \theta) d\theta \right]$$

$$= \frac{ah^3}{12} \int_0^{\pi} (5 - 6 \cos \theta + \cos^2 \theta) d\theta$$

$$= \frac{ah^3}{12} \left[ 5\theta - 6 \sin \theta + \frac{\theta}{2} + \frac{\sin 2\theta}{4} \right]_0^{\pi}$$

$$= \frac{11}{24} \pi ah^3$$

$$\int \bar{z}_{EL} dV = 2 \int_0^{\pi} a \cos \theta \left[ \frac{ah^2}{2}(1 - \cos \theta) d\theta \right]$$

$$= a^2 h^2 \left[ \sin \theta - \frac{\theta}{2} - \frac{\sin 2\theta}{4} \right]_0^{\pi}$$

$$= -\frac{1}{2} \pi a^2 h^2$$

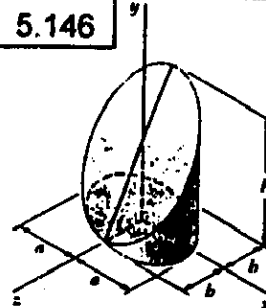
$$\text{Now.. } \bar{y}V = \int \bar{y}_{EL} dV: \quad \bar{y}(\pi ah^2) = \frac{11}{24} \pi ah^3$$

$$\text{OR } \bar{y} = \frac{11}{24}h$$

$$\text{AND } \bar{z}V = \int \bar{z}_{EL} dV: \quad \bar{z}(\pi ah^2) = -\frac{1}{2} \pi a^2 h^2$$

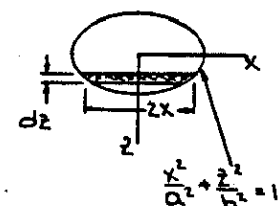
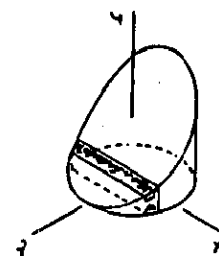
$$\text{OR } \bar{z} = -\frac{1}{2}a$$

# \* 5.146



GIVEN: THE PORTION OF AN ELLIPTICAL CYLINDER SHOWN  
FIND: LOCATION OF THE CENTROIDS

FIRST NOTE THAT SYMMETRY IMPLIES  $\bar{x} = 0$



CHOOSE AS THE ELEMENT OF VOLUME A VERTICAL SLICE OF WIDTH  $2x$ , THICKNESS  $dz$ , AND HEIGHT  $y$ . THEN

$$dV = 2xy dz, \quad \bar{y}_{EL} = \frac{1}{2}y, \quad \bar{z}_{EL} = z$$

$$\text{Now } x = \frac{a}{b} \sqrt{b^2 - z^2}$$

$$\text{THEN } V = \int_{-b}^b \left( 2 \frac{a}{b} \sqrt{b^2 - z^2} \right) \left[ \frac{h}{2b}(b - z) \right] dz$$

$$\text{LET } z = b \sin \theta, \quad dz = b \cos \theta d\theta$$

$$\text{THEN } V = \frac{ah}{b^2} \int_{-\pi/2}^{\pi/2} (b \cos \theta) (b(1 - \sin \theta)) b \cos \theta d\theta$$

$$= abh \int_{-\pi/2}^{\pi/2} (\cos^2 \theta - \sin \theta \cos^2 \theta) d\theta$$

$$= abh \left[ \frac{\theta}{2} + \frac{\sin 2\theta}{4} + \frac{1}{3} \cos^3 \theta \right]_{-\pi/2}^{\pi/2}$$

(CONTINUED)

# 5.146 CONTINUED

$$V = \frac{1}{2} \pi a b h$$

$$\text{AND } \int_{\text{EL}} dV = \int_{-b}^b \left[ \frac{1}{2} \pi \frac{h}{2b} (b-z) \left( \frac{2}{3} \frac{b}{b-z} \sqrt{b^2 - z^2} \right) \frac{h}{2b} (b-z) dz \right]$$

$$= \frac{1}{4} \frac{\pi h^2}{b^2} \int_{-b}^b (b-z)^2 \sqrt{b^2 - z^2} dz$$

$$\text{LET } z = b \sin \theta, \quad dz = b \cos \theta d\theta$$

$$\text{THEN } \int_{\text{EL}} dV = \frac{1}{4} \frac{\pi h^2}{b^2} \int_{-\pi/2}^{\pi/2} [b(1 - \sin \theta)]^2 (b \cos \theta) \times (b \cos \theta d\theta)$$

$$= \frac{1}{4} \pi a b h^2 \int_{-\pi/2}^{\pi/2} (\cos^2 \theta - 2 \sin \theta \cos^3 \theta + \sin^2 \theta \cos^5 \theta) d\theta$$

$$\text{NOW } \sin^2 \theta = \frac{1}{2} (1 - \cos 2\theta) \quad \cos^2 \theta = \frac{1}{2} (1 + \cos 2\theta)$$

$$\text{SO THAT } \sin^2 \theta \cos^5 \theta = \frac{1}{4} (1 - \cos^2 2\theta)$$

$$\text{THEN } \int_{\text{EL}} dV = \frac{1}{4} \pi a b h^2 \int_{-\pi/2}^{\pi/2} \left[ \cos^4 \theta - 2 \sin \theta \cos^3 \theta + \frac{1}{4} (1 - \cos^2 2\theta) \right] d\theta$$

$$= \frac{1}{4} \pi a b h^2 \left[ \left( \frac{\theta}{2} + \frac{\sin 2\theta}{4} \right) + \frac{1}{3} \cos^3 \theta + \frac{1}{4} \theta - 4 \left( \frac{\theta}{2} + \frac{\sin 4\theta}{8} \right) \right]_{-\pi/2}^{\pi/2}$$

$$= \frac{3}{32} \pi a b h^2$$

$$\text{ALSO } \int_{\text{EL}} dV = \int_{-b}^b \left[ \frac{1}{2} \pi \frac{h}{2b} \sqrt{b^2 - z^2} \left( \frac{h}{2b} (b-z) \right) dz \right]$$

$$= \frac{\pi h}{b^2} \int_{-b}^b z(b-z) \sqrt{b^2 - z^2} dz$$

$$\text{LET } z = b \sin \theta, \quad dz = b \cos \theta d\theta$$

$$\text{THEN } \int_{\text{EL}} dV = \frac{\pi h}{b^2} \int_{-\pi/2}^{\pi/2} (b \sin \theta) [b(1 - \sin \theta)] (b \cos \theta) \times (b \cos \theta d\theta)$$

$$= \pi a b h^2 \int_{-\pi/2}^{\pi/2} (\sin \theta \cos^3 \theta - \sin^2 \theta \cos^5 \theta) d\theta$$

$$\text{USING } \sin^2 \theta \cos^5 \theta = \frac{1}{4} (1 - \cos^2 2\theta) \text{ FROM ABOVE..}$$

$$\int_{\text{EL}} dV = \pi a b h^2 \int_{-\pi/2}^{\pi/2} \left[ \sin \theta \cos^3 \theta - \frac{1}{4} (1 - \cos^2 2\theta) \right] d\theta$$

$$= \pi a b h^2 \left[ -\frac{1}{3} \cos^3 \theta - \frac{1}{4} \theta + \frac{1}{4} \left( \frac{\theta}{2} + \frac{\sin 4\theta}{8} \right) \right]_{-\pi/2}^{\pi/2}$$

$$= -\frac{1}{8} \pi a b^2 h$$

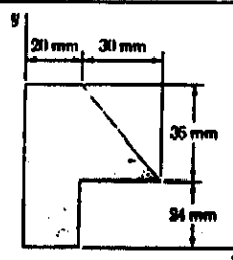
$$\text{NOW.. } \bar{y}V = \int_{\text{EL}} dV: \quad \bar{y} \left( \frac{1}{2} \pi a b h \right) = \frac{3}{32} \pi a b h^2$$

$$\text{OR } \bar{y} = \frac{3}{16} h$$

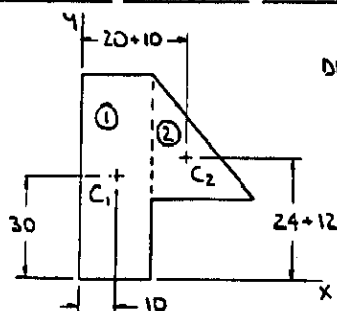
$$\text{AND } \bar{z}V = \int_{\text{EL}} dV: \quad \bar{z} \left( \frac{1}{2} \pi a b h \right) = -\frac{1}{8} \pi a b^2 h$$

$$\text{OR } \bar{z} = -\frac{1}{4} b$$

# 5.147



GIVEN: PLANE AREA  
SHOWN  
FIND:  $\bar{X}$  AND  $\bar{Y}$



DIMENSIONS IN MM

(CONTINUED)

# 5.147 CONTINUED

	A, mm <sup>2</sup>	$\bar{x}$ , mm	$\bar{y}$ , mm	$\bar{x}A$ , mm <sup>3</sup>	$\bar{y}A$ , mm <sup>3</sup>
1	20 × 36 = 720	10	30	12 000	36 000
2	$\frac{1}{2} \times 30 \times 24 = 360$	30	36	16 200	19 440
$\Sigma$	1080			28 200	55 440

$$\text{THEN } \bar{X} \Sigma A = \Sigma \bar{x} A$$

$$\bar{X} (1080) = 28 200$$

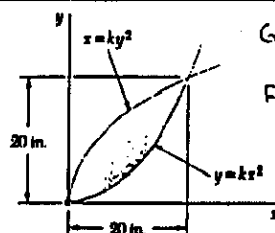
$$\text{OR } \bar{X} = 16.21 \text{ mm}$$

$$\text{AND } \bar{Y} \Sigma A = \Sigma \bar{y} A$$

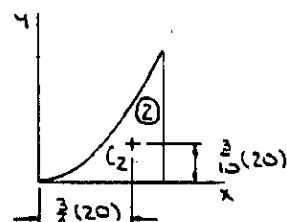
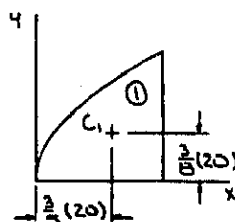
$$\bar{Y} (1080) = 55 440$$

$$\text{OR } \bar{Y} = 31.9 \text{ mm}$$

# 5.148



GIVEN: PLANE AREA  
SHOWN  
FIND:  $\bar{X}$  AND  $\bar{Y}$



	A, in <sup>2</sup>	$\bar{x}$ , in.	$\bar{y}$ , in.	$\bar{x}A$ , in <sup>3</sup>	$\bar{y}A$ , in <sup>3</sup>
1	$\frac{1}{3} (20 \times 20) = 800/3$	12	7.5	3200	2000
2	$-\frac{1}{3} (20 \times 20) = -800/3$	15	6	-2000	-800
$\Sigma$	400/3			1200	1200

$$\text{THEN } \bar{X} \Sigma A = \Sigma \bar{x} A$$

$$\bar{X} (400/3) = 1200$$

$$\text{OR } \bar{X} = 9.00 \text{ in.}$$

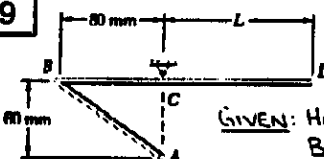
$$\text{AND } \bar{Y} \Sigma A = \Sigma \bar{y} A$$

$$\bar{Y} (400/3) = 1200$$

$$\text{OR } \bar{Y} = 9.00 \text{ in.}$$

NOTE: SYMMETRY IMPLIES  $\bar{X} = \bar{Y}$ , WHICH IS CONFIRMED BY THE ABOVE SOLUTION.

# 5.149



GIVEN: HOMOGENEOUS WIRE,  
BCD IS HORIZONTAL  
FIND: L

FIRST NOTE THAT FOR EQUILIBRIUM, THE CENTER OF GRAVITY OF THE WIRE MUST LIE ON A VERTICAL LINE THROUGH C. FURTHER, BECAUSE THE WIRE IS HOMOGENEOUS, THE CENTER OF GRAVITY OF THE WIRE WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING LINE. THUS,

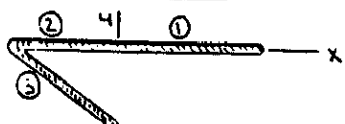
$$\bar{X} = 0 \quad (\text{SEE SKETCH ON THE NEXT PAGE})$$

$$\text{SO THAT } \Sigma \bar{x} L = 0$$

(CONTINUED)

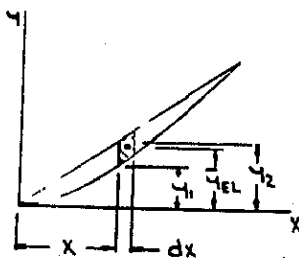
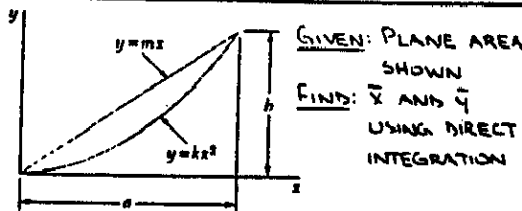


### 5.149 CONTINUED



THEN  $\frac{1}{2}(L) + (-40 \text{ mm})(80 \text{ mm}) + (-40 \text{ mm})(100 \text{ mm}) = 0$   
 OR  $L^2 = 14400 \text{ mm}^2$   
 OR  $L = 120.0 \text{ mm}$  ◀

### 5.150



AT  $(a, b)$   
 $y_1: b = ka^2$   
 OR  $k = \frac{b}{a^2}$   
 $y_2: b = ma$   
 OR  $m = \frac{b}{a}$   
 NOW...  $\bar{x}_{EL} = x$   
 $\bar{y}_{EL} = \frac{1}{2}(y_1 + y_2)$

AND  $dA = (y_2 - y_1)dx$   
 $= (\frac{b}{a}x - \frac{b}{a^2}x^2)dx$   
 $= \frac{b}{a^2}(ax - x^2)dx$

THEN  $A = \int dA = \int_0^a \frac{b}{a^2}(ax - x^2)dx = \frac{b}{a^2}[\frac{a}{2}x^2 - \frac{1}{3}x^3]_0^a$   
 $= \frac{1}{6}ab$

AND  $\bar{x}_{EL} \cdot dA = \int_0^a x[\frac{b}{a^2}(ax - x^2)]dx = \frac{b}{a^2}[\frac{a}{3}x^3 - \frac{1}{4}x^4]_0^a$   
 $= \frac{1}{12}a^2b$

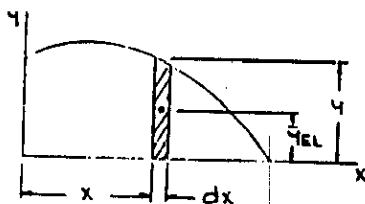
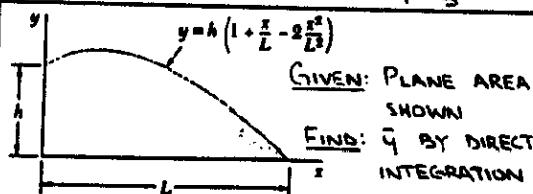
$\int \bar{y}_{EL} dA = \int \frac{1}{2}(y_1 + y_2)(y_2 - y_1)dx$   
 $= \int \frac{1}{2}(y_2^2 - y_1^2)dx$   
 $= \frac{1}{2} \int_0^a (\frac{b^2}{a^2}x^2 - \frac{b^2}{a^4}x^4)dx$   
 $= \frac{1}{2} \frac{b^2}{a^4} [\frac{a^2}{3}x^3 - \frac{1}{5}x^5]_0^a$   
 $= \frac{1}{15}ab^2$

$\bar{x}A = \bar{x}_{EL}DA: \bar{x}(\frac{1}{6}ab) = \frac{1}{12}a^2b$

$\bar{y}A = \bar{y}_{EL}DA: \bar{y}(\frac{1}{6}ab) = \frac{1}{15}ab^2$

$\bar{x} = \frac{1}{2}a$   
 $\bar{y} = \frac{2}{5}b$  ▶

### 5.151



(CONTINUED)

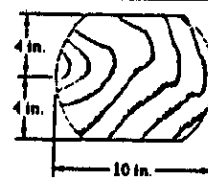
### 5.151 CONTINUED

HAVE  $dA = ydx = h(1 + \frac{x}{L} - 2\frac{x^2}{L^2})dx$   
 AND  $\bar{y}_{EL} = \frac{1}{2}y = \frac{1}{2}h(1 + \frac{x}{L} - 2\frac{x^2}{L^2})$

THEN  $A = \int dA = \int_0^L h(1 + \frac{x}{L} - 2\frac{x^2}{L^2})dx$   
 $= h[x + \frac{1}{2L}x^2 - \frac{2}{3L^2}x^3]_0^L = \frac{5}{6}hL$   
 AND  $\int \bar{y}_{EL} dA = \int_0^L \frac{1}{2}h(1 + \frac{x}{L} - 2\frac{x^2}{L^2})[h(1 + \frac{x}{L} - 2\frac{x^2}{L^2})]dx$   
 $= \frac{1}{2}h^2 \int_0^L (1 + 2\frac{x}{L} - 3\frac{x^2}{L^2} - 4\frac{x^3}{L^3} + 4\frac{x^4}{L^4})dx$   
 $= \frac{1}{2}h^2 [x + \frac{1}{L}x^2 - \frac{1}{L^2}x^3 - \frac{1}{L^3}x^4 + \frac{4}{5L^4}x^5]_0^L$   
 $= \frac{2}{5}h^2L$

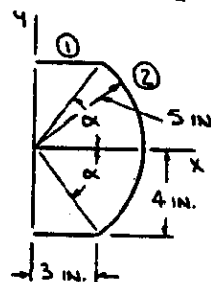
$\bar{y}A = \int \bar{y}_{EL} dA: \bar{y}(\frac{5}{6}hL) = \frac{2}{5}h^2L$   
 $\bar{y} = \frac{12}{25}h$   
 OR  $\bar{y} = 0.48h$  ▶

### 5.152



GIVEN: WOODEN SPHERE WITH TWO EQUAL CAPS REMOVED  
 FIND: SURFACE AREA OF BODY

THE SURFACE AREA CAN BE GENERATED BY ROTATING THE LINE SHOWN ABOUT THE  $y$  AXIS. APPLYING THE FIRST THEOREM OF PAPPUS-GULBINUS HAVE



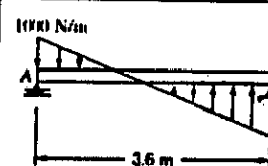
$A = 2\pi \bar{x}L = 2\pi \sum \bar{x}_i L_i$   
 $= 2\pi (\bar{x}_1 L_1 + \bar{x}_2 L_2)$

NOW  $\tan \alpha = \frac{4}{3}$   
 OR  $\alpha = 53.130^\circ$   
 THEN  $\bar{x}_2 = \frac{5 \sin \alpha \sin 53.130^\circ}{53.130^\circ} = \frac{\pi}{180^\circ}$

$= 4.3136 \text{ in.}$   
 AND  $L_2 = 2(53.130^\circ \cdot \frac{\pi}{180^\circ})(5 \text{ in.})$   
 $= 9.2729 \text{ in.}$

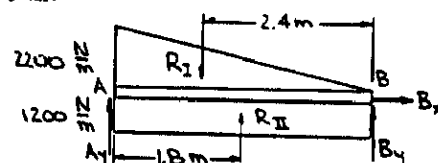
$\therefore A = 2\pi [2(\frac{3}{2} \text{ in.})(3 \text{ in.}) + (4.3136 \text{ in.})(9.2729 \text{ in.})]$   
 OR  $A = 308 \text{ in}^2$  ▶

### 5.153



GIVEN: BEAM AND LOADING SHOWN  
 FIND: REACTIONS AT THE SUPPORTS

FIRST REPLACE THE GIVEN LOADING WITH THE LOADING SHOWN BELOW. THE TWO LOADINGS ARE EQUIVALENT BECAUSE BOTH ARE DEFINED BY A LINEAR RELATION BETWEEN LOAD AND DISTANCE AND THE VALUES AT THE END POINTS ARE THE SAME.

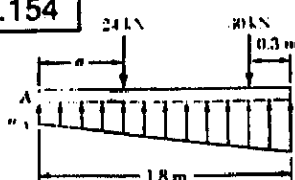


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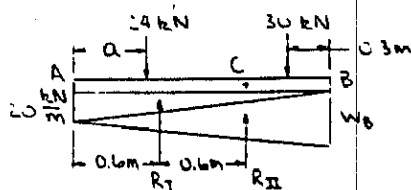
### 5.153 CONTINUED

HAVE...  $R_I = \frac{1}{2}(3.6 \text{ m})(2200 \frac{\text{N}}{\text{m}}) = 3960 \text{ N}$   
 $R_{II} = (3.6 \text{ m})(1200 \frac{\text{N}}{\text{m}}) = 4320 \text{ N}$   
 THEN...  $\sum F_x = 0: B_x = 0$   
 $\sum M_B = 0: -(3.6 \text{ m})A_y + (2.4 \text{ m})(3960 \text{ N}) - (1.8 \text{ m})(4320 \text{ N}) = 0$   
 OR  $A_y = 480 \text{ N}$   $A = 480 \text{ N} \uparrow$   
 $\sum F_y = 0: 480 \text{ N} - 3960 \text{ N} + 4320 \text{ N} + B_y = 0$   
 OR  $B_y = -840 \text{ N}$   $B = 840 \text{ N} \downarrow$

### 5.154

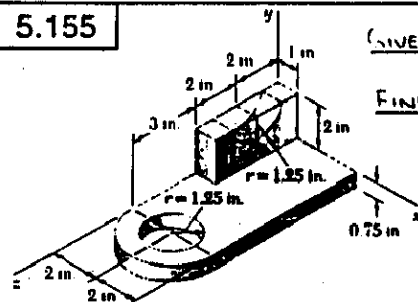


GIVEN: BEAM AND LOADING SHOWN.  
 $W_A = 20 \text{ kN/m}$   
 FIND: (a)  $A$   
 (b)  $W_B$



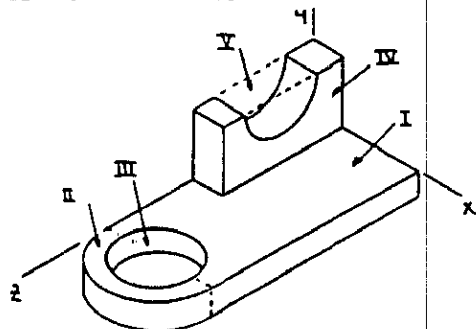
HAVE...  $R_I = \frac{1}{2}(1.8 \text{ m})(20 \frac{\text{kN}}{\text{m}}) = 18 \text{ kN}$   
 $R_{II} = \frac{1}{2}(1.8 \text{ m})(W_B \frac{\text{kN}}{\text{m}}) = 0.9 W_B \text{ kN}$   
 (a)  $\sum M_C = 0: (1.2 - 0.6) \text{ m} = 24 \text{ kN} \cdot 0.6 \text{ m} - 18 \text{ kN} \cdot 0.3 \text{ m} + 30 \text{ kN} \cdot 0.3 \text{ m} = 0$   
 OR  $A = 0.375 \text{ m}$   
 (b)  $\sum F_y = 0: -24 \text{ kN} + 18 \text{ kN} + 0.9 W_B \text{ kN} - 30 \text{ kN} = 0$   
 OR  $W_B = 40 \frac{\text{kN}}{\text{m}}$

### 5.155



GIVEN: MACHINE ELEMENT SHOWN  
 FIND:  $\bar{x}$

FIRST ASSUME THAT THE MACHINE ELEMENT IS HOMOGENEOUS SO THAT ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROIDS OF THE CORRESPONDING VOLUME.



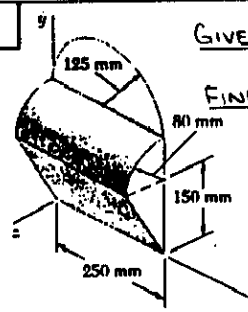
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### 5.155 CONTINUED

	$V, \text{in}^3$	$\bar{z}, \text{in.}$	$\bar{z}V, \text{in}^4$
I	$(4)(0.15)(7) = 21$	3.5	73.5
II	$\frac{1}{2}(27)(0.75) = 4.7124$	$7 + \frac{4.7124}{2} = 7.8488$	36.987
III	$-\pi(1.25)^2(0.75) = -3.6816$	7	-25.771
IV	$(1.7)(2)(4) = 13.6$	2	27.2
V	$-\frac{\pi}{2}(1.25)^2(1) = -2.4544$	2	-4.9088
$\Sigma$	27.576		95.807

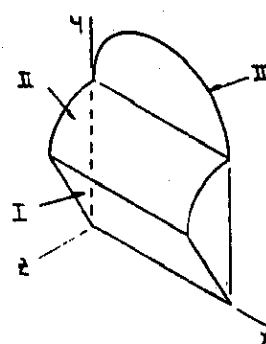
HAVE...  $\bar{z} \Sigma V = \Sigma \bar{z}V: \bar{z}(27.576 \text{ in}^3) = 95.807 \text{ in}^4$   
 OR  $\bar{z} = 3.47 \text{ in.}$

### 5.156



GIVEN: SHEET-METAL FORM SHOWN  
 FIND: LOCATION OF CENTER OF GRAVITY

FIRST ASSUME THAT THE SHEET METAL IS HOMOGENEOUS SO THAT THE CENTER OF GRAVITY OF THE FORM WILL COINCIDE WITH THE CENTROIDS OF THE CORRESPONDING AREA. NOW NOTE THAT SYMMETRY IMPLIES



$\bar{x} = 125 \text{ mm}$

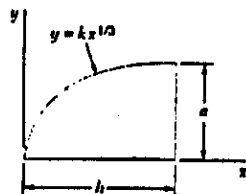
$\bar{y}_{II} = 150 + \frac{2 \times 80}{\pi}$   
 $= 200.93 \text{ mm}$   
 $\bar{z}_{II} = \frac{2 \times 80}{\pi}$   
 $= 50.930 \text{ mm}$

$\bar{y}_{III} = 230 + \frac{4 \times 125}{3\pi}$   
 $= 283.05 \text{ mm}$

	$A, \text{mm}^2$	$\bar{y}, \text{mm}$	$\bar{z}, \text{mm}$	$\bar{y}A, \text{mm}^3$	$\bar{z}A, \text{mm}^3$
I	$(250)(170) = 42500$	75	40	3187500	1700000
II	$\frac{1}{2}(80)(250) = 31416$	200.93	50.930	6312400	1600000
III	$\frac{\pi}{2}(125)^2 = 24544$	283.05	0	6947200	0
$\Sigma$	98460			16447100	3300000

HAVE...  $\bar{y} \Sigma A = \Sigma \bar{y}A: \bar{y}(98460 \text{ mm}^2) = 16447100 \text{ mm}^3$   
 OR  $\bar{y} = 167.0 \text{ mm}$   
 $\bar{z} \Sigma A = \Sigma \bar{z}A: \bar{z}(98460 \text{ mm}^2) = 3300000 \text{ mm}^3$   
 OR  $\bar{z} = 33.5 \text{ mm}$

5.157



GIVEN: VOLUME GENERATED BY ROTATING THE AREA SHOWN ABOUT THE X AXIS

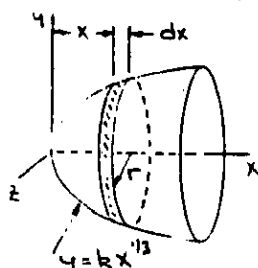
FIND: LOCATION OF THE CENTROIDS OF THE VOLUME

FIRST NOTE THAT SYMMETRY IMPLIES

$$\bar{y} = 0$$

$$\bar{z} = 0$$

CHOOSE AS THE ELEMENT OF VOLUME A DISK OF RADIUS  $r$  AND THICKNESS  $dx$ . THEN



$$dV = \pi r^2 dx \quad \bar{x}_{EL} = x$$

Now  $r = kx^{1/3}$  SO THAT

$$dV = \pi k^2 x^{2/3} dx$$

At  $x = h$ ,  $y = a$ :  $a = kh^{1/3}$

$$\text{OR } k = a^3/h$$

$$\text{THEN } dV = \pi \frac{a^2}{h^{1/3}} x^{2/3} dx$$

$$\text{AND } V = \int_0^h \pi \frac{a^2}{h^{1/3}} x^{2/3} dx$$

$$= \pi \frac{a^2}{h^{1/3}} \left[ \frac{3}{5} x^{5/3} \right]_0^h$$

$$= \frac{3}{5} \pi a^2 h$$

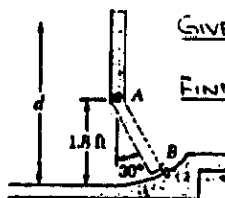
$$\text{ALSO } \int \bar{x}_{EL} dV = \int_0^h x \left( \pi \frac{a^2}{h^{1/3}} x^{2/3} dx \right) = \pi \frac{a^2}{h^{1/3}} \left[ \frac{3}{8} x^{8/3} \right]_0^h$$

$$= \frac{3}{8} \pi a^2 h^2$$

$$\text{Now } \bar{x} V = \int \bar{x} dV: \bar{x} \left( \frac{3}{5} \pi a^2 h \right) = \frac{3}{8} \pi a^2 h^2$$

$$\text{OR } \bar{x} = \frac{5}{8} h$$

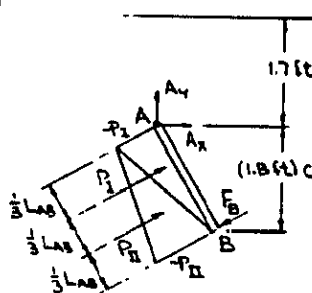
5.158



GIVEN: 1.8 ft x 1.8 ft GATE,

$d = 3.5$  ft, WATER

FIND: FORCE  $F_B$  EXERTED BY PIN AT B ON GATE



FIRST CONSIDER THE FORCE OF THE WATER ON THE GATE. HAVE

$$P = \frac{1}{2} A p$$

$$= \frac{1}{2} A (\gamma h)$$

THEN ...

$$P_1 = \frac{1}{2} (1.8 \text{ ft})^2 (62.4 \frac{\text{lb}}{\text{ft}^3}) (1.7 \text{ ft})$$

$$= 171.850 \text{ lb}$$

$$P_2 = \frac{1}{2} (1.8 \text{ ft})^2 (62.4 \frac{\text{lb}}{\text{ft}^3}) \times (1.7 + 1.8 \cos 30^\circ) \text{ ft}$$

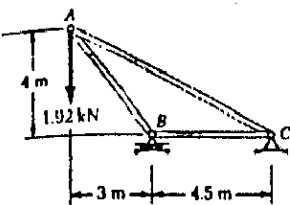
$$= 329.43 \text{ lb}$$

$$\text{Now } \sum M_A = 0: \left( \frac{1}{3} L \sin 30^\circ \right) P_1 + \left( \frac{2}{3} L \sin 30^\circ \right) P_2 - L \sin 30^\circ F_B = 0$$

$$\text{OR } \frac{1}{3} (171.850 \text{ lb}) + \frac{2}{3} (329.43 \text{ lb}) - F_B = 0$$

$$\text{OR } F_B = 276.90 \text{ lb}$$

$$F_B = 277 \text{ lb } \angle 30^\circ$$



**GIVEN:**  
TRUSS AND LOADING SHOWN.  
**FIND:**  
FORCE IN EACH MEMBER

**FREE BODY: ENTIRE TRUSS**

$$\begin{aligned} \sum F_x = 0: C_x = 0 \quad C_x = 0 \\ \sum M_B = 0: (1.92 \text{ kN})(3 \text{ m}) + C_y(4.5 \text{ m}) = 0 \\ C_y = -1.28 \text{ kN} \quad C_y = 1.28 \text{ kN} \uparrow \\ \sum F_y = 0: B - 1.92 \text{ kN} - 1.28 \text{ kN} = 0 \\ B = 3.20 \text{ kN} \uparrow \end{aligned}$$

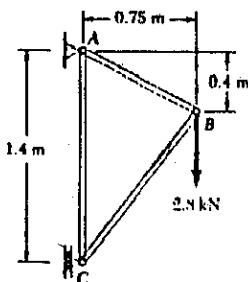
**FREE BODY: JOINT B**

$$\begin{aligned} \frac{F_{AB}}{5} = \frac{F_{BC}}{3} = \frac{3.20 \text{ kN}}{4} \\ F_{AB} = 4.00 \text{ kN} \quad \leftarrow \\ F_{BC} = 2.40 \text{ kN} \quad \leftarrow \end{aligned}$$

**FREE BODY: JOINT C**

$$\begin{aligned} \sum F_x = 0: -\frac{7.5}{8.5} F_{AC} + 2.40 \text{ kN} = 0 \\ F_{AC} = +2.72 \text{ kN} \quad F_{AC} = 2.72 \text{ kN} \quad \leftarrow \\ \sum F_y = 0: \frac{4}{8.5} (2.72 \text{ kN}) - 1.28 \text{ kN} = 0 \quad (\text{CHECKS}) \\ C_y = 1.28 \text{ kN} \end{aligned}$$

6.2



**GIVEN:**  
TRUSS AND LOADING SHOWN.  
**FIND:**  
FORCE IN EACH MEMBER

**FREE BODY: ENTIRE TRUSS**

$$\begin{aligned} \sum M_A = 0: C(1.4 \text{ m}) - (2.8 \text{ kN})(0.75 \text{ m}) = 0 \\ C = +1.500 \text{ kN} \quad C = 1.500 \text{ kN} \quad \leftarrow \\ \sum F_x = 0: A_x + 1.500 \text{ kN} = 0 \\ A_x = -1.500 \text{ kN} \quad A_x = 1.500 \text{ kN} \quad \leftarrow \\ \sum F_y = 0: A_y - 2.8 \text{ kN} = 0 \\ A_y = 2.8 \text{ kN} \uparrow \end{aligned}$$

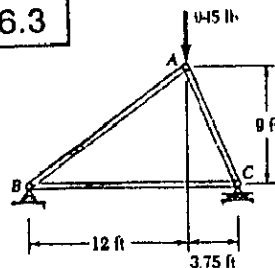
**FREE BODY: JOINT A**

$$\begin{aligned} \sum F_x = 0: \frac{7.5}{8.5} F_{AB} - 1.500 \text{ kN} = 0 \\ F_{AB} = 1.700 \text{ kN} \quad \leftarrow \\ \sum F_y = 0: 2.8 \text{ kN} - \frac{4}{8.5} (1.700 \text{ kN}) - F_{AC} = 0 \\ F_{AC} = 2.00 \text{ kN} \quad \leftarrow \end{aligned}$$

**FREE BODY: JOINT C**

$$\begin{aligned} \frac{F_{BC}}{1.25} = \frac{F_{AC}}{0.75} = \frac{1.500 \text{ kN}}{0.75} \\ F_{BC} = 2.50 \text{ kN} \quad \leftarrow \\ F_{AC} = 2.00 \text{ kN} \quad (\text{CHECKS}) \end{aligned}$$

6.3



**GIVEN:**  
TRUSS AND LOADING SHOWN  
**FIND:**  
FORCE IN EACH MEMBER

**FREE BODY: ENTIRE TRUSS**

$$\begin{aligned} \sum F_x = 0: B_x = 0 \\ \sum M_B = 0: C(15.75 \text{ ft}) - (145 \text{ lb})(12 \text{ ft}) = 0 \\ C = 720 \text{ lb} \uparrow \\ \sum F_y = 0: B_y + 720 \text{ lb} - 145 \text{ lb} = 0 \\ B_y = 225 \text{ lb} \uparrow \end{aligned}$$

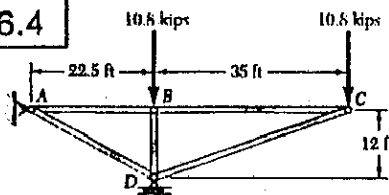
**FREE BODY: JOINT B**

$$\begin{aligned} \frac{F_{AB}}{5} = \frac{F_{BC}}{4} = \frac{225 \text{ lb}}{3} \\ F_{AB} = 375 \text{ lb} \quad \leftarrow \\ F_{BC} = 300 \text{ lb} \quad \leftarrow \end{aligned}$$

**FREE BODY: JOINT C**

$$\begin{aligned} \frac{F_{AC}}{9.75} = \frac{F_{BC}}{3.75} = \frac{720 \text{ lb}}{9} \\ F_{AC} = 780 \text{ lb} \quad \leftarrow \\ F_{BC} = 300 \text{ lb} \quad (\text{CHECKS}) \end{aligned}$$

6.4



**GIVEN:**  
TRUSS AND LOADING SHOWN.  
**FIND:**  
FORCE IN EACH MEMBER

**FREE BODY: TRUSS**

$$\begin{aligned} \sum F_x = 0: A_x = 0 \\ \sum M_A = 0: D(22.5) - (10.8 \text{ kips})(57.5) = 0 \\ D = 38.4 \text{ kips} \uparrow \\ \sum F_y = 0: A_y = 16.8 \text{ kips} \uparrow \end{aligned}$$

**FREE BODY: JOINT A**

$$\begin{aligned} \frac{F_{AB}}{22.5} = \frac{F_{AD}}{35.5} = \frac{16.8 \text{ kips}}{12} \\ F_{AB} = 31.5 \text{ kips} \quad \leftarrow \\ F_{AD} = 35.7 \text{ kips} \quad \leftarrow \end{aligned}$$

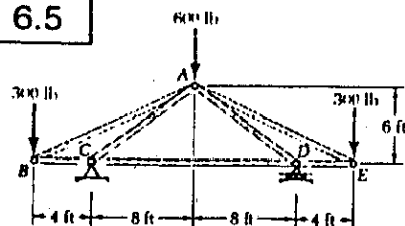
**FREE BODY: JOINT B**

$$\begin{aligned} \sum F_x = 0: F_{BC} = 31.5 \text{ kips} \quad \leftarrow \\ \sum F_y = 0: F_{BD} = 10.80 \text{ kips} \quad \leftarrow \end{aligned}$$

**FREE BODY: JOINT C**

$$\begin{aligned} \frac{F_{CD}}{37} = \frac{F_{BC}}{35} = \frac{10.8 \text{ kips}}{12} \\ F_{CD} = 33.3 \text{ kips} \quad \leftarrow \\ F_{BC} = 31.5 \text{ kips} \quad (\text{CHECKS}) \end{aligned}$$

6.5



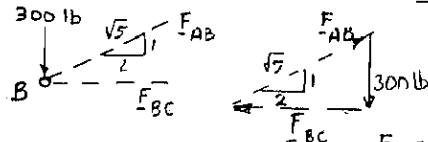
GIVEN:

TRUSS AND LOADING SHOWN

FIND:

FORCE IN EACH MEMBER

FREE BODY: TRUSS FROM THE SYMMETRY OF THE TRUSS AND LOADING, WE FIND  $C = D = 600 \text{ lb} \uparrow$



FREE BODY: JOINT B

$$\frac{F_{AB}}{\sqrt{5}} = \frac{F_{BC}}{2} = \frac{300 \text{ lb}}{1}$$

$$F_{AB} = 671 \text{ lb T}, F_{BC} = 600 \text{ lb C}$$

FREE BODY: JOINT C

$$F_{BC} = 600 \text{ lb}$$

$$+\uparrow \sum F_y = 0: \frac{3}{5} F_{AC} + 600 \text{ lb} = 0$$

$$F_{AC} = -1000 \text{ lb} \quad F_{AC} = 1000 \text{ lb C}$$

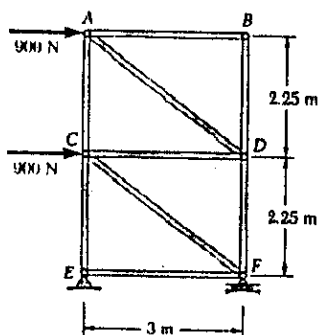
$$+\sum F_x = 0: \frac{4}{5} (-1000 \text{ lb}) + 600 \text{ lb} + F_{CD} = 0$$

$$F_{CD} = 200 \text{ lb T}$$

FROM SYMMETRY:

$$F_{AD} = F_{AC} = 1000 \text{ lb C}, F_{AE} = F_{AB} = 671 \text{ lb T}, F_{DE} = F_{BC} = 600 \text{ lb C}$$

6.6



GIVEN:

TRUSS AND LOADING SHOWN

FIND:

FORCE IN EACH MEMBER

FREE BODY: TRUSS

$$+\uparrow \sum M_E = 0:$$

$$F(3 \text{ m}) - (900 \text{ N})(2.25 \text{ m}) - (900 \text{ N})(4.5 \text{ m}) = 0$$

$$F = 2025 \text{ N} \uparrow$$

$$+\sum F_x = 0: E_x + 900 \text{ N} + 900 \text{ N} = 0$$

$$E_x = -1800 \text{ N} \quad E_x = 1800 \text{ N} \leftarrow$$

$$+\uparrow \sum F_y = 0: E_y + 2025 \text{ N} = 0$$

$$E_y = -2025 \text{ N} \quad E_y = 2025 \text{ N} \downarrow$$

WE NOTE THAT AB AND BD ARE ZERO-FORCE MEMBERS:  $F_{AB} = F_{BD} = 0$

FREE BODY: JOINT A

$$\frac{F_{AC}}{2.25} = \frac{F_{AD}}{3.75} = \frac{900 \text{ N}}{3}$$

$$F_{AC} = 675 \text{ N T}$$

$$F_{AD} = 1125 \text{ N C}$$

FREE BODY: JOINT D

$$\frac{F_{CD}}{3} = \frac{F_{DE}}{2.25} = \frac{1125 \text{ N}}{3.75}$$

$$F_{CD} = 900 \text{ N T}$$

$$F_{DE} = 675 \text{ N C}$$

CONTINUED

6.6 CONTINUED

FREE BODY:

$$E_x = 1800 \text{ N}$$

$$+\sum F_x = 0: F_{EF} - 1800 \text{ N} = 0$$

$$F_{EF} = 1800 \text{ N}$$

$$+\uparrow \sum F_y = 0: F_{CE} - 2025 \text{ N} = 0$$

$$F_{CE} = 2025 \text{ N}$$

FREE BODY:

$$F_{CF} = 2025 \text{ N}$$

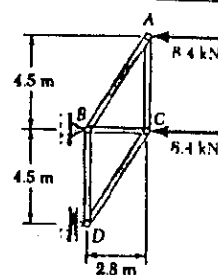
$$+\uparrow \sum F_y = 0: \frac{2.25}{3.75} F_{CF} + 2025 \text{ N} = 0$$

$$F_{CF} = -2250 \text{ N}$$

$$F_{CF} = 2250 \text{ N}$$

$$+\sum F_x = 0: -\frac{3}{3.75} (-2250 \text{ N}) - 1800 \text{ N} = 0$$

6.7



GIVEN:

TRUSS AND LOADING SHOWN

FIND:

FORCE IN EACH MEMBER

FREE BODY: TRUSS

$$+\uparrow \sum F_y = 0: B_y = 0$$

$$+\sum M_B = 0:$$

$$D(4.5 \text{ m}) + (8.4 \text{ kN})(4.5 \text{ m}) = 0$$

$$D = -8.4 \text{ kN}$$

$$+\sum F_x = 0:$$

$$B_x - 8.4 \text{ kN} - 8.4 \text{ kN} = 0$$

$$B_x = +16.8 \text{ kN}$$

FREE BODY:

$$\frac{F_{AB}}{5.3} = \frac{F_{AC}}{4.5}$$

$$F_{AB} = 13.5 \text{ kN}$$

$$F_{AC} = 11.25 \text{ kN}$$

FREE BODY: JOINT C

$$+\uparrow \sum F_y = 0: 13.50 \text{ kN} - \frac{4.5}{5.3} F_{CD} = 0$$

$$F_{CD} = +15.90 \text{ kN}$$

$$F_{CD} = 15.90 \text{ kN}$$

$$+\sum F_x = 0:$$

$$-F_{BC} - 8.4 \text{ kN} - \frac{2.8}{5.3} (15.90 \text{ kN}) = 0$$

$$F_{BC} = -16.80 \text{ kN}$$

$$F_{BC} = 16.80 \text{ kN}$$

FREE BODY: JOINT D

$$\frac{F_{BD}}{4.5} = \frac{F_{CD}}{2.8}$$

$$F_{BD} = 13.50 \text{ kN}$$

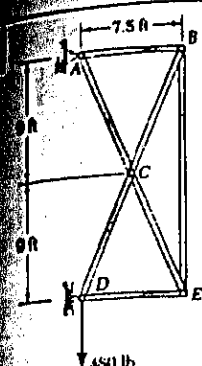
$$F_{BD} = 13.50 \text{ kN}$$

WE CAN ALSO WRITE THE PROPORTION

$$\frac{F_{BD}}{4.5} = \frac{15.90 \text{ kN}}{5.3}$$

$$F_{BD} = 13.50 \text{ kN}$$

(CHECK)



GIVEN:  
TRUSS AND LOADING SHOWN

FIND:  
FORCE IN EACH MEMBER

FREE BODY: TRUSS

$$+\uparrow \sum F_y = 0: A_y - 480 \text{ lb} = 0$$

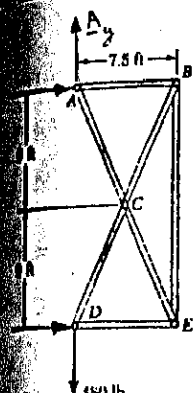
$$A_y = +480 \text{ lb} \quad H_y = 480 \text{ lb} \uparrow$$

$$+\circlearrowleft \sum M_A = 0: D(18 \text{ ft}) = 0$$

$$D = 0$$

$$+\rightarrow \sum F_x = 0: A_x + D = 0$$

$$A_x = 0$$



FREE BODY: JOINT A

$$\frac{F_{AB}}{7.5} = \frac{F_{AC}}{19.5} = \frac{480 \text{ lb}}{18}$$

$$F_{AB} = 200 \text{ lb C}$$

$$F_{AC} = 520 \text{ lb T}$$

FREE BODY: JOINT B

$$\frac{F_{BC}}{19.5} = \frac{F_{BE}}{18} = \frac{200 \text{ lb}}{7.5}$$

$$F_{BC} = 520 \text{ lb T}$$

$$F_{BE} = 480 \text{ lb C}$$

FREE BODY: JOINT C

SINCE THE FORCE POLYGON IS A RHOMBUS:

$$F_{CD} = 520 \text{ lb T}$$

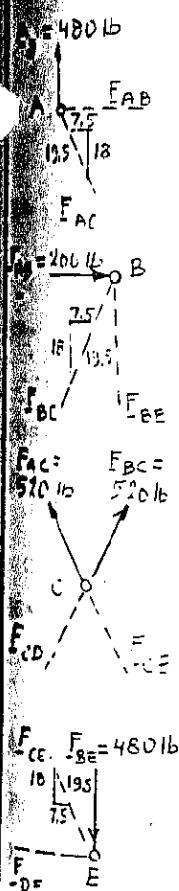
$$F_{CE} = 520 \text{ lb T}$$

FREE BODY: JOINT E

$$\frac{F_{DE}}{7.5} = \frac{F_{CE}}{19.5} = \frac{480 \text{ lb}}{18}$$

$$F_{DE} = 200 \text{ lb C}$$

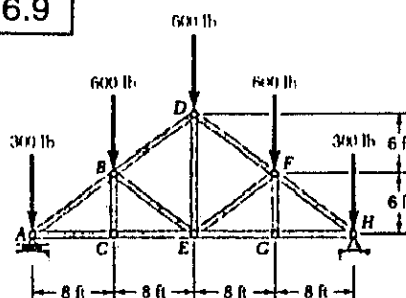
$$F_{CE} = 520 \text{ lb T (CHECKS)}$$



6.9

GIVEN:  
HOWE ROOF TRUSS  
LOADED AS SHOWN

FIND:  
FORCE IN EACH  
MEMBER

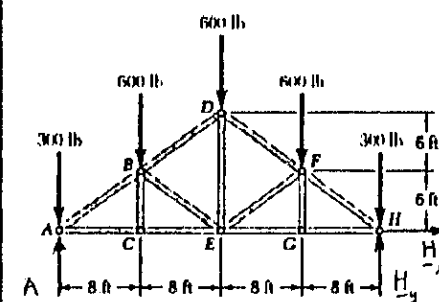


FREE BODY: TRUSS

$$\sum F_x = 0: H_x = 0$$

BECAUSE OF THE  
SYMMETRY OF THE  
TRUSS AND LOADING:  
 $A = H_y = \frac{1}{2} \text{ TOTAL LOAD}$

$$A = H_y = 1200 \text{ lb} \uparrow$$



FREE BODY: JOINT A

$$\frac{F_{AB}}{5} = \frac{F_{AC}}{4} = \frac{900 \text{ lb}}{3}$$

$$F_{AB} = 1500 \text{ lb C}$$

$$F_{AC} = 1200 \text{ lb T}$$

FREE BODY: JOINT C

BC IS A ZERO-FORCE MEMBER

$$F_{BC} = 0 \quad F_{CE} = 1200 \text{ lb T}$$

FREE BODY: JOINT B

$$+\rightarrow \sum F_x = 0: \frac{4}{5} F_{BD} + \frac{4}{5} F_{BE} + \frac{4}{5} (1500 \text{ lb}) = 0$$

$$\text{OR: } F_{BD} + F_{BE} = -1500 \text{ lb (1)}$$

$$+\uparrow \sum F_y = 0: \frac{3}{5} F_{BD} - \frac{3}{5} F_{BE} + \frac{3}{5} (1500 \text{ lb}) - 600 \text{ lb} = 0$$

$$\text{OR: } F_{BD} - F_{BE} = -500 \text{ lb (2)}$$

$$\text{ADD EQS. (1) AND (2): } 2F_{BD} = -2000 \text{ lb}$$

$$F_{BD} = 1000 \text{ lb C}$$

$$\text{SUBTRACT (2) FROM (1): } 2F_{BE} = -1000 \text{ lb}$$

$$F_{BE} = 500 \text{ lb C}$$

FREE BODY: JOINT D

$$+\rightarrow \sum F_x = 0: \frac{4}{5} (1000 \text{ lb}) + \frac{4}{5} F_{DF} = 0$$

$$F_{DF} = -1000 \text{ lb} \quad F_{DF} = 1000 \text{ lb C}$$

$$+\uparrow \sum F_y = 0: \frac{3}{5} (1000 \text{ lb}) - \frac{3}{5} (-1000 \text{ lb}) - 600 \text{ lb} - F_{DE} = 0$$

$$F_{DE} = +600 \text{ lb} \quad F_{DE} = 600 \text{ lb T}$$

BECAUSE OF THE SYMMETRY OF THE TRUSS AND  
LOADING, WE DEDUCE THAT

$$F_{EF} = F_{BE}$$

$$F_{EG} = F_{CE}$$

$$F_{FG} = F_{BC}$$

$$F_{FH} = F_{AE}$$

$$F_{GH} = F_{AC}$$

$$F_{EF} = 500 \text{ lb C}$$

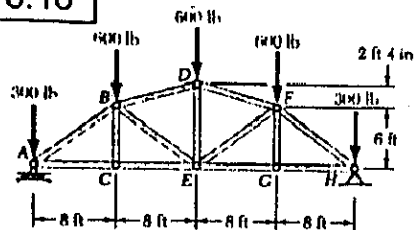
$$F_{EG} = 1200 \text{ lb T}$$

$$F_{FG} = 0$$

$$F_{FH} = 1500 \text{ lb C}$$

$$F_{GH} = 1200 \text{ lb T}$$

6.10



GIVEN:

GABLE ROOF  
TRUSS WITH LOADING  
SHOWN

FIND:  
FORCE IN EACH  
MEMBER.

FREE BODY: TRUSS

$$\sum F_x = 0: H_x = 0$$

BECAUSE OF THE  
SYMMETRY OF THE  
TRUSS AND LOADING:

$$A_x = H_x = \frac{1}{2} \text{ TOTAL LOAD}$$

$$A_x = H_x = 1200 \text{ lb} \uparrow$$

FREE BODY: JOINT A

$$\frac{F_{AB}}{5} = \frac{F_{AC}}{4} = \frac{900 \text{ lb}}{3}$$

$$F_{AB} = 1500 \text{ lb C}$$

$$F_{AC} = 1200 \text{ lb T}$$

FREE BODY: JOINT C

BC IS A ZERO-FORCE MEMBER

$$F_{BC} = 0 \quad F_{CE} = 1200 \text{ lb T}$$

FREE BODY: JOINT E

$$\pm \sum F_x = 0: \frac{24}{25} F_{BD} + \frac{4}{5} F_{BE} + \frac{1}{5} (1500 \text{ lb}) = 0$$

$$\text{OR } 24 F_{BD} + 20 F_{BE} = -30,000 \text{ lb (1)}$$

$$+\uparrow \sum F_y = 0: \frac{7}{25} F_{BD} - \frac{3}{5} F_{BE} + \frac{3}{5} (1500) - 600 = 0$$

$$\text{OR } 7 F_{BD} - 15 F_{BE} = -7,500 \text{ lb (2)}$$

MULTIPLY (1) BY 2, (2) BY 4, AND ADD:

$$100 F_{BD} = -120,000 \text{ lb}$$

$$F_{BD} = 1200 \text{ lb C}$$

MULTIPLY (1) BY 7, (2) BY -24, AND ADD:

$$500 F_{BE} = -30,000 \text{ lb}$$

$$F_{BE} = 60.0 \text{ lb C}$$

FREE BODY: JOINT D

$$\pm \sum F_x = 0: \frac{24}{25} (1200 \text{ lb}) + \frac{24}{25} F_{DE} = 0$$

$$F_{DE} = -1200 \text{ lb} \quad F_{DE} = 1200 \text{ lb C}$$

$$+\uparrow \sum F_y = 0:$$

$$\frac{7}{25} (1200 \text{ lb}) - \frac{7}{25} (-1200 \text{ lb}) - 600 \text{ lb} - F_{DF} = 0$$

$$F_{DE} = 72.0 \text{ lb} \quad F_{DE} = 72.0 \text{ lb T}$$

BECAUSE OF THE SYMMETRY OF THE TRUSS AND  
LOADING, WE DEDUCE THAT

$$F_{EF} = F_{BE}$$

$$F_{EF} = 60.0 \text{ lb C}$$

$$F_{EG} = F_{CE}$$

$$F_{EG} = 1200 \text{ lb T}$$

$$F_{FG} = F_{CG}$$

$$F_{FG} = 0$$

$$F_{FH} = F_{AB}$$

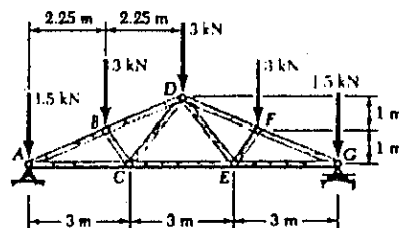
$$F_{FH} = 1500 \text{ lb C}$$

$$F_{GH} = F_{AC}$$

$$F_{GH} = 1200 \text{ lb T}$$

NOTE: COMPARE RESULTS WITH THOSE OF PROB. 6.7.

6.11



GIVEN:

FINK ROOF  
TRUSS WITH LOADING  
SHOWN.

FIND:  
FORCE IN  
MEMBER.

FREE BODY: TRUSS

$$\sum F_x = 0: A_x = 0$$

BECAUSE OF THE  
SYMMETRY OF THE  
TRUSS AND LOADING:

$$A_x = G_x = \frac{1}{2} \text{ TOTAL}$$

$$A_x = G_x = 6.00 \text{ kN}$$

FREE BODY: JOINT A

$$\frac{F_{AB}}{2.462} = \frac{F_{AC}}{2.25} = \frac{4.50 \text{ kN}}{2.25}$$

$$F_{AB} = 11.08 \text{ kN C}$$

$$F_{AC} = 10.125 \text{ kN T}$$

FREE BODY: JOINT B

$$\pm \sum F_x = 0:$$

$$\frac{3}{5} F_{BC} + \frac{2.25}{2.462} F_{BD} + \frac{2.25}{2.462} (11.08 \text{ kN}) = 0$$

$$+\uparrow \sum F_y = 0:$$

$$-\frac{4}{5} F_{BC} + \frac{F_{BD}}{2.462} + \frac{11.08 \text{ kN}}{2.462} - 3 \text{ kN} = 0$$

MULTIPLY EQ. (2) BY -2.25 AND ADD TO EQ. (1):

$$\frac{12}{5} F_{BC} + 6.75 \text{ kN} = 0 \quad F_{BC} = -2.8125 \text{ kN} \quad F_{BC} = 2.81 \text{ kN C}$$

MULTIPLY EQ. (1) BY 4, EQ. (2) BY 3, AND ADD:

$$\frac{12}{2.462} F_{BD} + \frac{12}{2.462} (11.08 \text{ kN}) - 9 \text{ kN} = 0$$

$$F_{BD} = -9.2335 \text{ kN}$$

$$F_{BD} = 9.23 \text{ kN C}$$

FREE BODY: JOINT C

$$+\uparrow \sum F_y = 0: \frac{4}{5} F_{CD} - \frac{4}{5} (2.8125 \text{ kN}) = 0$$

$$F_{CD} = 2.8125 \text{ kN}, \quad F_{CD} = 2.81 \text{ kN T}$$

$$\pm \sum F_x = 0:$$

$$F_{CE} - 10.125 \text{ kN} + \frac{3}{5} (2.8125 \text{ kN}) + \frac{3}{5} (2.8125 \text{ kN}) = 0$$

$$F_{CE} = +6.7500 \text{ kN} \quad F_{CE} = 6.75 \text{ kN T}$$

BECAUSE OF THE SYMMETRY OF THE TRUSS AND  
LOADING, WE DEDUCE THAT

$$F_{DE} = F_{CD}$$

$$F_{CD} = 2.81 \text{ kN T}$$

$$F_{DF} = F_{BD}$$

$$F_{DF} = 9.23 \text{ kN C}$$

$$F_{EF} = F_{BC}$$

$$F_{EF} = 2.81 \text{ kN C}$$

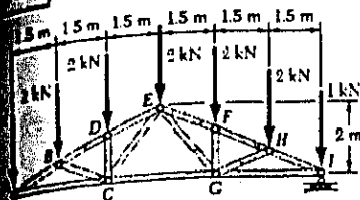
$$F_{EG} = F_{AC}$$

$$F_{EG} = 10.125 \text{ kN T}$$

$$F_{FG} = F_{AB}$$

$$F_{FG} = 11.08 \text{ kN C}$$

2



GIVEN:

FAN ROOF TRUSS  
AND LOADING  
SHOWN.

FIND:

FORCE IN EACH  
MEMBER.

FREE BODY: TRUSS

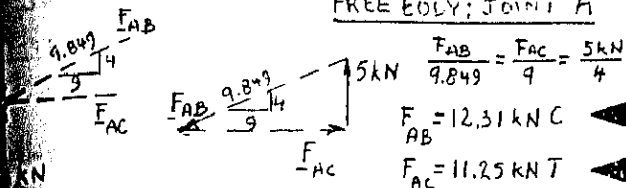
$$\sum F_x = 0: A_x = 0$$

FROM SYMMETRY OF  
TRUSS AND LOADING:

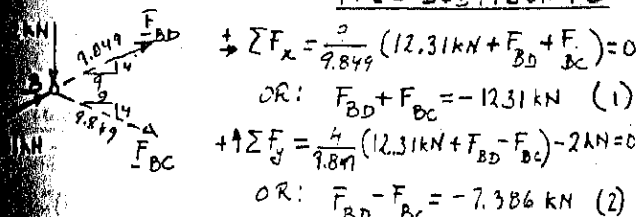
$$A_y = I_y = \frac{1}{2} \text{ TOTAL LOAD}$$

$$A_y = I_y = 6 \text{ kN} \uparrow$$

FREE BODY: JOINT A



FREE BODY: JOINT B



(1) AND (2):

$$F_{BD} = -19.70 \text{ kN}$$

SUBTRACT (2) FROM (1):

$$F_{BC} = -4.924 \text{ kN}$$

FREE BODY: JOINT D

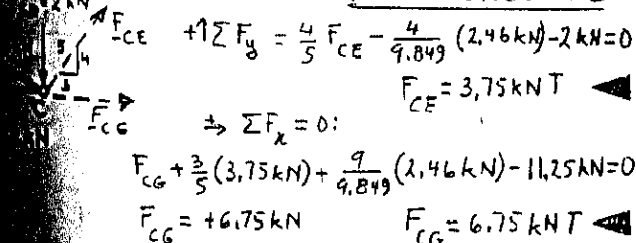
FROM FORCE

POLYGON:

$$F_{CD} = 2.00 \text{ kN} \leftarrow$$

$$F_{DE} = 9.85 \text{ kN} \leftarrow$$

FREE BODY: JOINT C



THE SYMMETRY OF THE TRUSS AND LOADING:

$$F_{ED} = F_{DE}$$

$$F_{EG} = F_{GE}$$

$$F_{FG} = F_{GF}$$

$$F_{GH} = F_{HG}$$

$$F_{HI} = F_{IH}$$

$$F_{FI} = F_{IF}$$

$$F_{FI} = F_{IF}$$

$$F_{EF} = 9.85 \text{ kN} \leftarrow$$

$$F_{EG} = 3.75 \text{ kN} \leftarrow$$

$$F_{FG} = 2.00 \text{ kN} \leftarrow$$

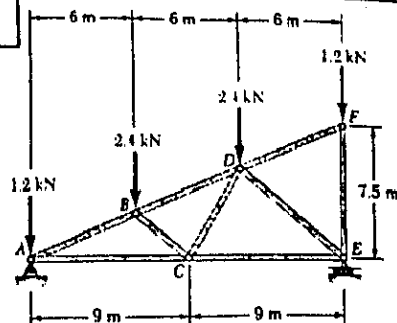
$$F_{GH} = 9.85 \text{ kN} \leftarrow$$

$$F_{HI} = 2.46 \text{ kN} \leftarrow$$

$$F_{FI} = 11.25 \text{ kN} \leftarrow$$

$$F_{FI} = 12.31 \text{ kN} \leftarrow$$

6.13



GIVEN:

ROOF TRUSS AND  
LOADING SHOWN.

FIND:

FORCE IN EACH  
MEMBER.

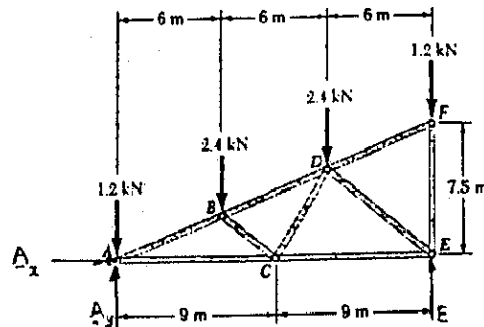
FREE BODY: TRUSS

$$\sum F_x = 0: A_x = 0$$

FROM SYMMETRY OF  
LOADING:

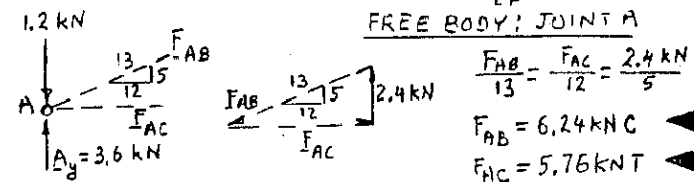
$$A_y = E_y = \frac{1}{2} \text{ TOTAL LOAD}$$

$$A_y = E_y = 3.6 \text{ kN} \uparrow$$

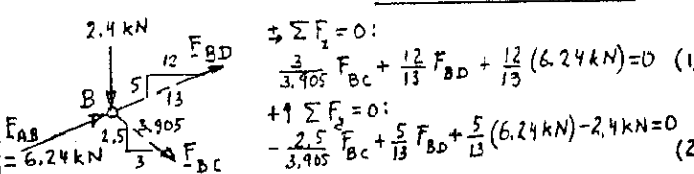
WE NOTE THAT DF IS A ZERO-FORCE MEMBER AND THAT  
EF IS ALIGNED WITH THE LOAD. THUS  $F_{DF} = 0$ 

$$F_{EF} = 1.2 \text{ kN} \leftarrow$$

FREE BODY: JOINT A



FREE BODY: JOINT B



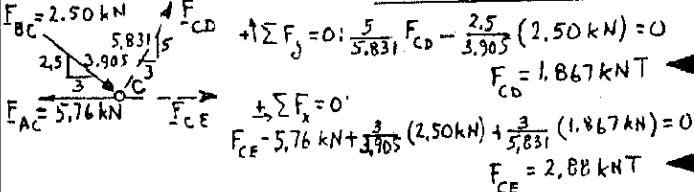
MULTIPLY (1) BY 2.5, (2) BY 3, AND ADD:

$$\frac{45}{13} F_{BD} + \frac{45}{13} (6.24 \text{ kN}) - 7.1 \text{ kN} = 0, F_{BD} = -4.16 \text{ kN}, F_{BD} = 4.16 \text{ kN} \leftarrow$$

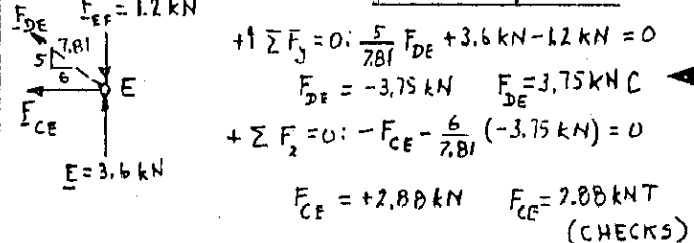
MULTIPLY (1) BY 5, (2) BY -12, AND ADD:

$$\frac{45}{3.905} F_{BC} + 28.8 \text{ kN} = 0, F_{BC} = -2.50 \text{ kN}, F_{BC} = 2.50 \text{ kN} \leftarrow$$

FREE BODY: JOINT C

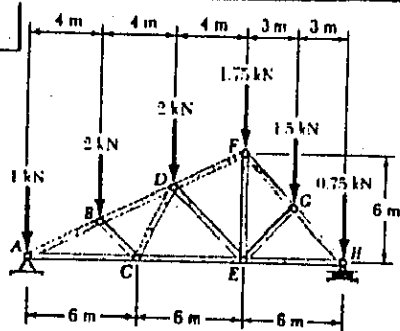


FREE BODY: JOINT E





6.14



GIVEN:

DOUBLE-PITCH  
ROOF TRUSS AND  
LOADING SHOWN.

FIND:

FORCE IN  
EACH MEMBER.

FREE BODY: TRUSS

$$+\sum M_A = 0:$$

$$H(18m) - (2kN)(4m)$$

$$- (2kN)(8m) - (1.75kN)(12m)$$

$$- (1.5kN)(15m) - (0.75kN)(18m)$$

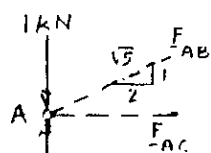
$$= 0 \quad H = 4.50 kN \uparrow$$

$$\sum F_x = 0: A_x = 0$$

$$\sum F_y = 0: A_y + H - 9 = 0$$

$$A_y = 9 - 4.50, A_y = 4.50 kN \uparrow$$

FREE BODY: JOINT A



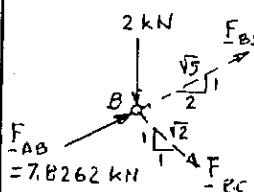
$$\frac{F_{AB}}{\sqrt{5}} = \frac{F_{AC}}{2} = \frac{3.50 kN}{1}$$

$$F_{AB} = 7.8262 kN \quad C$$

$$F_{AB} = 7.83 kN \quad C$$

$$F_{AC} = 7.00 kN \quad T$$

FREE BODY: JOINT B



$$+\sum F_z = 0:$$

$$\frac{2}{\sqrt{5}} F_{BD} + \frac{2}{\sqrt{5}} (7.8262 kN) + \frac{1}{\sqrt{2}} F_{BC} = 0$$

$$\text{OR: } F_{BD} + 0.79057 F_{BC} = -7.8262 kN \quad (1)$$

$$+\sum F_y = 0:$$

$$\frac{1}{\sqrt{5}} F_{BD} + \frac{1}{\sqrt{5}} (7.8262 kN) - \frac{1}{\sqrt{2}} F_{BC} - 2 kN = 0$$

$$\text{OR: } F_{BD} - 1.58114 F_{BC} = -3.3541 \quad (2)$$

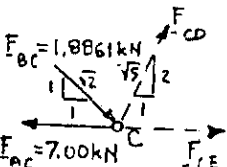
MULTIPLY (1) BY 2 AND ADD (2):

$$3F_{BD} = -19.0465, F_{BD} = -6.3355 kN \quad F_{BD} = 6.34 kN \quad C$$

SUBTRACT (2) FROM (1):

$$2.37111 F_{BC} = -4.4721, F_{BC} = -1.8861 kN \quad F_{BC} = 1.89 kN \quad C$$

FREE BODY: JOINT C



$$+\sum F_y = 0: \frac{2}{\sqrt{5}} F_{CD} - \frac{1}{\sqrt{2}} (1.8861 kN) = 0$$

$$F_{CD} = 1.4911 kN \quad F_{CD} = 1.49 kN \quad T$$

$$+\sum F_x = 0:$$

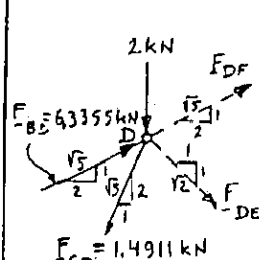
$$F_{CE} - 7.00 kN + \frac{1}{\sqrt{2}} (1.8861 kN) + \frac{1}{\sqrt{5}} (1.4911 kN) = 0$$

$$F_{CE} = 5.000 kN$$

CONTINUED

6.14 CONTINUED

FREE BODY: JOINT D



$$+\sum F_x = 0:$$

$$\frac{2}{\sqrt{5}} F_{DF} + \frac{1}{\sqrt{2}} F_{DE} + \frac{2}{\sqrt{5}} (6.3355 kN) - \frac{1}{\sqrt{5}} (1.4911 kN) = 0$$

$$\text{OR: } F_{DF} + 0.79057 F_{DE} = -5.5900$$

$$+\sum F_y = 0:$$

$$\frac{1}{\sqrt{5}} F_{DF} - \frac{1}{\sqrt{2}} F_{DE} + \frac{1}{\sqrt{5}} (6.3355 kN) - \frac{2}{\sqrt{5}} (1.4911 kN) - 2 kN = 0$$

$$\text{OR: } F_{DF} - 0.79057 F_{DE} = -1.1188$$

$$\text{ADD (1) AND (2): } 2F_{DF} = -6.7088 kN$$

$$F_{DF} = -3.3544 kN$$

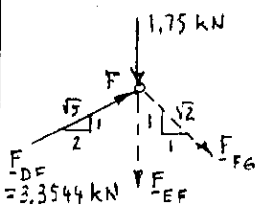
$$F_{DF} = 3.35 kN \quad C$$

$$\text{SUBTRACT (2) FROM (1): } 1.58114 F_{DE} = -4.4712 kN$$

$$F_{DE} = -2.8278 kN$$

$$F_{DE} = 2.83 kN \quad C$$

FREE BODY: JOINT F



$$+\sum F_x = 0: \frac{1}{\sqrt{2}} F_{FG} + \frac{2}{\sqrt{5}} (3.3544 kN) = 0$$

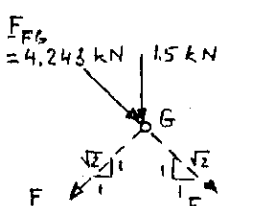
$$F_{FG} = -4.243 kN, F_{FG} = 4.24 kN \quad C$$

$$+\sum F_y = 0:$$

$$-F_{EF} - 1.75 kN + \frac{1}{\sqrt{5}} (3.3544 kN) - \frac{1}{\sqrt{2}} (-4.243 kN) = 0$$

$$F_{EF} = 2.750 kN \quad F_{EF} = 2.75 kN \quad T$$

FREE BODY: JOINT G



$$+\sum F_z = 0:$$

$$\frac{1}{\sqrt{2}} F_{GH} - \frac{1}{\sqrt{2}} F_{GE} + \frac{1}{\sqrt{2}} (4.243 kN) = 0$$

$$\text{OR: } F_{GH} - F_{GE} = -4.243 kN \quad (1)$$

$$+\sum F_y = 0:$$

$$- \frac{1}{\sqrt{2}} F_{GH} - \frac{1}{\sqrt{2}} F_{GE} - \frac{1}{\sqrt{2}} (4.243 kN) - 1.5 kN = 0$$

$$\text{OR: } F_{GH} + F_{GE} = -6.364 kN \quad (2)$$

$$\text{ADD (1) AND (2): } 2F_{GH} = -10.607$$

$$F_{GH} = -5.303$$

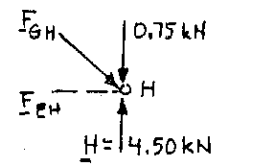
$$F_{GH} = 5.30 kN \quad C$$

$$\text{SUBTRACT (1) FROM (2): } 2F_{GE} = -2.121 kN$$

$$F_{GE} = -1.0605 kN$$

$$F_{GE} = 1.06 kN \quad C$$

FREE BODY: JOINT H



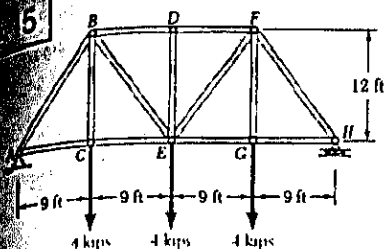
$$\frac{F_{EH}}{1} = \frac{3.75 kN}{1}$$

WE CAN ALSO WRITE:

$$\frac{F_{GH}}{\sqrt{2}} = \frac{3.75 kN}{1}$$

$$F_{GH} = 5.30 kN \quad (\text{CHECK})$$

5



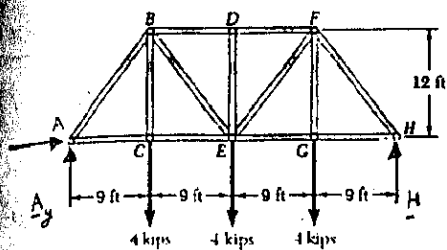
GIVEN:

PRATT BRIDGE  
TRUSS AND LOADING  
SHOWN.

FIND:

FORCE IN EACH  
MEMBER.

FREE BODY:  
TRUSS



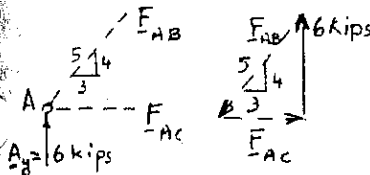
$$\sum F_x = 0: A_x = 0$$

$$\sum M_A = 0: H(36 \text{ ft}) - (4 \text{ kips})(9 \text{ ft}) - (4 \text{ kips})(18 \text{ ft}) - (4 \text{ kips})(27 \text{ ft}) = 0$$

$$H = 6 \text{ kips} \uparrow$$

$$\sum F_y = 0: A_y + 6 \text{ kips} - 12 \text{ kips} = 0 \quad A_y = 6 \text{ kips} \uparrow$$

FREE BODY: JOINT A

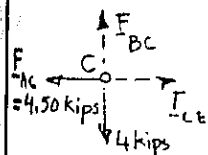


$$\frac{F_{AB}}{5} = \frac{F_{AC}}{3} = \frac{6 \text{ kips}}{4}$$

$$F_{AB} = 7.50 \text{ kips C}$$

$$F_{AC} = 4.50 \text{ kips T}$$

FREE BODY: JOINT C



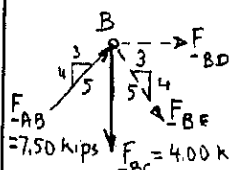
$$\sum F_x = 0:$$

$$F_{CE} = 4.50 \text{ kips T}$$

$$\sum F_y = 0:$$

$$F_{BC} = 4.00 \text{ kips T}$$

FREE BODY: JOINT B



$$+\uparrow \sum F_y = 0:$$

$$-\frac{4}{5} F_{BE} + \frac{4}{5} (7.50 \text{ kips}) - 4.00 \text{ kips} = 0$$

$$F_{BE} = 2.50 \text{ kips T}$$

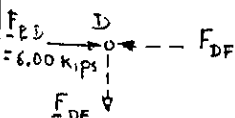
$$\pm \sum F_x = 0:$$

$$\frac{3}{5} (7.50 \text{ kips}) + \frac{3}{5} (2.50 \text{ kips}) + F_{BD} = 0$$

$$F_{BD} = -6.00 \text{ kips}$$

$$F_{BD} = 6.00 \text{ kips C}$$

FREE BODY: JOINT D



WE NOTE THAT DE IS A  
ZERO-FORCE MEMBER:  $F_{DE} = 0$

ALSO:

$$F_{DF} = 6.00 \text{ kips C}$$

FROM JOINT C:

$$F_{EF} = F_{BE}$$

$$F_{EG} = F_{CE}$$

$$F_{FG} = F_{BC}$$

$$F_{FH} = F_{AB}$$

$$F_{GH} = F_{AC}$$

$$F_{EF} = 2.50 \text{ kips T}$$

$$F_{EG} = 4.50 \text{ kips T}$$

$$F_{FG} = 4.00 \text{ kips T}$$

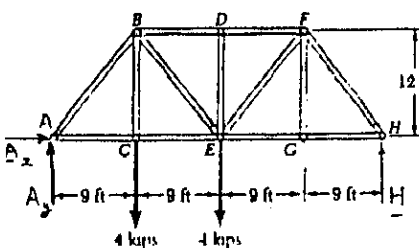
$$F_{FH} = 7.50 \text{ kips C}$$

$$F_{GH} = 4.50 \text{ kips T}$$

6.16

GIVEN: TRUSS OF PROB. 6.15, ASSUMING THAT  
THE LOAD APPLIED AT E HAS BEEN REMOVED.

FIND: FORCE IN EACH MEMBER.



FREE BODY: TRUSS

$$\sum F_x = 0: A_x = 0$$

$$\sum M_A = 0:$$

$$H(36 \text{ ft}) - (4 \text{ kips})(9 \text{ ft}) - (4 \text{ kips})(18 \text{ ft}) = 0$$

$$H = 3.00 \text{ kips} \uparrow$$

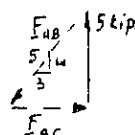
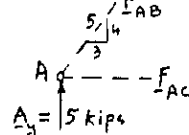
$$+\uparrow \sum F_y = 0: A_y = 5.00 \text{ kips} \uparrow$$

WE NOTE THAT DE AND FG ARE ZERO-FORCE MEMBERS.

THEREFORE:  $F_{DE} = 0$ ,  $F_{FG} = 0$ .

ALSO:  $F_{BD} = F_{DF}$  (1) AND  $F_{EG} = F_{GH}$  (2)

FREE BODY: JOINT A

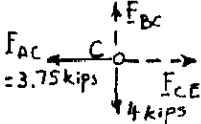


$$\frac{F_{AB}}{5} = \frac{F_{AC}}{3} = \frac{5 \text{ kips}}{4}$$

$$F_{AB} = 6.25 \text{ kips C}$$

$$F_{AC} = 3.75 \text{ kips T}$$

FREE BODY: JOINT C



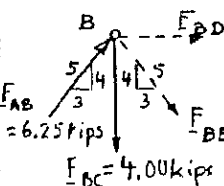
$$\sum F_x = 0:$$

$$F_{CE} = 3.75 \text{ kips T}$$

$$\sum F_y = 0:$$

$$F_{BC} = 4.00 \text{ kips T}$$

FREE BODY: JOINT B



$$+\uparrow \sum F_y = 0: \frac{4}{5} (6.25 \text{ kips}) - 4.00 \text{ kips} - \frac{4}{5} F_{BE} = 0$$

$$F_{BE} = 1.250 \text{ kips T}$$

$$\pm \sum F_x = 0: F_{BD} + \frac{3}{5} (6.25 \text{ kips}) + \frac{3}{5} (1.250 \text{ kips}) = 0$$

$$F_{BD} = -4.50 \text{ kips} \quad F_{BD} = 4.50 \text{ kips C}$$

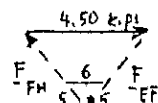
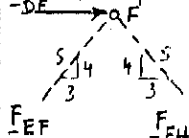
FREE BODY: JOINT F

WE RECALL THAT  $F_{FS} = 0$ , AND FROM (1) THAT

$$F_{DF} = F_{BD}$$

$$F_{DF} = 4.50 \text{ kips C}$$

$$F_{DE} = 4.50 \text{ kips}$$

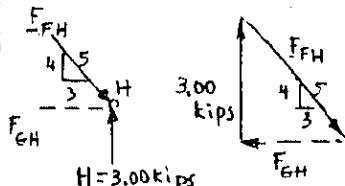


$$\frac{F_{FE}}{5} = \frac{F_{FH}}{5} = \frac{4.50 \text{ kips}}{6}$$

$$F_{FE} = 3.75 \text{ kips T}$$

$$F_{FH} = 3.75 \text{ kips C}$$

FREE BODY: JOINT H



$$\frac{F_{GH}}{3} = \frac{3.00 \text{ kips}}{4}$$

$$F_{GH} = 2.25 \text{ kips T}$$

$$\text{ALSO: } \frac{F_{FH}}{5} = \frac{3.00 \text{ kips}}{4}$$

$$F_{FH} = 3.75 \text{ kips C}$$

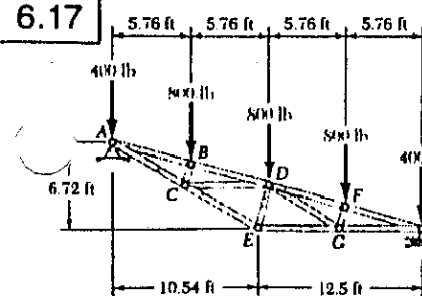
(CHECKS)

$$F_{EG} = 2.25 \text{ kips T}$$

FROM EQ. (2):

$$F_{EG} = F_{GH}$$

6.17

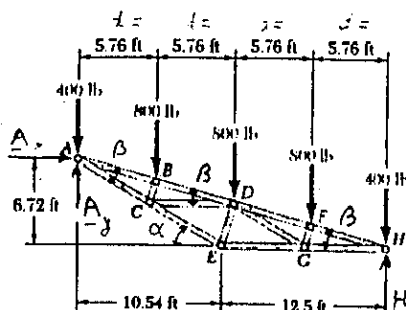


GIVEN:

INVERTED HOWE ROOF TRUSS AND LOADING SHOWN.

FIND:

FORCE IN MEMBER DE AND IN MEMBERS TO THE LEFT OF DE.



FREE BODY: TRUSS

$$\sum F_x = 0: A_x = 0$$

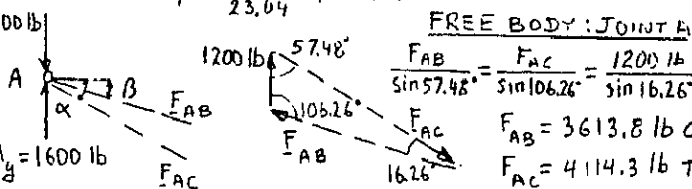
$$+\sum M_H = 0:$$

$$(400 \text{ lb})(4d) + (800 \text{ lb})(3d) + (800 \text{ lb})(2d) + (800 \text{ lb})d - A_y(4d) = 0$$

$$A_y = 1600 \text{ lb} \uparrow$$

$$\text{ANGLES: } \tan \alpha = \frac{6.72}{10.54} \quad \alpha = 32.52^\circ$$

$$\tan \beta = \frac{6.72}{23.04} \quad \beta = 16.26^\circ$$



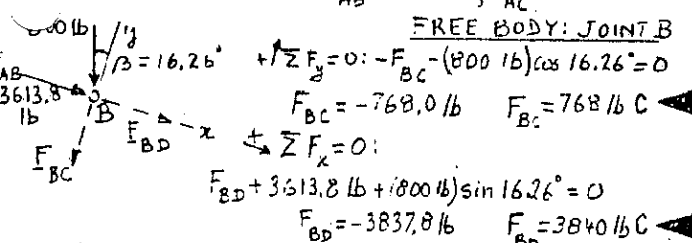
FREE BODY: JOINT A

$$\frac{F_{AB}}{\sin 57.48^\circ} = \frac{F_{AC}}{\sin 106.26^\circ} = \frac{1200 \text{ lb}}{\sin 16.26^\circ}$$

$$F_{AB} = 3613.8 \text{ lb C}$$

$$F_{AC} = 4114.3 \text{ lb T}$$

$$F_{AB} = 3610 \text{ lb C}, F_{AC} = 4110 \text{ lb T}$$



FREE BODY: JOINT B

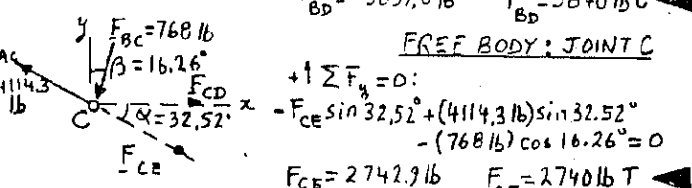
$$+\sum F_y = 0: -F_{BC} - (800 \text{ lb}) \cos 16.26^\circ = 0$$

$$F_{BC} = -768.0 \text{ lb} \quad F_{BC} = 768 \text{ lb C}$$

$$+\sum F_x = 0:$$

$$F_{BD} + 3613.8 \text{ lb} + (800 \text{ lb}) \sin 16.26^\circ = 0$$

$$F_{BD} = -3837.8 \text{ lb} \quad F_{BD} = 3840 \text{ lb C}$$



FREE BODY: JOINT C

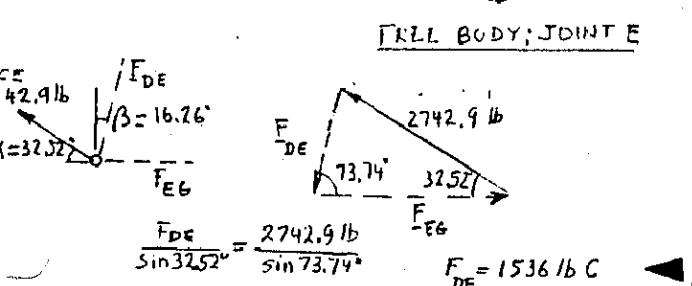
$$+\sum F_y = 0:$$

$$-F_{CE} \sin 32.52^\circ + (4114.3 \text{ lb}) \sin 32.52^\circ - (768 \text{ lb}) \cos 16.26^\circ = 0$$

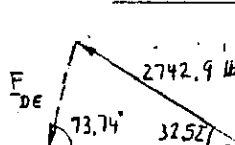
$$F_{CE} = 2742.9 \text{ lb} \quad F_{CE} = 2740 \text{ lb T}$$

$$\sum F_x = 0: F_{CD} - (4114.3 \text{ lb}) \cos 32.52^\circ + (2742.9 \text{ lb}) \cos 32.52^\circ - (768 \text{ lb}) \sin 16.26^\circ = 0$$

$$F_{CD} = 1371.4 \text{ lb} \quad F_{CD} = 1371 \text{ lb T}$$



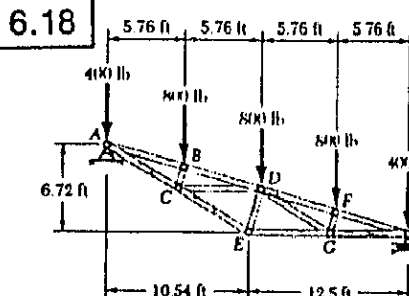
FREE BODY: JOINT E



$$\frac{F_{DE}}{\sin 32.52^\circ} = \frac{2742.9 \text{ lb}}{\sin 73.74^\circ}$$

$$F_{DE} = 1536 \text{ lb C}$$

6.18

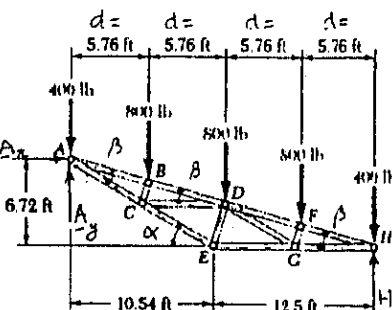


GIVEN:

INVERTED HOWE ROOF TRUSS AND LOADING SHOWN.

FIND:

FORCE IN MEMBERS TO THE RIGHT OF DE.



FREE BODY: TRUSS

$$+\sum M_A = 0:$$

$$H(4d) - (800 \text{ lb})d$$

$$- (800 \text{ lb})(2d)$$

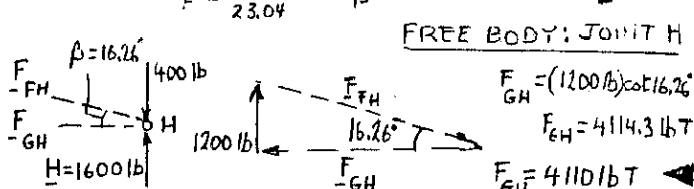
$$- (800 \text{ lb})(3d)$$

$$- (400 \text{ lb})(4d) = 0$$

$$H = 1600 \text{ lb} \uparrow$$

$$\text{ANGLES: } \tan \alpha = \frac{6.72}{10.54} \quad \alpha = 32.52^\circ$$

$$\tan \beta = \frac{6.72}{23.04} \quad \beta = 16.26^\circ$$



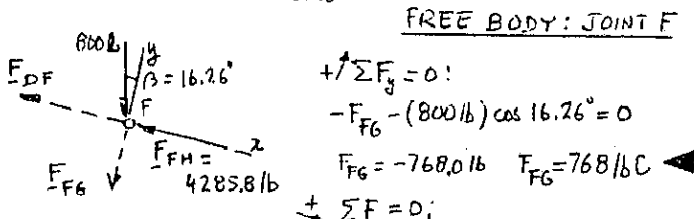
FREE BODY: JOINT H

$$F_{GH} = (1200 \text{ lb}) \cot 16.26^\circ$$

$$F_{GH} = 4114.3 \text{ lb T}$$

$$F_{GH} = 4110 \text{ lb T}$$

$$F_{FH} = \frac{1200 \text{ lb}}{\sin 16.26^\circ} = 4285.8 \text{ lb} \quad F_{FH} = 4290 \text{ lb C}$$



FREE BODY: JOINT F

$$+\sum F_y = 0:$$

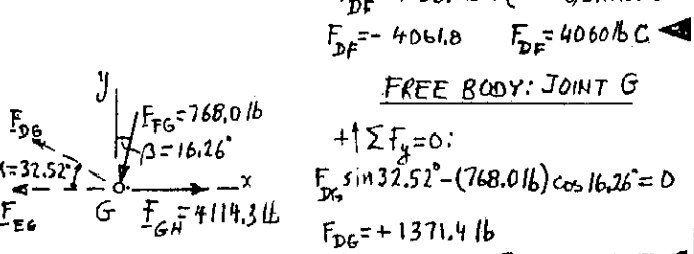
$$-F_{FG} - (800 \text{ lb}) \cos 16.26^\circ = 0$$

$$F_{FG} = -768.0 \text{ lb} \quad F_{FG} = 768 \text{ lb C}$$

$$+\sum F_x = 0:$$

$$-F_{DF} - 4285.8 \text{ lb} + (800 \text{ lb}) \sin 16.26^\circ = 0$$

$$F_{DF} = -4060.8 \quad F_{DF} = 4060 \text{ lb C}$$



FREE BODY: JOINT G

$$+\sum F_y = 0:$$

$$F_{DG} \sin 32.52^\circ - (768.0 \text{ lb}) \cos 16.26^\circ = 0$$

$$F_{DG} = +1371.4 \text{ lb}$$

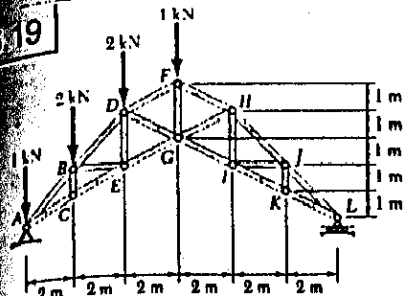
$$F_{DG} = 1371 \text{ lb T}$$

$$+\sum F_x = 0: -F_{EG} + 4114.3 \text{ lb} - (768.0 \text{ lb}) \sin 16.26^\circ - (1371.4 \text{ lb}) \cos 32.52^\circ = 0$$

$$F_{EG} = +2742.9 \text{ lb}$$

$$F_{EG} = 2740 \text{ lb T}$$

6.19



GIVEN:  
SCISSORS ROOF  
TRUSS AND LOADING  
SHOWN.  
FIND:  
FORCE IN MEMBER  
TO THE LEFT OF  
FG.

FREE BODY: TRUSS

$$\sum F_x = 0: \quad A_x = 0$$

$$\sum M_L = 0:$$

$$\begin{aligned} & (1 \text{ kN})(12 \text{ m}) \\ & + (2 \text{ kN})(10 \text{ m}) \\ & + (2 \text{ kN})(4 \text{ m}) \\ & + (1 \text{ kN})(6 \text{ m}) \\ & - A_y(12 \text{ m}) = 0 \\ & A_y = 4.50 \text{ kN} \uparrow \end{aligned}$$

WE NOTE THAT BC IS A ZERO-FORCE MEMBER:  $F_{BC} = 0$   
ALSO:  $F_{CE} = F_{AC}$  (1)

FREE BODY: JOINT A

$$\sum F_x = 0: \quad \frac{1}{\sqrt{2}} F_{AB} + \frac{1}{\sqrt{2}} F_{AC} = 0 \quad (2)$$

$$+\uparrow \sum F_y = 0: \quad \frac{1}{\sqrt{2}} F_{AB} + \frac{1}{\sqrt{2}} F_{AC} + 3.50 \text{ kN} = 0 \quad (3)$$

MULTIPLY (3) BY -2 AND ADD (2):

$$-\frac{1}{\sqrt{2}} F_{AB} - 7 \text{ kN} = 0 \quad F_{AB} = 9.90 \text{ kN C}$$

SUBTRACT (3) FROM (2):

$$\frac{1}{\sqrt{2}} F_{AC} - 3.50 \text{ kN} = 0, \quad F_{AC} = 7.826 \text{ kN}, \quad F_{AC} = 7.83 \text{ kN T}$$

$$\text{FROM (1): } F_{CE} = F_{AC} = 7.826 \text{ kN} \quad F_{CE} = 7.83 \text{ kN T}$$

FREE BODY: JOINT B

$$+\uparrow \sum F_y = 0: \quad \frac{1}{\sqrt{2}} F_{BD} + \frac{1}{\sqrt{2}} (9.90 \text{ kN}) - 2 \text{ kN} = 0$$

$$F_{BD} = -7.071 \text{ kN} \quad F_{BD} = 7.07 \text{ kN C}$$

$$+\rightarrow \sum F_x = 0: \quad F_{BE} + \frac{1}{\sqrt{2}} (9.90 - 7.071) \text{ kN} = 0$$

$$F_{BE} = -2.000 \text{ kN} \quad F_{BE} = 2.00 \text{ kN C}$$

FREE BODY: JOINT E

$$+\rightarrow \sum F_x = 0: \quad \frac{2}{\sqrt{5}} (F_{EG} - 7.826 \text{ kN}) + 2.00 \text{ kN} = 0$$

$$F_{EG} = 5.590 \text{ kN} \quad F_{EG} = 5.59 \text{ kN T}$$

$$+\uparrow \sum F_y = 0: \quad F_{DE} - \frac{1}{\sqrt{5}} (7.826 - 5.590) \text{ kN} = 0$$

$$F_{DE} = 1.000 \text{ kN} \quad F_{DE} = 1.000 \text{ kN T}$$

FREE BODY: JOINT D

$$+\rightarrow \sum F_x = 0: \quad \frac{2}{\sqrt{5}} (F_{DF} + F_{DG}) + \frac{1}{\sqrt{2}} (7.071 \text{ kN}) = 0$$

$$\text{OR: } F_{DF} + F_{DG} = -5.590 \text{ kN} \quad (4)$$

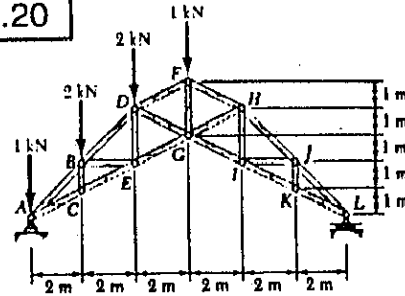
$$+\uparrow \sum F_y = 0: \quad \frac{1}{\sqrt{5}} (F_{DF} - F_{DG}) + \frac{1}{\sqrt{2}} (7.071 \text{ kN}) - 2 \text{ kN} - 1 \text{ kN} = 0$$

$$\text{OR: } F_{DF} - F_{DG} = -4.472 \quad (5)$$

$$\text{ADD (4) AND (5): } 2 F_{DF} = -10.062 \text{ kN} \quad F_{DF} = 5.03 \text{ kN C}$$

$$\text{SUBTRACT (5) FROM (4): } 2 F_{DG} = -1.1180 \text{ kN} \quad F_{DG} = 0.559 \text{ kN C}$$

6.20



GIVEN:  
SCISSORS ROOF  
TRUSS AND LOADING  
SHOWN.  
FIND:  
FORCE IN MEMBER  
FG AND IN MEMBERS  
TO THE RIGHT OF  
FG.

FREE BODY: TRUSS

$$+\rightarrow \sum M_A = 0:$$

$$L(12 \text{ m}) - (2 \text{ kN})(2 \text{ m})$$

$$- (2 \text{ kN})(4 \text{ m})$$

$$- (1 \text{ kN})(6 \text{ m}) = 0$$

$$L = 1.500 \text{ kN} \uparrow$$

$$\text{ANGLES: } \tan \alpha = \frac{1}{2} \quad \alpha = 45^\circ$$

$$\tan \beta = \frac{1}{2} \quad \beta = 26.57^\circ$$

ZERO-FORCE MEMBERS:

EXAMINING SUCCESSIVELY JOINTS K, J, AND I, WE  
NOTE THAT THE FOLLOWING MEMBERS TO THE RIGHT OF  
FG ARE ZERO-FORCE MEMBERS: JK, IJ, AND HI.  
THUS:  $F_{HI} = F_{IJ} = F_{JK} = 0$

WE ALSO NOTE THAT

$$F_{GI} = F_{IK} = F_{KL} \quad (1) \quad \text{AND} \quad F_{HJ} = F_{JL} \quad (2)$$

FREE BODY: JOINT L

$$\frac{F_{JL}}{\sin 116.57^\circ} = \frac{F_{KL}}{\sin 45^\circ} = \frac{1.500 \text{ kN}}{\sin 18.43^\circ}$$

$$F_{JL} = 4.2436 \text{ kN} \quad F_{JL} = 4.24 \text{ kN C}$$

$$F_{KL} = 3.35 \text{ kN T}$$

$$\text{FROM EQ. (1): } F_{GI} = F_{IK} = F_{KL} \quad F_{GI} = F_{IK} = 3.35 \text{ kN T}$$

$$\text{FROM EQ. (2): } F_{HJ} = F_{JL} = 4.2436 \text{ kN}, \quad F_{HJ} = 4.24 \text{ kN C}$$

FREE BODY: JOINT H

$$\frac{F_{FH}}{\sin 108.43^\circ} = \frac{F_{GH}}{\sin 18.43^\circ} = \frac{4.2436}{\sin 53.14^\circ}$$

$$F_{FH} = 5.03 \text{ kN C} \quad F_{GH} = 1.677 \text{ kN T}$$

FREE BODY: JOINT F

$$+\rightarrow \sum F_x = 0:$$

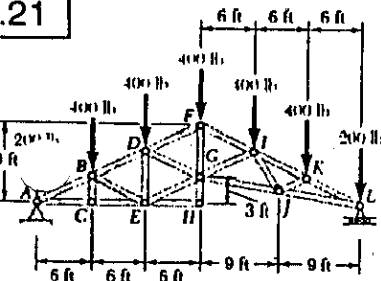
$$- F_{DF} \cos 26.57^\circ - (5.03 \text{ kN}) \cos 26.57^\circ = 0$$

$$F_{DF} = -5.03 \text{ kN}$$

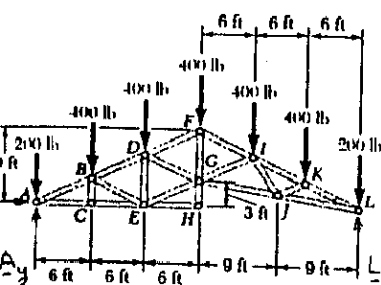
$$+\uparrow \sum F_y = 0:$$

$$- F_{FG} - 1 \text{ kN} + (5.03 \text{ kN}) \sin 26.57^\circ - (-5.03 \text{ kN}) \sin 26.57^\circ = 0$$

$$F_{FG} = +3.500 \text{ kN} \quad F_{FG} = 3.50 \text{ kN T}$$



**GIVEN:**  
STUDIO ROOF TRUSS  
AND LOADING SHOWN.  
**FIND:**  
FORCE IN MEMBERS  
TO THE LEFT OF LINE  
FGH.



**FREE BODY: TRUSS**

$$\sum F_x = 0: A_x = 0$$

BECAUSE OF SYMMETRY  
OF LOADING:

$$A_y = L = \frac{1}{2} \text{ TOTAL LOAD}$$

$$A_y = L = 1200 \text{ lb} \uparrow$$

0-FORCE MEMBERS. EXAMINING JOINTS C AND H,  
CONCLUDE THAT BC, EH, AND GH ARE ZERO-FORCE  
MEMBERS. THUS:

$$F_{BC} = F_{EH} = 0$$

$$F_{CE} = F_{AC} \quad (1)$$

**FREE BODY: JOINT A**

$$\frac{F_{AB}}{\sqrt{5}} = \frac{F_{AC}}{2} = \frac{1000 \text{ lb}}{1}$$

$$F_{AC} = 2236 \text{ lb C}$$

$$F_{AB} = 2240 \text{ lb C}$$

$$F_{AC} = 2000 \text{ lb T}$$

$$\text{FROM (1): } F_{CE} = 2000 \text{ lb T}$$

**FREE BODY: JOINT B**

$$\sum F_x = 0: \frac{2}{\sqrt{5}} F_{BD} + \frac{2}{\sqrt{5}} F_{BE} + \frac{2}{\sqrt{5}} (2236 \text{ lb}) = 0$$

$$\text{OR: } F_{BD} + F_{BE} = -2236 \text{ lb} \quad (2)$$

$$\sum F_y = 0: \frac{1}{\sqrt{5}} F_{BD} - \frac{1}{\sqrt{5}} F_{BE} + \frac{1}{\sqrt{5}} (2236 \text{ lb}) - 400 \text{ lb} = 0$$

$$\text{OR: } F_{BD} - F_{BE} = -1342 \text{ lb} \quad (3)$$

$$\text{AND (3): } 2 F_{BD} = -3578 \text{ lb}$$

$$\text{ACT (3) FROM (1): } 2 F_{BE} = -894 \text{ lb}$$

$$F_{BD} = 1789 \text{ lb C}$$

$$F_{BE} = 447 \text{ lb C}$$

**FREE BODY: JOINT E**

$$\sum F_x = 0: \frac{2}{\sqrt{5}} F_{EG} + \frac{2}{\sqrt{5}} (447 \text{ lb}) - 2000 \text{ lb} = 0$$

$$F_{EG} = 1789 \text{ lb T}$$

$$\sum F_y = 0: F_{DE} + \frac{1}{\sqrt{5}} (1789 \text{ lb}) - \frac{1}{\sqrt{5}} (447 \text{ lb}) = 0$$

$$F_{DE} = -600 \text{ lb}$$

$$F_{DE} = 600 \text{ lb C}$$

**FREE BODY: JOINT D**

$$\sum F_x = 0: \frac{2}{\sqrt{5}} F_{DF} + \frac{2}{\sqrt{5}} F_{DG} + \frac{2}{\sqrt{5}} (1789 \text{ lb}) = 0$$

$$\text{OR: } F_{DF} + F_{DG} = -1789 \text{ lb} \quad (4)$$

$$\sum F_y = 0: \frac{1}{\sqrt{5}} F_{DF} - \frac{1}{\sqrt{5}} F_{DG} + \frac{1}{\sqrt{5}} (1789 \text{ lb}) + 600 \text{ lb} - 400 \text{ lb} = 0$$

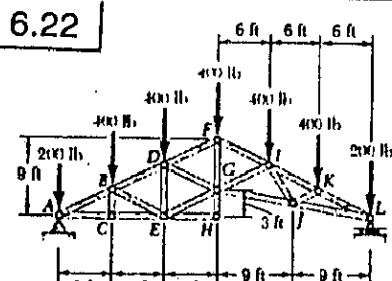
$$\text{OR: } F_{DF} - F_{DG} = -2236 \text{ lb} \quad (5)$$

$$\text{AND (5): } 2 F_{DF} = -4025 \text{ lb}$$

$$F_{DF} = 2010 \text{ lb C}$$

$$\text{ACT (5) FROM (4): } 2 F_{DG} = 447 \text{ lb}$$

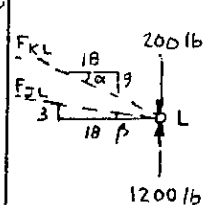
$$F_{DG} = 224 \text{ lb T}$$



**GIVEN:**

STUDIO ROOF TRUSS  
AND LOADING SHOWN.  
**FIND:**  
FORCE IN FG AND  
IN MEMBERS TO THE  
RIGHT OF FG.

**REACTION AT L:** BECAUSE OF THE SYMMETRY OF THE  
LOADING,  $L = \frac{1}{2} \text{ TOTAL LOAD}$ ,  $L = 1200 \text{ lb} \uparrow$   
(SEE F.B. DIAGRAM TO THE LEFT FOR MORE DETAILS.)



$$\alpha = \tan^{-1} \frac{9}{18} = 26.57^\circ$$

$$\beta = \tan^{-1} \frac{3}{18} = 9.46^\circ$$

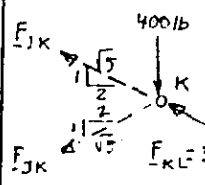
$$\frac{F_{JL}}{\sin 63.43^\circ} = \frac{F_{KL}}{\sin 99.46^\circ} = \frac{1000 \text{ lb}}{\sin 17.11^\circ}$$

$$F_{JL} = 3040 \text{ lb T}$$

$$F_{KL} = 3352.7 \text{ lb C}$$

$$F_{KL} = 3350 \text{ lb C}$$

**FREE BODY: JOINT K**



$$\sum F_x = 0: -\frac{2}{\sqrt{5}} F_{JK} - \frac{2}{\sqrt{5}} F_{KL} - \frac{2}{\sqrt{5}} (3352.7 \text{ lb}) = 0$$

$$\text{OR: } F_{JK} + F_{KL} = -3352.7 \text{ lb} \quad (1)$$

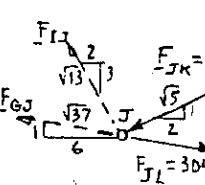
$$\sum F_y = 0: \frac{1}{\sqrt{5}} F_{JK} - \frac{1}{\sqrt{5}} F_{KL} + \frac{1}{\sqrt{5}} (3352.7) - 400 = 0$$

$$\text{OR: } F_{JK} - F_{KL} = -2458.3 \text{ lb} \quad (2)$$

$$\text{ADD (1) AND (2): } 2 F_{JK} = -5811.0, F_{JK} = -2905.5 \text{ lb}, F_{JK} = 2910 \text{ lb C}$$

$$\text{SUBTRACT (2) FROM (1): } 2 F_{KL} = -894.4, F_{KL} = -447.2 \text{ lb}, F_{KL} = 447 \text{ lb C}$$

**FREE BODY: JOINT J**



$$\sum F_x = 0: -\frac{2}{\sqrt{5}} F_{IJ} - \frac{2}{\sqrt{5}} F_{JK} - \frac{2}{\sqrt{5}} (3040 \text{ lb}) - \frac{2}{\sqrt{5}} (447.2) = 0$$

$$\text{OR: } F_{IJ} + F_{JK} = -3040 \text{ lb} \quad (3)$$

$$\sum F_y = 0: \frac{1}{\sqrt{5}} F_{IJ} - \frac{1}{\sqrt{5}} F_{JK} + \frac{1}{\sqrt{5}} (3040 \text{ lb}) - \frac{1}{\sqrt{5}} (447.2) = 0$$

$$\text{OR: } F_{IJ} - F_{JK} = -2458.3 \text{ lb} \quad (4)$$

$$\text{MULTIPLY (4) BY 6 AND ADD TO (3):}$$

$$\frac{16}{\sqrt{5}} F_{IJ} - \frac{6}{\sqrt{5}} (447.2) = 0, F_{IJ} = 360.54 \text{ lb}$$

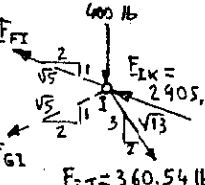
$$F_{IJ} = 361 \text{ lb T}$$

$$\text{MULTIPLY (3) BY 3, (4) BY 2, AND ADD:}$$

$$-\frac{16}{\sqrt{5}} (F_{JK} - 3040) - \frac{6}{\sqrt{5}} (447.2) = 0, F_{JK} = 2431.7 \text{ lb}$$

$$F_{JK} = 2430 \text{ lb T}$$

**FREE BODY: JOINT I**



$$\sum F_x = 0: -\frac{2}{\sqrt{5}} F_{HI} - \frac{2}{\sqrt{5}} F_{IJ} - \frac{2}{\sqrt{5}} (2905.5) + \frac{2}{\sqrt{5}} (360.54) = 0$$

$$\text{OR: } F_{HI} + F_{IJ} = -2681.9 \text{ lb} \quad (5)$$

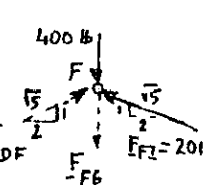
$$\sum F_y = 0: \frac{1}{\sqrt{5}} F_{HI} - \frac{1}{\sqrt{5}} F_{IJ} + \frac{1}{\sqrt{5}} (2905.5) - \frac{3}{\sqrt{5}} (360.54) - 400 = 0$$

$$\text{OR: } F_{HI} - F_{IJ} = -1340.3 \text{ lb} \quad (6)$$

$$\text{ADD (5) AND (6): } 2 F_{HI} = -4022.2, F_{HI} = -2011.1 \text{ lb}, F_{HI} = 2010 \text{ lb C}$$

$$\text{SUBTRACT (6) FROM (5): } 2 F_{IJ} = -1341.6 \text{ lb}, F_{IJ} = 671 \text{ lb C}$$

**FREE BODY: JOINT F**

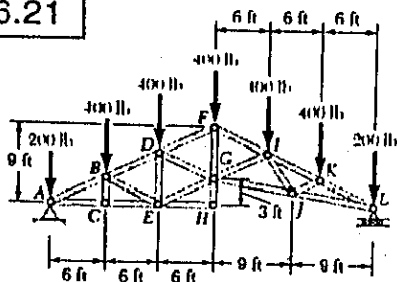


$$\text{FROM } \sum F_x = 0: F_{EF} = F_{FG} = 2011.1 \text{ lb C}$$

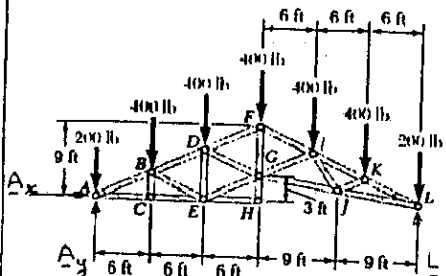
$$\sum F_y = 0: -F_{FG} - 400 \text{ lb} + 2 \left( \frac{1}{\sqrt{5}} (2011.1) \right) = 0$$

$$F_{FG} = 1400 \text{ lb}, F_{FG} = 1400 \text{ lb T}$$

6.21



**GIVEN:**  
STUDIO ROOF TRUSS  
AND LOADING SHOWN.  
**FIND:**  
FORCE IN MEMBERS  
TO THE LEFT OF LINE  
FGH.



**FREE BODY: TRUSS**

$$\sum F_x = 0: A_x = 0$$

BECAUSE OF SYMMETRY  
OF LOADING:

$$A_y = L = \frac{1}{2} \text{TOTAL LOAD}$$

$$A_y = L = 1200 \text{ lb} \uparrow$$

**ZERO-FORCE MEMBERS.** EXAMINING JOINTS C AND H,  
WE CONCLUDE THAT BC, EH, AND GH ARE ZERO-FORCE  
MEMBERS. THUS:

$$\text{ALSO: } F_{CE} = F_{AC} \quad (1)$$

$$F_{BC} = F_{EH} = 0$$

**FREE BODY: JOINT A**

$$\frac{F_{AB}}{\sqrt{5}} = \frac{F_{AC}}{2} = \frac{1000 \text{ lb}}{1}$$

$$F_{AB} = 2236 \text{ lb C}$$

$$F_{AC} = 2000 \text{ lb T}$$

$$\text{FROM EQ. (1): } F_{CE} = 2000 \text{ lb T}$$

**FREE BODY: JOINT B**

$$\sum F_x = 0: \frac{2}{\sqrt{5}} F_{BD} + \frac{2}{\sqrt{5}} F_{BE} + \frac{2}{\sqrt{5}} (2236 \text{ lb}) = 0$$

$$\text{OR: } F_{BD} + F_{BE} = -2236 \text{ lb} \quad (2)$$

$$\sum F_y = 0: \frac{1}{\sqrt{5}} F_{BD} - \frac{1}{\sqrt{5}} F_{BE} + \frac{1}{\sqrt{5}} (2236 \text{ lb}) - 400 \text{ lb} = 0$$

$$\text{OR: } F_{BD} - F_{BE} = -1342 \text{ lb} \quad (3)$$

$$\text{ADD (2) AND (3): } 2 F_{BD} = -3578 \text{ lb}$$

$$\text{SUBTRACT (3) FROM (2): } 2 F_{BE} = -894 \text{ lb}$$

$$\sum F_x = 0: \frac{2}{\sqrt{5}} F_{EG} + \frac{2}{\sqrt{5}} (447 \text{ lb}) - 2000 \text{ lb} = 0$$

$$F_{EG} = 1789 \text{ lb T}$$

$$\sum F_y = 0: F_{DE} + \frac{1}{\sqrt{5}} (1789 \text{ lb}) - \frac{1}{\sqrt{5}} (447 \text{ lb}) = 0$$

$$F_{DE} = -600 \text{ lb} \quad F_{DE} = 600 \text{ lb C}$$

**FREE BODY: JOINT D**

$$\sum F_x = 0: \frac{2}{\sqrt{5}} F_{DF} + \frac{2}{\sqrt{5}} F_{DG} + \frac{2}{\sqrt{5}} (1789 \text{ lb}) = 0$$

$$\text{OR: } F_{DF} + F_{DG} = -1789 \text{ lb} \quad (4)$$

$$\sum F_y = 0: \frac{1}{\sqrt{5}} F_{DF} - \frac{1}{\sqrt{5}} F_{DG} + \frac{1}{\sqrt{5}} (1789 \text{ lb}) + 600 \text{ lb} - 400 \text{ lb} = 0$$

$$\text{OR: } F_{DF} - F_{DG} = -2236 \text{ lb} \quad (5)$$

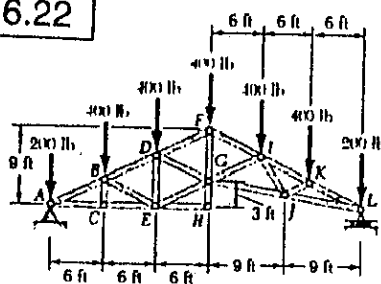
$$\text{ADD (4) AND (5): } 2 F_{DF} = -4025 \text{ lb}$$

$$F_{DF} = 2010 \text{ lb C}$$

$$\text{SUBTRACT (5) FROM (4): } 2 F_{DG} = 447 \text{ lb}$$

$$F_{DG} = 224 \text{ lb T}$$

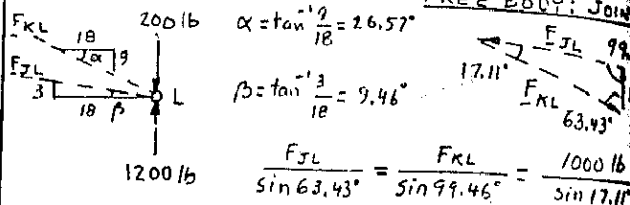
6.22



**GIVEN:**  
STUDIO ROOF  
AND LOADING  
**FIND:**  
FORCE IN F<sub>6</sub>  
IN MEMBERS  
RIGHT OF F<sub>6</sub>.

**REACTION AT L:** BECAUSE OF THE SYMMETRY OF  
LOADING,  $L = \frac{1}{2} \text{TOTAL LOAD}$ ,  $L = 1200 \text{ lb} \uparrow$   
(SEE F.B. DIAGRAM TO THE LEFT FOR MORE DETAILS)

**FREE BODY: JOINT**

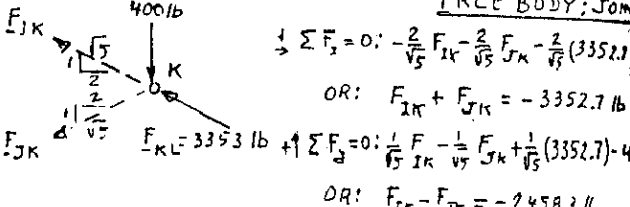


$$\frac{F_{JL}}{\sin 63.43^\circ} = \frac{F_{KL}}{\sin 99.46^\circ} = \frac{1000 \text{ lb}}{\sin 17.11^\circ}$$

$$F_{JL} = 3040 \text{ lb T}$$

$$F_{KL} = 3352.7 \text{ lb C} \quad F_{KL} = 3350 \text{ lb C}$$

**FREE BODY: JOINT**



$$\sum F_x = 0: -\frac{2}{\sqrt{5}} F_{JK} - \frac{2}{\sqrt{5}} F_{KL} - \frac{2}{\sqrt{5}} (3352.7 \text{ lb}) = 0$$

$$\text{OR: } F_{JK} + F_{KL} = -3352.7 \text{ lb}$$

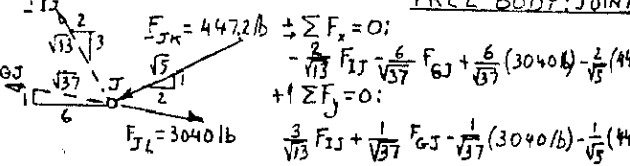
$$\sum F_y = 0: \frac{1}{\sqrt{5}} F_{JK} - \frac{1}{\sqrt{5}} F_{KL} + \frac{1}{\sqrt{5}} (3352.7 \text{ lb}) - 400 \text{ lb} = 0$$

$$\text{OR: } F_{JK} - F_{KL} = -2458.3 \text{ lb}$$

$$\text{ADD (1) AND (2): } 2 F_{JK} = -5811 \text{ lb}, F_{JK} = -2905.5 \text{ lb}, F_{JK} = 2910 \text{ lb C}$$

$$\text{SUBTRACT (2) FROM (1): } 2 F_{KL} = -8944 \text{ lb}, F_{KL} = -4472 \text{ lb}, F_{KL} = 4470 \text{ lb C}$$

**FREE BODY: JOINT**



$$\sum F_x = 0: -\frac{2}{\sqrt{5}} F_{JK} - \frac{2}{\sqrt{5}} F_{JL} - \frac{2}{\sqrt{5}} (3040 \text{ lb}) - \frac{2}{\sqrt{5}} (4472 \text{ lb}) = 0$$

$$\text{OR: } F_{JK} + F_{JL} = -3605.4 \text{ lb}$$

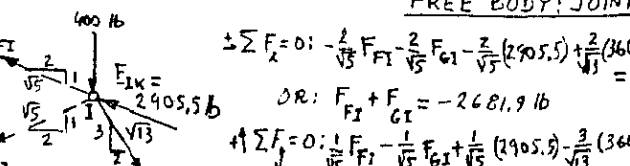
$$\sum F_y = 0: \frac{1}{\sqrt{5}} F_{JK} - \frac{1}{\sqrt{5}} F_{JL} + \frac{1}{\sqrt{5}} (3040 \text{ lb}) - \frac{1}{\sqrt{5}} (4472 \text{ lb}) = 0$$

$$\text{OR: } F_{JK} - F_{JL} = -1340.3 \text{ lb}$$

$$\text{ADD (5) AND (6): } 2 F_{JK} = -4022.2 \text{ lb}, F_{JK} = -2011.1 \text{ lb}, F_{JK} = 2010 \text{ lb C}$$

$$\text{SUBTRACT (6) FROM (5): } 2 F_{JL} = -1341.6 \text{ lb}, F_{JL} = 671 \text{ lb C}$$

**FREE BODY: JOINT**



$$\sum F_x = 0: -\frac{1}{\sqrt{5}} F_{IK} - \frac{1}{\sqrt{5}} F_{IL} - \frac{1}{\sqrt{5}} (2905.5 \text{ lb}) - \frac{1}{\sqrt{5}} (2010 \text{ lb}) = 0$$

$$\text{OR: } F_{IK} + F_{IL} = -2681.9 \text{ lb}$$

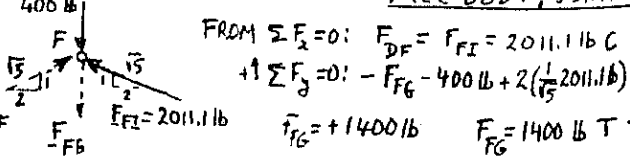
$$\sum F_y = 0: \frac{1}{\sqrt{5}} F_{IK} - \frac{1}{\sqrt{5}} F_{IL} + \frac{1}{\sqrt{5}} (2905.5 \text{ lb}) - \frac{1}{\sqrt{5}} (2010 \text{ lb}) - 400 \text{ lb} = 0$$

$$\text{OR: } F_{IK} - F_{IL} = -1340.3 \text{ lb}$$

$$\text{ADD (5) AND (6): } 2 F_{IK} = -4022.2 \text{ lb}, F_{IK} = -2011.1 \text{ lb}, F_{IK} = 2010 \text{ lb C}$$

$$\text{SUBTRACT (6) FROM (5): } 2 F_{IL} = -1341.6 \text{ lb}, F_{IL} = 671 \text{ lb C}$$

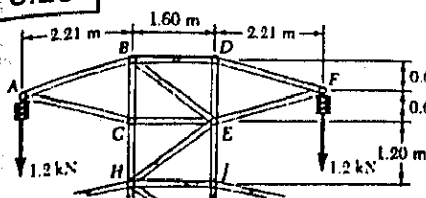
**FREE BODY: JOINT**



$$\sum F_x = 0: F_{FI} = F_{FL} = 2010 \text{ lb C}$$

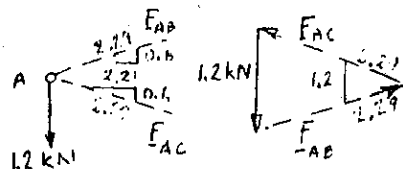
$$\sum F_y = 0: -F_{FG} - 400 \text{ lb} + 2 \left( \frac{1}{\sqrt{5}} (2010 \text{ lb}) \right) = 0$$

6.23



GIVEN: TOP CHORD  
OF POWER TRANSMISSION  
LINE TOWER  
AND LOADING SHOWN.  
FIND: FORCE IN  
MEMBERS LOCATED  
ABOVE HJ.

FREE BODY: JOINT A

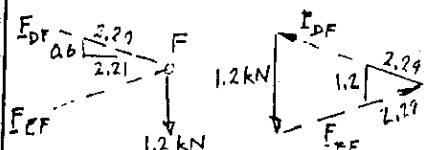


$$\frac{F_{AB}}{2.21} = \frac{F_{AH}}{2.21} = \frac{1.2 \text{ kN}}{1.2}$$

$$F_{AB} = 2.21 \text{ kN T}$$

$$F_{AH} = 2.21 \text{ kN C}$$

FREE BODY: JOINT F

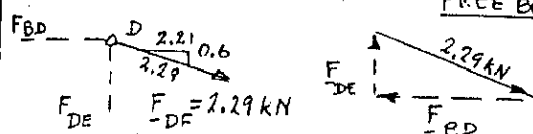


$$\frac{F_{FD}}{2.21} = \frac{F_{FE}}{2.21} = \frac{1.2 \text{ kN}}{2.1}$$

$$F_{FD} = 2.29 \text{ kN T}$$

$$F_{FE} = 2.29 \text{ kN C}$$

FREE BODY: JOINT D

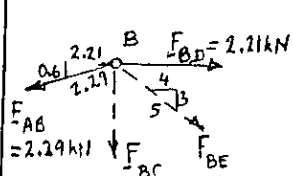


$$\frac{F_{DB}}{2.21} = \frac{F_{DE}}{0.6} = \frac{2.21 \text{ kN}}{2.29}$$

$$F_{DB} = 2.21 \text{ kN T}$$

$$F_{DE} = 0.600 \text{ kN C}$$

FREE BODY: JOINT B



$$\pm \sum F_x = 0:$$

$$\frac{4}{5} F_{BE} + 2.21 \text{ kN} - \frac{2.21}{2.29} (2.29 \text{ kN}) = 0$$

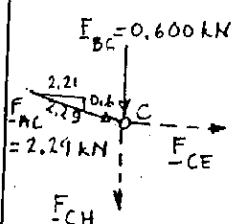
$$F_{BE} = 0$$

$$+\uparrow \sum F_y = 0:$$

$$-F_{BC} - \frac{3}{5}(0) - \frac{0.6}{2.29} (2.29 \text{ kN}) = 0$$

$$F_{BC} = -0.600 \text{ kN}, F_{BC} = 0.600 \text{ kN C}$$

FREE BODY: JOINT C



$$\pm \sum F_x = 0:$$

$$F_{CE} + \frac{2.21}{2.29} (2.29 \text{ kN}) = 0$$

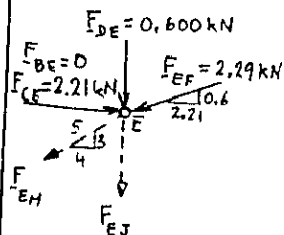
$$F_{CE} = -2.21 \text{ kN}, F_{CE} = 2.21 \text{ kN C}$$

$$+\uparrow \sum F_y = 0:$$

$$-F_{CH} - 0.600 \text{ kN} - \frac{0.6}{2.29} (2.29 \text{ kN}) = 0$$

$$F_{CH} = -1.200 \text{ kN}, F_{CH} = 1.200 \text{ kN C}$$

FREE BODY: JOINT E



$$\pm \sum F_x = 0:$$

$$2.21 \text{ kN} - \frac{2.21}{2.29} (2.29 \text{ kN}) - \frac{4}{5} F_{EH} = 0$$

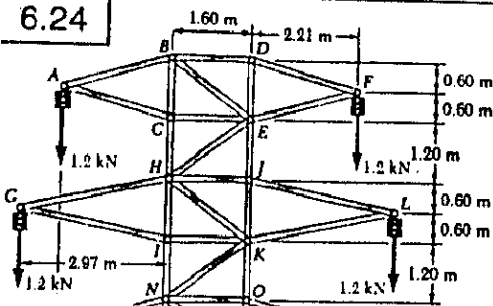
$$F_{EH} = 0$$

$$+\uparrow \sum F_y = 0:$$

$$-F_{EJ} - 0.600 \text{ kN} - \frac{0.6}{2.29} (2.29 \text{ kN}) - 0 = 0$$

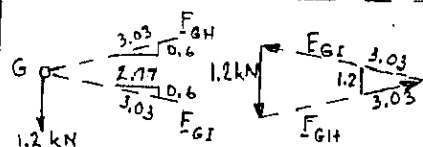
$$F_{EJ} = -1.200 \text{ kN}, F_{EJ} = 1.200 \text{ kN C}$$

6.24



GIVEN: POWER  
TRANSMISSION  
LINE TOWER  
LOADING SHOWN  
WITH  $F_{CH} =$   
1.2 kN AND  
FIND: FORCE  
MEMBERS BETWEEN  
HJ AND N

FREE BODY: JOINT G

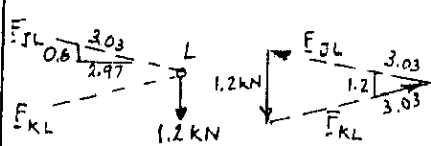


$$\frac{F_{GH}}{3.03} = \frac{F_{GI}}{3.03} = \frac{1.2 \text{ kN}}{1.2}$$

$$F_{GH} = 3.03 \text{ kN T}$$

$$F_{GI} = 3.03 \text{ kN C}$$

FREE BODY: JOINT J

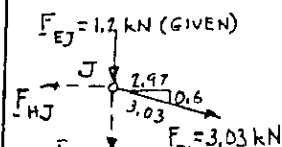


$$\frac{F_{JL}}{3.03} = \frac{F_{JK}}{3.03} = \frac{1.2 \text{ kN}}{1.2}$$

$$F_{JL} = 3.03 \text{ kN T}$$

$$F_{JK} = 3.03 \text{ kN C}$$

FREE BODY: JOINT H



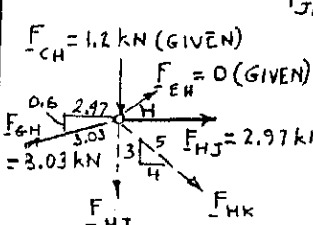
$$\pm \sum F_x = 0: -F_{HJ} + \frac{2.97}{3.03} (3.03 \text{ kN}) = 0$$

$$F_{HJ} = 2.97 \text{ kN T}$$

$$+\uparrow \sum F_y = 0: -F_{JK} - 1.2 \text{ kN} - \frac{0.6}{3.03} (3.03 \text{ kN}) = 0$$

$$F_{JK} = -1.800 \text{ kN}, F_{JK} = 1.800 \text{ kN C}$$

FREE BODY: JOINT I



$$\pm \sum F_x = 0:$$

$$\frac{4}{5} F_{IK} + 2.97 \text{ kN} - \frac{2.97}{3.03} (3.03 \text{ kN}) = 0$$

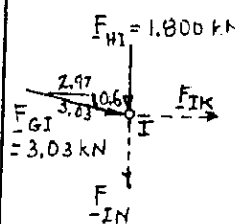
$$F_{IK} = 0$$

$$+\uparrow \sum F_y = 0:$$

$$-F_{HI} - 1.2 \text{ kN} - \frac{0.6}{3.03} (3.03 \text{ kN}) - \frac{3}{5}(0) = 0$$

$$F_{HI} = -1.800 \text{ kN}, F_{HI} = 1.800 \text{ kN C}$$

FREE BODY: JOINT K



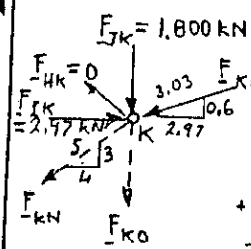
$$\pm \sum F_x = 0: F_{IK} + \frac{2.97}{3.03} (3.03 \text{ kN}) = 0$$

$$F_{IK} = -2.97 \text{ kN}, F_{IK} = 2.97 \text{ kN C}$$

$$+\uparrow \sum F_y = 0: -F_{IN} - 1.800 \text{ kN} - \frac{0.6}{3.03} (3.03 \text{ kN}) = 0$$

$$F_{IN} = -2.40 \text{ kN}, F_{IN} = 2.40 \text{ kN C}$$

FREE BODY: JOINT L



$$\pm \sum F_x = 0:$$

$$-\frac{4}{5} F_{KN} + 2.97 \text{ kN} - \frac{2.97}{3.03} (3.03 \text{ kN}) = 0$$

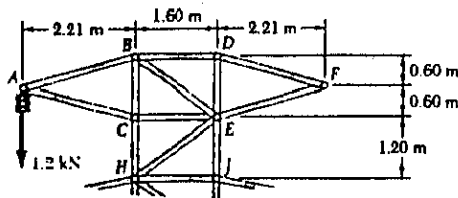
$$F_{KN} = 0$$

$$+\uparrow \sum F_y = 0:$$

$$-F_{KO} - \frac{0.6}{3.03} (3.03 \text{ kN}) - 1.800 \text{ kN} - \frac{3}{5}(0) = 0$$

$$F_{KO} = -2.40 \text{ kN}, F_{KO} = 2.40 \text{ kN C}$$

6.25

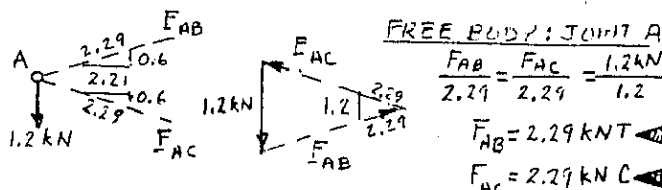


GIVEN: TOP PORTION OF LOWER TRANSMISSION LINE OF PRB. 6.23, ASSUMING THAT CABLES ON RIGHT-HAND SIDE ARE MISSING. (LOADING WILL BE AS SHOWN ABOVE.)  
FIND: FORCE IN MEMBERS LOCATED ABOVE HJ.

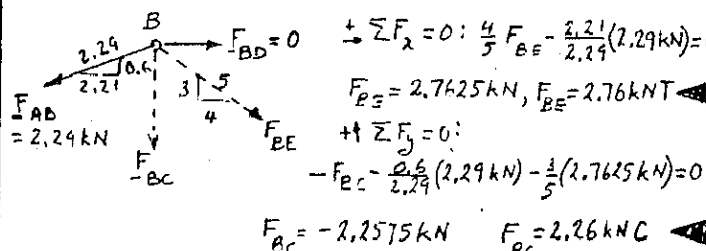
ZERO-FORCE MEMBERS.

CONSIDERING JOINT F, WE NOTE THAT DF AND EF ARE ZERO-FORCE MEMBERS:  
 $F_{DF} = F_{EF} = 0$

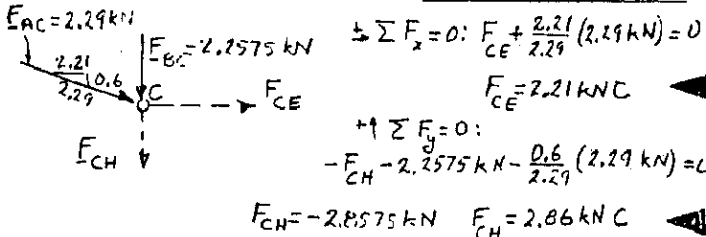
CONSIDERING NEXT JOINT D, WE NOTE THAT BD AND DE ARE ZERO-FORCE MEMBERS:  
 $F_{BD} = F_{DE} = 0$



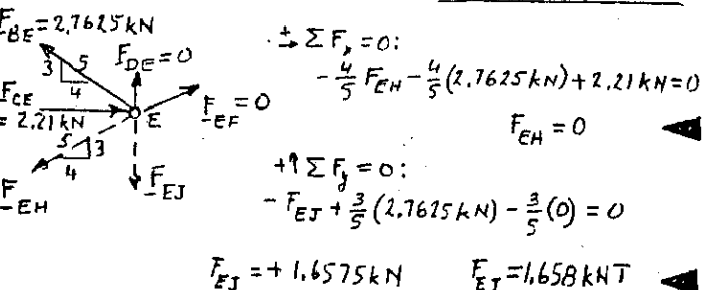
FREE BODY: JOINT A  
 $\frac{F_{AB}}{2.29} = \frac{F_{AC}}{2.29} = \frac{1.2}{1.2}$   
 $F_{AB} = 2.29 \text{ kN T}$   
 $F_{AC} = 2.29 \text{ kN C}$



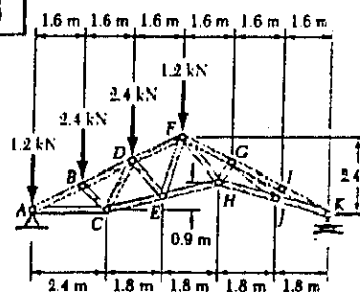
FREE BODY: JOINT C



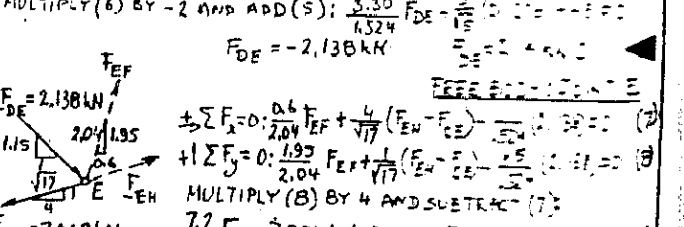
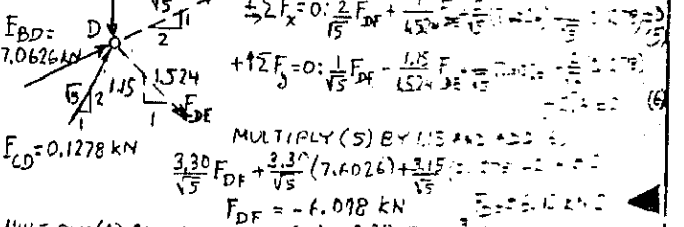
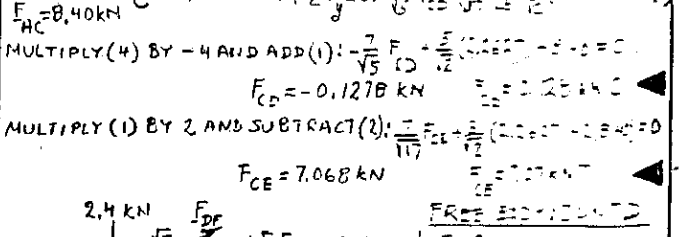
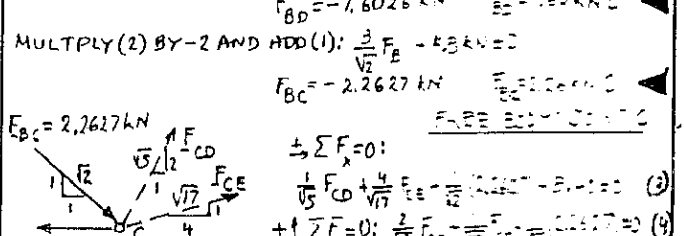
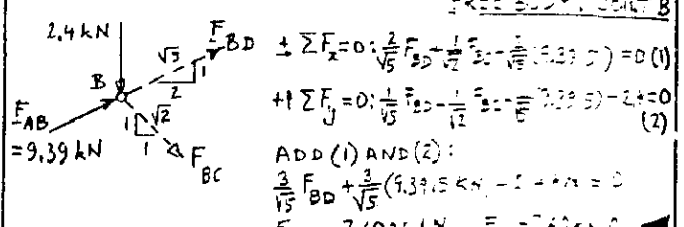
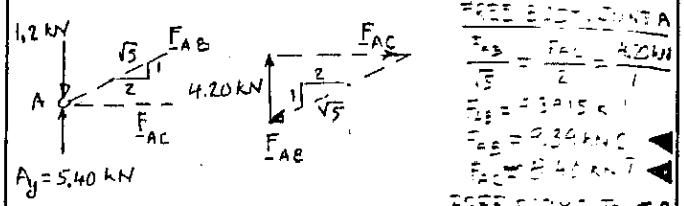
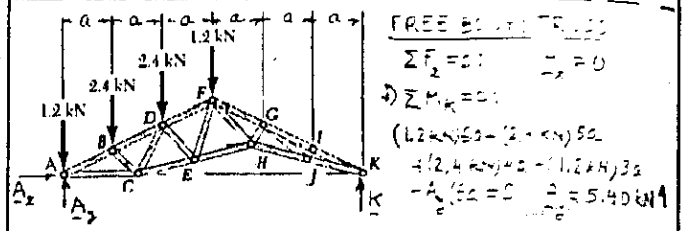
FREE BODY: JOINT E



6.26

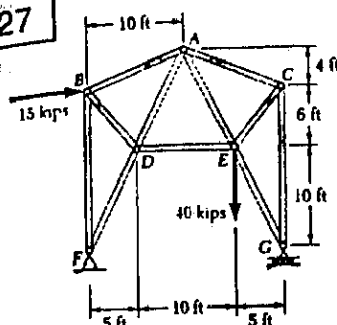


GIVEN:  
VERTICAL LOAD  
7.5 kN AT JOINT D  
DOWN  
FIND:  
FORCE IN MEMBERS  
CONNECTING  
THAT IS A THRU H





6.27



GIVEN:

TRUSS AND LOADING SHOWN.

FIND:

FORCE IN EACH MEMBER OF THE TRUSS.

FREE BODY: TRUSS

$$\rightarrow \sum M_F = 0:$$

$$G(20 \text{ ft}) - (15 \text{ kips})(16 \text{ ft}) - (40 \text{ kips})(15 \text{ ft}) = 0$$

$$G = 42 \text{ kips} \uparrow$$

$$\rightarrow \sum F_x = 0: F_x + 15 \text{ kips} = 0$$

$$F_x = 15 \text{ kips} \leftarrow$$

$$\uparrow \sum F_y = 0: F_y - 40 \text{ kips} + 42 \text{ kips} = 0$$

$$F_y = 2 \text{ kips} \downarrow$$

FREE BODY: JOINT F

$$\rightarrow \sum F_x = 0: \frac{1}{\sqrt{5}} F_{DF} - 15 \text{ kips} = 0$$

$$F_{DF} = 33.54 \text{ kips}, F_{DF} = 33.5 \text{ kips T}$$

$$\uparrow \sum F_y = 0: F_{BF} - 2 \text{ kips} + \frac{2}{\sqrt{5}} (33.54 \text{ kips}) = 0$$

$$F_{BF} = -28.00 \text{ kips}, F_{BF} = 28.0 \text{ kips C}$$

FREE BODY: JOINT B

$$\rightarrow \sum F_x = 0: \frac{5}{\sqrt{29}} F_{AB} + \frac{5}{\sqrt{61}} F_{BD} + 15 \text{ kips} = 0 \quad (1)$$

$$\uparrow \sum F_y = 0: \frac{2}{\sqrt{29}} F_{AB} - \frac{6}{\sqrt{61}} F_{BD} + 28 \text{ kips} = 0 \quad (2)$$

MULTIPLY (1) BY 6, (2) BY 5, AND ADD:

$$\frac{40}{\sqrt{29}} F_{AB} + 230 \text{ kips} = 0 \quad F_{AB} = -30.96 \text{ kips}$$

$$F_{AB} = 31.0 \text{ kips C}$$

MULTIPLY (1) BY 2, (2) BY -5, AND ADD:

$$\frac{40}{\sqrt{61}} F_{BD} - 110 \text{ kips} = 0, F_{BD} = 21.48 \text{ kips}, F_{BD} = 21.5 \text{ kips T}$$

FREE BODY: JOINT D

$$\uparrow \sum F_y = 0: \frac{2}{\sqrt{5}} F_{AD} - \frac{2}{\sqrt{5}} (33.54) + \frac{6}{\sqrt{61}} (21.48) = 0$$

$$F_{AD} = 15.09 \text{ kips T}$$

$$\rightarrow \sum F_x = 0:$$

$$F_{DE} + \frac{1}{\sqrt{5}} (15.09 - 33.54) - \frac{5}{\sqrt{61}} (21.48) = 0$$

$$F_{DE} = 22.0 \text{ kips T}$$

FREE BODY: JOINT A

$$\rightarrow \sum F_x = 0: \frac{5}{\sqrt{29}} F_{AC} + \frac{1}{\sqrt{5}} F_{AE} + \frac{5}{\sqrt{29}} (30.96) - \frac{1}{\sqrt{5}} (15.09) = 0 \quad (3)$$

$$\uparrow \sum F_y = 0: -\frac{2}{\sqrt{29}} F_{AC} - \frac{2}{\sqrt{5}} F_{AE} + \frac{2}{\sqrt{29}} (30.96) - \frac{2}{\sqrt{5}} (15.09) = 0 \quad (4)$$

MULTIPLY (3) BY 2 AND ADD (4):

$$\frac{8}{\sqrt{29}} F_{AC} + \frac{12}{\sqrt{5}} (30.96) - \frac{4}{\sqrt{5}} (15.09) = 0$$

$$F_{AC} = -28.27 \text{ kips}, F_{AC} = 28.3 \text{ kips C}$$

$$F_{AE} = 9.50 \text{ kips T}$$

(CONTINUED)

6.27 CONTINUED

FREE BODY: JOINT C

FROM FORCE TRIANGLE

$$\frac{F_{CE}}{\sqrt{61}} = \frac{F_{CG}}{8} = \frac{28.27 \text{ kips}}{\sqrt{29}}$$

$$F_{CE} = 41.0 \text{ kips T}$$

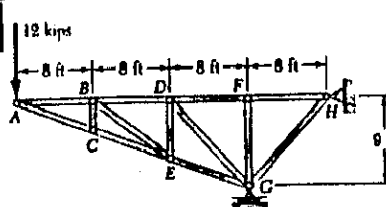
$$F_{CG} = 42.0 \text{ kips C}$$

FREE BODY: JOINT G

$$F_{EG} = 0$$

$$\uparrow \sum F_y = 0: 42 \text{ kips} - 42 \text{ kips} = 0 \quad (\text{CHECKS})$$

6.28



GIVEN:

TRUSS AND LOADING SHOWN.

FIND:

FORCE IN EACH MEMBER

FREE BODY: TRUSS

$$\rightarrow \sum F_x = 0: H_x = 0$$

$$\uparrow \sum M_G = 0:$$

$$(12 \text{ kips})(24 \text{ ft}) + H_y(8 \text{ ft}) = 0$$

$$H_y = -36 \text{ kips}, H_y = 36 \text{ kips} \downarrow$$

$$\sum F_y = 0: G = 48 \text{ kips} \uparrow$$

ZERO-FORCE MEMBERS

$$\text{JOINT F: } F_{DF} = F_{FH} \quad (1)$$

AND

$$F_{FG} = 0$$

$$\text{JOINT C: } F_{AC} = F_{CE} \quad (2)$$

AND

$$F_{BC} = 0$$

$$\text{JOINT B: } F_{AB} = F_{BD} \quad (3)$$

AND

$$F_{BE} = 0$$

$$\text{JOINT E: } F_{CE} = F_{EG} \quad (4)$$

AND

$$F_{DE} = 0$$

$$\text{JOINT D: } F_{BD} = F_{DF} \quad (5)$$

AND

$$F_{DG} = 0$$

FREE BODY: JOINT A

$$\frac{F_{AB}}{8} = \frac{F_{AC}}{\sqrt{73}} = \frac{12 \text{ kips}}{3}$$

$$F_{AB} = 32.0 \text{ kips T}$$

$$F_{AC} = 34.2 \text{ kips C}$$

FROM EQS. (3), (5), AND (1):

$$F_{BD} = F_{DF} = F_{FH} = 32.0 \text{ kips T}$$

FROM EQS. (2) AND (4):

$$F_{CE} = F_{EG} = 34.2 \text{ kips C}$$

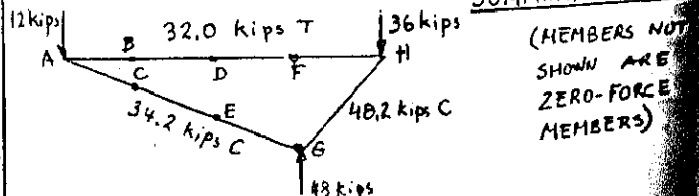
FREE BODY: JOINT H

$$\uparrow \sum F_y = 0: -\frac{9}{12.04} F_{GH} - 36 \text{ kips} = 0$$

$$F_{GH} = -48.16 \text{ kips}, F_{GH} = 48.2 \text{ kips C}$$

$$\rightarrow \sum F_x = -32 \text{ kips} - \frac{8}{12.04} (-48.16 \text{ kips}) = 0 \quad (\text{CHECKS})$$

SUMMARY OF FORCES

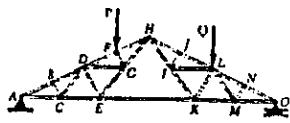


(MEMBERS NOT SHOWN ARE ZERO-FORCE MEMBERS)

6.29

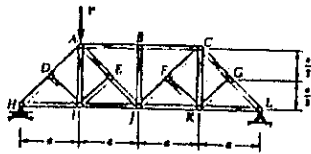
DETERMINE WHETHER THE TRUSSES OF PROBS. 6.31a, 6.32a, AND 6.33a ARE SIMPLE TRUSSES.

TRUSS OF PROB. 6.31a



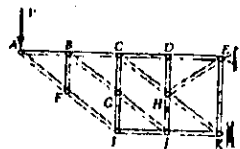
STARTING WITH TRIANGLE ABC AND ADDING TWO MEMBERS AT A TIME, WE OBTAIN JOINTS D, E, G, F, AND H, BUT CANNOT GO FURTHER. THUS, THIS TRUSS IS NOT A SIMPLE TRUSS

TRUSS OF PROB. 6.32a



STARTING WITH TRIANGLE HDI AND ADDING TWO MEMBERS AT A TIME, WE OBTAIN SUCCESSIVELY JOINTS A, E, J, AND B, BUT CANNOT GO FURTHER. THUS, THIS TRUSS IS NOT A SIMPLE TRUSS

TRUSS OF PROB. 6.33a

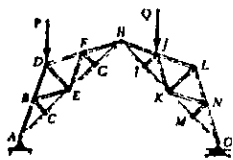


STARTING WITH TRIANGLE EHK AND ADDING TWO MEMBERS AT A TIME, WE OBTAIN SUCCESSIVELY JOINTS D, J, C, G, I, B, F, AND A, THUS COMPLETING THE TRUSS. THEREFORE, THIS TRUSS IS A SIMPLE TRUSS

6.30

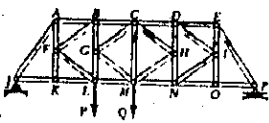
DETERMINE WHETHER THE TRUSSES OF PROBLEMS 6.31b, 6.32b, AND 6.33b ARE SIMPLE TRUSSES.

TRUSS OF PROB. 6.31b



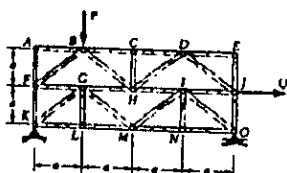
STARTING WITH TRIANGLE ABC AND ADDING TWO MEMBERS AT A TIME, WE OBTAIN SUCCESSIVELY JOINTS E, D, F, G, AND H, BUT CANNOT GO FURTHER. THUS, THIS TRUSS IS NOT A SIMPLE TRUSS

TRUSS OF PROB. 6.32b



STARTING WITH TRIANGLE CGI AND ADDING TWO MEMBERS AT A TIME, WE OBTAIN SUCCESSIVELY JOINTS B, L, F, A, K, J, THEN H, D, N, I, E, O, AND P, THUS COMPLETING THE TRUSS. THEREFORE, THIS TRUSS IS A SIMPLE TRUSS

TRUSS OF PROB. 6.33b

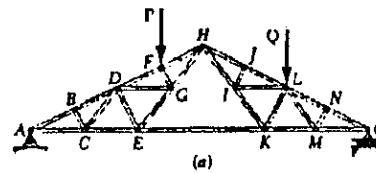


STARTING WITH TRIANGLE GLM AND ADDING TWO MEMBERS AT A TIME, WE OBTAIN JOINTS K AND F BUT CANNOT CONTINUE. STARTING INSTEAD WITH TRIANGLE BCH, WE OBTAIN JOINT D BUT CANNOT CONTINUE. THUS, THIS TRUSS IS NOT A SIMPLE TRUSS

6.31

DETERMINE THE ZERO-FORCE MEMBERS IN EACH OF THE TRUSSES SHOWN FOR THE GIVEN LOADING

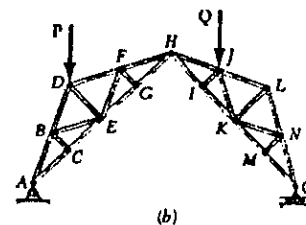
TRUSS (a)



FB: JOINT B:  $F_{BC} = 0$   
FB: JOINT C:  $F_{CD} = 0$   
FB: JOINT J:  $F_{IJ} = 0$   
FB: JOINT I:  $F_{IL} = 0$   
FB: JOINT N:  $F_{MN} = 0$   
FB: JOINT M:  $F_{LM} = 0$

THE ZERO-FORCE MEMBERS, THEREFORE, ARE BC, CD, IJ, IL, LM, MN

TRUSS (b)



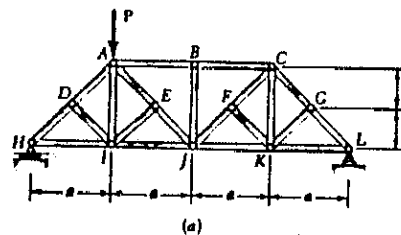
FB: JOINT C:  $F_{BC} = 0$   
FB: JOINT B:  $F_{BE} = 0$   
FB: JOINT G:  $F_{FG} = 0$   
FB: JOINT F:  $F_{EF} = 0$   
FB: JOINT E:  $F_{DE} = 0$   
FB: JOINT I:  $F_{IJ} = 0$   
FB: JOINT M:  $F_{MN} = 0$   
FB: JOINT N:  $F_{KN} = 0$

THE ZERO-FORCE MEMBERS, THEREFORE, ARE BC, BE, DE, EF, FG, IJ, KN, MN

6.32

DETERMINE THE ZERO-FORCE MEMBERS IN EACH OF THE TRUSSES SHOWN FOR THE GIVEN LOADING

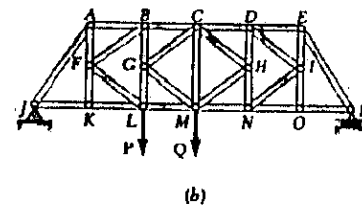
TRUSS (a)



FB: JOINT B:  $F_{BJ} = 0$   
FB: JOINT D:  $F_{DI} = 0$   
FB: JOINT E:  $F_{EI} = 0$   
FB: JOINT I:  $F_{AI} = 0$   
FB: JOINT F:  $F_{FK} = 0$   
FB: JOINT G:  $F_{GK} = 0$   
FB: JOINT K:  $F_{CK} = 0$

THE ZERO-FORCE MEMBERS, THEREFORE, ARE AI, BJ, CK, DI, EI, FK, GK

TRUSS (b)

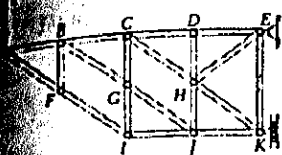


FB: JOINT K:  $F_{FK} = 0$   
FB: JOINT O:  $F_{IO} = 0$

THE ZERO-FORCE MEMBERS, THEREFORE, ARE FK AND IO

ALL OTHER MEMBERS ARE EITHER IN TENSION OR COMPRESSION.

DETERMINE THE ZERO-FORCE MEMBERS IN EACH OF THE TRUSSES SHOWN FOR THE GIVEN LOADING.

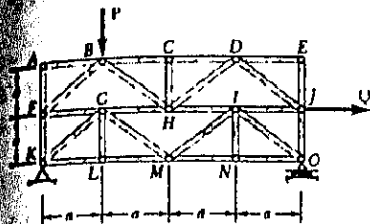


(a)

TRUSS (a)

FE: JOINT F:  $F_{BF} = 0$   
 FE: JOINT E:  $F_{BE} = 0$   
 FE: JOINT C:  $F_{GC} = 0$   
 FE: JOINT D:  $F_{DH} = 0$   
 FE: JOINT J:  $F_{HJ} = 0$   
 FE: JOINT H:  $F_{JH} = 0$

THE ZERO-FORCE MEMBERS, THEREFORE, ARE  
BF, BG, DH, EH, GJ, HJ



位

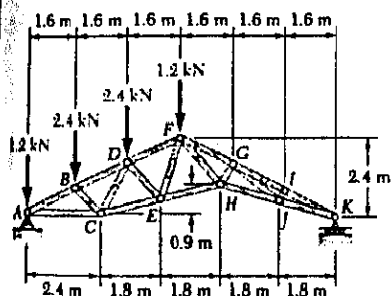
TRUSS(b)

$\overline{FB: \text{JOINT A: } F_{AB} = F_{AF} = 0}$   
 $\overline{FB: \text{JOINT C: } F_{CH} = 0}$   
 $\overline{FB: \text{JOINT E: } F_{DE} = F_{EJ} = 0}$   
 $\overline{FB: \text{JOINT L: } F_{GL} = 0}$   
 $\overline{FB: \text{JOINT N: } F_{TN} = 0}$

THE ZERO-FORCE MEMBERS, THEREFORE, ARE  
AB, AF, CH, DE, EJ, GL, IN

6.34

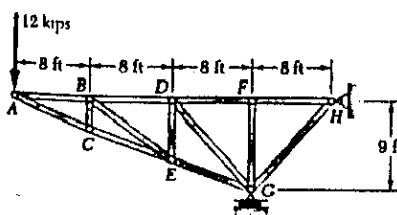
DETERMINE THE ZERO-FORCE MEMBERS IN THE TRUSS OF (a) PROB. 6.26, (b) PROB. 6.28



(c) TRUSS OF FIG. 6.26

FB: JOINT I:  $F_{IJ} = 0$   
 FB: JOINT J:  $F_{GJ} = 0$   
 FB: JOINT G:  $F_{GH} = 0$

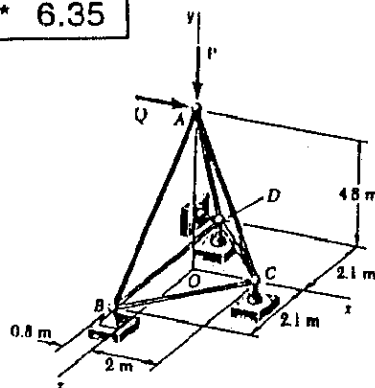
THE ZERO-FORCE  
MEMBERS, THEREFORE,  
ARE  
GH, GJ, IJ



FB: JOINT C:	$F_{BC} = 0$
FB: JOINT B:	$F_{BE} = 0$
FB: JOINT E:	$F_{DE} = 0$
FB: JOINT D:	$F_{DB} = 0$
FB: JOINT F:	$F_{FC} = 0$

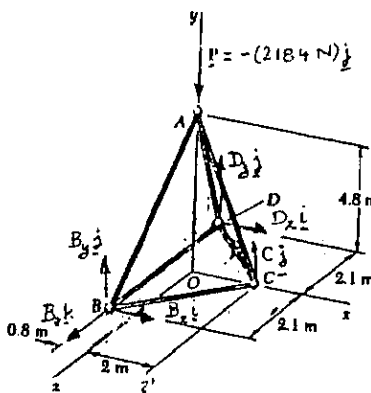
THE ZERO-FACTOR PRINCIPLES, THEREFORE ARE  
BC, BE, DE, DG, FG

★ 6.35



G.M. 11/2

TRUSS SHOWING, WITH  
 $P = (-2184 \text{ N}) \hat{j}$   
 $Q = 0$   
FIND:  
 FORCE IN EACH  
 MEMBER.



FREE BODY : TRUSS  
FROM SYMMETRY:

$$\begin{aligned} D_x &= B_x \text{ AND } D_y = B_y \\ \Sigma F_x &= 0: 2B_x = 0 \\ B_x &= D_x = 0 \\ \Sigma F_z &= 0: B_z = 0 \\ \Sigma M_{CG} &= 0: \\ -2B_y(2.8m) + (218+N)(2m) &= 0 \\ B_y &= 780 \text{ N} \\ \text{THUS: } B &= (780 \text{ N}) \end{aligned}$$

FREE BODY: A

$F_{AB} = F_{AB} \frac{AB}{AB} = \frac{F_{AB}}{5.30} (-0.8\hat{i} - 4.8\hat{j} + 2.1\hat{k})$   
 $F_{AC} = F_{AC} \frac{AC}{AC} = \frac{F_{AC}}{5.20} (2\hat{i} - 4.8\hat{j})$   
 $F_{AD} = F_{AD} \frac{AD}{AD} = \frac{F_{AD}}{5.30} (-0.8\hat{i} - 4.8\hat{j} - 2.1\hat{k})$

$\Sigma \vec{F} = 0: F_{AB} + F_{AC} + F_{AD} - (2184 \text{ N})\hat{j} = 0$

SUBSTITUTING FOR  $F_{AB}$ ,  $F_{AC}$ ,  $F_{AD}$ , AND EQUATING TO ZERO THE COEFFICIENTS OF  $\hat{i}$ ,  $\hat{j}$ ,  $\hat{k}$ :

(C)  $-\frac{0.8}{5.30} (F_{AB} + F_{AD}) + \frac{2}{5.20} F_{AC} = 0$  (1)

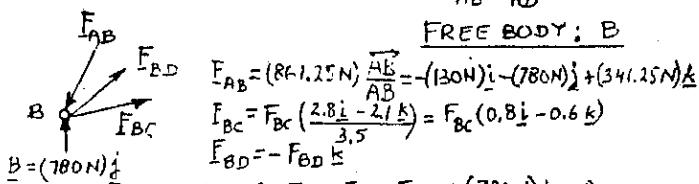
(D)  $-\frac{4.8}{5.30} (F_{AB} + F_{AD}) - \frac{4.8}{5.20} F_{AC} - 2184 \text{ N} = 0$  (2)

(E)  $\frac{2.1}{5.30} (F_{AB} - F_{AD}) = 0 \quad F_{AD} = F_{AB}$

MULTIPLY (1) BY -6 AND ADD (2):  
 $-(16.8/5.20) F_{AC} - 2184 N = 0, F_{AC} = -676 N, F_{AC} = 676 N$   
 SUBSTITUTE FOR  $F_{AC}$  AND  $F_{AD}$  IN (1):  
 $-(0.8/5.30) 2 F_{AB} + (2/5.20)(-676 N) = 0, F_{AB} = -861.25 N$

$$F_{AB} = F_{AD} = 861 \text{ N}$$

FREE BODY: B



SUBSTITUTING FOR  $E_{AB}$ ,  $E_{CC}$ ,  $F_{BD}$  AND EQUATING TO ZERO THE COEFFICIENTS OF  $i$  AND  $k$ :

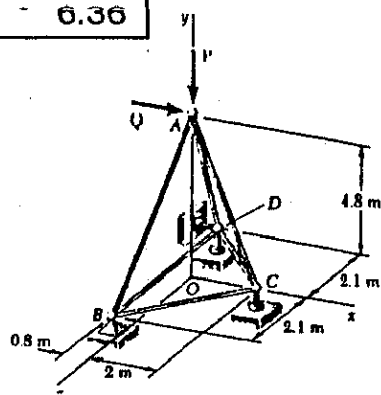
(b)  $341.25 \text{ N} - 0.6 F_{BC} - F_{BD} = 0$   
 $F_{BD} = 341.25 - 0.6(162.5) = 243.75 \text{ N}$   $F_{BD} = 244 \text{ N}$

FROM SYMMETRY:  $F_{CD} = F_{BC}$

$$F = 162,5 \text{ NT}$$

1

6.36



GIVEN:

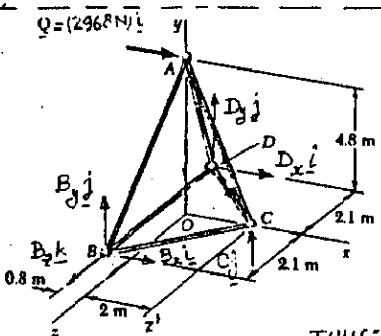
TRUSS SHOWN, WITH

$$P=0$$

$$Q = (2968 \text{ N})\mathbf{i}$$

FIND:

FORCE IN EACH MEMBER



FREE BODY: TRUSS

FROM SYMMETRY:

$$D_x = B_x \text{ AND } D_y = B_y$$

$$\sum F_x = 0: 2B_x + 2968 \text{ N} = 0$$

$$B_x = D_x = -1484 \text{ N}$$

$$\sum M_{CZ} = 0$$

$$-2B_y(2.8 \text{ m}) - (2968 \text{ N})(4.8 \text{ m}) = 0$$

$$B_y = -2544 \text{ N}$$

$$\text{THUS: } \underline{B} = -(1484 \text{ N})\mathbf{i} - (2544 \text{ N})\mathbf{j}$$

FREE BODY: A

$$\begin{aligned} F_{AB} &= F_{AB} \frac{\vec{AB}}{AB} = \frac{F_{AB}}{5.30} (-0.8\mathbf{i} - 4.8\mathbf{j} + 2.1\mathbf{k}) \\ F_{AC} &= F_{AC} \frac{\vec{AC}}{AC} = \frac{F_{AC}}{5.20} (2\mathbf{i} - 4.8\mathbf{j}) \\ F_{AD} &= F_{AD} \frac{\vec{AD}}{AD} = \frac{F_{AD}}{5.30} (-0.8\mathbf{i} - 4.8\mathbf{j} - 2.1\mathbf{k}) \end{aligned}$$

$$\sum \mathbf{F} = 0: F_{AB} + F_{AC} + F_{AD} + (2968 \text{ N})\mathbf{i} = 0$$

SUBSTITUTING FOR  $F_{AB}$ ,  $F_{AC}$ ,  $F_{AD}$  AND EQUATING TO ZERO THE COEFFICIENTS OF  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$ :

$$(1) \quad -\frac{0.8}{5.30} (F_{AB} + F_{AD}) + \frac{2}{5.20} F_{AC} + 2968 \text{ N} = 0$$

$$(2) \quad -\frac{4.8}{5.30} (F_{AB} + F_{AD}) - \frac{4.8}{5.20} F_{AC} = 0$$

$$(3) \quad \frac{2.1}{5.30} (F_{AB} - F_{AD}) = 0 \quad F_{AB} = F_{AD}$$

MULTIPLY (1) BY -6 AND ADD (2):

$$-(16.8/5.20) F_{AC} - 6(2968 \text{ N}) = 0, F_{AC} = -5512 \text{ N}, F_{AC} = 5510 \text{ N}$$

SUBSTITUTE FOR  $F_{AC}$  AND  $F_{AD}$  IN (2):

$$-(4.8/5.30) 2F_{AB} - (4.8/5.20) (-5512 \text{ N}) = 0, F_{AB} = +2809 \text{ N}$$

$$F_{AB} = F_{AD} = 2810 \text{ N}$$

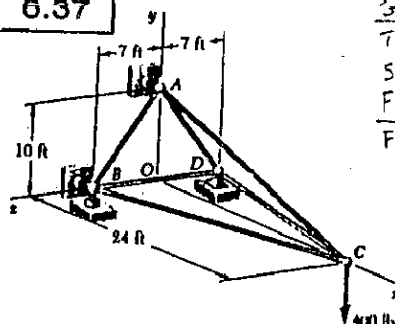
FREE BODY: B

$$\begin{aligned} F_{BA} &= (2810 \text{ N}) \frac{\vec{BA}}{BA} = (424 \text{ N})\mathbf{i} + (2544 \text{ N})\mathbf{j} - (1113 \text{ N})\mathbf{k} \\ F_{BC} &= F_{BC} \frac{\vec{BC}}{BC} = F_{BC} (0.8\mathbf{i} - 0.6\mathbf{k}) \\ F_{BD} &= -F_{BD} \mathbf{k} \end{aligned}$$

$$\sum \mathbf{F} = 0: F_{BA} + F_{BC} + F_{BD} - (1484 \text{ N})\mathbf{i} - (2544 \text{ N})\mathbf{j} = 0$$

SUBSTITUTING FOR  $F_{BA}$ ,  $F_{BC}$ ,  $F_{BD}$  AND EQUATING TO ZERO THE COEFFICIENTS OF  $\mathbf{i}$  AND  $\mathbf{k}$ :

6.37

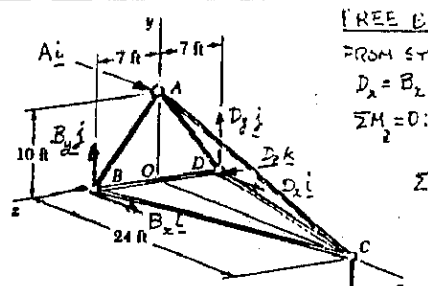


GIVEN:

TRUSS AND LOADING SHOWN

FIND:

FORCE IN EACH MEMBER



FREE BODY: TRUSS

FROM SYMMETRY:

$$D_x = B_x \text{ AND } D_y = B_y$$

$$\sum M_z = 0: -A(10 \text{ ft}) - (400 \text{ lb})(24 \text{ ft}) = 0$$

$$A = -960 \text{ lb}$$

$$\sum F_x = 0: B_x + D_x + A = 0$$

$$2B_x - 960 \text{ lb} = 0, B_x = 480 \text{ lb}$$

$$\sum F_y = 0: B_y - D_y - 400 \text{ lb} = 0$$

$$2B_y = 400 \text{ lb}$$

$$B_y = +200 \text{ lb}$$

$$\text{THUS: } \underline{B} = (480 \text{ lb})\mathbf{i} + (200 \text{ lb})\mathbf{j}$$

FREE BODY: C

$$\begin{aligned} F_{CA} &= F_{AC} \frac{\vec{CA}}{CA} = \frac{F_{AC}}{26} (-24\mathbf{i} + 10\mathbf{j}) \\ F_{CB} &= F_{BC} \frac{\vec{CB}}{CB} = \frac{F_{BC}}{25} (-24\mathbf{i} + 7\mathbf{k}) \\ F_{CD} &= F_{CD} \frac{\vec{CD}}{CD} = \frac{F_{CD}}{25} (-24\mathbf{i} - 7\mathbf{k}) \end{aligned}$$

$$\sum \mathbf{F} = 0: F_{CA} + F_{CB} + F_{CD} - (400 \text{ lb})\mathbf{j} = 0$$

SUBSTITUTING FOR  $F_{CA}$ ,  $F_{CB}$ ,  $F_{CD}$  AND EQUATING TO ZERO THE COEFFICIENTS OF  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$ :

$$(1) \quad -\frac{24}{26} F_{AC} - \frac{24}{25} (F_{BC} + F_{CD}) = 0$$

$$(2) \quad \frac{10}{26} F_{AC} - 400 \text{ lb} = 0 \quad F_{AC} = 1040 \text{ lb}$$

$$(3) \quad \frac{7}{25} (F_{BC} - F_{CD}) = 0 \quad F_{CD} = F_{BC}$$

SUBSTITUTE FOR  $F_{AC}$  AND  $F_{CD}$  IN EQ. (1):

$$-\frac{24}{26} (1040 \text{ lb}) - \frac{24}{25} (2F_{BC}) = 0 \quad F_{BC} = -500 \text{ lb}$$

$$F_{BC} = F_{CD} = 500 \text{ lb}$$

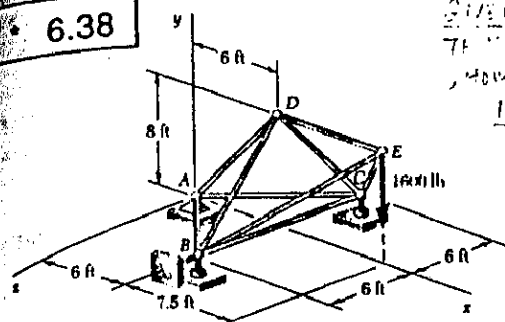
FREE BODY: B

$$\begin{aligned} F_{BC} &= (500 \text{ lb}) \frac{\vec{CB}}{CB} = -(480 \text{ lb})\mathbf{i} + (140 \text{ lb})\mathbf{k} \\ F_{BA} &= F_{AB} \frac{\vec{BA}}{BA} = \frac{F_{AB}}{12.21} (10\mathbf{j} - 7\mathbf{k}) \\ F_{BD} &= -F_{BD} \mathbf{k} \end{aligned}$$

$$\sum \mathbf{F} = 0: F_{BA} + F_{BD} + F_{BC} + (480 \text{ lb})\mathbf{i} + (200 \text{ lb})\mathbf{j} = 0$$

SUBSTITUTING FOR  $F_{BA}$ ,  $F_{BD}$ ,  $F_{BC}$  AND EQUATING TO ZERO THE COEFFICIENTS OF  $\mathbf{j}$  AND  $\mathbf{k}$ :

6.38



6.38  
FIND THE FORCE IN EACH MEMBER.

FIND:

FORCE IN EACH MEMBER.

FREE BODY: TRUSS

FROM SYMMETRY:  $A_z = B_z = 0$ 

$$\sum F_x = 0: A_x = 0$$

$$\sum M_{BC} = 0:$$

$$A_y(6 \text{ ft}) + (1600 \text{ lb})(7.5 \text{ ft}) = 0$$

$$A_y = -2000 \text{ lb}$$

$$A = -(2000 \text{ lb})\hat{j}$$

FROM SYMMETRY:

$$B_y = C$$

$$\sum F_y = 0:$$

$$2B_y - 2000 \text{ lb} - 1600 \text{ lb} = 0$$

$$B_y = 1800 \text{ lb} \quad B = (1800 \text{ lb})\hat{j}$$

FREE BODY: A

$$\sum F = 0: F_{AB} + F_{AC} + F_{AD} - (2000 \text{ lb})\hat{j} = 0$$

$$F_{AB} \frac{\hat{i} + \hat{j}}{\sqrt{2}} + F_{AC} \frac{\hat{i} - \hat{j}}{\sqrt{2}} + F_{AD}(0.6\hat{i} + 0.8\hat{j}) - (2000 \text{ lb})\hat{j} = 0$$

FACTORIZING  $\hat{i}$ ,  $\hat{j}$ , AND  $\hat{k}$  AND EQUATING THEIR COEFFICIENTS TO ZERO

$$\frac{1}{\sqrt{2}} F_{AB} + \frac{1}{\sqrt{2}} F_{AC} + 0.6 F_{AD} = 0 \quad (1)$$

$$0.8 F_{AD} - 2000 \text{ lb} = 0$$

$$F_{AD} = 2500 \text{ lb T}$$

$$\frac{1}{\sqrt{2}} F_{AB} - \frac{1}{\sqrt{2}} F_{AC} = 0$$

$$F_{AC} = F_{AB}$$

SUBSTITUTE FOR  $F_{AD}$  AND  $F_{AC}$  INTO (1):

$$\frac{2}{\sqrt{2}} F_{AB} + 0.6(2500 \text{ lb}) = 0, F_{AB} = -1060.7 \text{ lb}, F_{AB} = F_{AC} = 1061 \text{ lb C}$$

FREE BODY: B

$$F_{BA} = F_{AB} \frac{\hat{BA}}{\hat{BA}} = + (1060.7 \text{ lb}) \frac{\hat{i} + \hat{j}}{\sqrt{2}} = (750 \text{ lb})(\hat{i} + \hat{j})$$

$$F_{BC} = -F_{BC} \hat{k}, F_{BD} = F_{BD}(0.8\hat{j} - 0.6\hat{k})$$

$$B = (1800 \text{ lb})\hat{j} \quad F_{BE} = F_{BE} \frac{\hat{BE}}{\hat{BE}} = \frac{F_{BE}}{12.5} (7.5\hat{i} + 8\hat{j} - 6\hat{k})$$

$$\sum F = 0: F_{BA} + F_{BC} + F_{BD} + F_{BE} + (1800 \text{ lb})\hat{j} = 0$$

SUBSTITUTE FOR  $F_{BA}$ ,  $F_{BC}$ ,  $F_{BD}$ ,  $F_{BE}$  AND EQUATE TO ZERO THE COEFF. OF  $\hat{i}$ ,  $\hat{j}$ ,  $\hat{k}$ :

$$(1) \quad 750 \text{ lb} + (7.5/12.5) F_{BE} = 0, F_{BE} = -1250 \text{ lb}, F_{BE} = 1250 \text{ lb C}$$

$$(2) \quad 0.8 F_{BD} + (8/12.5)(-1250 \text{ lb}) + 1800 \text{ lb} = 0, F_{BD} = 1250 \text{ lb C}$$

$$(3) \quad 750 \text{ lb} - F_{BC} - 0.6(-1250 \text{ lb}) - \frac{6}{12.5}(-1250 \text{ lb}) = 0$$

$$F_{BC} = 2100 \text{ lb T}$$

FROM SYMMETRY:  $F_{CE} = F_{CD} = 1250 \text{ lb C}$ 

FREE BODY: D

$$\sum F = 0: F_{DA} + F_{DB} + F_{DC} + F_{DE} = 0$$

$$F_{DA}$$

$$F_{DB}$$

$$F_{DC}$$

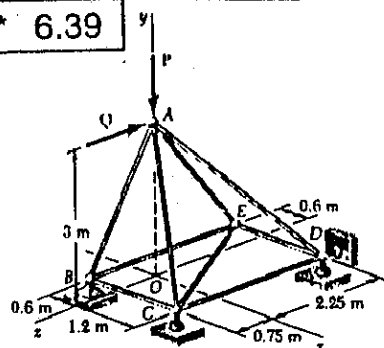
$$F_{DE}$$

WE NOW SUBSTITUTE FOR  $F_{DA}$ ,  $F_{DB}$ ,  $F_{DC}$  AND EQUATE TO ZERO THE COEFFICIENT OF  $\hat{i}$ . ONLY  $F_{DE}$  CONTAINS  $\hat{i}$ AND ITS COEFFICIENT IS  $-0.6 F_{AD} = -0.6(2500 \text{ lb}) = -1500 \text{ lb}$ 

$$(1) \quad -1500 \text{ lb} + F_{DE} = 0$$

$$F_{DE} = 1500 \text{ lb T}$$

6.39



GIVEN:

TRUSS SHOWN WITH

$$P = -(1200 \text{ N})\hat{j}$$

$$Q = 0$$

FIND:

FORCE IN EACH MEMBER

FREE BODY: TRUSS

$$\sum M_B = 0:$$

$$1.8 \hat{i} \times C \hat{j}$$

$$+ (1.8 \hat{i} - 3 \hat{k}) \times (D_y \hat{j} + D_z \hat{k})$$

$$+ (0.6 \hat{i} - 0.75 \hat{k}) \times (-1200 \hat{j}) = 0$$

$$1.8 C \hat{k} + 1.8 D_y \hat{k} - 1.8 D_z \hat{j}$$

$$+ 3 D_y \hat{i} - 720 \hat{k} - 900 \hat{i} = 0$$

$$\text{EQUATE TO ZERO THE COEFFICIENTS OF } \hat{i}, \hat{j}, \hat{k}:$$

$$(1) \quad 3 D_y - 900 = 0, D_y = 300 \text{ N}$$

$$(2) \quad D_z = 0, D = (300 \text{ N})\hat{j}$$

$$(3) \quad 1.8 C + 1.8(300) - 720 = 0, C = (100 \text{ N})\hat{k}$$

$$B = (800 \text{ N})\hat{j}$$

$$\sum F = 0: B + 300 \hat{j} + 100 \hat{k} - 1200 \hat{j} = 0$$

FREE BODY: B

$$\sum F = 0: F_{BA} + F_{BC} + F_{BE} + (800 \text{ N})\hat{j} = 0, \text{ WITH}$$

$$F_{BA} = F_{AB} \frac{\hat{BA}}{\hat{BA}} = \frac{F_{AB}}{3.15} (0.6 \hat{i} + 3 \hat{j} - 0.75 \hat{k})$$

$$F_{BC} = F_{BC} \hat{i} \quad F_{BE} = -F_{BE} \hat{k}$$

$$B = (800 \text{ N})\hat{j}$$

SUBSTITUTE AND EQUATE TO ZERO THE COEFF. OF  $\hat{i}$ ,  $\hat{j}$ ,  $\hat{k}$ :

$$(1) \quad (3/3.15) F_{AB} + 800 \text{ N} = 0, F_{AB} = -840 \text{ N}, F_{AB} = 840 \text{ N C}$$

$$(2) \quad (0.6/3.15)(-840 \text{ N}) + F_{BC} = 0 \quad F_{BC} = 160 \text{ N T}$$

$$(3) \quad (-0.75/3.15)(-840 \text{ N}) - F_{BE} = 0 \quad F_{BE} = 200 \text{ N T}$$

FREE BODY: C

$$\sum F = 0: F_{CA} + F_{CB} + F_{CD} + F_{CE} + (100 \text{ N})\hat{k} = 0, \text{ WITH}$$

$$F_{CA} = F_{AC} \frac{\hat{CA}}{\hat{CA}} = \frac{F_{AC}}{3.317} (-1.2 \hat{i} + 3 \hat{j} - 0.75 \hat{k})$$

$$F_{CB} = -(160 \text{ N})\hat{i} \quad F_{CD} = -F_{CD} \hat{k} \quad F_{CE} = F_{CE} \frac{\hat{CE}}{\hat{CE}} = \frac{F_{CE}}{3.499} (-1.8 \hat{i} - 3 \hat{k})$$

SUBSTITUTE AND EQUATE TO ZERO THE COEFF. OF  $\hat{i}$ ,  $\hat{j}$ ,  $\hat{k}$ :

$$(1) \quad (3/3.317) F_{AC} + 100 \text{ N} = 0, F_{AC} = -110.57 \text{ N}, F_{AC} = 110.6 \text{ N C}$$

$$(2) \quad -\frac{1.2}{3.317}(-110.57) - 160 - \frac{1.8}{3.499} F_{CE} = 0, F_{CE} = 233.3, F_{CE} = 233 \text{ N C}$$

$$(3) \quad -\frac{0.75}{3.317}(-110.57) - F_{CD} - \frac{3}{3.499}(-233.3) = 0 \quad F_{CD} = 225 \text{ N T}$$

FREE BODY: D

$$\sum F = 0: F_{DA} + F_{DC} + F_{DE} + (300 \text{ N})\hat{j} = 0, \text{ WITH}$$

$$F_{DA} = F_{AD} \frac{\hat{DA}}{\hat{DA}} = \frac{F_{AD}}{3.937} (-1.2 \hat{i} + 3 \hat{j} + 2.25 \hat{k})$$

$$F_{DC} = F_{CD} \hat{k} = (225 \text{ N})\hat{k} \quad F_{DE} = -F_{DE} \hat{i}$$

$$D = (300 \text{ N})\hat{j}$$

SUBSTITUTE AND EQUATE TO ZERO THE COEFF. OF  $\hat{i}$ ,  $\hat{j}$ ,  $\hat{k}$ :

$$(1) \quad (3/3.937) F_{AD} + 300 \text{ N} = 0, F_{AD} = -393.7 \text{ N}, F_{AD} = 394 \text{ N C}$$

$$(2) \quad (-1.2/3.937)(-393.7 \text{ N}) - F_{DE} = 0 \quad F_{DE} = 120 \text{ N T}$$

$$(3) \quad (2.25/3.937)(-393.7 \text{ N}) + 225 \text{ N} = 0 \quad (\text{CHECKS})$$

FREE BODY: E

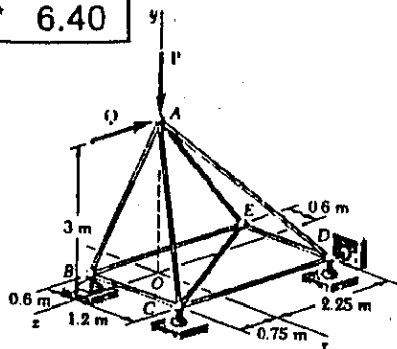
MEMBER AE IS THE ONLY MEMBER AT E

WHICH DOES NOT LIE IN THE XZ PLANE.

THEREFORE, IT IS A ZERO-FORCE MEMBER.

$$F_{AE} = 0$$

6.40



GIVEN:

TRUSS SHOWN WITH

$$P=0$$

$$Q=(-900\text{ N})\mathbf{j}$$

FIND:

FORCE IN EACH MEMBER.

FREE BODY: TRUSS

$$\begin{aligned}\sum \mathbf{M}_B = 0: & 1.8\mathbf{i} \times C\mathbf{j} \\ & + (1.8\mathbf{j} - 3\mathbf{k}) \times (D\mathbf{j} + D\mathbf{k}) \\ & + (0.6\mathbf{i} + 3\mathbf{j} - 0.75\mathbf{k}) \times (-900\text{ N})\mathbf{j} = 0\end{aligned}$$

$$1.8C\mathbf{k} + 1.8D\mathbf{k} - 1.8D\mathbf{j} + 3D\mathbf{j} + 540\mathbf{j} - 2700\mathbf{k} = 0$$

EQUATE TO ZERO THE COEFF. OF  $\mathbf{i}, \mathbf{j}, \mathbf{k}$ :

$$3D - 2700 = 0 \quad D = 900\text{ N}$$

$$-1.8D + 540 = 0 \quad D = 300\text{ N}$$

$$1.8C + 1.8D = 0, \quad C = -D = -900\text{ N}$$

$$\text{THUS: } C = -(900\text{ N})\mathbf{j}; \quad D = (900\text{ N})\mathbf{j} + (300\text{ N})\mathbf{k}$$

$$\sum \mathbf{F} = 0: \quad B - 900\mathbf{j} + 900\mathbf{j} + 300\mathbf{k} - 900\mathbf{k} = 0 \quad B = (600\text{ N})\mathbf{k}$$

FREE BODY: B

SINCE  $B$  IS ALIGNED WITH MEMBER BE:

$$F_{AB} = F_{BC} = 0, \quad F_{BE} = 600\text{ N T}$$

FREE BODY: C

$$\sum \mathbf{F} = 0: \quad F_{CA} + F_{CD} + F_{CE} - (900\text{ N})\mathbf{j} = 0, \quad \text{WITH}$$

$$F_{CA} = F_{AC} \quad \frac{F_{AC}}{CA} = \frac{F_{AC}}{3.317} \quad (-1.2\mathbf{i} + 3\mathbf{j} - 0.75\mathbf{k})$$

$$F_{CD} = -F_{DC} \quad F_{CE} = F_{EC} \quad \frac{F_{EC}}{CE} = \frac{F_{EC}}{3.437} \quad (-1.8\mathbf{i} - 3\mathbf{k})$$

SUBSTITUTE AND EQUATE TO ZERO THE COEFF. OF  $\mathbf{j}, \mathbf{i}, \mathbf{k}$ :

$$\textcircled{2} \quad (3/3.317)F_{AC} - 900\text{ N} = 0, \quad F_{AC} = 995.1\text{ N}, \quad F_{AC} = 995\text{ N T}$$

$$\textcircled{1} \quad -\frac{1.2}{3.317}(995.1) - \frac{1.8}{3.437}F_{CE} = 0, \quad F_{CE} = -699.8\text{ N}, \quad F_{CE} = 700\text{ N C}$$

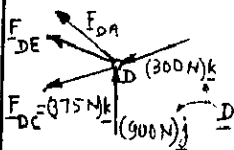
$$\textcircled{3} \quad -\frac{0.75}{3.317}(995.1) - F_{CD} - \frac{3}{3.437}(-699.8) = 0 \quad F_{CD} = 375\text{ N T}$$

FREE BODY: D

$$\sum \mathbf{F} = 0: \quad F_{DA} + F_{DC} + (375\text{ N})\mathbf{k} + (900\text{ N})\mathbf{j} + (300\text{ N})\mathbf{k} = 0$$

$$\text{WITH } F_{DA} = F_{AD} \quad \frac{F_{AD}}{DA} = \frac{F_{AD}}{3.437} \quad (-1.2\mathbf{i} + 3\mathbf{j} + 2.25\mathbf{k})$$

$$\text{AND } F_{DE} = -F_{ED}$$

SUBSTITUTE AND EQUATE TO ZERO THE COEFF. OF  $\mathbf{i}, \mathbf{j}, \mathbf{k}$ :

$$\textcircled{2} \quad (3/3.437)F_{AD} + 900\text{ N} = 0, \quad F_{AD} = -1181.1\text{ N}, \quad F_{AD} = 1181\text{ N C}$$

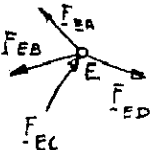
$$\textcircled{1} \quad -(1.2/3.437)(-1181.1\text{ N}) - F_{DE} = 0 \quad F_{DE} = 360\text{ N T}$$

$$\textcircled{3} \quad (2.25/3.437)(-1181.1\text{ N}) + 375\text{ N} + 300\text{ N} = 0 \quad (\text{CHECKS})$$

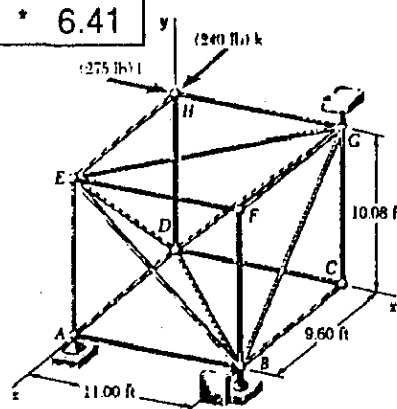
FREE BODY: E

MEMBER AE IS THE ONLY MEMBER AT E WHICH DOES NOT LIE IN THE XZ PLANE. THEREFORE, IT IS A ZERO-FORCE MEMBER.

$$F_{AE} = 0$$



6.41



GIVEN:

TRUSS AND LOADING SHOWN

(a) CHECK THAT TRUSS IS A SIMPLE TRUSS, COMPLETELY CONSTRAINED, AND REACTIONS STATICALLY DETERMINABLE

(b) FIND:

FORCE IN EACH OF THE SIX MEMBERS JOINED AT E.

- (a) CHECK SIMPLE TRUSS, (1) START WITH TETRAHEDRON BEFG  
(2) ADD MEMBER, BD, ED, GD JOINING AT D.  
(3) ADD MEMBERS BA, DA, EA JOINING AT A.  
(4) ADD MEMBERS DH, EH, GH JOINING AT H.  
(5) ADD MEMBERS BC, DC, GC JOINING AT C

TRUSS HAS BEEN COMPLETED; IT IS A SIMPLE TRUSS

FREE BODY: TRUSS

CHECK CONSTRAINTS AND REACTIONS:

SIX UNKNOWN REACTIONS - OK - HOWEVER SUPPORTS AT A AND B CONSTRAIN TRUSS TO ROTATE ABOUT AB AND SUPPORT AT G PREVENTS SUCH A ROTATION. THUS TRUSS IS COMPLETELY CONSTRAINED AND REACTIONS ARE STATICALLY DETERMINABLE

DETERMINATION OF REACTIONS:

$$\sum \mathbf{M}_A = 0: 11\mathbf{i} \times (B\mathbf{j} + B\mathbf{k}) + (11\mathbf{i} - 9.6\mathbf{k}) \times G\mathbf{j} + (10.08\mathbf{j} - 9.6\mathbf{k}) \times (275\mathbf{i} + 240\mathbf{k}) = 0$$

$$11B\mathbf{j} - 11B\mathbf{k} + 11G\mathbf{k} + 9.6G\mathbf{j} - (10.08 \times 275)\mathbf{k} + (10.08 \times 240)\mathbf{i} - (9.6 \times 275)\mathbf{j} = 0$$

EQUATE TO ZERO THE COEFF. OF  $\mathbf{i}, \mathbf{j}, \mathbf{k}$ :

$$\textcircled{2} \quad 9.6G + (10.08 \times 240) = 0 \quad G = -252\text{ lb} \quad G = (-252\text{ lb})\mathbf{j}$$

$$\textcircled{1} \quad -11B - (9.6)(275) = 0 \quad B = -240\text{ lb}$$

$$\textcircled{3} \quad 11B + 11(-252) - (10.08 \times 275) = 0, \quad B = 504\text{ lb}$$

$$B = (504\text{ lb})\mathbf{j} - (240\text{ lb})\mathbf{k}$$

$$\sum \mathbf{F} = 0: \quad A + (504\text{ lb})\mathbf{j} - (240\text{ lb})\mathbf{k} - (252\text{ lb})\mathbf{j} + (275\text{ lb})\mathbf{i} + (240\text{ lb})\mathbf{k} = 0$$

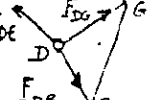
$$A = (-275\text{ lb})\mathbf{i} - (252\text{ lb})\mathbf{j}$$

ZERO-FORCE MEMBERS

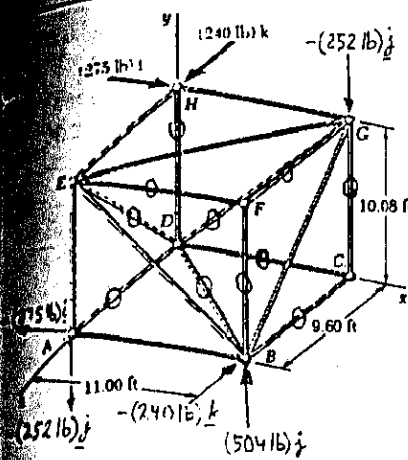
THE DETERMINATION OF THESE MEMBERS WILL FACILITATE OUR SOLUTION

FB: C. WRITING  $\sum F_x = 0, \sum F_y = 0, \sum F_z = 0$  YIELDS  $F_{BC} = F_{CD} = F_{CG} = 0$ FB: E. WRITING  $\sum F_x = 0, \sum F_y = 0, \sum F_z = 0$  YIELDS  $F_{BE} = F_{EF} = F_{EG} = 0$ FB: A. SINCE  $A_x = 0$ , WRITING  $\sum F_x = 0$  YIELDS  $F_{AD} = 0$ FB: H. WRITING  $\sum F_z = 0$  YIELDS  $F_{DH} = 0$ FB: D. SINCE  $F_{AD} = F_{CD} = F_{DH} = 0$ , WE NEED CONSIDER ONLY MEMBERS DB, DE, AND DG.SINCE  $F_{DE}$  IS THE ONLY FORCE NOT CONTAINED IN PLANE BDG, IT MUST BE ZERO. SIMILAR REASONING SHOWS THAT THE OTHER TWO FORCES ARE ALSO ZERO  $F_{BD} = F_{DE} = F_{DG} = 0$ 

(CONTINUED)



# 6.41 CONTINUED



THE RESULTS OBTAINED FOR THE REACTIONS AT THE SUPPORTS AND THE ZERO-FORCE MEMBERS ARE SHOWN ON THE ADJACENT FIGURE.

ZERO-FORCE MEMBERS ARE INDICATED BY A ZERO ("0").

(b) FORCE IN EACH OF THE MEMBERS JOINED AT E

WE ALREADY FOUND THAT

$$F_{DE} = F_{EF} = 0$$

FREE BODY: A  $\sum F_y = 0$  YIELDS  $F_{AE} = 252 \text{ lb T}$

FREE BODY: H  $\sum F_x = 0$  YIELDS  $F_{EH} = 240 \text{ lb C}$

FREE BODY: E

$$\sum \vec{F} = 0: F_{EB} + F_{EG} + (240 \text{ lb})\vec{i} - (252 \text{ lb})\vec{j} = 0$$

$$\frac{F_{EB}(11\vec{i} - 10.08\vec{j})}{14.92} + \frac{F_{EG}(11\vec{i} - 9.6\vec{j})}{14.6} + 240\vec{i} - 252\vec{j} = 0$$

EQUATE TO ZERO THE COEFF. OF  $\vec{i}$  AND  $\vec{j}$ :

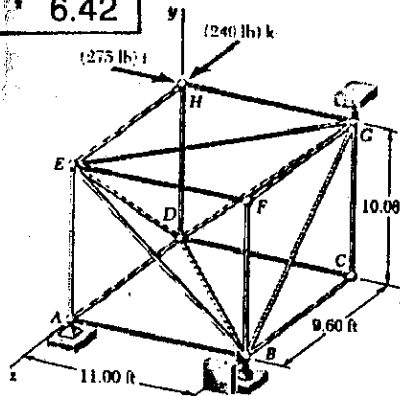
$$\textcircled{1} -(10.08/14.92)F_{EB} - 252 = 0$$

$$F_{EB} = 373 \text{ lb C}$$

$$\textcircled{2} -(9.6/14.6)F_{EG} + 240 = 0$$

$$F_{EG} = 365 \text{ lb T}$$

## 6.42



GIVEN:

TRUSS AND LOADING SHOWN.

(a) CHECK THAT TRUSS IS SIMPLE TRUSS, COMPLETELY CONSTRAINED, AND REACTIONS STATICALLY DETERMINATE

(b) FIND: FORCE IN EACH OF THE SIX MEMBERS JOINED AT G.

SEE SOLUTION OF PROB. 6.41 FOR PART (a) AND FOR REACTIONS AND ZERO-FORCE MEMBERS

(b) FORCE IN EACH OF THE MEMBERS JOINED AT G.

WE ALREADY KNOW (SEE FIG. AT TOP OF PAGE) THAT

$$F_{CG} = F_{DG} = F_{FG} = 0$$

FREE BODY: H  $\sum F_x = 0$  YIELDS  $F_{GH} = 275 \text{ lb C}$

FREE BODY: G

$$\sum \vec{F} = 0: F_{GB} + F_{GE} + (275 \text{ lb})\vec{i} - (252 \text{ lb})\vec{j} = 0$$

$$\frac{F_{GB}(-10.08\vec{j} + 9.6\vec{i})}{13.92} + \frac{F_{GE}(-11\vec{i} + 9.6\vec{j})}{14.6} + 275\vec{i} - 252\vec{j} = 0$$

EQUATE TO ZERO THE COEFF. OF  $\vec{i}$  AND  $\vec{j}$ :

$$\textcircled{1} -(11/14.6)F_{GE} + 275 = 0$$

$$F_{GE} = 365 \text{ lb T}$$

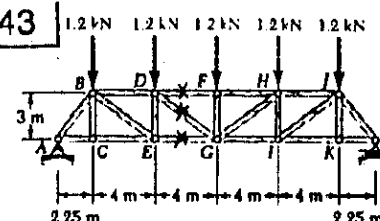
$$\textcircled{2} -(10.08/13.92)F_{GB} - 252 = 0$$

$$F_{GB} = 348 \text{ lb C}$$

$$\textcircled{3} (9.6/13.92)(-348) + (9.6/14.6)(365) = 0$$

$$\text{(CHECKS)}$$

## 6.43



GIVEN: MANSARD ROOF TRUSS AND LOADING SHOWN.

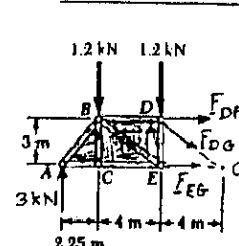
FIND:

FORCE IN MEMBERS DF, DG, AND EG.

REACTIONS AT SUPPORTS

BECAUSE OF THE SYMMETRY OF THE TRUSS AND LOADING,  $A_x = 0$ ,  $A_y = L_y = \frac{1}{2}(\text{TOTAL LOAD}) = \frac{1}{2}(6 \text{ kN})$   $A = L = 3 \text{ kN} \uparrow$

WE PASS A SECTION THROUGH DF, DG, AND EG AND USE THE FREE BODY SHOWN.



$$\sum M_G = 0:$$

$$(1.2 \text{ kN})(8 \text{ m}) + (1.2 \text{ kN})(4 \text{ m}) - (3 \text{ kN})(10.25 \text{ m}) - F_{DF}(3 \text{ m}) = 0$$

$$F_{DF} = -5.45 \text{ kN}, F_{DF} = 5.45 \text{ kN C}$$

$$\sum F_y = 0:$$

$$3 \text{ kN} - 1.2 \text{ kN} - 1.2 \text{ kN} - \frac{3}{5}F_{DG} = 0$$

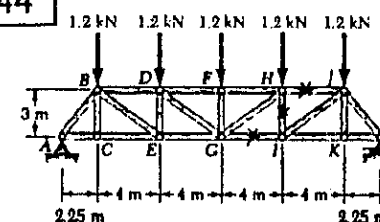
$$F_{DG} = +1.00 \text{ kN}, F_{DG} = 1.00 \text{ kN T}$$

$$\sum M_D = 0:$$

$$(1.2 \text{ kN})(4 \text{ m}) - (3 \text{ kN})(6.25 \text{ m}) + F_{EG}(3 \text{ m}) = 0$$

$$F_{EG} = +4.65 \text{ kN}, F_{EG} = 4.65 \text{ kN T}$$

## 6.44



GIVEN: MANSARD ROOF TRUSS AND LOADING SHOWN

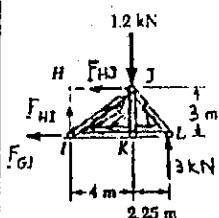
FIND:

FORCE IN MEMBERS GI, HI, AND HJ

REACTIONS AT SUPPORTS

BECAUSE OF THE SYMMETRY OF THE TRUSS AND LOADING,  $A_x = 0$ ,  $A_y = L_y = \frac{1}{2}(\text{TOTAL LOAD}) = \frac{1}{2}(6 \text{ kN})$   $A = L = 3 \text{ kN} \uparrow$

WE PASS A SECTION THROUGH GI, HI, AND HJ AND USE THE FREE BODY SHOWN.



$$\sum M_H = 0:$$

$$(3 \text{ kN})(6.25 \text{ m}) - (1.2 \text{ kN})(4 \text{ m}) - F_{GI}(3 \text{ m}) = 0$$

$$F_{GI} = +4.65 \text{ kN}, F_{GI} = 4.65 \text{ kN T}$$

$$\sum F_y = 0:$$

$$F_{HI} - 1.2 \text{ kN} + 3 \text{ kN} = 0$$

$$F_{HI} = -1.80 \text{ kN}, F_{HI} = 1.80 \text{ kN C}$$

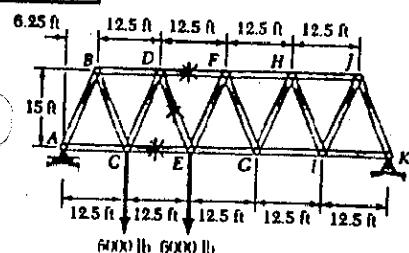
$$\sum M_I = 0:$$

$$F_{HJ}(3 \text{ m}) - (1.2 \text{ kN})(4 \text{ m}) + (3 \text{ kN})(6.25 \text{ m}) = 0$$

$$F_{HJ} = -4.65 \text{ kN}, F_{HJ} = 4.65 \text{ kN C}$$

$$\text{CHECK: } \sum F_x = 4.65 \text{ kN} - 4.65 \text{ kN} = 0$$

6.45



GIVEN: WARREN  
BRIDGE TRUSS AND  
LOADING SHOWN.  
FIND:  
FORCE IN MEMBERS  
CE, DE, AND DF.

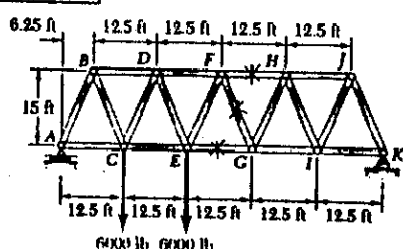
FREE BODY: TRUSS

$$\begin{aligned} \sum F_x = 0: K_x &= 0 \\ + \sum M_A = 0: K_y(62.5 \text{ ft}) &- (6000 \text{ lb})(12.5 \text{ ft}) &- (6000 \text{ lb})(25 \text{ ft}) = 0 \\ K_y &= K_2 = 3600 \text{ lb} \uparrow \\ + \sum F_y = 0: A + 3600 \text{ lb} &- 6000 \text{ lb} - 6000 \text{ lb} = 0 \\ A &= 8400 \text{ lb} \uparrow \end{aligned}$$

WE PASS A SECTION THROUGH MEMBERS CE, DE, AND DF.  
AND USE THE FREE BODY SHOWN.

$$\begin{aligned} + \sum M_D = 0: & F_{CE}(15 \text{ ft}) - (8400 \text{ lb})(18.75 \text{ ft}) + (6000 \text{ lb})(6.25 \text{ ft}) = 0 \\ F_{CE} &= +8000 \text{ lb} \quad F_{CE} = 8000 \text{ lb T} \\ + \sum F_y = 0: & 8400 \text{ lb} - 6000 \text{ lb} - \frac{15}{16.25} F_{DE} = 0 \\ F_{DE} &= +2600 \text{ lb} \quad F_{DE} = 2600 \text{ lb T} \\ + \sum M_E = 0: & 6000 \text{ lb}(12.5 \text{ ft}) - (8400 \text{ lb})(15 \text{ ft}) - F_{DF}(15 \text{ ft}) = 0 \\ F_{DF} &= -9000 \text{ lb} \quad F_{DF} = 9000 \text{ lb C} \end{aligned}$$

6.46



GIVEN: WARREN  
BRIDGE TRUSS AND  
LOADING SHOWN.  
FIND:  
FORCE IN MEMBERS  
EG, FG, AND FH.

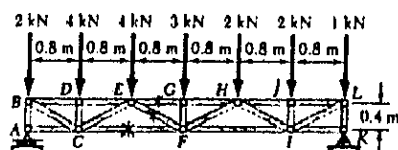
SEE SOLUTION OF PROB. 6.45 FOR FREE-BODY  
DIAGRAM OF TRUSS AND DETERMINATION OF REACTION.

$$A = 8400 \text{ lb}, K = 3600 \text{ lb} \uparrow$$

WE PASS A SECTION THROUGH MEMBERS EG, FG, AND FH,  
AND USE THE FREE BODY SHOWN.

$$\begin{aligned} + \sum M_F = 0: & (3600 \text{ lb})(31.25 \text{ ft}) - F_{EG}(15 \text{ ft}) = 0 \\ F_{EG} &= +7500 \text{ lb} \quad F_{EG} = 7500 \text{ lb T} \\ + \sum F_y = 0: & \frac{15}{16.25} F_{FG} + 3600 \text{ lb} = 0 \\ F_{FG} &= -3900 \text{ lb} \quad F_{FG} = 3900 \text{ lb C} \\ + \sum M_G = 0: & F_{FH}(15 \text{ ft}) + (3600 \text{ lb})(25 \text{ ft}) = 0 \\ F_{FH} &= -6000 \text{ lb} \quad F_{FH} = 6000 \text{ lb C} \end{aligned}$$

6.47



GIVEN:  
FLOOR TRUSS WITH  
LOADING SHOWN.  
FIND:  
FORCE IN MEMBERS  
CF, EF, AND EK.

FREE BODY: TRUSS

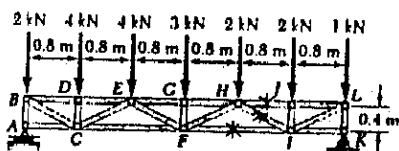
$$+ \sum F_x = 0: K_x = 0$$

$$\begin{aligned} + \sum M_A = 0: & K_y(4.8 \text{ m}) - (4 \text{ kN})(0.8 \text{ m}) - (4 \text{ kN})(1.6 \text{ m}) - (3 \text{ kN})(2.4 \text{ m}) \\ & - (2 \text{ kN})(3.2 \text{ m}) - (2 \text{ kN})(4 \text{ m}) - (1 \text{ kN})(4.8 \text{ m}) = 0 \\ K_y &= 7.5 \text{ kN} \quad \text{THUS: } K = 7.5 \text{ kN} \uparrow \\ + \sum F_y = 0: & A + 7.5 \text{ kN} - 18 \text{ kN} = 0 \quad A = 10.5 \text{ kN} \quad A = 10.5 \text{ kN} \uparrow \end{aligned}$$

WE PASS A SECTION THROUGH MEMBERS CF, EF, AND EG AND USE  
THE FREE BODY SHOWN.

$$\begin{aligned} + \sum M_E = 0: & F_{CF}(0.4 \text{ m}) - (10.5 \text{ kN})(1.6 \text{ m}) + (2 \text{ kN})(1.6 \text{ m}) + (4 \text{ kN})(0.8 \text{ m}) = 0 \\ F_{CF} &= +26.0 \text{ kN} \quad F_{CF} = 26.0 \text{ kN T} \\ + \sum F_y = 0: & 10.5 \text{ kN} - 2 \text{ kN} - 4 \text{ kN} - 4 \text{ kN} - \frac{1}{\sqrt{5}} F_{EF} = 0 \\ F_{EF} &= +1.118 \text{ kN} \quad F_{EF} = 1.118 \text{ kN T} \\ + \sum M_F = 0: & (2 \text{ kN})(2.4 \text{ m}) + (4 \text{ kN})(1.6 \text{ m}) + (4 \text{ kN})(0.8 \text{ m}) - (10.5 \text{ kN})(2.4 \text{ m}) \\ & - F_{EG}(0.4 \text{ m}) = 0 \\ F_{EG} &= -27.0 \text{ kN} \quad F_{EG} = 27.0 \text{ kN C} \end{aligned}$$

6.48



GIVEN:  
FLOOR TRUSS WITH  
LOADING SHOWN.  
FIND:  
FORCE IN MEMBERS  
FI, HI, AND HJ.

SEE SOLUTION OF PROB. 6.47 FOR FREE-BODY DIAGRAM  
OF TRUSS AND DETERMINATION OF REACTIONS.

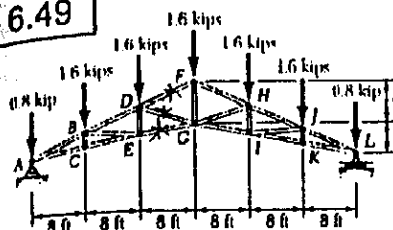
$$A = 10.5 \text{ kN} \uparrow, K = 7.5 \text{ kN} \uparrow$$

WE PASS A SECTION THROUGH MEMBERS FI, HI, AND HJ,  
AND USE THE FREE BODY SHOWN.

$$\begin{aligned} + \sum M_H = 0: & (7.5 \text{ kN})(1.6 \text{ m}) - (2 \text{ kN})(0.8 \text{ m}) - (1 \text{ kN})(1.6 \text{ m}) - F_{FI}(0.4 \text{ m}) = 0 \\ F_{FI} &= +22.0 \text{ kN} \quad F_{FI} = 22.0 \text{ kN T} \\ + \sum F_y = 0: & \frac{1}{\sqrt{5}} F_{HI} - 2 \text{ kN} - 1 \text{ kN} + 7.5 \text{ kN} = 0 \\ F_{HI} &= -10.06 \text{ kN} \quad F_{HI} = 10.06 \text{ kN C} \\ + \sum M_I = 0: & F_{HJ}(0.4 \text{ m}) + (7.5 \text{ kN})(0.8 \text{ m}) - (1 \text{ kN})(0.8 \text{ m}) = 0 \\ F_{HJ} &= -13.00 \text{ kN} \quad F_{HJ} = 13.00 \text{ kN C} \end{aligned}$$



6.49



GIVEN: HOWE TRUSS WITH LOADING SHOWN.  
FIND: FORCE IN MEMBERS DE, J-I, AND EG.

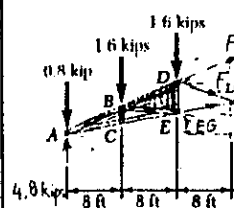
REACTIONS AT SUPPORTS:

BECAUSE OF SYMMETRY OF LOADING:

$$A_x = 0, A_y = L = \frac{1}{2}(\text{TOTAL LOAD}) = \frac{1}{2}(9.60 \text{ kips}) = 4.80 \text{ kips}$$

$$A = L = 4.80 \text{ kips} \uparrow$$

WE PASS A SECTION THROUGH MEMBERS DE, J-I, AND EG, AND USE THE FREE BODY SHOWN.



WE SLIDE  $F_{DE}$  TO APPLY IT AT F.

$$\begin{aligned} +\circlearrowleft \sum M_G = 0: & (0.8 \text{ kip})(24 \text{ ft}) \\ & + (1.6 \text{ kip})(16 \text{ ft}) + (1.6 \text{ kip})(8 \text{ ft}) \\ & - (4.8 \text{ kip})(24 \text{ ft}) - \frac{B F_{DE}}{\sqrt{B^2 + 3.5^2}}(6 \text{ ft}) = 0 \end{aligned}$$

$$F_{DE} = -10.98 \text{ kips}, F_{DE} = 10.98 \text{ kips} \leftarrow$$

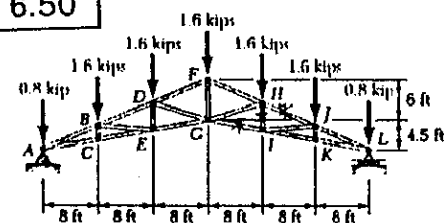
$$\begin{aligned} +\circlearrowleft \sum M_H = 0: & - (1.6 \text{ kip})(8 \text{ ft}) \\ & - (1.6 \text{ kip})(16 \text{ ft}) - \frac{2.5 F_{DE}}{\sqrt{8^2 + 2.5^2}}(16 \text{ ft}) \\ & - \frac{B F_{EG}}{\sqrt{B^2 + 2.5^2}}(7 \text{ ft}) = 0 \end{aligned}$$

$$F_{EG} = -3.35 \text{ kips}, F_{EG} = 3.35 \text{ kips} \leftarrow$$

$$+\circlearrowleft \sum M_D = 0: (0.8 \text{ kip})(16 \text{ ft}) + (1.6 \text{ kip})(8 \text{ ft}) - (4.8 \text{ kip})(16 \text{ ft}) + \frac{B F_{EG}}{\sqrt{B^2 + 1.5^2}}(4 \text{ ft}) = 0$$

$$F_{EG} = +13.02 \text{ kips}, F_{EG} = 13.02 \text{ kips} \leftarrow$$

6.50

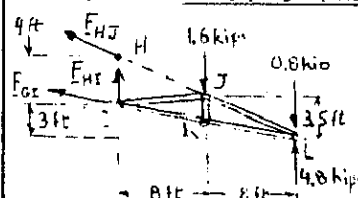


GIVEN: HOWE TRUSS WITH LOADING SHOWN.  
FIND: FORCE IN MEMBERS GI, HI, AND HJ.

REACTIONS AT SUPPORTS BECAUSE OF SYMMETRY OF LOADING:

$$A_x = 0, A_y = L = \frac{1}{2}(\text{TOTAL LOAD}) = \frac{1}{2}(9.60 \text{ kips}) = 4.80 \text{ kips}, A = L = 4.80 \text{ kips} \uparrow$$

WE PASS A SECTION THROUGH MEMBERS GI, HI, AND HJ, AND USE THE FREE BODY SHOWN.



$$\begin{aligned} +\circlearrowleft \sum M_H = 0: & - \frac{16 F_{GI}}{\sqrt{16^2 + 3^2}}(4 \text{ ft}) \\ & + (4.8 \text{ kip})(16 \text{ ft}) - (0.8 \text{ kip})(16 \text{ ft}) \\ & - (1.6 \text{ kip})(8 \text{ ft}) = 0 \end{aligned}$$

$$F_{GI} = +13.02 \text{ kips}, F_{GI} = 13.02 \text{ kips} \leftarrow$$

$$+\circlearrowleft \sum M_L = 0: (1.6 \text{ kip})(8 \text{ ft}) - F_{HI}(16 \text{ ft}) = 0, F_{HI} = +0.800 \text{ kips}$$

$$F_{HI} = 0.800 \text{ kips} \leftarrow$$

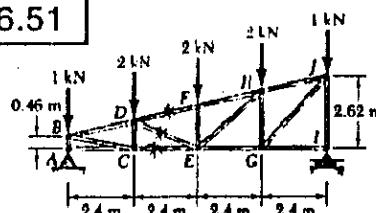
WE SLIDE  $F_{HJ}$  TO APPLY IT AT H.

$$+\circlearrowleft \sum M_I = 0: \frac{8 F_{HJ}}{\sqrt{8^2 + 3.5^2}}(4 \text{ ft}) + (4.8 \text{ kip})(16 \text{ ft}) - (1.6 \text{ kip})(8 \text{ ft}) - (0.8 \text{ kip})(16 \text{ ft}) = 0$$

$$F_{HJ} = -13.17 \text{ kips}$$

$$F_{HJ} = 13.17 \text{ kips} \leftarrow$$

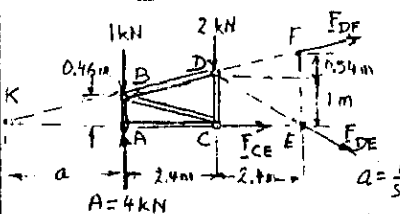
6.51



GIVEN: PITCHED FLAT ROOF TRUSS WITH LOADING SHOWN.  
FIND: FORCE IN MEMBERS CE, DE, AND DF.

REACTIONS AT SUPPORTS: BECAUSE OF THE SYMMETRY OF THE LOADING,  $A_x = 0, A_y = I = \frac{1}{2}(\text{TOTAL LOAD}) = \frac{1}{2}(8 \text{ kN})$   $A = I = 4 \text{ kN} \uparrow$

WE PASS A SECTION THROUGH MEMBERS CE, DE, AND DF, AND USE THE FREE BODY SHOWN. (WE MOVED  $F_{DE}$  TO E AND  $F_{DF}$  TO F.)



$$\text{SLOPE BJ} = \frac{2.16 \text{ m}}{9.6 \text{ m}} = \frac{9}{40}$$

$$\text{SLOPE DE} = \frac{-1 \text{ m}}{2.4 \text{ m}} = \frac{-5}{12}$$

$$a = \frac{0.46 \text{ m}}{\text{SLOPE BJ}} = \frac{0.46 \text{ m}}{9/40} = 2.0444 \text{ m}$$

$$A = 4 \text{ kN}$$

$$+\circlearrowleft \sum M_D = 0: F_{CE}(1 \text{ m}) + (1 \text{ kN})(2.4 \text{ m}) - (4 \text{ kN})(2.4 \text{ m}) = 0$$

$$F_{CE} = +7.20 \text{ kN}$$

$$F_{CE} = 7.20 \text{ kN} \leftarrow$$

$$+\circlearrowleft \sum M_K = 0: (4 \text{ kN})(2.0444 \text{ m}) - (1 \text{ kN})(2.0444 \text{ m}) - (2 \text{ kN})(4.4444 \text{ m}) - \left(\frac{5}{13} F_{DE}\right)(6.8444 \text{ m}) = 0$$

$$F_{DE} = -1.047 \text{ kN}$$

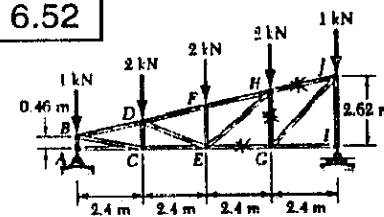
$$F_{DE} = 1.047 \text{ kN} \leftarrow$$

$$+\circlearrowleft \sum M_E = 0: (1 \text{ kN})(4.8 \text{ m}) + (2 \text{ kN})(2.4 \text{ m}) - (4 \text{ kN})(4.8 \text{ m}) - \left(\frac{40}{41} F_{DF}\right)(1.54 \text{ m}) = 0$$

$$F_{DF} = -6.39 \text{ kN}$$

$$F_{DF} = 6.39 \text{ kN} \leftarrow$$

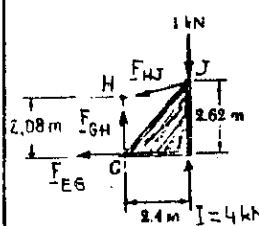
6.52



GIVEN: PITCHED FLAT ROOF TRUSS WITH LOADING SHOWN.  
FIND: FORCE IN MEMBERS EG, GH, AND HJ.

REACTIONS AT SUPPORTS: BECAUSE OF THE SYMMETRY OF THE LOADING,  $A_x = 0, A_y = I = \frac{1}{2}(\text{TOTAL LOAD}) = \frac{1}{2}(8 \text{ kN})$   $A = I = 4 \text{ kN} \uparrow$

WE PASS A SECTION THROUGH MEMBERS EG, GH, AND HJ, AND USE THE FREE BODY SHOWN.



$$+\circlearrowleft \sum M_H = 0:$$

$$(4 \text{ kN})(2.4 \text{ m}) - (1 \text{ kN})(2.4 \text{ m}) - F_{EG}(2.08 \text{ m}) = 0$$

$$F_{EG} = +3.4615 \text{ kN}, F_{EG} = 3.46 \text{ kN} \leftarrow$$

$$+\circlearrowleft \sum M_J = 0:$$

$$- F_{GH}(2.4 \text{ m}) - F_{EG}(2.62 \text{ m}) = 0$$

$$F_{GH} = - \frac{2.62}{2.4} (3.4615 \text{ kN})$$

$$F_{GH} = -3.7788 \text{ kN}, F_{GH} = 3.78 \text{ kN} \leftarrow$$

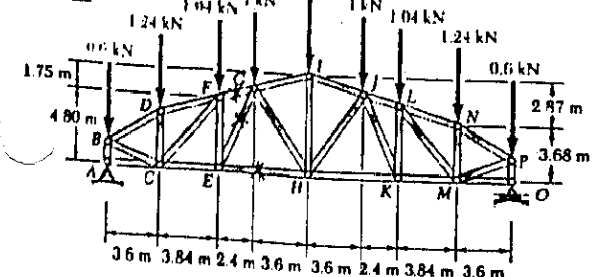
$$+\circlearrowleft \sum F = 0:$$

$$- F_{EG} - \frac{2.4}{2.46} F_{HJ} = 0$$

$$F_{HJ} = - \frac{2.46}{2.4} F_{EG} = - \frac{2.46}{2.4} (3.4615 \text{ kN})$$

$$F_{HJ} = -3.548 \text{ kN}, F_{HJ} = 3.55 \text{ kN} \leftarrow$$

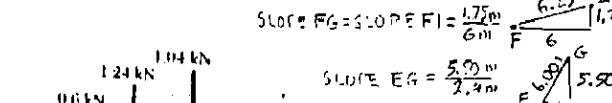
6.53



GIVEN: MARKET ROOF TRUSS WITH LOADING SHOWN.  
FIND: FORCE IN MEMBERS FE, EG, AND EH.

REACTIONS AT SUPPORTS: REACTION OF THE SUPPORTS OF THE TRUSS:  $A_y = 0$ ,  $A_x = 0$ ,  $A_z = 4.48 \text{ kN}$

WE PASS A SECTION THROUGH MEMBERS FE, EG, AND EH, AND USE THE FREE BODY SHOWN



SLOPE EG = SLOPE FI =  $\frac{1.75}{6}$   $\frac{6.25}{6}$   $\frac{I}{1.75}$

SLOPE EG =  $\frac{5.50}{2.4}$   $\frac{5.50}{2.4}$   $\frac{G}{5.50}$

$\sum M_F = 0: (0.6 \text{ kN})(7.44 \text{ m}) + (1.24 \text{ kN})(3.84 \text{ m}) - (4.48 \text{ kN})(7.44 \text{ m}) - (0.6 \text{ kN})(4.80 \text{ m}) = 0$

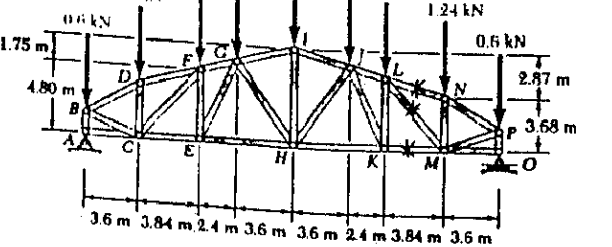
$F_{EG} = -5.231 \text{ kN}$ ,  $F_{EG} = 5.23 \text{ kN C}$

$\sum M_E = 0: F_{EH}(5.50 \text{ m}) + (0.6 \text{ kN})(9.84 \text{ m}) + (1.24 \text{ kN})(6.74 \text{ m}) + (1.04 \text{ kN})(2.4 \text{ m}) - (4.48 \text{ kN})(9.84 \text{ m}) = 0$

$F_{EH} = 5.08 \text{ kN T}$

$\sum F_y = 0: \frac{5.50}{6.25} F_{EG} + \frac{4.75}{6.25} (-5.231 \text{ kN}) + 4.48 \text{ kN} - 0.6 \text{ kN} - 1.24 \text{ kN} - 1.04 \text{ kN} = 0$

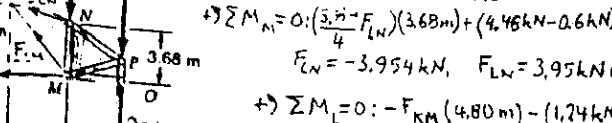
$F_{EG} = -0.147 \text{ kN}$ ,  $F_{EG} = 0.147 \text{ kN C}$



GIVEN: MARKET ROOF TRUSS WITH LOADING SHOWN.  
FIND: FORCE IN MEMBERS KN, LM, AND LN.

BECAUSE OF SYMMETRY OF LOADING,  $O = \frac{1}{2}(\text{LOAD})$ ,  $O = 4.48 \text{ kN}$

WE PASS A SECTION THROUGH MEMBERS KN, LM, AND LN, AND USE FREE BODY SHOWN



$\sum M_N = 0: (\frac{3.84}{4} F_{LN}) (3.68 \text{ m}) + (4.48 \text{ kN} - 0.6 \text{ kN})(3.68 \text{ m}) = 0$

$F_{LN} = -3.954 \text{ kN}$ ,  $F_{LN} = 3.95 \text{ kN C}$

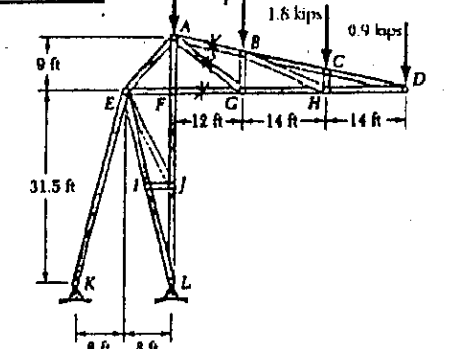
$\sum M_L = 0: -F_{KN}(4.80 \text{ m}) - (1.24 \text{ kN})(3.84 \text{ m}) + (4.48 \text{ kN} - 0.6 \text{ kN})(7.44 \text{ m}) = 0$

$F_{KN} = +5.02 \text{ kN}$ ,  $F_{KN} = 5.02 \text{ kN T}$

$\sum F_y = 0: \frac{4.80}{6.147} F_{LM} + \frac{1.12}{6.147} (-3.954 \text{ kN}) - 1.24 \text{ kN} - 0.6 \text{ kN} + 4.48 \text{ kN} = 0$

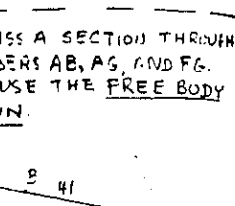
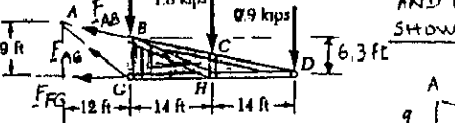
$F_{LM} = -1.963 \text{ kN}$ ,  $F_{LM} = 1.963 \text{ kN C}$

6.55



GIVEN: STADIUM ROOF TRUSS WITH LOADING SHOWN.  
FIND: FORCE IN MEMBERS AB, AG, AND FG.

WE PASS A SECTION THROUGH MEMBERS AB, AG, AND FG, AND USE THE FREE BODY SHOWN



$\sum M_G = 0: (\frac{40}{41} F_{AB})(6.3 \text{ ft}) - (1.8 \text{ kips})(14 \text{ ft}) - (0.9 \text{ kips})(28 \text{ ft}) = 0$

$F_{AB} = +8.20 \text{ kips}$ ,  $F_{AB} = 8.20 \text{ kips T}$

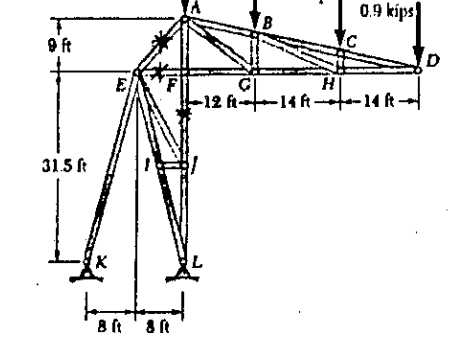
$\sum M_D = 0: -(\frac{3}{5} F_{AG})(28 \text{ ft}) + (1.8 \text{ kips})(28 \text{ ft}) + (1.8 \text{ kips})(14 \text{ ft}) = 0$

$F_{AG} = +4.50 \text{ kips}$ ,  $F_{AG} = 4.50 \text{ kips T}$

$\sum M_A = 0: -F_{FG}(9 \text{ ft}) - (1.8 \text{ kips})(12 \text{ ft}) - (1.8 \text{ kips})(26 \text{ ft}) - (0.9 \text{ kips})(40 \text{ ft}) = 0$

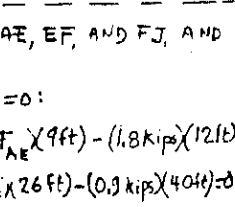
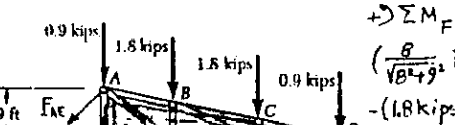
$F_{FG} = -11.60 \text{ kips}$ ,  $F_{FG} = 11.60 \text{ kips C}$

6.56



GIVEN: STADIUM ROOF TRUSS WITH LOADING SHOWN.  
FIND: FORCE IN MEMBERS AE, EF, AND FJ.

WE PASS A SECTION THROUGH MEMBERS AE, EF, AND FJ, AND USE THE FREE BODY SHOWN



$\sum M_F = 0: (\frac{8}{\sqrt{8^2+9^2}} F_{AE})(9 \text{ ft}) - (1.8 \text{ kips})(12 \text{ ft}) - (1.8 \text{ kips})(26 \text{ ft}) - (0.9 \text{ kips})(40 \text{ ft}) = 0$

$F_{AE} = +17.46 \text{ kips}$ ,  $F_{AE} = 17.46 \text{ kips T}$

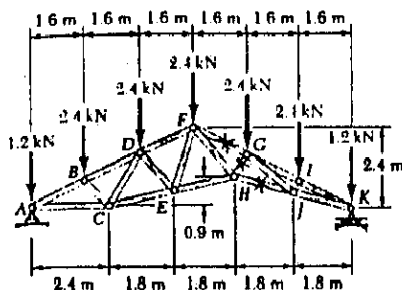
$\sum M_A = 0: -F_{EF}(9 \text{ ft}) - (1.8 \text{ kips})(12 \text{ ft}) - (1.8 \text{ kips})(26 \text{ ft}) - (0.9 \text{ kips})(40 \text{ ft}) = 0$

$F_{EF} = -11.60 \text{ kips}$ ,  $F_{EF} = 11.60 \text{ kips C}$

$\sum M_E = 0: -F_{FJ}(8 \text{ ft}) - (0.9 \text{ kips})(8 \text{ ft}) - (1.8 \text{ kips})(20 \text{ ft}) - (1.8 \text{ kips})(34 \text{ ft}) - (0.9 \text{ kips})(48 \text{ ft}) = 0$

$F_{FJ} = -18.45 \text{ kips}$ ,  $F_{FJ} = 18.45 \text{ kips C}$

6.57



**GIVEN:** VAULTED ROOF TRUSS WITH LOADING SHOWN.

**FIND:** FORCE IN MEMBERS FG, GH, AND HJ.

BECAUSE OF THE SYMMETRY OF THE LOADING,  $A = K = 7.20 \text{ kN} \uparrow$   
WE PASS A SECTION THROUGH MEMBERS FG, GH, AND HJ, AND USE THE FREE BODY SHOWN.

$$\sum M_H = 0: \left(\frac{2}{\sqrt{5}} F_{FG}\right)(0.7 \text{ m}) + \left(\frac{1}{\sqrt{5}} F_{FG}\right)(0.4 \text{ m}) - (2.4 \text{ kN})(0.9 \text{ m}) - (2.4 \text{ kN})(2 \text{ m}) - (1.2 \text{ kN})(3.6 \text{ m}) + (7.2 \text{ kN})(3.6 \text{ m}) = 0$$

$$\frac{1.6}{\sqrt{5}} F_{FG} = -15.84$$

$$F_{FG} = 19.68 \text{ kN C}$$

$$\sum M_K = 0: \left(\frac{4}{\sqrt{5}} F_{GH}\right)(1.6 \text{ m}) + \left(\frac{7}{\sqrt{5}} F_{GH}\right)(3.2 \text{ m}) + (2.4 \text{ kN})(3.2 \text{ m}) + (2.4 \text{ kN})(1.6 \text{ m}) = 0$$

$$(28.8/\sqrt{5}) F_{GH} = -11.52$$

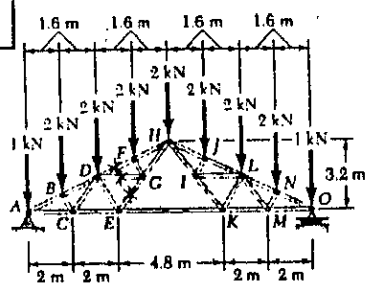
$$F_{GH} = 3.22 \text{ kN C}$$

$$\sum M_G = 0: -\left(\frac{4}{\sqrt{5}} F_{HJ}\right)(1.15 \text{ m}) + \left(\frac{1}{\sqrt{5}} F_{HJ}\right)(1.4 \text{ m}) - (2.4 \text{ kN})(1.6 \text{ m}) + (6 \text{ kN})(3.2 \text{ m}) = 0$$

$$-(3.2/\sqrt{5}) F_{HJ} = -15.36$$

$$F_{HJ} = 19.79 \text{ kN T}$$

6.58



**GIVEN:**

FINK ROOF TRUSS WITH LOADING SHOWN.

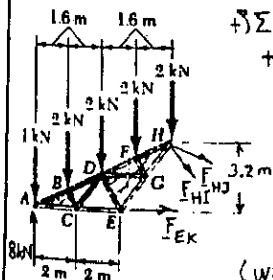
**FIND:**

FORCE IN MEMBERS DF, FG, AND EG.

BECAUSE OF THE SYMMETRY OF THE LOADING,  $A = O = 8 \text{ kN} \uparrow$   
WE NEXT DETERMINE  $F_{EK}$  FROM THE F.B. DIAGRAM OF PANEL AEH:

$$\sum M_H = 0: (1 \text{ kN})(6.4 \text{ m}) + (2 \text{ kN})(4.8 \text{ m}) + (2 \text{ kN})(3.2 \text{ m}) + (2 \text{ kN})(1.6 \text{ m}) + F_{EK}(3.2 \text{ m}) - (8 \text{ kN})(6.4 \text{ m}) = 0$$

$$F_{EK} = 8.00 \text{ kN T}$$



**FREE BODY: PANEL ADE**

(WE SLIDE  $F_{EG}$  TO APPLY IT AT G)

$$\sum M_G = 0: (8 \text{ kN})(1.6 \text{ m}) - (8 \text{ kN})(5.2 \text{ m}) + (1 \text{ kN})(5.2 \text{ m}) + (2 \text{ kN})(3.6 \text{ m}) + (2 \text{ kN})(2 \text{ m}) - \left(\frac{1}{\sqrt{5}} F_{DF}\right)(2 \text{ m}) = 0$$

$$F_{DF} = -13.86 \text{ kN}$$

$$F_{DF} = 13.86 \text{ kN C}$$

$$\sum M_H = 0: F_{DG}(1.6 \text{ m}) + (8 \text{ kN})(3.2 \text{ m}) - (8 \text{ kN})(6.4 \text{ m}) + (1 \text{ kN})(6.4 \text{ m}) + (2 \text{ kN})(4.8 \text{ m}) + (2 \text{ kN})(3.2 \text{ m}) = 0$$

$$F_{DG} = 2.00 \text{ kN}$$

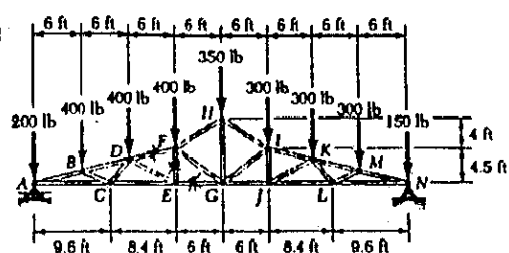
$$F_{DG} = 2.00 \text{ kN T}$$

$$\sum M_E = 0: \left(\frac{4}{\sqrt{5}} F_{EG}\right)(2 \text{ m}) + (8 \text{ kN})(1.6 \text{ m}) - (8 \text{ kN})(3.2 \text{ m}) + (1 \text{ kN})(5.2 \text{ m}) + (2 \text{ kN})(1.6 \text{ m}) = 0$$

$$F_{EG} = +4.00 \text{ kN}$$

$$F_{EG} = 4.00 \text{ kN T}$$

6.59



**GIVEN:** DUOPITCH ROOF TRUSS WITH LOADING SHOWN.

**FIND:** FORCE IN MEMBERS DF, EF, AND EG.

**FREE BODY: TRUSS**

$$\sum F_x = 0: N_x = 0$$

$$\sum M_A = 0: (200 \text{ lb})(8 \text{ ft}) + (400 \text{ lb})(7 \text{ ft} + 6 \text{ ft} + 5 \text{ ft}) + (350 \text{ lb})(4 \text{ ft}) + (300 \text{ lb})(3 \text{ ft} + 2 \text{ ft} + \text{ft}) - A(8 \text{ ft}) = 0$$

$$A = 1500 \text{ lb} \uparrow$$

$$\sum F_y = 0: 1500 \text{ lb} - 200 \text{ lb} - 3(400 \text{ lb}) - 350 \text{ lb} - 3(300 \text{ lb}) - 150 \text{ lb} + N_y = 0$$

$$N_y = 1300 \text{ lb}$$

$$N_x = 1300 \text{ lb} \uparrow$$

WE PASS A SECTION THROUGH DF, EF, AND EG, AND USE THE FREE BODY SHOWN.

$$\sum M_E = 0: (200 \text{ lb})(18 \text{ ft}) + (400 \text{ lb})(12 \text{ ft}) + (400 \text{ lb})(6 \text{ ft}) - (1500 \text{ lb})(18 \text{ ft}) - \left(\frac{18}{18^2 + 4.5^2}\right) F_{DF}(4.5 \text{ ft}) = 0$$

$$F_{DF} = -3711 \text{ lb}$$

$$F_{DF} = 3710 \text{ lb C}$$

$$\sum M_A = 0: F_{EF}(18 \text{ ft}) - (400 \text{ lb})(6 \text{ ft}) - (400 \text{ lb})(12 \text{ ft}) = 0$$

$$F_{EF} = +400 \text{ lb}$$

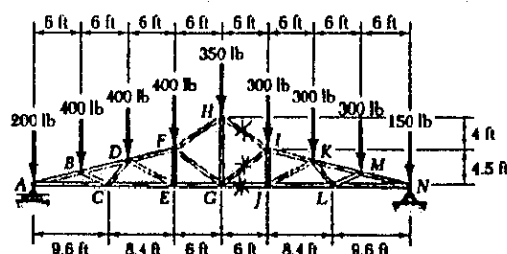
$$F_{EF} = 400 \text{ lb T}$$

$$\sum M_F = 0: F_{EG}(4.5 \text{ ft}) - (1500 \text{ lb})(18 \text{ ft}) + (200 \text{ lb})(18 \text{ ft}) + (400 \text{ lb})(12 \text{ ft}) + (400 \text{ lb})(6 \text{ ft}) = 0$$

$$F_{EG} = +3600 \text{ lb}$$

$$F_{EG} = 3600 \text{ lb T}$$

6.60



**GIVEN:** DUOPITCH ROOF TRUSS WITH LOADING SHOWN.

**FIND:** FORCE IN MEMBERS HI, GI, AND GJ.

SEE SOLUTION OF PROB. 6.59 FOR REACTIONS:  $A = 1500 \text{ lb} \uparrow$ ,  $N = 1300 \text{ lb} \uparrow$

WE PASS A SECTION THROUGH HI, GI, AND GJ, AND USE THE FREE BODY SHOWN. (WE APPLY  $F_{HI}$  AT H.)

$$\sum M_G = 0: \left(\frac{6}{\sqrt{6^2 + 4.5^2}} F_{HI}\right)(8.5 \text{ ft}) + (1300 \text{ lb})(2 \text{ ft} + 4 \text{ ft}) - (300 \text{ lb})(6 \text{ ft}) - (300 \text{ lb})(12 \text{ ft}) - (300 \text{ lb})(18 \text{ ft}) - (150 \text{ lb})(24 \text{ ft}) = 0$$

$$F_{HI} = -2375.4 \text{ lb}$$

$$F_{HI} = 2375 \text{ lb C}$$

$$\sum M_I = 0: (1300 \text{ lb})(18 \text{ ft}) - (300 \text{ lb})(6 \text{ ft}) - (300 \text{ lb})(12 \text{ ft}) - (150 \text{ lb})(18 \text{ ft}) - F_{GJ}(4.5 \text{ ft}) = 0$$

$$F_{GJ} = +3400 \text{ lb}$$

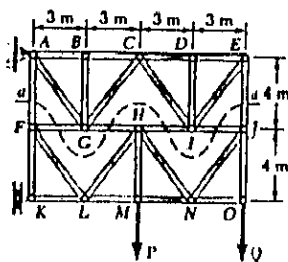
$$F_{GJ} = 3400 \text{ lb T}$$

$$\sum F_x = 0: -\frac{4}{5} F_{GI} - \frac{6}{\sqrt{6^2 + 4.5^2}} (-2375.4 \text{ lb}) - 3400 \text{ lb} = 0$$

$$F_{GI} = -1179.1 \text{ lb}$$

$$F_{GI} = 1179 \text{ lb C}$$

6.61



GIVEN:

TRUSS SHOWN WITH

$$P = Q = 1.2 \text{ kN}$$

FIND:

FORCE IN MEMBERS

AF AND EJ

(USE SECTION aa)

FREE BODY: ENTIRE TRUSS

$$+\circlearrowleft \sum M_A = 0:$$

$$K(8\text{m}) - (1.2\text{ kN})(6\text{m}) - (1.2\text{ kN})(12\text{m}) = 0$$

$$K = +2.70\text{ kN}$$

$$K = 2.70\text{ kN} \rightarrow$$

FREE BODY: LOWER PORTION

$$+\circlearrowleft \sum M_F = 0:$$

$$F_{EJ}(12\text{m}) + (2.70\text{ kN})(4\text{m}) - (1.2\text{ kN})(6\text{m}) - (1.2\text{ kN})(12\text{m}) = 0$$

$$F_{EJ} = +0.900\text{ kN}$$

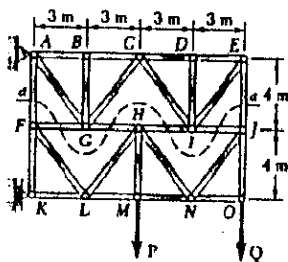
$$F_{EJ} = 0.900\text{ kN} \leftarrow$$

$$+\uparrow \sum F_y = 0: F_{AF} + 0.9\text{ kN} - 1.2\text{ kN} - 1.2\text{ kN} = 0$$

$$F_{AF} = +1.500\text{ kN}$$

$$F_{AF} = 1.500\text{ kN} \leftarrow$$

6.62



GIVEN:

TRUSS SHOWN WITH

$$P = 1.2\text{ kN}, Q = 0$$

FIND:

FORCE IN MEMBERS

AF AND EJ

(USE SECTION aa)

FREE BODY: ENTIRE TRUSS

$$+\circlearrowleft \sum M_A = 0:$$

$$K(8\text{m}) - (1.2\text{ kN})(6\text{m}) = 0$$

$$K = +0.900\text{ kN}$$

$$K = 0.900\text{ kN} \rightarrow$$

FREE BODY: LOWER PORTION

$$+\circlearrowleft \sum M_F = 0:$$

$$F_{EJ}(12\text{m}) + (0.900\text{ kN})(4\text{m}) - (1.2\text{ kN})(6\text{m}) = 0$$

$$F_{EJ} = +0.300\text{ kN}$$

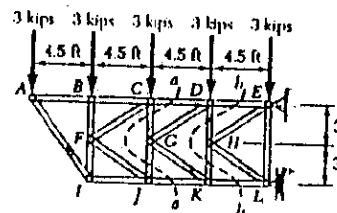
$$F_{EJ} = 0.300\text{ kN} \leftarrow$$

$$+\uparrow \sum F_y = 0:$$

$$F_{AF} + 0.300\text{ kN} - 1.2\text{ kN} = 0$$

$$F_{AF} = +0.900\text{ kN}, F_{AF} = 0.900\text{ kN} \leftarrow$$

6.63



GIVEN:

TRUSS AND LOADING SHOWN

FIND:

FORCE IN MEMBERS CD AND JK

(USE SECTION aa)

FREE BODY:

PORTION OF TRUSS SHOWN

$$+\circlearrowleft \sum M_C = 0:$$

$$F_{JK}(6\text{ ft}) + (3\text{ kips})(9\text{ ft}) + (3\text{ kips})(4.5\text{ ft}) = 0$$

$$F_{JK} = -6.75\text{ kips}$$

$$F_{JK} = 6.75\text{ kips} \leftarrow$$

$$+\rightarrow \sum F_x = 0:$$

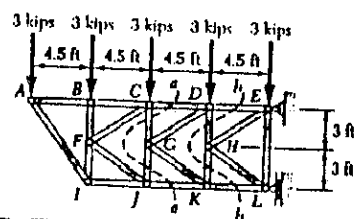
$$F_{CD} + F_{JK} = 0$$

$$F_{CD} - 6.75\text{ kips} = 0$$

$$F_{CD} = +6.75\text{ kips}$$

$$F_{CD} = 6.75\text{ kips} \rightarrow$$

6.64



GIVEN:

TRUSS AND LOADING SHOWN

FIND:

FORCE IN MEMBERS DE AND KL

(USE SECTION b-b)

FREE BODY:

PORTION OF TRUSS SHOWN

$$+\circlearrowleft \sum M_D = 0:$$

$$F_{KL}(6\text{ ft}) + (3\text{ kips})(12.5\text{ ft}) + (3\text{ kips})(9\text{ ft}) + (3\text{ kips})(4.5\text{ ft}) = 0$$

$$F_{KL} = -13.50\text{ kips}$$

$$F_{KL} = 13.50\text{ kips} \leftarrow$$

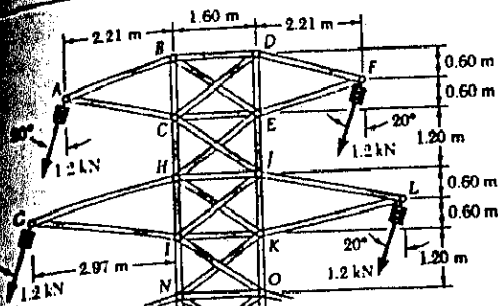
$$+\rightarrow \sum F_x = 0:$$

$$F_{DE} + F_{KL} = 0$$

$$F_{DE} - 13.50\text{ kips} = 0$$

$$F_{DE} = +13.50\text{ kips}, F_{DE} = 13.50\text{ kips} \rightarrow$$

# AND 6.66

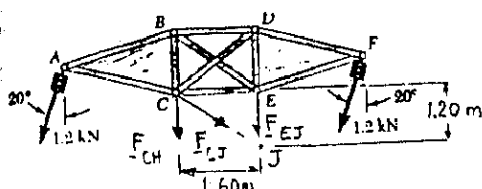


6.65 POWER TRANSMISSION LINE TOWER AND LOADING SHOWN.

85 FIND: (a) WHICH OF THE COUNTERS CJ AND HE IS ACTING  
(b) THE FORCE IN THAT COUNTER

FREE BODY: PORTION ABDFEC OF TOWER

ASSUME THAT COUNTER CJ IS ACTING AND SHOW THE FORCES EXERTED BY THAT COUNTER AND BY MEMBER CH



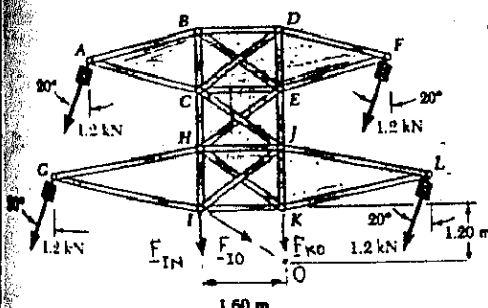
$$\sum F_x = 0; \frac{4}{5} F_{CJ} - 2(1.2 \text{ kN}) \sin 20^\circ = 0 \quad F_{CJ} = +1.026 \text{ kN}$$

SINCE CJ IS FOUND TO BE IN TENSION, OUR ASSUMPTION WAS CORRECT. THUS, THE ANSWERS ARE

- (a) CJ  
(b) 1.026 kN T

6.66 FIND: (a) WHICH OF THE COUNTERS IO AND KN IS ACTING  
(b) THE FORCE IN THAT COUNTER.

FREE BODY: PORTION OF TOWER SHOWN



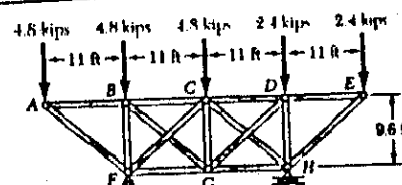
WE ASSUME THAT COUNTER IO IS ACTING AND SHOW THE FORCES EXERTED BY THAT COUNTER AND BY MEMBERS IN AND KO.

$$\sum F_x = 0; \frac{4}{5} F_{IO} - 4(1.2 \text{ kN}) \sin 20^\circ = 0 \quad F_{IO} = +2.05 \text{ kN}$$

SINCE IO IS FOUND TO BE IN TENSION, OUR ASSUMPTION WAS CORRECT. THUS, THE ANSWERS ARE

- (a) IO  
(b) 2.05 kN T

## 6.67



GIVEN:

TRUSS AND LOADING SHOWN.

FIND:

FORCES IN THE COUNTERS ACTING UNDER THIS LOADING

FREE BODY: TRUSS

$$\begin{aligned} \sum F_x = 0; F_x &= 0 \\ + \sum M_H = 0; & 4.8(3a) + 4.8(2a) + 4.8a - 2.4a - F_y(2a) = 0 \\ F_y &= +13.20 \text{ kips} \\ F &= 13.20 \text{ kips} \end{aligned}$$

$$+ \sum F_y = 0; H + 13.20 \text{ kips} - 3(4.8 \text{ kips}) - 2(2.4 \text{ kips}) = 0$$

$$H = +6.00 \text{ kips} \quad H = 6.00 \text{ kips}$$

FREE BODY: ABF

WE ASSUME THAT COUNTER BG IS ACTING

$$+ \sum F_y = 0; -\frac{9.6}{14.6} F_{BG} + 13.20 - 2(4.8) = 0$$

$$F_{BG} = +5.475 \quad F_{BG} = 5.48 \text{ kips T}$$

SINCE BG IS IN TENSION, OUR ASSUMPTION WAS CORRECT

FREE BODY: DEH

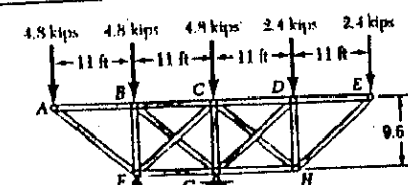
WE ASSUME THAT COUNTER DG IS ACTING.

$$+ \sum F_y = 0; -\frac{9.6}{14.6} F_{DG} + 6.00 - 2(2.4) = 0$$

$$F_{DG} = +1.825 \quad F_{DG} = 1.825 \text{ kips T}$$

SINCE DG IS IN TENSION, O.K.

## 6.68



GIVEN:

TRUSS AND LOADING SHOWN.

FIND:

FORCES IN THE COUNTERS ACTING UNDER THIS LOADING

FREE BODY: TRUSS

$$\begin{aligned} \sum F_x = 0; F_x &= 0 \\ + \sum M_G = 0; & -F_y a + 4.8(2a) + 4.8a - 2.4a - 2.4(2a) = 0 \\ F_y &= 7.20, F = 7.20 \text{ kips} \end{aligned}$$

FREE BODY: ABF

WE ASSUME THAT COUNTER CF IS ACTING.

$$+ \sum F_y = 0; \frac{9.6}{14.6} F_{CF} + 7.20 - 2(4.8) = 0$$

$$F_{CF} = +3.65 \quad F_{CF} = 3.65 \text{ kips T}$$

SINCE CF IS IN TENSION, O.K.

FREE BODY: DEH

WE ASSUME THAT COUNTER CH IS ACTING

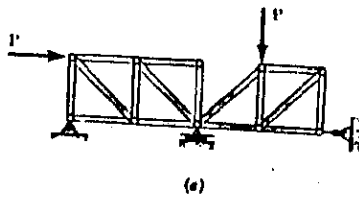
$$+ \sum F_x = 0; \frac{9.6}{14.6} F_{CH} - 1(2.4 \text{ kips}) = 0$$

$$F_{CH} = +7.30 \quad F_{CH} = 7.30 \text{ kips T}$$

SINCE CH IS IN TENSION, O.K.

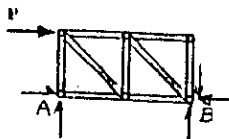
CLASSIFY EACH STRUCTURE AS COMPLETELY, PARTIALLY, OR IMPROPERLY CONSTRAINED. IF COMPLETELY CONSTRAINED, FURTHER CLASSIFY AS DETERMINATE OR INDETERMINATE.

### STRUCTURE (a)

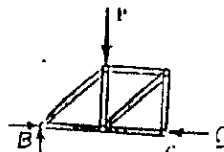


NUMBER OF MEMBERS:  
 $m = 16$   
 NUMBER OF JOINTS:  
 $n = 10$   
 REACTION COMPONENTS:  
 $r = 4$   
 $m + r = 20$      $2n = 20$   
 THUS:  $m + r = 2n$  <

TO DETERMINE WHETHER THE STRUCTURE IS ACTUALLY COMPLETELY CONSTRAINED AND DETERMINATE, WE MUST TRY TO FIND THE REACTIONS AT THE SUPPORTS. WE DIVIDE THE STRUCTURE INTO TWO SIMPLE TRUSSES AND DRAW THE FREE-BODY DIAGRAM OF EACH TRUSS.



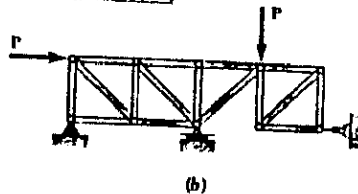
THIS IS A PROPERLY SUPPORTED SIMPLE TRUSS - O.K.



THIS IS AN IMPROPERLY SUPPORTED SIMPLE TRUSS. (REACTION AT C PASSES THROUGH B, THUS, E.G.  $\sum M_B = 0$  CANNOT BE SATISFIED.)

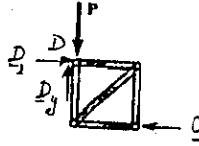
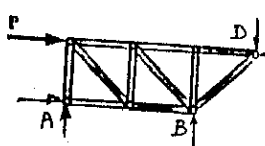
STRUCTURE IS PARTIALLY CONSTRAINED

### STRUCTURE (b)



$m = 16$   
 $n = 10$   
 $r = 4$   
 $m + r = 20$      $2n = 20$   
 THUS:  $m + r = 2n$  <

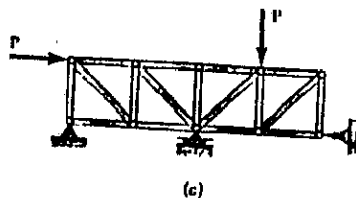
WE MUST AGAIN TRY TO FIND THE REACTIONS AT THE SUPPORTS, DIVIDING THE STRUCTURE AS SHOWN



BOTH PORTIONS ARE SIMPLY SUPPORTED SIMPLE TRUSSES

STRUCTURE IS COMPLETELY CONSTRAINED AND DETERMINATE

### STRUCTURE (c)

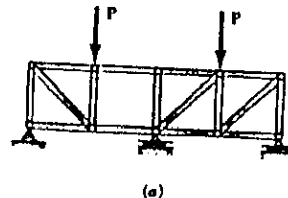


$m = 17$   
 $n = 11$   
 $r = 4$   
 $m + r = 21$      $2n = 22$   
 THUS:  $m + r < 2n$  <

THIS IS A SIMPLE TRUSS WITH AN EXTRA JOINT WHICH CAUSES REACTION AND FORCE MEMBERS TO BE INDETERMINATE. STRUCTURE IS PARTIALLY CONSTRAINED AND INDETERMINATE

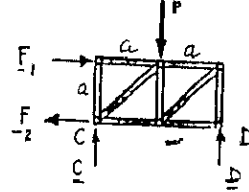
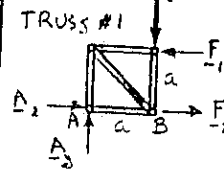
6.70 GIVEN: THE THREE STRUCTURES SHOWN. CLASSIFY EACH STRUCTURE AS COMPLETELY, PARTIALLY, OR IMPROPERLY CONSTRAINED. IF COMPLETELY CONSTRAINED, FURTHER CLASSIFY AS DETERMINATE OR INDETERMINATE.

### STRUCTURE (a)



NUMBER OF MEMBERS:  
 $m = 16$   
 NUMBER OF JOINTS:  
 $n = 10$   
 REACTION COMPONENTS:  
 $r = 4$   
 $m + r = 20$      $2n = 20$   
 THUS:  $m + r = 2n$

TO DETERMINE WHETHER THE STRUCTURE IS ACTUALLY COMPLETELY CONSTRAINED AND DETERMINATE, WE MUST TRY TO FIND THE REACTIONS AT THE SUPPORTS. WE DIVIDE THE STRUCTURE INTO TWO SIMPLE TRUSSES AND DRAW THE FREE-BODY DIAGRAM OF EACH TRUSS



FREE BODY: TRUSS #1

$\sum M_A = 0: F_1 a - P a = 0$      $F_1 = P$   
 $\sum F_y = 0: A_y - P = 0$      $A_y = P$

FREE BODY: TRUSS #2

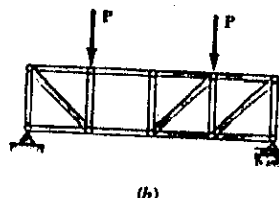
$\sum M_C = 0: D(2a) - P a - P a = 0$      $D = P$   
 $\sum F_x = 0: F_1 - F_2 = 0$      $F_2 = F_1$      $F_2 = P$   
 $\sum F_y = 0: C - P + P = 0$      $C = 0$

FREE BODY: TRUSS #1

$\sum F_x = 0: A_2 - F_1 + F_2 = 0$      $A_2 = 0$

SINCE ALL UNKNOWN REACTIONS HAVE BEEN FOUND AND ALL EQUATIONS SATISFIED,

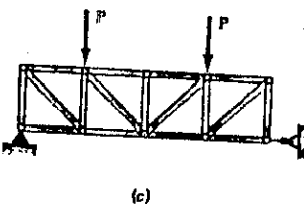
STRUCTURE IS COMPLETELY CONSTRAINED AND DETERMINATE



### STRUCTURE (b)

$m = 16$ ,  $n = 10$ ,  $r = 3$   
 $m + r = 19$      $2n = 20$   
 THUS:  $m + r < 2n$

STRUCTURE IS PARTIALLY CONSTRAINED



### STRUCTURE (c)

$m = 17$ ,  $n = 11$ ,  $r = 4$   
 $m + r = 21$ ,  $2n = 22$   
 THUS:  $m + r < 2n$

HOWEVER, WE NOTE THAT THE STRUCTURE IS A SIMPLE TRUSS WHICH IS IMPROPERLY CONSTRAINED, SINCE THE REACTION AT D PASSES THROUGH A, RESULTING IN  $\sum M_A \neq 0$ .

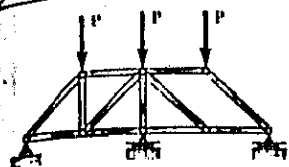
STRUCTURE IS IMPROPERLY CONSTRAINED

6.71

GIVEN: THE THREE STRUCTURES SHOWN.  
CLASSIFY EACH STRUCTURE AS COMPLETELY, PARTIALLY,

OR IMPROPERLY CONSTRAINED. IF COMPLETELY CONSTRAINED,  
FURTHER CLASSIFY AS DETERMINATE OR INDETERMINATE.

STRUCTURE (a)



(a)

NUMBER OF MEMBERS:

$$m = 12$$

NUMBER OF JOINTS:

$$n = 8$$

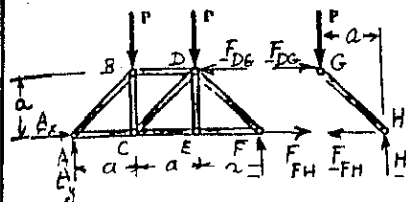
REACTION COMPONENTS:

$$r = 4$$

$$m + r = 16 \quad 2n = 16$$

$$\text{THUS: } m + r = 2n$$

TO DETERMINE WHETHER THE STRUCTURE IS ACTUALLY COMPLETELY  
CONSTRAINED AND DETERMINATE, WE MUST TRY TO FIND  
THE REACTIONS AT THE SUPPORTS. WE PASS A SECTION  
AND OBTAIN THE SIMPLE TRUSS ABCDEF AND MEMBER GH.



FREE BODY: GH

$$\sum M_H = 0:$$

$$Pa - F_{DG}a = 0$$

$$F_{DG} = P$$

$$\sum F_x = 0: F_{FH} = F_{DG} = P$$

$$\sum F_y = 0: H = P$$

FREE BODY: TRUSS ABCDEF

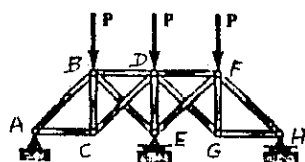
$$\sum F_x = 0: A_x + F_{FH} - F_{DG} = 0 \quad A_x + P - P = 0 \quad A_x = 0$$

$$\sum M_A = 0: F(3a) + Pa - P(2a) - P(1a) = 0 \quad F = \frac{2}{3}P$$

$$\sum F_y = 0: A_y - P - P + \frac{2}{3}P = 0 \quad A_y = \frac{4}{3}P$$

SINCE ALL UNKNOWN HAVE BEEN FOUND AND ALL EQUATIONS  
SATISFIED.

STRUCTURE IS COMPLETELY CONSTRAINED AND DETERMINATE



(b)

STRUCTURE (b)

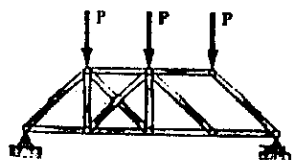
$$m = 13, n = 8, r = 4$$

$$m + r = 17 \quad 2n = 16$$

$$\text{THUS: } m + r > 2n$$

MOOREOVER, WE NOTE THAT  
STRUCTURE IS A SIMPLE TRUSS  
(FOLLOW LETTERING TO CHECK THIS)

STRUCTURE IS COMPLETELY CONSTRAINED AND INDETERMINATE



(c)

STRUCTURE (c)

$$m = 13, n = 8, r = 3$$

$$m + r = 16 \quad 2n = 16$$

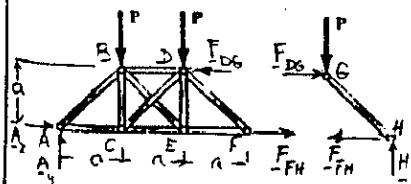
$$\text{THUS: } m + r = 2n$$

WE PASS A SECTION AND  
OBTAIN THE TWO FREE BODIES  
SHOWN.

FREE BODY: FG

WE RECALL FROM PART (a)  
THAT

$$F_{DG} = F_{FH} = H = P$$



FREE BODY: ABCDEF

$$\sum M_A = F_{DG}a - Pa - P(2a) = Pa - 3Pa = -2Pa \neq 0$$

THIS EQUILIBRIUM EQUATION IS NOT SATISFIED, THEREFORE

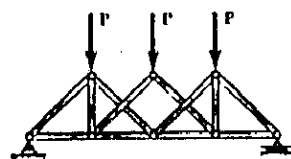
STRUCTURE IS IMPROPERLY CONSTRAINED

6.72

GIVEN: THE THREE STRUCTURES SHOWN

CLASSIFY EACH STRUCTURE AS COMPLETELY,  
PARTIALLY, OR IMPROPERLY CONSTRAINED. IF COMPLETELY  
CONSTRAINED, FURTHER CLASSIFY AS DETERMINATE  
OR INDETERMINATE.

STRUCTURE (a)



(a)

NUMBER OF MEMBERS:

$$m = 12$$

NUMBER OF JOINTS:

$$n = 8$$

REACTION COMPONENTS:

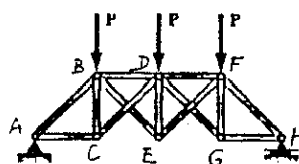
$$r = 3$$

$$m + r = 15 \quad 2n = 16$$

$$\text{THUS: } m + r < 2n$$

STRUCTURE IS PARTIALLY CONSTRAINED

STRUCTURE (b)



(b)

$$m = 13, n = 8$$

$$r = 3$$

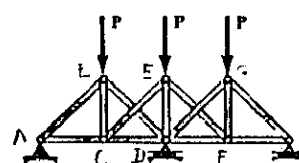
$$m + r = 16 \quad 2n = 16$$

$$\text{THUS: } m + r = 2n$$

TO VERIFY THAT THE STRUCTURE IS ACTUALLY COMPLETELY  
CONSTRAINED AND DETERMINATE, WE OBSERVE THAT IT IS  
A SIMPLE TRUSS (FOLLOW LETTERING TO CHECK THIS) AND  
THAT IT IS SIMPLY SUPPORTED BY A PIN-AND-BRACKET  
AND A ROLLER. THUS:

STRUCTURE IS COMPLETELY CONSTRAINED AND DETERMINATE.

STRUCTURE (c)



(c)

$$m = 13, n = 8$$

$$r = 4$$

$$m + r = 17 \quad 2n = 16$$

$$\text{THUS: } m + r > 2n$$

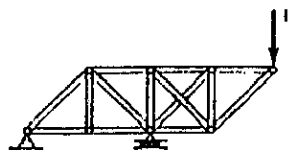
STRUCTURE IS COMPLETELY  
CONSTRAINED AND  
INDETERMINATE

THIS RESULT CAN BE VERIFIED BY OBSERVING THAT THE  
STRUCTURE IS A SIMPLE TRUSS (FOLLOW LETTERING TO  
CHECK THIS), THEREFORE RIGID, AND THAT ITS SUPPORTS  
INVOLVE 4 UNKNOWN.

6.73

GIVEN: THE THREE STRUCTURES SHOWN CLASSIFY EACH STRUCTURE AS COMPLETELY, PARTIALLY, OR IMPROPERLY CONSTRAINED. IF COMPLETELY CONSTRAINED, FURTHER CLASSIFY AS DETERMINATE OR INDETERMINATE.

## STRUCTURE (a)



NUMBER OF MEMBERS:  
 $m = 14$

NUMBER OF JOINTS:  
 $n = 8$

REACTION COMPONENTS:  
 $r = 3$

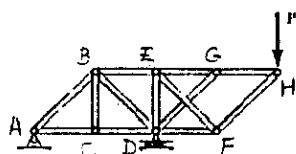
$$m + r = 17 \quad 2n = 16$$

$$\text{THUS: } m + r > 2n$$

STRUCTURE IS COMPLETELY CONSTRAINED AND INDETERMINATE

THIS RESULT CAN BE VERIFIED BY OBSERVING THAT THE STRUCTURE IS AN OVERRIGID TRUSS (ONE EXTRA MEMBER).

## STRUCTURE (b)



$m = 13, n = 8$   
 $r = 3$

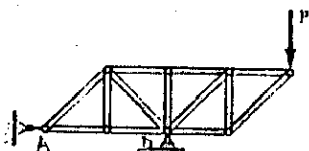
$$m + r = 16 \quad 2n = 16$$

$$\text{THUS: } m + r = 2n$$

WE OBSERVE THAT THE STRUCTURE IS A SIMPLE TRUSS (FOLLOW LETTERING TO CHECK THIS) AND THAT IT IS SIMPLY SUPPORTED BY A PIN AND ROLLER AND A ROLLER. THUS:

STRUCTURE IS COMPLETELY CONSTRAINED AND DETERMINATE

## STRUCTURE (c)



$m = 13, n = 8$   
 $r = 3$

$$m + r = 16 \quad 2n = 16$$

$$\text{THUS: } m + r = 2n$$

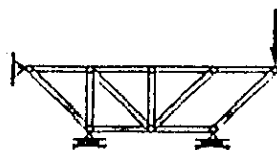
WE OBSERVE THAT THE STRUCTURE IS A SIMPLE TRUSS, BUT THAT IT IS IMPROPERLY CONSTRAINED, SINCE THE REACTION AT A PASSES THROUGH THE SUPPORT D. THE EQUATION  $\sum M_D = 0$ , THEREFORE, IS NOT SATISFIED.

THUS: STRUCTURE IS IMPROPERLY CONSTRAINED

6.74

GIVEN: THE THREE STRUCTURES SHOWN CLASSIFY EACH STRUCTURE AS COMPLETELY, PARTIALLY, OR IMPROPERLY CONSTRAINED. IF COMPLETELY CONSTRAINED, FURTHER CLASSIFY AS DETERMINATE OR INDETERMINATE.

## STRUCTURE (a)



NUMBER OF MEMBERS:  
 $m = 12$

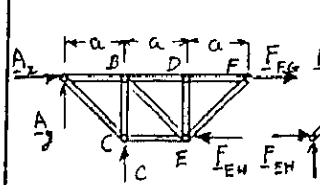
NUMBER OF JOINTS:  
 $n = 8$

REACTION COMPONENTS:  
 $r = 4$

$$m + r = 16 \quad 2n = 16$$

$$\text{THUS: } m + r = 2n$$

TO VERIFY WHETHER OR NOT THE STRUCTURE IS COMPLETELY CONSTRAINED AND DETERMINATE, WE PASS A SECTION AND CONSIDER THE FREE BODIES ABCDEF (A SIMPLE TRUSS) AND G.



FREE BODY: GH

$$+\circlearrowleft \sum M_H = 0: F_{FG}a - Pa = 0$$

$$F_{FG} = P$$

$$+\circlearrowleft \sum F_x = 0: F_{EH} - F_{FG} = 0$$

$$F_{EH} = F_{FG} = P$$

$$+\uparrow \sum F_y = 0: H - P = 0 \quad H = P$$

FREE BODY: TRUSS ABCDEF

$$+\circlearrowleft \sum M_A = 0: Ca - F_{EH}a = 0$$

$$C = F_{EH} = P$$

$$+\circlearrowleft \sum F_x = 0: A_x + F_{FG} - F_{EH} = 0$$

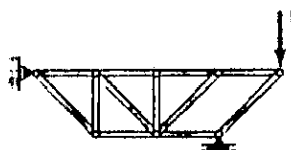
$$A_x = 0$$

$$+\uparrow \sum F_y = 0: A_y + C = 0 \quad A_y = -C = -P$$

SINCE ALL UNKNOWN HAVE BEEN FOUND AND ALL EQUATIONS SATISFIED,

STRUCTURE IS COMPLETELY CONSTRAINED AND DETERMINATE

## STRUCTURE (b)



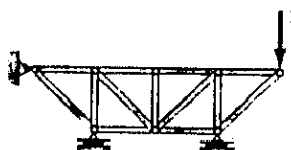
$m = 12, n = 8$   
 $r = 3$

$$m + r = 15 \quad 2n = 16$$

$$\text{THUS: } m + r < 2n$$

STRUCTURE IS PARTIALLY CONSTRAINED

## STRUCTURE (c)



$m = 13, n = 8$   
 $r = 4$

$$m + r = 17 \quad 2n = 16$$

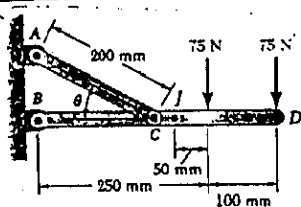
$$\text{THUS: } m + r > 2n$$

WE OBSERVE THAT THE STRUCTURE IS A SIMPLE TRUSS AND THAT ITS SUPPORTS INVOLVE 4 UNKNOWN (INSTEAD OF 3 FOR A SIMPLY SUPPORTED TRUSS), THUS

STRUCTURE IS COMPLETELY CONSTRAINED AND INDETERMINATE



6.75



FIND: FORCE  
IN AC AND  
REACTION  
AT B WHEN  
(a)  $\theta = 30^\circ$   
(b)  $\theta = 60^\circ$

(a)  $\theta = 30^\circ$

SINCE  $AC = 200 \text{ mm}$   
 $BC = (200) \cos 30^\circ = 173.2 \text{ mm}$

$\sum M_B = 0:$   
 $-75 \text{ N}(250 \text{ mm}) - 75 \text{ N}(350 \text{ mm}) + F_{AC} \sin 30^\circ (173.2 \text{ mm}) = 0$   
 $F_{AC} = 519.6 \text{ N}; F_{AC} = 520 \text{ N T.}$

$\sum F_x = 0: B_x - (519.6 \text{ N}) \cos 30^\circ = 0 \quad B_x = 450 \text{ N} \rightarrow$   
 $\sum F_y = 0: B_y + (519.6 \text{ N}) \sin 30^\circ - 75 \text{ N} - 75 \text{ N} = 0$   
 $B_y = -109.8 \text{ N} \quad B_y = 109.8 \text{ N} \downarrow$   
 $B = 463 \text{ N} \angle 13.7^\circ$

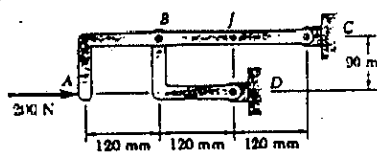
(b)  $\theta = 60^\circ$

SINCE  $AC = 200 \text{ mm}$   
 $BC = (200) \cos 60^\circ = 100 \text{ mm}$

$\sum M_B = 0:$   
 $-75 \text{ N}(250 \text{ mm}) - 75 \text{ N}(350 \text{ mm}) + F_{AC} \sin 60^\circ (100 \text{ mm}) = 0$   
 $F_{AC} = 519.6 \text{ N}; F_{AC} = 520 \text{ N T.}$

$\sum F_x = 0: B_x - (519.6 \text{ N}) \cos 60^\circ = 0 \quad B_x = 259.8 \text{ N} \rightarrow$   
 $\sum F_y = 0: B_y + (519.6 \text{ N}) \sin 60^\circ - 75 \text{ N} - 75 \text{ N} = 0$   
 $B_y = -300 \text{ N} \quad B_y = 300 \text{ N} \downarrow$   
 $B = 397 \text{ N} \angle 49.1^\circ$

6.76



FIND: FORCE  
ACTING ON  
MEMBER ABC  
(a) AT B,  
(b) AT C.

NOTE THAT BD IS A TWO-FORCE MEMBER. FORCE B IS DIRECTED ALONG DB.

(a)

$\sum M_C = 0: (200 \text{ N})(90 \text{ mm}) - \frac{3}{5} B (240 \text{ mm}) = 0$   
 $B = 125 \text{ N}$   
 $B = 125 \text{ N} \angle 36.9^\circ$

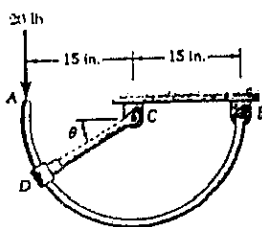
(b)

$\sum F_x = 0: C_x + 200 \text{ N} - \frac{4}{5} (125 \text{ N}) = 0$   
 $C_x = -100 \text{ N} \quad C_x = 100 \text{ N} \leftarrow$

$\sum F_y = 0: C_y + \frac{3}{5} (125 \text{ N}) = 0$   
 $C_y = -75 \text{ N} \quad C_y = 75 \text{ N} \downarrow$

$C = 125 \text{ N} \angle 36.9^\circ$

6.77

GIVEN:  $\theta = 30^\circ$ 

FIND:  
(a) FORCE IN CD,  
(b) REACTION AT B.

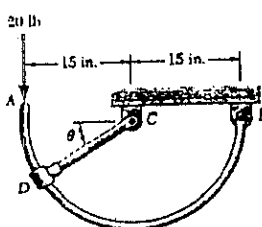
NOTE THAT DC IS A TWO-FORCE MEMBER.  $F_{CD}$  MUST BE DIRECTED ALONG DC.

(a)  $\sum M_B = 0: (20 \text{ lb})(2R) - (F_{CD} \sin 30^\circ) R = 0$   
 $F_{CD} = 80 \text{ lb} \quad F_{CD} = 80 \text{ lb T}$

(b)  $\sum M_C = 0: (20 \text{ lb}) R + (B_y) R = 0$   
 $B_y = -20 \text{ lb} \quad B_y = 20 \text{ lb} \downarrow$

$\sum F_x = 0: F_{CD} \cos 30^\circ + B_x = 0$   
 $(80 \text{ lb})(\cos 30^\circ) + B_x = 0$   
 $B_x = -69.28 \text{ lb} \quad B_x = 69.28 \text{ lb} \leftarrow$   
 $B = 72.1 \text{ lb} \angle 16.1^\circ$

6.78

GIVEN:  $\theta = 150^\circ$ 

FIND:  
(a) FORCE IN CD  
(b) REACTION AT B.

NOTE THAT CD IS A TWO-FORCE MEMBER.  $F_{CD}$  MUST BE DIRECTED ALONG DC.

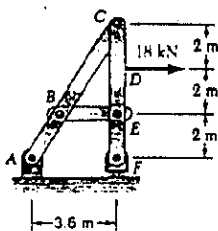
(a)  $\sum M_B = 0: (20 \text{ lb})(2R) - (F_{CD} \sin 30^\circ) R = 0$   
 $F_{CD} = 80 \text{ lb} \quad F_{CD} = 80 \text{ lb T}$

(b)  $\sum M_C = 0: (20 \text{ lb}) R + (B_y) R = 0$   
 $B_y = -20 \text{ lb} \quad B_y = 20 \text{ lb} \downarrow$

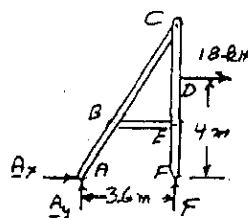
$\sum F_x = 0: -F_{CD} \cos 30^\circ + B_x = 0$   
 $-(20 \text{ lb}) \cos 30^\circ + B_x = 0$   
 $B_x = 69.28 \text{ lb} \quad B_x = 69.28 \text{ lb} \rightarrow$   
 $B = 72.1 \text{ lb} \angle 16.1^\circ$

6.29

6.79



FIND: COMPONENTS OF FORCES ACTING ON MEMBER ABC.



FREE BODY: ENTIRE FRAME

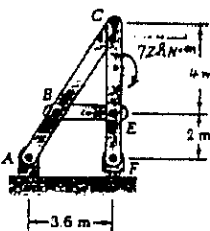
$$\begin{aligned} \pm \sum F_x = 0: A_x + 18 \text{ kN} &= 0 \\ A_x &= -18 \text{ kN} \quad A_x = 18 \text{ kN} \leftarrow \\ + \sum M_E = 0: & \\ -(18 \text{ kN})(4 \text{ m}) - A_y(3.6 \text{ m}) &= 0 \\ A_y &= -20 \text{ kN} \quad A_y = 20 \text{ kN} \uparrow \\ + \sum F_y = 0: 20 \text{ kN} + F &= 0 \\ F &= -20 \text{ kN} \quad F = 20 \text{ kN} \uparrow \end{aligned}$$

FREE BODY: MEMBER ABC

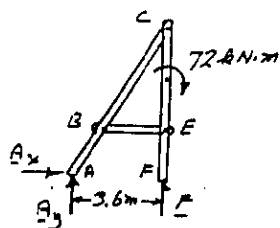
NOTE: BE IS A TWO-FORCE MEMBER. THUS  $B$  IS DIRECTED ALONG LINE BE.

$$\begin{aligned} + \sum M_C = 0: & \\ B(4 \text{ m}) - (18 \text{ kN})(6 \text{ m}) + (20 \text{ kN})(3.6 \text{ m}) &= 0 \\ B &= 9 \text{ kN} \quad B = 9 \text{ kN} \rightarrow \\ \pm \sum F_x = 0: C_x - 18 \text{ kN} + 9 \text{ kN} &= 0 \\ C_x &= 9 \text{ kN} \quad C_x = 9 \text{ kN} \rightarrow \\ + \sum F_y = 0: C_y - 20 \text{ kN} &= 0 \\ C_y &= 20 \text{ kN} \quad C_y = 20 \text{ kN} \uparrow \end{aligned}$$

6.80



GIVEN:  $M = 72 \text{ kN} \cdot \text{m}$ .  
FIND: COMPONENTS OF FORCES ACTING ON MEMBER ABC.



FREE BODY: ENTIRE FRAME

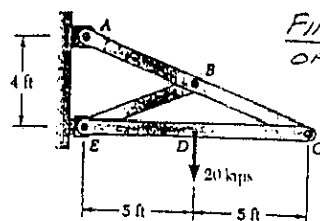
$$\begin{aligned} \pm \sum F_x = 0: A_x &= 0 \\ + \sum M_F = 0: & \\ -72 \text{ kN} \cdot \text{m} - A_y(3.6 \text{ m}) &= 0 \\ A_y &= -20 \text{ kN} \quad A_y = 20 \text{ kN} \uparrow \\ A &= 20 \text{ kN} \uparrow \end{aligned}$$

FREE BODY: MEMBER ABC

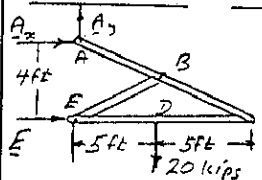
NOTE: BE IS A TWO-FORCE MEMBER. THUS  $B$  IS DIRECTED ALONG LINE BE.

$$\begin{aligned} + \sum M_C = 0: & \\ B(4 \text{ m}) + (20 \text{ kN})(3.6 \text{ m}) &= 0 \\ B &= -18 \text{ kN} \quad B = 18 \text{ kN} \leftarrow \\ \pm \sum F_x = 0: -18 \text{ kN} + C_x &= 0 \\ C_x &= 18 \text{ kN} \quad C_x = 18 \text{ kN} \rightarrow \\ + \sum F_y = 0: C_y - 20 \text{ kN} &= 0 \\ C_y &= 20 \text{ kN} \quad C_y = 20 \text{ kN} \uparrow \end{aligned}$$

6.81



FIND: COMPONENTS OF FORCES ACTING ON MEMBER ABC.



FREE BODY: ENTIRE FRAME

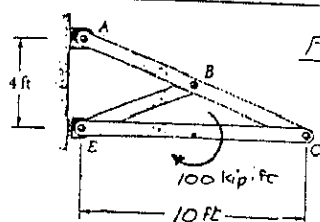
$$\begin{aligned} + \sum M_E = 0: -A_x(4) - (20 \text{ kips})(5) &= 0 \\ A_x &= -25 \text{ kips} \quad A_x = 25 \text{ kips} \leftarrow \\ + \sum F_y = 0: A_y - 20 \text{ kips} &= 0 \\ A_y &= 20 \text{ kips} \quad A_y = 20 \text{ kips} \uparrow \end{aligned}$$

FREE BODY: MEMBER ABC

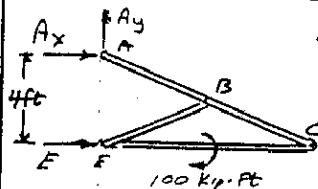
NOTE: BE IS A TWO-FORCE MEMBER. THUS  $B$  IS DIRECTED ALONG LINE BE. AND  $B_x = \frac{3}{4} B$ .

$$\begin{aligned} + \sum M_C = 0: (25 \text{ kips})(4 \text{ ft}) - (20 \text{ kips})(10 \text{ ft}) &+ B_x(2 \text{ ft}) + B_y(5 \text{ ft}) = 0 \\ -100 \text{ kips} \cdot \text{ft} + B_x(2 \text{ ft}) + \frac{3}{4} B_x(5 \text{ ft}) &= 0 \\ B_x &= 25 \text{ kips} \quad B_x = 25 \text{ kips} \leftarrow \\ B_y &= \frac{3}{4} B_x = \frac{3}{4}(25) = 18.75 \text{ kips} \quad B_y = 10 \text{ kips} \uparrow \\ \pm \sum F_x = 0: C_x - 25 \text{ kips} - 25 \text{ kips} &= 0 \\ C_x &= 50 \text{ kips} \quad C_x = 50 \text{ kips} \rightarrow \\ + \sum F_y = 0: C_y + 20 \text{ kips} - 10 \text{ kips} &= 0 \\ C_y &= -10 \text{ kips} \quad C_y = 10 \text{ kips} \downarrow \end{aligned}$$

6.82



FIND: COMPONENTS OF FORCES ACTING ON MEMBER ABC.



FREE BODY: ENTIRE FRAME

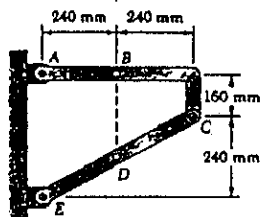
$$\begin{aligned} + \sum F_x = 0: A_x &= 0 \\ + \sum M_E = 0: & \\ -A_x(4 \text{ ft}) - 100 \text{ kips} \cdot \text{ft} &= 0 \\ A_x &= -25 \text{ kips} \quad A_x = 25 \text{ kips} \leftarrow \\ A &= 25 \text{ kips} \leftarrow \end{aligned}$$

FREE BODY: MEMBER ABC

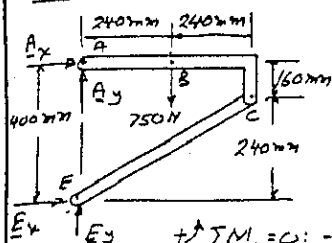
NOTE: BE IS A TWO-FORCE MEMBER. THUS  $B$  IS DIRECTED ALONG LINE BE AND  $B_y = \frac{3}{4} B_x$ .

$$\begin{aligned} + \sum M_C = 0: (25 \text{ kips})(4 \text{ ft}) + B_x(2 \text{ ft}) + B_y(5 \text{ ft}) &= 0 \\ 100 \text{ kips} \cdot \text{ft} + B_x(2 \text{ ft}) + \frac{3}{4} B_x(5 \text{ ft}) &= 0 \\ B_x &= -25 \text{ kips} \quad B_x = 25 \text{ kips} \leftarrow \\ B_y &= \frac{3}{4} B_x = \frac{3}{4}(-25) = -18.75 \text{ kips} \quad B_y = 10 \text{ kips} \uparrow \\ \pm \sum F_x = 0: -25 \text{ kips} + 25 \text{ kips} + C_x &= 0 \\ C_x &= 0 \\ + \sum F_y = 0: 10 \text{ kips} + C_y &= 0 \\ C_y &= -10 \text{ kips} \quad C_y = 10 \text{ kips} \downarrow \\ C &= 10 \text{ kips} \downarrow \end{aligned}$$

6.83



FIND: COMPONENTS OF REACTIONS AT A AND E IF A 750 N  $\downarrow$  FORCE IS APPLIED (a) AT B, (b) AT D.

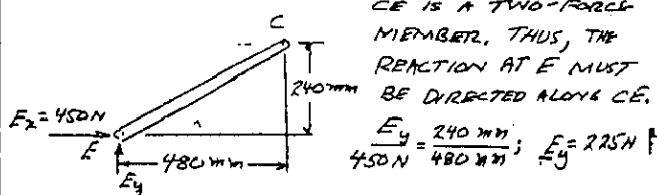


FREE-BODY: ENTIRE FRAME

THE FOLLOWING ANALYSIS IS VALID FOR BOTH (a) AND (b) SINCE POSITION OF LOAD ON ITS LINE OF ACTION IS INVARIABLE.

$$\begin{aligned} \sum M_E = 0: & -(750\text{ N})(240\text{ mm}) - A_x(400\text{ mm}) = 0 \\ A_x = -950\text{ N} & \quad A_x = 450\text{ N} \leftarrow \\ \sum F_x = 0: & E_x - 450\text{ N} = 0; \quad E_x = 450\text{ N} \rightarrow \\ \sum F_y = 0: & A_y + E_y - 750\text{ N} = 0 \end{aligned} \quad (1)$$

(a) LOAD APPLIED AT B. FREE BODY: MEMBER CE

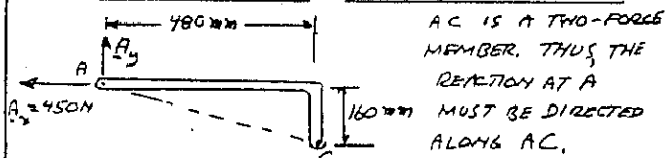


$$\text{FROM EQ. (1): } A_y + 225 - 750 = 0; \quad A_y = 525\text{ N} \uparrow$$

THUS, REACTIONS ARE:

$$\begin{aligned} A_x = 450\text{ N} \leftarrow, \quad A_y = 525\text{ N} \uparrow \\ E_x = 450\text{ N} \rightarrow, \quad E_y = 225\text{ N} \uparrow \end{aligned}$$

(b) LOAD APPLIED AT D. FREE BODY: MEMBER AC



$$\frac{A_y}{450\text{ N}} = \frac{160\text{ mm}}{480\text{ mm}}$$

$$A_y = 150\text{ N} \uparrow$$

$$\text{FROM EQ. (1): } A_y + E_y - 750\text{ N} = 0$$

$$150\text{ N} + E_y - 750\text{ N} = 0$$

$$E_y = 600\text{ N}$$

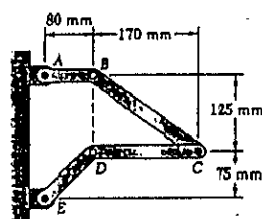
$$E_y = 600\text{ N} \uparrow$$

THUS, REACTIONS ARE:

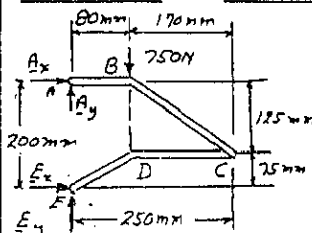
$$A_x = 450\text{ N} \leftarrow, \quad A_y = 150\text{ N} \uparrow$$

$$E_x = 450\text{ N} \rightarrow, \quad E_y = 600\text{ N} \uparrow$$

6.84



FIND: COMPONENTS OF REACTIONS AT A AND E IF A 750 N  $\downarrow$  FORCE IS APPLIED (a) AT B, (b) AT D.

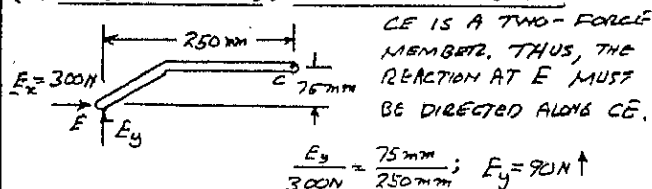


FREE BODY: ENTIRE FRAME

THE FOLLOWING ANALYSIS IS VALID FOR BOTH (a) AND (b) SINCE POSITION OF LOAD ON ITS LINE OF ACTION IS INVARIABLE.

$$\begin{aligned} \sum M_E = 0: & -(750\text{ N})(60\text{ mm}) - A_x(200\text{ mm}) = 0 \\ A_x = -300\text{ N} & \quad A_x = 300\text{ N} \leftarrow \\ \sum F_x = 0: & E_x - 300\text{ N} = 0; \quad E_x = 300\text{ N} \rightarrow \\ \sum F_y = 0: & A_y + E_y - 750\text{ N} = 0 \end{aligned} \quad (1)$$

(a) LOAD APPLIED AT B. FREE BODY: MEMBER CE



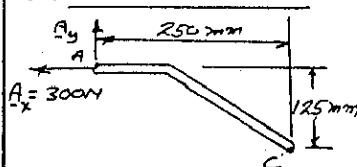
$$\frac{E_y}{300\text{ N}} = \frac{75\text{ mm}}{250\text{ mm}}; \quad E_y = 90\text{ N} \uparrow$$

$$\text{FROM EQ. (1): } A_y + 90\text{ N} - 750\text{ N} = 0; \quad A_y = 660\text{ N} \uparrow$$

THUS REACTIONS ARE:

$$\begin{aligned} A_x = 300\text{ N} \leftarrow, \quad A_y = 660\text{ N} \uparrow \\ E_x = 300\text{ N} \rightarrow, \quad E_y = 90\text{ N} \uparrow \end{aligned}$$

(b) LOAD APPLIED AT D. FREE BODY: MEMBER AC



AC IS A TWO-FORCE MEMBER. THUS, THE REACTION AT A MUST BE DIRECTED ALONG AC.

$$\frac{A_y}{300\text{ N}} = \frac{125\text{ mm}}{250\text{ mm}}$$

$$A_y = 150\text{ N} \uparrow$$

$$\text{FROM EQ. (1): } A_y + E_y - 750\text{ N} = 0$$

$$150\text{ N} + E_y - 750\text{ N} = 0$$

$$E_y = 600\text{ N}$$

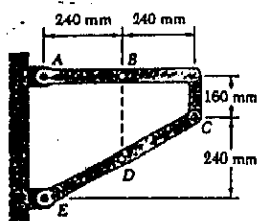
$$E_y = 600\text{ N} \uparrow$$

THUS, REACTIONS ARE:

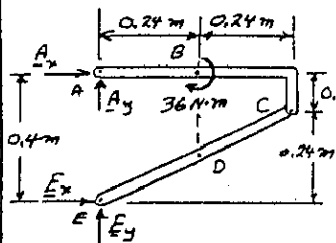
$$A_x = 300\text{ N} \leftarrow, \quad A_y = 150\text{ N} \uparrow$$

$$E_x = 300\text{ N} \rightarrow, \quad E_y = 600\text{ N} \uparrow$$

6.85



FIND: COMPONENTS OF REACTIONS AT A AND E IF A  $36 \text{ N}\cdot\text{m}$  COUPLE IS APPLIED (a) AT B, (b) AT D.

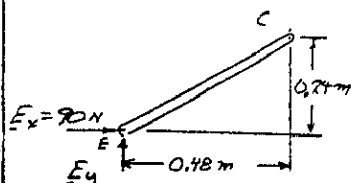


FREE BODY: ENTIRE FRAME

THE FOLLOWING ANALYSIS IS VALID FOR BOTH (a) AND (b) SINCE THE POINT OF APPLICATION OF THE COUPLE IS IMMATRIAL.

$$\begin{aligned} \uparrow \Sigma M_E = 0: & -36 \text{ N}\cdot\text{m} - A_x(0.4 \text{ m}) = 0 \\ & A_x = -90 \text{ N} \quad A_x = 90 \text{ N} \leftarrow \\ \rightarrow \Sigma F_x = 0: & -90 + E_x = 0 \\ & E_x = 90 \text{ N} \quad E_x = 90 \text{ N} \rightarrow \\ \uparrow \Sigma F_y = 0: & A_y + E_y = 0 \end{aligned} \quad (1)$$

(a) COUPLE APPLIED AT B. FREE BODY: MEMBER CE



AC IS A TWO-FORCE MEMBER. THUS, THE REACTION AT E MUST BE DIRECTED ALONG EC.

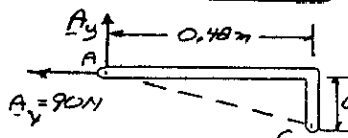
$$\frac{E_y}{90 \text{ N}} = \frac{0.24 \text{ m}}{0.48 \text{ m}}; \quad E_y = 45 \text{ N} \uparrow$$

$$\text{FROM EQ(1): } A_y + 45 \text{ N} = 0 \\ A_y = -45 \text{ N} \quad A_y = 45 \text{ N} \downarrow$$

THUS, REACTIONS ARE

$$A_x = 90 \text{ N} \leftarrow, A_y = 45 \text{ N} \downarrow \\ E_x = 90 \text{ N} \rightarrow, E_y = 45 \text{ N} \uparrow$$

(b) COUPLE APPLIED AT D. FREE BODY: MEMBER AC



AC IS A TWO-FORCE MEMBER. THUS, THE REACTION AT A MUST BE DIRECTED ALONG AC.

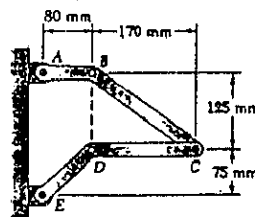
$$\frac{A_y}{90 \text{ N}} = \frac{0.16 \text{ m}}{0.48 \text{ m}}; \quad A_y = 30 \text{ N} \uparrow$$

$$\text{FROM EQ(1): } A_y + E_y = 0 \\ 30 \text{ N} + E_y = 0 \\ E_y = -30 \text{ N} \quad E_y = 30 \text{ N} \downarrow$$

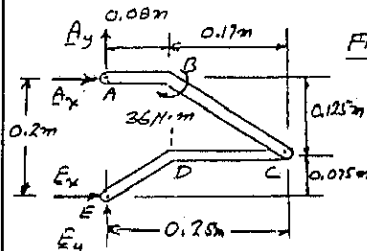
THUS, REACTIONS ARE:

$$A_x = 90 \text{ N} \leftarrow, A_y = 30 \text{ N} \uparrow \\ E_x = 90 \text{ N} \rightarrow, E_y = 30 \text{ N} \downarrow$$

6.86



FIND: COMPONENTS OF REACTIONS AT A AND E IF A  $36 \text{ N}\cdot\text{m}$  COUPLE IS APPLIED (a) AT B, (b) AT D.

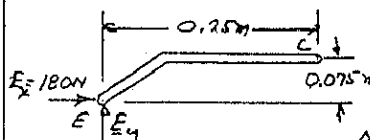


FREE BODY: ENTIRE FRAME

THE FOLLOWING ANALYSIS IS VALID FOR BOTH (a) AND (b) SINCE THE POINT OF APPLICATION OF THE COUPLE IS IMMATRIAL.

$$\begin{aligned} \uparrow \Sigma M_E = 0: & -36 \text{ N}\cdot\text{m} - A_x(0.2 \text{ m}) = 0 \\ & A_x = -180 \text{ N} \quad A_x = 180 \text{ N} \leftarrow \\ \rightarrow \Sigma F_x = 0: & -180 + E_x = 0 \\ & E_x = 180 \text{ N} \quad E_x = 180 \text{ N} \rightarrow \\ \uparrow \Sigma F_y = 0: & A_y + E_y = 0 \end{aligned} \quad (1)$$

(a) COUPLE APPLIED AT B. FREE BODY: MEMBER CE



AC IS A TWO-FORCE MEMBER. THUS, THE REACTION AT E MUST BE DIRECTED ALONG EC.

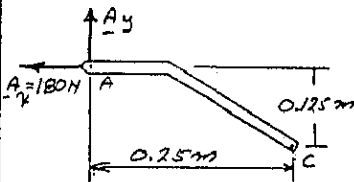
$$\frac{E_y}{180 \text{ N}} = \frac{0.075 \text{ m}}{0.25 \text{ m}} \quad E_y = 54 \text{ N} \uparrow$$

$$\text{FROM EQ(1): } A_y + 54 \text{ N} = 0 \\ A_y = -54 \text{ N} \quad A_y = 54 \text{ N} \downarrow$$

THUS, REACTIONS ARE

$$A_x = 180 \text{ N} \leftarrow, A_y = 54 \text{ N} \downarrow \\ E_x = 180 \text{ N} \rightarrow, E_y = 54 \text{ N} \uparrow$$

(b) COUPLE APPLIED AT D. FREE BODY: MEMBER AC



AC IS A TWO-FORCE MEMBER. THUS, THE REACTION AT A MUST BE DIRECTED ALONG AC.

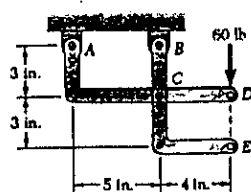
$$\frac{A_y}{180 \text{ N}} = \frac{0.125 \text{ m}}{0.25 \text{ m}} \quad A_y = 90 \text{ N} \uparrow$$

$$\text{FROM EQ(1): } A_y + E_y = 0 \\ 90 \text{ N} + E_y = 0 \\ E_y = -90 \text{ N} \quad E_y = 90 \text{ N} \downarrow$$

THUS, REACTIONS ARE

$$A_x = 180 \text{ N} \leftarrow, A_y = 90 \text{ N} \uparrow \\ E_x = 180 \text{ N} \rightarrow, E_y = 90 \text{ N} \downarrow$$

6.87



FIND: THE COMPONENTS OF THE REACTIONS AT A AND B. WHEN THE 60-LB LOAD IS (a) APPLIED AT D, (b) APPLIED AT E.

FREE BODY:

ENTIRE FRAME

THE FOLLOWING ANALYSIS IS VALID FOR BOTH (a) AND (b) SINCE THE POSITION

OF THE LOAD ALONG ITS LINE OF ACTION IS IMMATERIAL.

$$\uparrow \Sigma M_B = 0: -A_y(5\text{ in.}) - (60\text{ lb})(4\text{ in.}) = 0$$

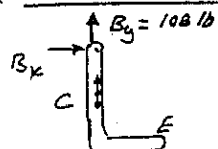
$$A_y = -48\text{ lb} \quad A_y = 48\text{ lb} \downarrow$$

$$\uparrow \Sigma F_y = 0: -60\text{ lb} + B_y - 48\text{ lb} = 0$$

$$B_y = 108\text{ lb} \quad B_y = 108\text{ lb} \uparrow$$

$$\rightarrow \Sigma F_x = 0: A_x + B_x = 0$$

(a) LOAD APPLIED AT D. FREE BODY: MEMBER BCE



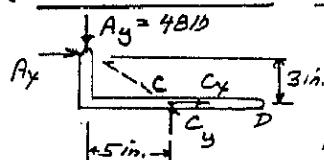
BCE IS A TWO-FORCE MEMBER. THUS, REACTION AT B IS:  $B = 108\text{ lb} \uparrow$  AND  $B_x = 0$

$$\text{FROM EQ(1): } A_x + B_x = 0; A_x + 0 = 0 \quad A_x = 0$$

THUS, REACTIONS ARE

$$A = 48\text{ lb} \downarrow, B = 108\text{ lb} \uparrow$$

(b) LOAD APPLIED AT E. FREE BODY: MEMBER ACD



ACD IS A TWO-FORCE MEMBER. THUS, THE REACTION AT A MUST BE DIRECTED ALONG AC

$$\frac{48\text{ lb}}{A_x} = \frac{3\text{ in.}}{5\text{ in.}} \quad A_x = 80\text{ lb} \rightarrow$$

$$\text{FROM EQ(1): } A_x + B_x = 0$$

$$80\text{ lb} + B_x = 0$$

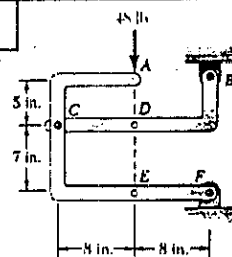
$$B_x = -80\text{ lb} \quad B_x = 80\text{ lb} \leftarrow$$

THUS, REACTIONS ARE:

$$A_x = 80\text{ lb} \rightarrow, A_y = 48\text{ lb} \downarrow$$

$$B_x = 80\text{ lb} \leftarrow, B_y = 108\text{ lb} \uparrow$$

6.88



FIND: THE COMPONENTS OF THE REACTIONS AT B AND F WHEN THE 48-LB LOAD IS APPLIED (a) AT A, (b) AT D, (c) AT E.

FREE BODY:

ENTIRE FRAME

THE FOLLOWING ANALYSIS IS VALID FOR (a), (b), AND (c), SINCE THE POSITION OF THE LOAD ALONG ITS LINE OF ACTION IS IMMATERIAL

$$\uparrow \Sigma M_F = 0: (48\text{ lb})(8\text{ in.}) - B_x(12\text{ in.}) = 0$$

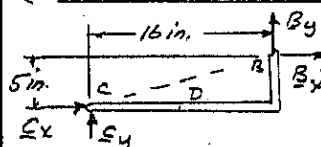
$$B_x = 32\text{ lb} \quad B_x = 32\text{ lb} \rightarrow$$

$$\rightarrow \Sigma F_x = 0: 32\text{ lb} + F_x = 0$$

$$F_x = -32\text{ lb} \quad F_x = 32\text{ lb} \leftarrow$$

$$\uparrow \Sigma F_y = 0: B_y + F_y - 48\text{ lb} = 0$$

(a) LOAD APPLIED AT A. FREE BODY: MEMBER CDB



CDB IS A TWO-FORCE MEMBER. THUS, THE REACTION AT B MUST BE DIRECTED ALONG BC.

$$\frac{B_y}{32\text{ lb}} = \frac{5\text{ in.}}{16\text{ in.}} \quad B_y = 10\text{ lb} \uparrow$$

$$\text{FROM EQ(1): } 10\text{ lb} + F_y - 48\text{ lb} = 0$$

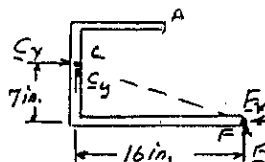
$$F_y = 38\text{ lb} \quad F_y = 38\text{ lb} \uparrow$$

THUS, REACTIONS ARE:

$$B_x = 32\text{ lb} \rightarrow, B_y = 10\text{ lb} \uparrow$$

$$F_x = 32\text{ lb} \leftarrow, F_y = 38\text{ lb} \uparrow$$

(b) LOAD APPLIED AT D. FREE BODY: MEMBER ACF



ACF IS A TWO-FORCE MEMBER. THUS, THE REACTION AT F MUST BE DIRECTED ALONG CF.

$$\frac{F_y}{32\text{ lb}} = \frac{7\text{ in.}}{16\text{ in.}} \quad F_y = 14\text{ lb} \uparrow$$

$$\text{FROM EQ(1): } B_y + 14\text{ lb} - 48\text{ lb} = 0$$

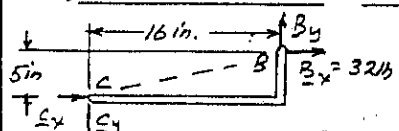
$$B_y = 34\text{ lb} \quad B_y = 34\text{ lb} \uparrow$$

THUS REACTIONS ARE:

$$B_x = 32\text{ lb} \leftarrow, B_y = 34\text{ lb} \uparrow$$

$$F_x = 32\text{ lb} \rightarrow, F_y = 14\text{ lb} \uparrow$$

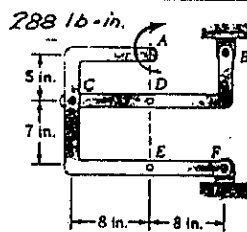
(c) LOAD APPLIED AT E. FREE BODY: MEMBER CDB



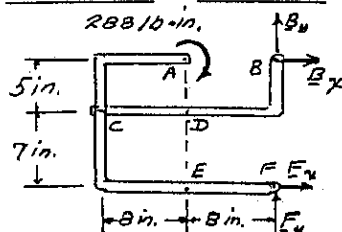
THIS IS THE SAME FREE BODY AS IN PART (a).

REACTIONS ARE SAME AS (a)

6.89



FIND: THE COMPONENTS OF THE REACTIONS AT B AND F WHEN THE 288-lb-in. COUPLE IS APPLIED (a) AT A, (b) AT D, (c) AT E.



FREE BODY:

ENTIRE FRAME

THE FOLLOWING ANALYSIS IS VALID FOR (a), (b), AND (c), SINCE THE POINT OF APPLICATION OF THE COUPLE IS IMMATERIAL.

$$+\circlearrowleft \Sigma M_F = 0: -288 \text{ lb-in.} - B_x(12 \text{ in.}) = 0$$

$$B_x = -241b \quad B_x = 241b \leftarrow$$

$$+\rightarrow \Sigma F_x = 0: -241b + F_x = 0$$

$$F_x = 241b$$

$$F_x = 241b \rightarrow$$

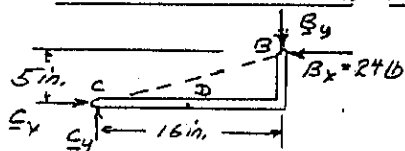
$$+\uparrow \Sigma F_y = 0: B_y + F_y = 0$$

(1)

(a) COUPLE APPLIED AT A.

FREE BODY: MEMBER CDB

CDB IS A TWO-FORCE MEMBER, THUS REACTION AT B MUST BE DIRECTED ALONG BC.



$$\frac{B_y}{241b} = \frac{5 \text{ in.}}{16 \text{ in.}}$$

$$B_y = 7.51b \downarrow$$

$$\text{FROM EQ. (1): } -7.51b + F_y = 0$$

$$F_y = 7.51b$$

$$F_y = 7.51b \uparrow$$

THUS, REACTIONS ARE:

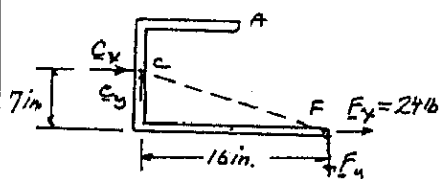
$$B_x = 241b \leftarrow, B_y = 7.51b \downarrow$$

$$F_x = 241b \rightarrow, F_y = 7.51b \uparrow$$

(b) COUPLE APPLIED AT D.

FREE BODY: MEMBER ACF

ACF IS A TWO-FORCE MEMBER, THUS, THE REACTION AT F MUST BE DIRECTED ALONG CF.



$$\frac{F_y}{241b} = \frac{7 \text{ in.}}{16 \text{ in.}}$$

$$F_y = 10.51b \downarrow$$

$$\text{FROM EQ. (1): } B_y - 10.51b = 0$$

$$B_y = +10.51b$$

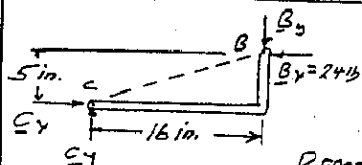
$$B_y = 10.51b \uparrow$$

THUS, REACTIONS ARE:

$$B_x = 241b \leftarrow, B_y = 10.51b \uparrow$$

$$F_x = 241b \rightarrow, F_y = 10.51b \downarrow$$

(c) COUPLE APPLIED AT E. FREE BODY: MEMBER CDB.

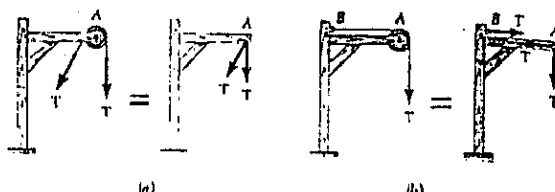


THIS IS THE SAME FREE BODY AS IN PART (a).

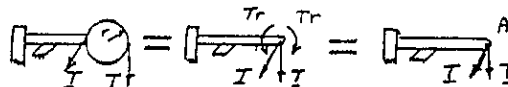
REACTIONS ARE SAME AS IN (a)

6.90

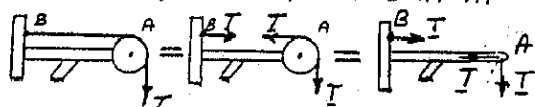
SHOW THAT THE LOADINGS SHOWN ARE EQUIVALENT IN (a) AND THEN IN (b).



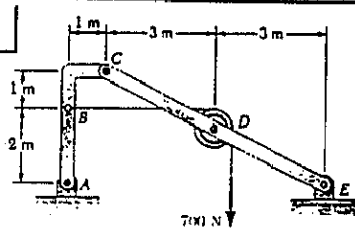
(a) REPLACE EACH FORCE BY A FORCE-COUPLE SYSTEM



(b) CUT CABLE AS SHOWN AND REPLACE FORCES ON PULLEY BY EQUIVALENT FORCES AT A.



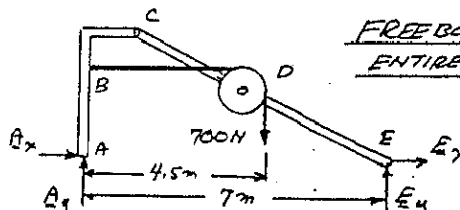
6.91



GIVEN: RADIUS OF PULLEY = 0.5 m. FIND: COMPONENTS OF REACTIONS AT A AND E.

FREE BODY:

ENTIRE ASSEMBLY



$$+\circlearrowleft \Sigma M_A = 0: E_y(7 \text{ m}) - (700 \text{ N})(4.5 \text{ m}) = 0$$

$$E_y = 450 \text{ N}$$

$$E_y = 450 \text{ N} \uparrow$$

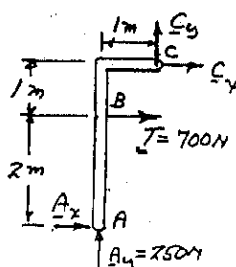
$$+\uparrow \Sigma F_y = 0: A_y + 450 \text{ N} - 700 \text{ N} = 0$$

$$A_y = 250 \text{ N}$$

$$A_y = 250 \text{ N} \uparrow$$

$$+\rightarrow \Sigma F_x = 0: A_x + E_x = 0$$

(1)



FREE BODY:

MEMBER ABC

$$+\circlearrowleft \Sigma M_C = 0:$$

$$(700 \text{ N})(1 \text{ m}) + A_x(3 \text{ m})$$

$$- (250 \text{ N})(1 \text{ m}) = 0$$

$$A_x = -150 \text{ N}$$

$$A_x = 150 \text{ N} \leftarrow$$

FROM EQ. (1):

$$A_x + E_x = 0$$

$$-150 \text{ N} + E_x = 0$$

$$E_x = 150 \text{ N}$$

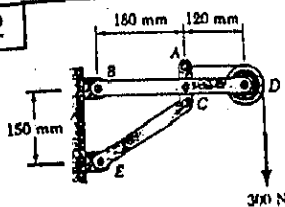
$$E_x = 150 \text{ N} \rightarrow$$

THUS, REACTIONS ARE:

$$A_x = 150 \text{ N} \leftarrow, A_y = 250 \text{ N} \uparrow$$

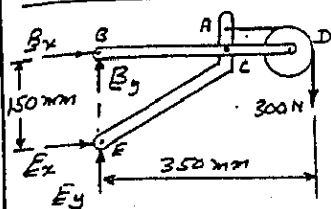
$$E_x = 150 \text{ N} \rightarrow, E_y = 450 \text{ N} \uparrow$$

6.92



GIVEN: RADIUS OF PULLEY = 50 mm.

FIND: COMPONENTS OF REACTIONS AT B AND E.



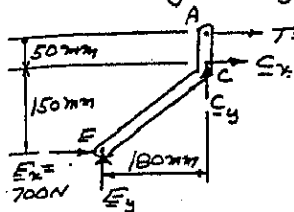
FREE BODY: ENTIRE ASSEMBLY

$$+\uparrow \Sigma M_E = 0: -(300 \text{ N})(350 \text{ mm}) - B_x(150 \text{ mm}) = 0$$

$$B_x = -700 \text{ N} \quad B_x = 700 \text{ N} \leftarrow$$

$$+\Sigma F_x = 0: -700 \text{ N} + E_x = 0 \quad E_x = 700 \text{ N} \quad E_x = 700 \text{ N} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: B_y + E_y - 300 \text{ N} = 0 \quad (1)$$



FREE BODY: MEMBER ACE

$$+\uparrow \Sigma M_C = 0: (700 \text{ N})(150 \text{ mm}) - (300 \text{ N})(50 \text{ mm}) - E_y(180 \text{ mm}) = 0$$

$$E_y = 500 \text{ N} \quad E_y = 500 \text{ N} \uparrow$$

FROM EQ. (1):  $B_y + 500 \text{ N} - 300 \text{ N} = 0$   
 $B_y = -200 \text{ N} \quad B_y = 200 \text{ N} \uparrow$

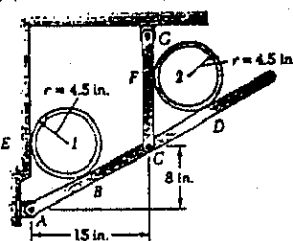
THUS, REACTIONS ARE:

$$B_x = 700 \text{ N} \leftarrow, \quad B_y = 200 \text{ N} \uparrow$$

$$E_x = 700 \text{ N} \rightarrow, \quad E_y = 500 \text{ N} \uparrow$$

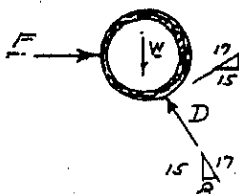
6.93 and 6.94

GIVEN: PIPES WEIGH  $30 \text{ lb/ft}$   
 FRAMES SPACED AT 7.5 ft.  
 FIND: COMPONENTS OF REACTIONS AT A AND G.



FREE BODY: PIPE 2.

$$W = (30 \text{ lb/ft})(7.5 \text{ ft}) = 225 \text{ lb}$$



$$\frac{W}{F} = \frac{D}{B} = \frac{225 \text{ lb}}{15} = \frac{F}{17}$$

$$F = 120 \text{ lb} \rightarrow$$

$$D = 225 \text{ lb} \uparrow$$

GEOMETRY OF PIPE 2

$$r = 4.5 \text{ in.}$$

BY SYMMETRY:  $CF = CD \quad (1)$

EQUATE HORIZONTAL DISTANCES

$$r + \frac{8}{17}r = CD \left( \frac{15}{17} \right)$$

$$\frac{25}{17}r = CD \left( \frac{15}{17} \right)$$

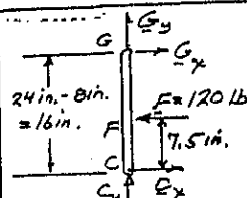
$$CD = \frac{25}{15}r = \frac{5}{3}r$$

FROM EQ. (1)  $CF = \frac{5}{3}r = \frac{5}{3}(4.5 \text{ in.})$

$$CF = 7.5 \text{ in.}$$

(CONTINUED)

6.93 and 6.94 CONTINUED



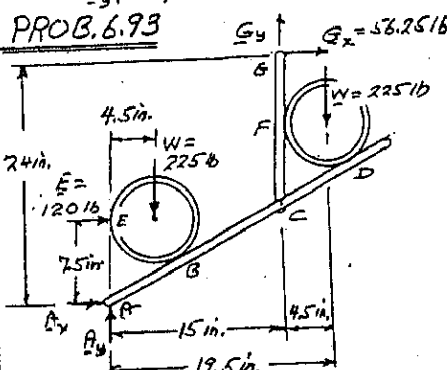
FREE BODY: MEMBER CFG

$$+\uparrow \Sigma M_C = 0$$

$$(120 \text{ lb})(7.5 \text{ in.}) - G_x(16 \text{ in.}) = 0$$

$$G_x = 56.25 \text{ lb} \quad G_x = 56.3 \text{ lb} \rightarrow$$

PROB. 6.93



FREE BODY: FRAME AND PIPES

NOTE: PIPE 2 IS SIMILAR TO PIPE 1.  
 $AE = CF = 7.5 \text{ in.}$   
 $E = F = 120 \text{ lb}$

$$+\uparrow \Sigma M_A = 0: G_y(15 \text{ in.}) - (56.25 \text{ lb})(24 \text{ in.}) - (225 \text{ lb})(4.5 \text{ in.}) - (225 \text{ lb})(19.5 \text{ in.}) - (120 \text{ lb})(7.5 \text{ in.}) = 0$$

$$G_y = 570 \text{ lb} \quad G_y = 570 \text{ lb} \uparrow$$

$$+\Sigma F_x = 0: A_x + 120 \text{ lb} + 56.25 \text{ lb} = 0$$

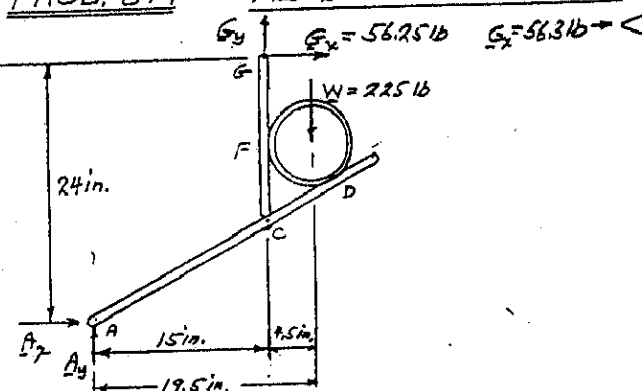
$$A_x = -176.25 \text{ lb} \quad A_x = 176.3 \text{ lb} \leftarrow$$

$$+\uparrow \Sigma F_y = 0: A_y + 570 \text{ lb} - 225 \text{ lb} - 225 \text{ lb} = 0$$

$$A_y = -60 \text{ lb} \quad A_y = 60 \text{ lb} \downarrow$$

PROB. 6.94

FREE BODY: FRAME AND PIPE 2



$$+\uparrow \Sigma M_A = 0:$$

$$G_y(15 \text{ in.}) - (56.25 \text{ lb})(24 \text{ in.}) - (225 \text{ lb})(19.5 \text{ in.}) = 0$$

$$G_y = 382.5 \text{ lb} \quad G_y = 383 \text{ lb} \uparrow$$

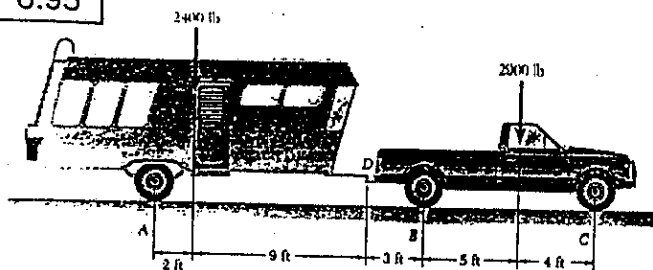
$$+\Sigma F_x = 0: A_x + 56.25 \text{ lb}$$

$$A_x = -56.25 \text{ lb} \quad A_x = 56.3 \text{ lb} \leftarrow$$

$$+\uparrow \Sigma F_y = 0: A_y + 382.5 \text{ lb} - 225 \text{ lb} = 0$$

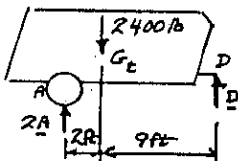
$$A_y = -157.5 \text{ lb} \quad A_y = 157.5 \text{ lb} \downarrow$$

6.95



FIND: (a) REACTIONS AT EACH OF THE SIX WHEELS.  
(b) ADDITIONAL LOAD ON EACH WHEEL DUE TO THE TRAILER

(a)



FREE BODY: TRAILER  
(WE SHALL DENOTE BY A, B, C THE REACTION AT ONE WHEEL)

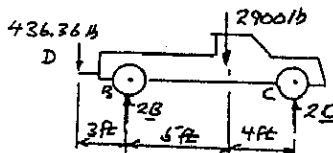
$$+\sum M_A = 0: -(2400 \text{ lb})(2 \text{ ft}) + D(11 \text{ ft}) = 0$$

$$D = 436.36 \text{ lb}$$

$$+\sum F_y = 0: 2A - 2400 \text{ lb} + 436.36 \text{ lb} = 0$$

$$A = 981.82 \text{ lb}$$

$$A = 982 \text{ lb} \uparrow$$



FREE BODY: TRUCK

$$+\sum M_B = 0:$$

$$(436.36 \text{ lb})(3 \text{ ft}) - (2900 \text{ lb})(5 \text{ ft}) + 2C(9 \text{ ft}) = 0$$

$$C = 732.83 \text{ lb} \quad C = 733 \text{ lb} \uparrow$$

$$+\sum F_y = 0: 2B - 436.36 \text{ lb} - 2900 \text{ lb} + 2(732.83 \text{ lb}) = 0$$

$$B = 935.35 \text{ lb}$$

$$B = 935 \text{ lb} \uparrow$$

(b) ADDITIONAL LOAD ON TRUCK WHEELS

USE FREE BODY DIAGRAM OF TRUCK WITHOUT 2900 lb.

$$+\sum M_B = 0: (436.36 \text{ lb})(3 \text{ ft}) + 2C(9 \text{ ft}) = 0$$

$$C = -72.73 \text{ lb}$$

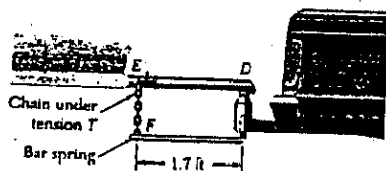
$$\Delta C = -72.7 \text{ lb}$$

$$+\sum F_y = 0: 2B - 436.36 \text{ lb} - 2(-72.73 \text{ lb}) = 0$$

$$B = 290.9 \text{ lb}$$

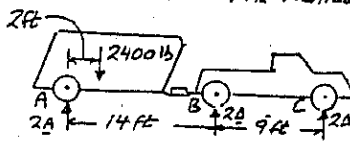
$$\Delta B = +291 \text{ lb}$$

6.96



FIND: (a) TENSION IN EACH CHAIN FOR EQUAL ADDITIONAL LOAD ON TRUCK WHEELS. (b) REACTION AT EACH WHEEL.

(a) WE SHALL FIRST FIND THE ADDITIONAL REACTION "D" AT EACH WHEEL DUE TO THE TRAILER. FREE BODY DIAGRAM



(SAME Δ AT EACH TRUCK WHEEL)

$$+\sum M_A = 0: -(2400 \text{ lb})(2 \text{ ft})$$

$$+ 2\Delta(14 \text{ ft}) + 2\Delta(23 \text{ ft}) = 0$$

$$\Delta = 64.86 \text{ lb}$$

$$+\sum F_y = 0: 2A - 2400 \text{ lb} + 4(64.86 \text{ lb}) = 0; A = 1070 \text{ lb}; A = 1077 \text{ lb} \uparrow$$

(CONTINUED)

6.96 CONTINUED

FREE BODY: TRUCK

(TRAILER LOADING ONLY)

$$+\sum M_D = 0$$

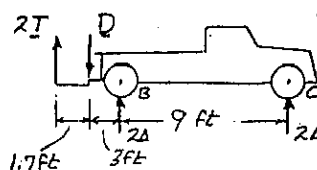
$$2\Delta(12 \text{ ft}) + 2\Delta(3 \text{ ft}) - 2T(17 \text{ ft})$$

$$T = 8.824 \Delta$$

$$= 8.824(64.86 \text{ lb})$$

$$T = 572.3 \text{ lb}$$

$$T = 572 \text{ lb}$$



FREE BODY: TRUCK

(TRUCK WEIGHT ONLY)

$$+\sum M_B = 0:$$

$$-(2900 \text{ lb})(5 \text{ ft}) + 2C(9 \text{ ft}) = 0$$

$$C = 805.6 \text{ lb} \quad C = 806 \text{ lb} \uparrow$$

$$+\sum F_y = 0: 2B - 2900 \text{ lb} + 2(805.6 \text{ lb}) = 0$$

$$B = 644.4 \text{ lb}$$

$$B = 644.4 \text{ lb} \uparrow$$

ACTUAL REACTIONS

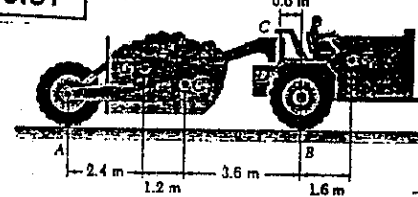
$$B = B' + \Delta = 644.4 \text{ lb} + 64.86 \text{ lb} = 709.2 \text{ lb} \quad B = 709 \text{ lb} \uparrow$$

$$C = C' + \Delta = 805.6 \text{ lb} + 64.86 \text{ lb} = 870.4 \text{ lb} \quad C = 870 \text{ lb} \uparrow$$

$$(FROM PART a): \quad A = 1077 \text{ lb} \uparrow$$

6.97

$$CD = 0.15 \text{ m}$$



GIVEN:  $m_1 = 45 \text{ kg}$

$$m_2 = 8 \text{ kg}$$

$$m_3 = 10 \text{ kg}$$

FIND: (a) REACTIONS

AT EACH OF 4 WHEELS

(b) FORCES ACTING ON

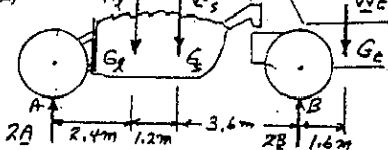
TRACTOR AT C AND D.

$$W_1 = m_1 g = (45 \text{ kg})(9.81 \text{ m/s}^2) = 441.45 \text{ N}$$

$$W_2 = m_2 g = (8 \text{ kg})(9.81 \text{ m/s}^2) = 78.48 \text{ N}$$

$$W_3 = m_3 g = (10 \text{ kg})(9.81 \text{ m/s}^2) = 98.1 \text{ N}$$

(a)



FREE BODY:

ENTIRE MACHINE

$$+\sum M_A = 0: 2B(2.2 \text{ m}) - W_1(2.4 \text{ m}) - W_2(3.6 \text{ m}) - W_3(3.8 \text{ m}) = 0$$

$$2B(2.2 \text{ m}) - (441.45 \text{ N})(2.4 \text{ m}) - (78.48 \text{ N})(3.6 \text{ m}) - (98.1 \text{ N})(3.8 \text{ m}) = 0$$

$$B = 153.14 \text{ N}$$

$$B = 153.1 \text{ N} \uparrow$$

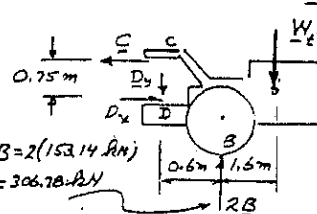
$$+\sum F_y = 0: 2A + 2(153.14 \text{ N}) - 441.45 \text{ N} - 78.48 \text{ N} - 98.1 \text{ N} = 0$$

$$A = 155.87 \text{ N}$$

$$A = 155.9 \text{ N} \uparrow$$

(b)

FREE BODY: TRACTOR



$$2B = 2(153.14 \text{ N})$$

$$= 306.28 \text{ N}$$

$$+\sum M_D = 0: C(0.75 \text{ m}) + (306.28 \text{ N})(0.6 \text{ m}) - (98.1 \text{ N})(2.2 \text{ m}) = 0$$

$$C = 42.74 \text{ N} \quad C = 42.7 \text{ N} \uparrow$$

$$+\sum F_x = 0: -42.74 \text{ N} + D_x = 0; \quad D_x = 42.74 \text{ N} \rightarrow$$

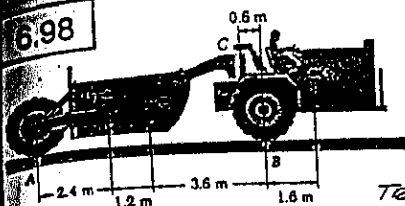
$$+\sum F_y = 0: 306.28 \text{ N} - 98.1 \text{ N} - D_y = 0; \quad D_y = 208.68 \text{ N} \uparrow$$

$$208.68 \text{ N}$$

$$D = 213 \text{ N} \angle 78.4^\circ$$



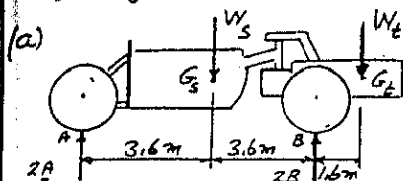
6.98



GIVEN:  $m_s = 8 \text{ Mg}$   
 $m_L = 10 \text{ Mg}$   
 (LOAD REMOVED)  
 FIND: (a) REACTIONS AT EACH OF 4 WHEELS.  
 (b) FORCES ACTING ON TRACTOR AT C AND D.

$$W_s = m_s g = (8 \text{ Mg})(9.81 \text{ m/s}^2) = 78.48 \text{ kN}$$

$$W_L = m_L g = (10 \text{ Mg})(9.81 \text{ m/s}^2) = 98.1 \text{ kN}$$



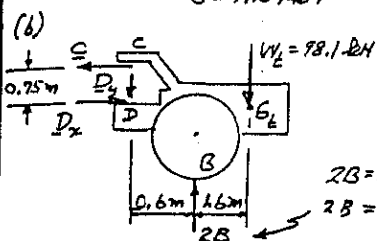
FREE BODY:  
ENTIRE MACHINE

$$+\circlearrowleft \Sigma M_A = 0: -2A(7.2\text{m}) + (78.48\text{kN})(3.6\text{m}) - (98.1\text{kN})(1.6\text{m}) = 0$$

$$A = B = 77.2 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: 2(77.2\text{kN}) + 2B - 78.48\text{kN} - 98.1\text{kN} = 0$$

$$B = 79.57 \text{ kN} \quad C = 79.6 \text{ kN}$$



FREE BODY:  
TRACTOR

$$+\circlearrowleft \Sigma M_D = 0: C(0.75\text{m}) + (159.14\text{kN})(0.6\text{m}) - (98.1\text{kN})(2.2\text{m}) = 0$$

$$C = 160.4 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: 159.14\text{kN} - 98.1\text{kN} - D_y = 0$$

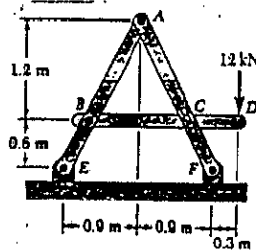
$$D_y = 61.04 \text{ kN}$$

$$+\rightarrow \Sigma F_x = 0: -160.4\text{kN} + D_x = 0$$

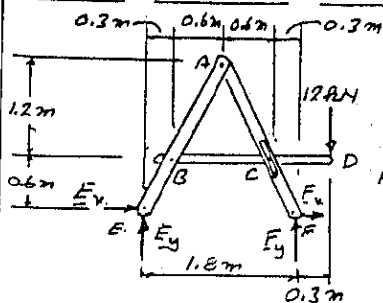
$$D_x = 160.4 \text{ kN}$$

$$D = 171.6 \text{ kN} \angle 20.8^\circ$$

6.99



FIND: COMPONENTS OF ALL FORCES ACTING ON MEMBER ABE.



FREE BODY:  
ENTIRE FRAME

$$+\circlearrowleft \Sigma M_E = 0: F_y(1.8\text{m}) - (12\text{kN})(2.1\text{m}) = 0$$

$$F_y = 14 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: E_y + 14\text{kN} - 12\text{kN} = 0$$

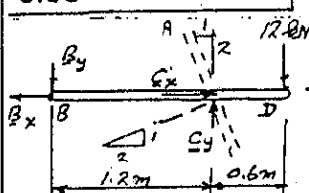
$$E_y = -2 \text{ kN}$$

$$F_y = 14 \text{ kN}$$

(CONTINUED)

6.99 CONTINUED

FREE BODY: MEMBER BCD



$$+\circlearrowleft \Sigma M_B = 0: C_y(1.2\text{m}) - (12\text{kN})(1.8\text{m}) = 0$$

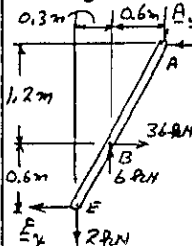
$$C_y = 18 \text{ kN}$$

$$\frac{C_y}{C_x} = \frac{1}{2}; \frac{18\text{kN}}{C_x} = \frac{1}{2}; C_x = 36 \text{ kN}$$

$$+\rightarrow \Sigma F_x = 0: -B_x + 36\text{kN} = 0 \quad B_x = 36 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: -B_y + 18\text{kN} - 12\text{kN} = 0 \quad B_y = 6 \text{ kN}$$

FREE BODY: MEMBER ABE



$$+\circlearrowleft \Sigma M_A = 0: -E_x(1.8\text{m}) + (2\text{kN})(0.7\text{m}) + (36\text{kN})(1.2\text{m}) - (6\text{kN})(0.6\text{m}) = 0$$

$$E_x = 23.2 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: -A_y + 6\text{kN} - 2\text{kN} = 0 \quad A_y = 4 \text{ kN}$$

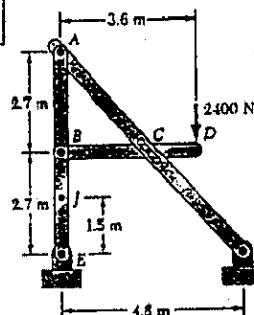
$$+\rightarrow \Sigma F_x = 0: -A_x + 36\text{kN} - 23.2\text{kN} = 0 \quad A_x = 13 \text{ kN}$$

FORCES ACTING ON ABE:

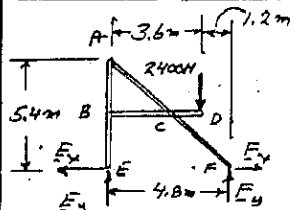
$$A_x = 13 \text{ kN} \rightarrow, A_y = 4 \text{ kN} \uparrow; B_x = 36 \text{ kN} \rightarrow, B_y = 6 \text{ kN} \uparrow$$

$$E_x = 13 \text{ kN} \leftarrow, E_y = 2 \text{ kN} \downarrow$$

6.100



FIND: COMPONENTS OF ALL FORCES ACTING ON MEMBER ABE.

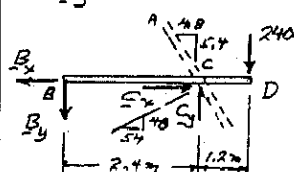


FREE BODY: ENTIRE FRAME

$$+\circlearrowleft \Sigma M_A = 0: (2400\text{N})(12\text{m}) - F_y(4.8\text{m}) = 0$$

$$F_y = 600 \text{ N}$$

$$F_y = 600 \text{ N} \uparrow$$



FREE BODY: MEMBER BCD

$$+\circlearrowleft \Sigma M_B = 0: C_y(2.4\text{m}) - (2400\text{N})(3.1\text{m}) = 0$$

$$C_y = +3600 \text{ N}$$

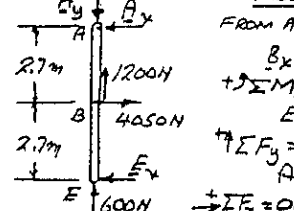
$$\frac{C_x}{C_y} = \frac{54}{48}; \frac{C_x}{3600\text{N}} = \frac{54}{48}$$

$$C_x = +4050 \text{ N}$$

$$+\rightarrow \Sigma F_x = 0: -B_x + 4050\text{N} = 0 \quad B_x = +4050 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: -B_y + 3600\text{N} - 2400\text{N} = 0 \quad B_y = +1200 \text{ N}$$

FREE BODY: MEMBER ABE

FROM ABOVE:  $F_y = 600 \text{ N} \uparrow$ 

$$B_x = 4050 \text{ N} \rightarrow, B_y = 1200 \text{ N} \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: (4050\text{N})(2.7\text{m}) - E_x(5.4\text{m}) = 0$$

$$E_x = 2025 \text{ N}$$

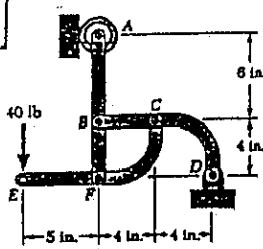
$$+\uparrow \Sigma F_y = 0: -A_y + 1200\text{N} + 600\text{N} = 0$$

$$A_y = +1800 \text{ N}$$

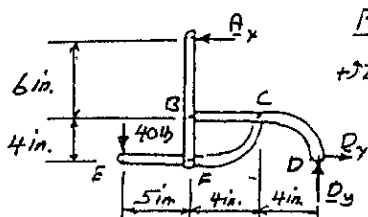
$$+\rightarrow \Sigma F_x = 0: -A_x + 4050\text{N} - 2025\text{N} = 0$$

$$A_x = +2025 \text{ N}$$

6.101



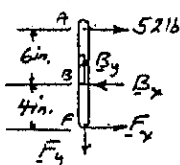
FIND: COMPONENTS OF FORCES ACTING ON MEMBER CDE AT C AND F.



FREE BODY: ENTIRE FRAME

$$+\circlearrowleft \Sigma M_D = 0: (40 \text{ lb})(13 \text{ in.}) + A_x(10 \text{ in.}) = 0$$

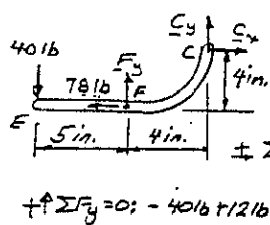
$$A_x = -52 \text{ lb}, A_x = 52 \text{ lb} \rightarrow$$



FREE BODY: MEMBER ABF

$$+\circlearrowleft \Sigma M_B = 0: -(52 \text{ lb})(6 \text{ in.}) + F_x(4 \text{ in.}) = 0$$

$$F_x = +78 \text{ lb}$$



FREE BODY: MEMBER CDE

FROM ABOVE:  $F_x = 78 \text{ lb}$

$$+\circlearrowleft \Sigma M_C = 0: (40 \text{ lb})(9 \text{ in.}) - (78 \text{ lb})(4 \text{ in.}) - F_y(4 \text{ in.}) = 0$$

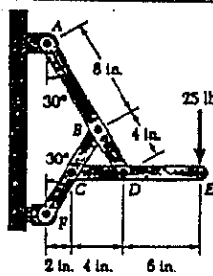
$$F_y = +12 \text{ lb}, F_y = 12 \text{ lb} \uparrow$$

$$\pm \Sigma F_x = 0: C_x - 78 \text{ lb} = 0$$

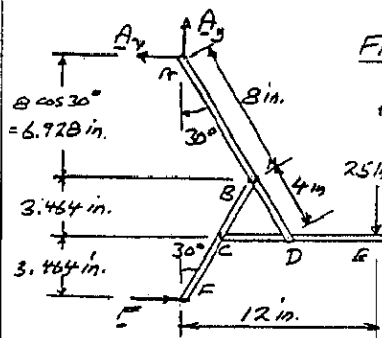
$$C_x = +78 \text{ lb}, C_x = 78 \text{ lb} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: -40 \text{ lb} + 12 \text{ lb} + C_y = 0; C_y + 28 \text{ lb}, C_y = 28 \text{ lb} \uparrow$$

6.102



FIND: COMPONENTS OF FORCES ACTING ON MEMBER CDE AT C AND D.



FREE BODY: ENTIRE FRAME

$$+\uparrow \Sigma F_y = 0: A_y - 25 \text{ lb} = 0$$

$$A_y = 25 \text{ lb}, A_y = 25 \text{ lb} \uparrow$$

$$+\circlearrowleft \Sigma M_D = 0: A_x(6.928 + 2 \times 3.464) - (25 \text{ lb})(12 \text{ in.}) = 0$$

$$A_x = 21.65 \text{ lb}, A_x = 21.65 \text{ lb} \leftarrow$$

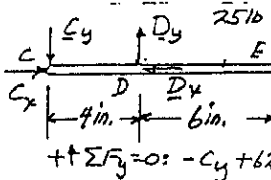
$$\pm \Sigma F_x = 0: F - 21.65 \text{ lb} = 0$$

$$F = 21.65 \text{ lb}, F = 21.65 \text{ lb} \rightarrow$$

(CONTINUED)

6.102 CONTINUED

FREE BODY: MEMBER CDE



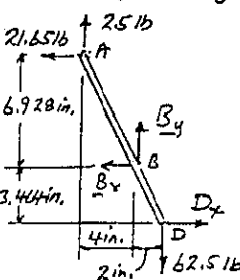
$$+\circlearrowleft \Sigma M_C = 0:$$

$$D_y(4 \text{ in.}) - (25 \text{ lb})(10 \text{ in.}) = 0$$

$$D_y = +62.5 \text{ lb}, D_y = 62.5 \text{ lb} \uparrow$$

$$+\uparrow \Sigma F_y = 0: -C_y + 62.5 \text{ lb} - 25 \text{ lb} = 0$$

$$C_y = +37.5 \text{ lb}, C_y = 37.5 \text{ lb} \downarrow$$



FREE BODY: MEMBER ABD

$$+\circlearrowleft \Sigma M_B = 0:$$

$$D_x(3.464 \text{ in.}) + (21.65 \text{ lb})(6.928 \text{ in.}) - (25 \text{ lb})(4 \text{ in.}) - (62.5 \text{ lb})(2 \text{ in.}) = 0$$

$$D_x = +21.65 \text{ lb}$$

RETURN TO

FREE BODY: MEMBER CDE

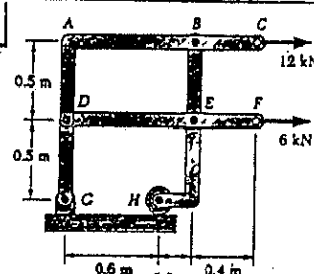
FROM ABOVE:

$$D_x = +21.65 \text{ lb}, D_x = 21.65 \text{ lb} \leftarrow$$

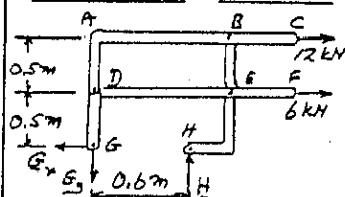
$$\pm \Sigma F_x = 0: C_x - 21.65 \text{ lb} = 0$$

$$C_x = +21.65 \text{ lb}, C_x = 21.65 \text{ lb} \rightarrow$$

6.103



FIND: COMPONENTS OF FORCES ACTING ON MEMBER DABC AT B AND D.



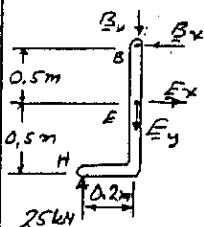
FREE BODY:

ENTIRE FRAME

$$+\circlearrowleft \Sigma M_D = 0:$$

$$H(0.6 \text{ m}) - (12 \text{ kN})(1 \text{ m}) - (6 \text{ kN})(0.5 \text{ m}) = 0$$

$$H = 25 \text{ kN}, H = 25 \text{ kN} \uparrow$$

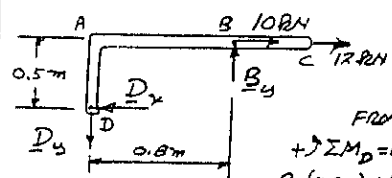


FREE BODY: MEMBER BEH

$$+\circlearrowleft \Sigma M_E = 0:$$

$$B_x(0.5 \text{ m}) - (25 \text{ kN})(0.2 \text{ m}) = 0$$

$$B_x = +10 \text{ kN}$$



FREE BODY:

MEMBER DABC

FROM ABOVE:  $B_x = 10 \text{ kN}$

$$+\circlearrowleft \Sigma M_D = 0:$$

$$-B_y(0.8 \text{ m}) + (10 \text{ kN} + 12 \text{ kN})(0.5 \text{ m}) = 0$$

$$B_y = +13.75 \text{ kN}, B_y = 13.75 \text{ kN} \uparrow$$

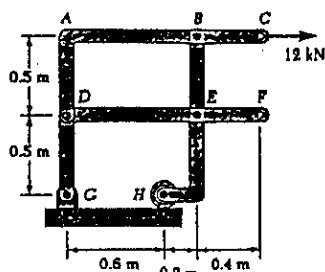
$$\pm \Sigma F_x = 0: -D_x + 10 \text{ kN} + 12 \text{ kN} = 0$$

$$D_x = +22 \text{ kN}, D_x = 22 \text{ kN} \leftarrow$$

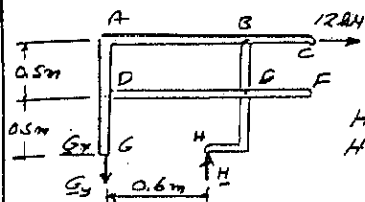
$$+\uparrow \Sigma F_y = 0: -D_y + 13.75 \text{ kN} = 0$$

$$D_y = +13.75 \text{ kN}, D_y = 13.75 \text{ kN} \uparrow$$

6.104



FIND:  
COMPONENTS OF  
FORCES ACTING  
ON MEMBER DABC  
AT B AND D.



FREE BODY:

ENTIRE FRAME:

$$+\uparrow \Sigma M_G = 0$$

$$H(0.6 \text{ m}) - (12 \text{ kN})(1 \text{ m}) = 0$$

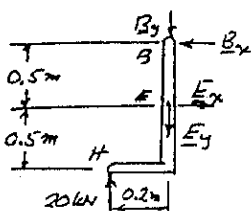
$$H = 20 \text{ kN} \quad H = 20 \text{ kN} \uparrow$$

FREE BODY: MEMBER BEH

$$+\uparrow \Sigma M_E = 0$$

$$B_x(0.5 \text{ m}) - (20 \text{ kN})(0.2 \text{ m}) = 0$$

$$B_x = +8 \text{ kN}$$



FREE BODY:

MEMBER DABC

FROM ABOVE  $B_x = 8 \text{ kN}$ 

$$+\uparrow \Sigma M_D = 0:$$

$$-B_y(0.5 \text{ m}) + (8 \text{ kN}) + (12 \text{ kN})(0.5 \text{ m}) = 0$$

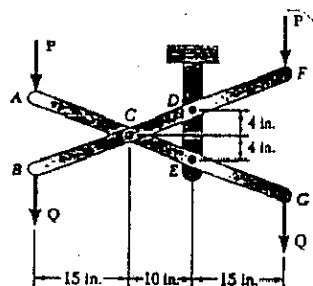
$$B_y = +12.5 \text{ kN} \quad B_y = 12.5 \text{ kN} \uparrow$$

$$+\rightarrow \Sigma F_x = 0: -D_x + 8 \text{ kN} + 12 \text{ kN} = 0$$

$$D_x = +20 \text{ kN} \quad D_x = 20 \text{ kN} \leftarrow$$

$$+\uparrow \Sigma F_y = 0: -D_y + 12.5 \text{ kN} = 0 \quad D_y = +12.5 \text{ kN} \quad D_y = 12.5 \text{ kN} \downarrow$$

6.105 and 6.106



FIND: COMPONENTS  
OF FORCES ACTING ON  
(a) MEMBER BCDF  
AT C AND D,  
(b) MEMBER ACEG AT E.

PROB. 6.105: WHEN  
 $P = 15 \text{ lb}$  AND  $Q = 65 \text{ lb}$ .  
PROB. 6.106: WHEN  
 $P = 25 \text{ lb}$  AND  $Q = 55 \text{ lb}$ .

FREE BODY:

ENTIRE FRAME

$$+\uparrow \Sigma M_A = 0: (P+Q)(25 \sin)$$

$$-(P+Q)(15 \sin) - D_x(8 \text{ in.}) = 0$$

$$D_x = (P+Q) \frac{10}{8}$$

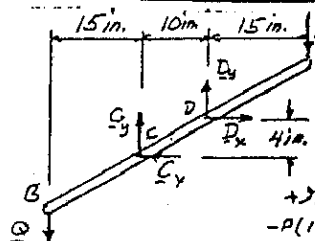
$$D_x = (P+Q) \frac{10}{8} \rightarrow$$

$$+\rightarrow \Sigma F_x = 0: -E_x + (P+Q) \frac{10}{8} = 0$$

$$E_x = (P+Q) \frac{10}{8} \quad E_x = (P+Q) \frac{10}{8} \leftarrow$$

(CONTINUED)

6.105 and 6.106 CONTINUED



FREE BODY: MEMBER BCDF

FROM ABOVE:

$$D_x = (P+Q) \frac{10}{8} \rightarrow$$

$$+\rightarrow \Sigma F_x = 0: -C_x + D_x = 0$$

$$C_x = (P+Q) \frac{10}{8} \leftarrow$$

$$+\uparrow \Sigma M_D = 0: -(P+Q) \frac{10}{8} (4 \text{ in.})$$

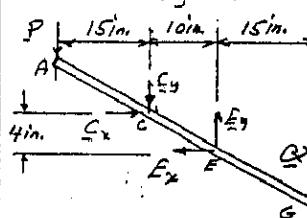
$$-P(15 \text{ in.}) + Q(25 \text{ in.}) + C_y(10 \text{ in.})$$

$$C_y = \frac{1}{10}(-20P + 20Q)$$

$$C_y = 2Q - 2P \quad C_y = 2Q - 2P \uparrow$$

$$+\uparrow \Sigma F_y = 0: D_y + (2Q - 2P) - P - Q = 0$$

$$D_y = -Q + 3P \quad D_y = -Q + 3P \uparrow$$



FREE BODY: MEMBER ACEG

FROM ABOVE

$$E_x = (P+Q) \frac{10}{8} \leftarrow$$

$$\text{AND } C_y = 2Q - P$$

$$+\rightarrow \Sigma F_x = 0: E_x - C_y - P - Q = 0$$

$$E_x - (2Q - P) - P - Q = 0$$

$$E_x = 3Q - P; \quad E_x = 3Q - P \leftarrow$$

PROB. 6.105:  $P = 15 \text{ lb}$  AND  $Q = 65 \text{ lb}$ 

$$C_x = (P+Q) \frac{10}{8} = (15+65) \frac{10}{8} = +100 \text{ lb} \quad C_x = 100 \text{ lb} \leftarrow$$

$$C_y = 2Q - 2P = 2(65) - 2(15) = +100 \text{ lb} \quad C_y = 100 \text{ lb} \uparrow$$

$$D_x = (P+Q) \frac{10}{8} = (15+65) \frac{10}{8} = +100 \text{ lb} \quad D_x = 100 \text{ lb} \rightarrow$$

$$D_y = -Q + 3P = -65 + 3(15) = -20 \text{ lb} \quad D_y = 20 \text{ lb} \downarrow$$

$$E_x = (P+Q) \frac{10}{8} = (15+15) \frac{10}{8} = +100 \text{ lb} \quad E_x = 100 \text{ lb} \leftarrow$$

$$E_y = 3Q - P = 3(65) - 15 = +180 \text{ lb} \quad E_y = 180 \text{ lb} \uparrow$$

PROB. 6.106:  $P = 25 \text{ lb}$  AND  $Q = 55 \text{ lb}$ 

$$C_x = (P+Q) \frac{10}{8} = (25+55) \frac{10}{8} = +100 \text{ lb} \quad C_x = 100 \text{ lb} \leftarrow$$

$$C_y = 2Q - 2P = 2(55) - 2(25) = +60 \text{ lb} \quad C_y = 60 \text{ lb} \uparrow$$

$$D_x = (P+Q) \frac{10}{8} = (25+55) \frac{10}{8} = +100 \text{ lb} \quad D_x = 100 \text{ lb} \rightarrow$$

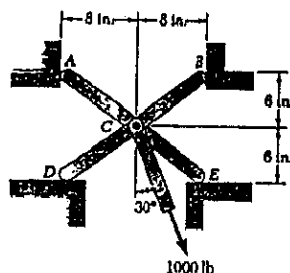
$$D_y = -Q + 3P = -55 + 3(25) = +20 \text{ lb} \quad D_y = 20 \text{ lb} \downarrow$$

$$E_x = (P+Q) \frac{10}{8} = (25+55) \frac{10}{8} = +100 \text{ lb} \quad E_x = 100 \text{ lb} \leftarrow$$

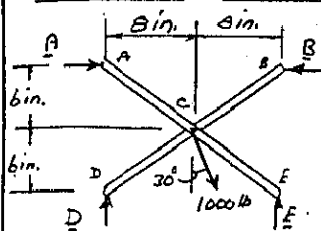
$$E_y = 3Q - P = 3(55) - 25 = +140 \text{ lb} \quad E_y = 140 \text{ lb} \uparrow$$



6.110

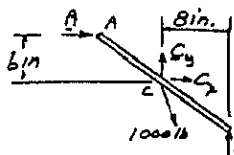


FIND  
REACTIONS  
AT A, B, D, AND E.



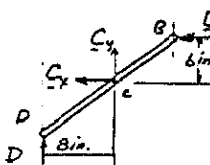
FREE BODY: ENTIRE FRAME

$$\begin{aligned} +\sum F_x = 0: & A - B + (1000 \text{ lb}) \sin 30^\circ = 0 \\ & A - B + 500 = 0 \quad (1) \\ +\sum F_y = 0: & D + E - (1000 \text{ lb}) \cos 30^\circ = 0 \\ & D + E - 866.03 = 0 \quad (2) \end{aligned}$$



FREE BODY: MEMBER ACE

$$\begin{aligned} +\sum M_C = 0: & -A(6 \text{ in}) + E(6 \text{ in}) = 0 \\ & E = \frac{3}{2}A \quad (3) \end{aligned}$$



FREE BODY: MEMBER BCD

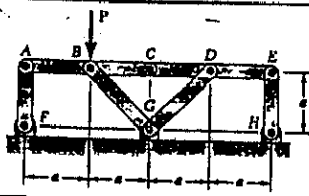
$$\begin{aligned} +\sum M_C = 0: & -D(6 \text{ in}) + B(6 \text{ in}) = 0 \\ & D = \frac{3}{2}B \quad (4) \end{aligned}$$

SUBSTITUTE E AND D FROM (3) AND (4) INTO (2):

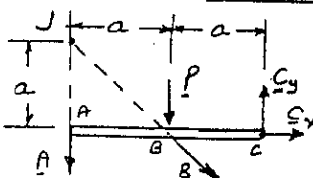
$$\begin{aligned} \frac{3}{2}A + \frac{3}{2}B - 866.06 &= 0 \\ A + B - 1154.7 &= 0 \quad (5) \\ A - B + 500 &= 0 \quad (6) \end{aligned}$$

$$\begin{aligned} (5) + (6) \quad 2A - 654.7 &= 0; & A &= 327.4 \text{ lb} \\ (5) - (6) \quad 2B - 1654.7 &= 0; & B &= 827.4 \text{ lb} \\ (4) \quad D = \frac{3}{2}(827.4) &= 620.5 \text{ lb} \\ (3) \quad E = \frac{3}{2}(327.4) &= 245.5 \text{ lb} \end{aligned}$$

6.111

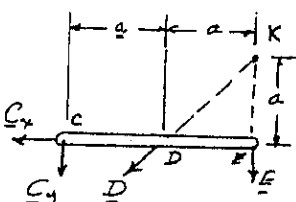


FIND: THE  
FORCE IN  
EACH LINK



FREE BODY: MEMBER ABC

$$\begin{aligned} +\sum M_J = 0: & -Pa + C_y(2a) + C_x(a) = 0 \quad (1) \end{aligned}$$



FREE BODY: MEMBER CDE

$$\begin{aligned} +\sum M_K = 0: & C_y(2a) - C_x(a) = 0 \\ & C_x = 2C_y \quad (2) \end{aligned}$$

6.111 CONTINUED

$$\begin{aligned} (2) - (1): & -Pa + C_y(2a) + 2C_y(a) = 0; & C_y &= +\frac{1}{4}P \\ (2): & C_x = 2C_y = 2\left(\frac{1}{4}P\right); & C_x &= +\frac{1}{2}P \end{aligned}$$

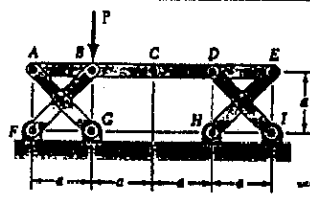
RETURN TO FREE BODY OF ABC:

$$\begin{aligned} +\sum F_x = 0: & C_x + \frac{1}{2}B = 0; & \frac{1}{2}P + \frac{1}{2}B &= 0; & B &= -\frac{P}{2} & F_{BG} &= \frac{P}{2} \text{ comp.} \\ +\sum M_B = 0: & C_y(a) + A(a) & \frac{1}{4}Pa + Aa &= 0; & A &= -\frac{P}{4} & F_{AF} &= \frac{P}{4} \text{ comp.} \end{aligned}$$

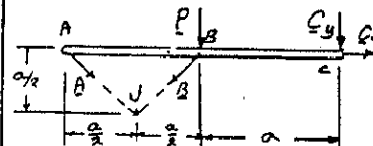
RETURN TO FREE BODY OF CDE:

$$\begin{aligned} +\sum F_x = 0: & -C_x - \frac{1}{2}D = 0 \\ & -\frac{P}{2} - \frac{1}{2}D = 0; & D &= -\frac{P}{2}; & F_{DE} &= \frac{P}{2} \text{ comp.} \\ +\sum M_D = 0: & C_y(a) - E(a) = 0 \\ & \frac{1}{4}Pa - Ea = 0; & E &= \frac{P}{4}; & F_{EH} &= \frac{P}{4} \text{ comp.} \end{aligned}$$

6.112

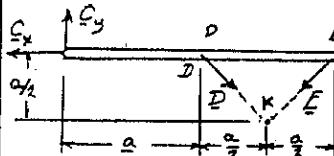


FIND: THE  
FORCE IN  
EACH LINK.



FREE BODY: MEMBER ABC

$$\begin{aligned} +\sum M_J = 0: & C_x\left(\frac{1}{2}a\right) + C_y\left(\frac{3}{2}a\right) + P\left(\frac{a}{2}\right) = 0 \\ & C_x + 3C_y + P = 0 \quad (1) \end{aligned}$$



FREE BODY: MEMBER CDE

$$\begin{aligned} +\sum M_K = 0: & C_x\left(\frac{a}{2}\right) - C_y\left(\frac{3a}{2}\right) = 0 \\ & C_x = 3C_y \quad (2) \end{aligned}$$

$$\begin{aligned} (2) - (1): & +3C_y + 3C_y + P = 0; & 6C_y + P &= 0 & C_y &= -\frac{P}{6} \\ (2): & C_x = 3C_y = 3\left(-\frac{P}{6}\right) & C_x &= -\frac{P}{2} \end{aligned}$$

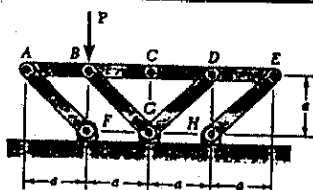
RETURN TO FREE BODY OF ABC:

$$\begin{aligned} +2\sum M_B = 0: & -\frac{P}{\sqrt{2}}(a) + C_y(a) = 0 \\ & -\frac{P}{\sqrt{2}}a - \frac{P}{6}a = 0; & A &= -\frac{\sqrt{3}}{6}P & F_{AG} &= \frac{\sqrt{3}}{6}P \text{ comp.} \\ +2\sum M_A = 0: & \frac{B}{\sqrt{2}}(a) + C_y(2a) + P(a) = 0 \\ & \frac{B}{\sqrt{2}}(a) - \frac{P}{6}(2a) + P(a) = 0 \\ & B = -\frac{2\sqrt{3}}{3}P & F_{BF} &= \frac{2\sqrt{3}}{3}P \text{ comp.} \end{aligned}$$

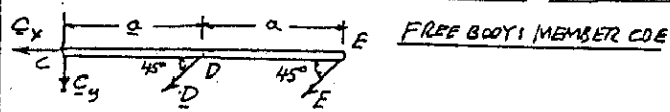
RETURN TO FREE BODY OF CDE:

$$\begin{aligned} +\sum M_E = 0: & \frac{D}{\sqrt{2}}(a) - C_y(2a) = 0 \\ & \frac{D}{\sqrt{2}}(a) - \left(-\frac{P}{6}\right)(2a) = 0 \\ & D = -\frac{\sqrt{2}}{3}P & F_{DI} &= \frac{\sqrt{2}}{3}P \text{ comp.} \\ +2\sum M_D = 0: & \frac{F}{\sqrt{2}}(a) + C_y(a) = 0 \\ & \frac{F}{\sqrt{2}}(a) - \frac{P}{6}(a) = 0 \\ & F = \frac{\sqrt{2}}{6}P & F_{EH} &= \frac{\sqrt{2}}{6}P \text{ comp.} \end{aligned}$$

6.113



FIND: THE  
FORCE IN  
EACH LINK



FREE BODY: MEMBER CDE

$$\pm \Sigma F_x = 0: C_x + \frac{D}{\sqrt{2}} + \frac{E}{\sqrt{2}} = 0$$

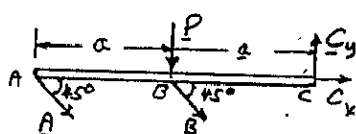
$$C_x - \frac{2E}{\sqrt{2}} + \frac{E}{\sqrt{2}} = 0$$

$$C_x = \frac{E}{\sqrt{2}}$$

$$+\uparrow \Sigma F_y = 0: C_y + \frac{D}{\sqrt{2}} + \frac{E}{\sqrt{2}} = 0$$

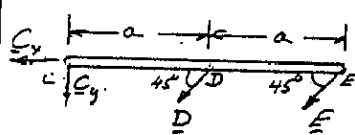
$$C_y - \frac{2E}{\sqrt{2}} + \frac{E}{\sqrt{2}} = 0$$

$$C_y = \frac{E}{\sqrt{2}}$$



FREE BODY:  
MEMBER ABC

$$\Delta 45^\circ \Sigma F = 0: -\frac{P}{\sqrt{2}} + \frac{C_x}{\sqrt{2}} + \frac{C_y}{\sqrt{2}} = 0: C_x + C_y = +P \quad (1)$$



FREE BODY:  
MEMBER CDE

$$45^\circ \Delta \Sigma F = 0: +\frac{C_x}{\sqrt{2}} - \frac{C_y}{\sqrt{2}} = 0 \quad C_x = C_y \quad (2)$$

$$(2) \rightarrow (1) \quad C_y + C_y = P; \quad C_y = \frac{P}{2} \quad C_x = \frac{P}{2}$$

$$+\uparrow \Sigma M_D = 0: C_y(a) - \frac{E}{\sqrt{2}}(a) = 0$$

$$E = \sqrt{2} C_y = \frac{\sqrt{2}}{2} P$$

$$F_{EA} = \frac{\sqrt{2}}{2} P \text{ ten.}$$

$$+\uparrow \Sigma M_E = 0: \frac{D}{\sqrt{2}}(a) + C_y(2a) = 0$$

$$D = -2\sqrt{2} C_y = -2\sqrt{2} \frac{P}{2}$$

$$F_{DE} = \sqrt{2} P \text{ comp.}$$

RETURN TO FREE BODY OF ABC

$$+\uparrow \Sigma M_B = 0: \frac{A}{\sqrt{2}}(a) + C_y(a) = 0$$

$$A = -\sqrt{2} C_y = -\frac{\sqrt{2}}{2} P$$

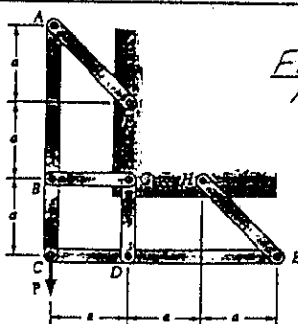
$$F_{AB} = \frac{\sqrt{2}}{2} P \text{ ten.}$$

$$+\uparrow \Sigma M_A = 0: \frac{B}{\sqrt{2}}(a) + Pa - C_y(2a) = 0$$

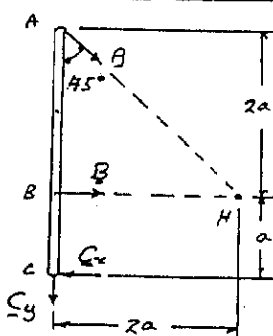
$$B = \sqrt{2} (P - \frac{P}{2} \cdot 2) = 0$$

$$F_{BC} = 0$$

6.114



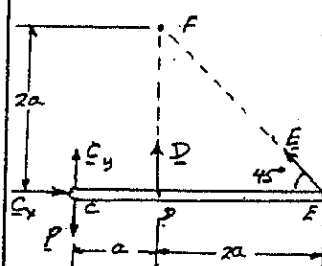
FIND: THE  
FORCE IN  
EACH LINK



FREE BODY: MEMBER ABC

$$+\uparrow \Sigma M_A = 0: C_x(a) - C_y(2a) = 0$$

$$C_x = 2C_y$$



FREE BODY: MEMBER CDE

$$+\uparrow \Sigma M_D = 0$$

$$C_x(2a) - C_y(a) + P(a) = 0$$

$$2C_x - C_y + P = 0$$

$$2(2C_y) - C_y + P = 0$$

$$C_y = -\frac{1}{3} P$$

$$C_x = 2C_y: C_x = -\frac{2}{3} P$$

$$\pm \Sigma F = 0: C_x - \frac{E}{\sqrt{2}} = 0; \quad -\frac{2}{3} P - \frac{E}{\sqrt{2}} = 0$$

$$E = -\frac{2\sqrt{2}}{3} P$$

$$F_{EH} = \frac{2\sqrt{2}}{3} P \text{ comp.}$$

$$+\uparrow \Sigma M_E = 0: D(2a) + C_y(3a) - P(3a) = 0$$

$$D(2a) - \frac{P}{3}(3a) - P(3a) = 0$$

$$D = +2P$$

$$F_{DE} = 2P \text{ ten.}$$

RETURN TO FREE BODY OF ABC

$$+\uparrow \Sigma F_y = 0: \frac{A}{\sqrt{2}} + C_y = 0$$

$$\frac{A}{\sqrt{2}} - \frac{P}{3} = 0$$

$$A = +\frac{\sqrt{2}}{3} P$$

$$F_{AF} = \frac{\sqrt{2}}{3} P \text{ ten.}$$

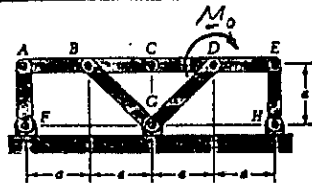
$$+\uparrow \Sigma M_A = 0: B(2a) - C_x(3a) = 0$$

$$B(2a) + \frac{2}{3} P(3a) = 0$$

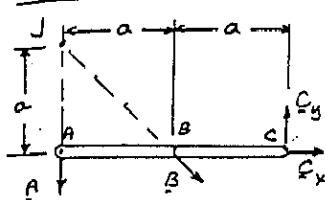
$$B = -P$$

$$F_{BE} = P \text{ comp.}$$

6.115



FIND: THE  
FORCE IN  
EACH LINK

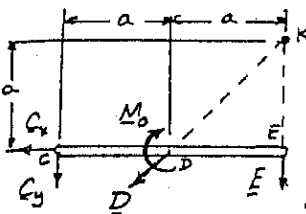


FREE BODY:  
MEMBER ABC

$$+\sum M_A = 0$$

$$C_y(2a) + C_x(a) = 0$$

$$C_x = -2C_y$$



FREE BODY: MEMBER CDE

$$+\sum M_C = 0:$$

$$C_y(2a) - C_x(a) - M_D = 0$$

$$C_y(2a) - (-2C_y)(a) - M_D = 0$$

$$C_y = M_D/4a$$

$$C_x = -2C_y; C_x = -M_D/2a$$

$$+\sum F_x = 0: \frac{D}{\sqrt{2}} + C_x = 0; \frac{D}{\sqrt{2}} - \frac{M_D}{2a} = 0$$

$$D = \frac{M_D}{\sqrt{2}a} \quad F_{DE} = \frac{M_D}{\sqrt{2}a} \text{ ten.}$$

$$+\sum M_D = 0: E(a) - C_y(a) + M_D = 0$$

$$E(a) - \left(\frac{M_D}{4a}\right)(a) + M_D = 0$$

$$E = -\frac{3}{4} \frac{M_D}{a} \quad F_{EH} = \frac{3}{4} \frac{M_D}{a} \text{ comp.}$$

RETURN TO FREE BODY OF ABC

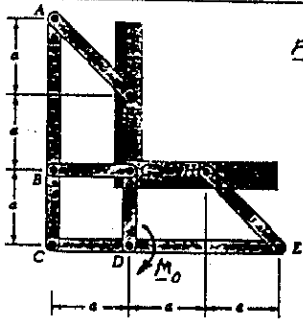
$$+\sum F_x = 0: \frac{B}{\sqrt{2}} + C_x = 0; \frac{B}{\sqrt{2}} - \frac{M_D}{2a} = 0$$

$$B = \frac{M_D}{\sqrt{2}a} \quad F_{BG} = \frac{M_D}{\sqrt{2}a} \text{ ten.}$$

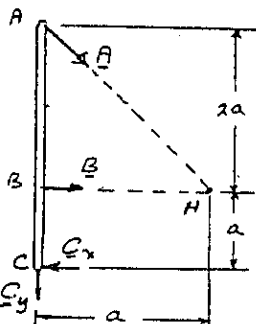
$$+\sum M_B = 0: A(a) + C_y(a); A(a) + \frac{M_D}{4a}(a) = 0$$

$$A = -\frac{M_D}{4a} \quad F_{AF} = \frac{M_D}{4a} \text{ comp.}$$

6.116



FIND: THE FORCE  
IN EACH LINK



FREE BODY: MEMBER ABC

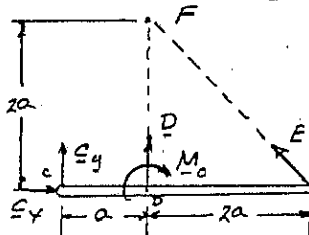
$$+\sum M_A = 0:$$

$$C_x(a) - C_y(2a) = 0$$

$$C_x = 2C_y$$

(CONTINUED)

6.116 CONTINUED



FREE BODY: MEMBER CDE

$$+\sum M_C = 0:$$

$$C_x(2a) - C_y(a) - M_D = 0$$

$$(2C_y)(2a) - C_y(a) - M_D = 0$$

$$C_y = \frac{M_D}{3a}$$

$$C_x = 2C_y; C_x = \frac{2M_D}{3a}$$

$$+\sum F_x = 0: C_x - \frac{E}{\sqrt{2}} = 0; \frac{2M_D}{3a} - \frac{E}{\sqrt{2}} = 0$$

$$E = \frac{2\sqrt{2}}{3} \frac{M_D}{a} \quad F_{EH} = \frac{2\sqrt{2}}{3} \frac{M_D}{a}$$

$$+\sum F_y = 0: D + \frac{E}{\sqrt{2}} + C_y = 0$$

$$D + \frac{2\sqrt{2}}{3} \frac{M_D}{a} \frac{1}{\sqrt{2}} + \frac{M_D}{3a} = 0$$

$$D = -\frac{M_D}{a} \quad F_{DG} = \frac{M_D}{a} \text{ comp.}$$

RETURN TO FREE BODY OF ABC

$$+\sum F_y = 0: \frac{A}{\sqrt{2}} + C_y = 0; \frac{A}{\sqrt{2}} + \frac{M_D}{3a} = 0$$

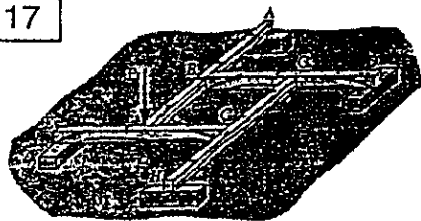
$$A = -\frac{\sqrt{2}}{3} \frac{M_D}{a} \quad F_{AF} = \frac{\sqrt{2}}{3} \frac{M_D}{a} \text{ comp.}$$

$$+\sum M_B = 0: B(2a) - C_x(3a) = 0$$

$$B(2a) - \left(\frac{2}{3} \frac{M_D}{a}\right)(3a) = 0$$

$$B = +\frac{M_D}{a} \quad F_{BG} = \frac{M_D}{a} \text{ ten.}$$

6.117



FIND: THE  
VERTICAL  
REACTIONS  
AT A, D, E, AND H

WE SHALL DRAW A FREE BODY OF EACH MEMBER.  
FORCE P WILL BE APPLIED TO MEMBER EFG.

STARTING WITH MEMBER ABF, WE SHALL  
EXPRESS ALL FORCES IN TERMS OF REACTION A.

MEMBER ABF

$$+\sum M_A = 0: A(2a) - B_y(a) = 0; B_y = 2A$$

$$+\sum M_B = 0: -F_y(a) + A(a) = 0; F_y = A$$

MEMBER BCD

$$+\sum M_B = 0: -(2A)(a) + D(a) = 0; D = 2A \quad (1)$$

$$+\sum M_D = 0: -(2A)(2a) + C_y(a) = 0; C_y = 4A$$

MEMBER CGH

$$+\sum M_C = 0: -(4A)(a) + H(a) = 0; H = 4A \quad (2)$$

$$+\sum M_H = 0: -(4A)(2a) + G_y(a) = 0; G_y = 8A$$

MEMBER EFG

$$+\sum M_F = 0: -(8A)(a) + E(a) = 0$$

$$E = 8A \quad (3)$$

$$+\sum F_y = 0:$$

$$E - A + 8A - P = 0$$

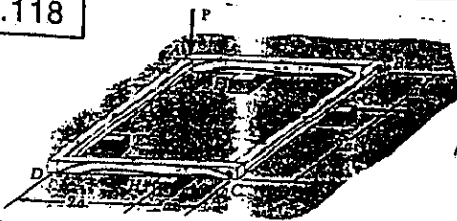
$$8A - A + 8A - P = 0; A = \frac{P}{15}$$

SUBSTITUTE  $A = \frac{P}{15}$  INTO EQS. (1), (2), AND (3):

ANSWERS:

$$A = \frac{1}{15} P; D = \frac{2}{15} P; H = \frac{4}{15} P; E = \frac{8}{15} P$$

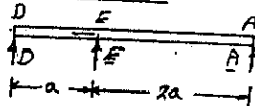
6.118



FIND: THE  
VERTICAL  
REACTIONS  
AT A, D, E, AND H.

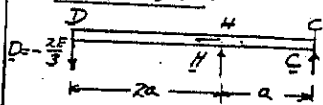
WE SHALL DRAW THE FREE BODY OF EACH MEMBER.  
FORCE  $P$  WILL BE APPLIED TO MEMBER AFB.  
STARTING WITH MEMBER AED, WE SHALL EXPRESS  
ALL FORCES IN TERMS OF REACTION  $E$ .

MEMBER AED:



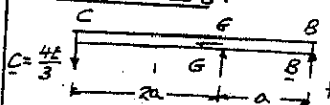
$$\begin{aligned} +\sum M_D = 0: & A(3a) + E(a) = 0 \\ & A = -E/3 \\ +\sum M_A = 0: & -D(3a) - E(2a) = 0 \\ & D = -2E/3 \end{aligned}$$

MEMBER DHC:



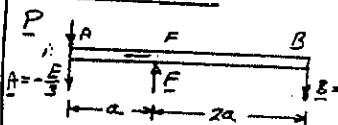
$$\begin{aligned} +\sum M_C = 0: & (-2E/3)(3a) - H(a) = 0 \\ & H = -2E \\ +\sum M_H = 0: & (-2E/3)(2a) + C(a) = 0 \\ & C = +4E/3 \end{aligned} \quad (1)$$

MEMBER CGB:



$$\begin{aligned} +\sum M_B = 0: & (+4E/3)(3a) - G(a) = 0 \\ & G = +4E \\ +\sum M_G = 0: & (+4E/3)(2a) + B(a) = 0 \\ & B = -8E/3 \end{aligned} \quad (2)$$

MEMBER AFB:



$$\begin{aligned} +\sum F_y = 0: & F - A - B - P = 0 \\ & P - (-E/3) - (-8E/3) - P = 0 \\ & F = P - 3E \end{aligned} \quad (3)$$

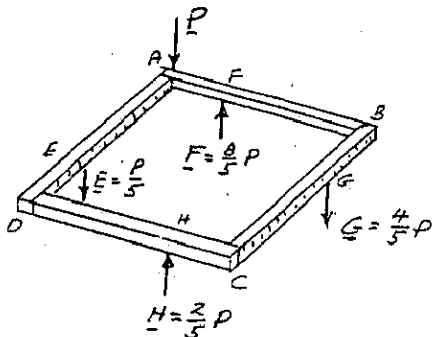
$$\begin{aligned} +\sum M_A = 0: & F(a) - B(3a) = 0 \\ & (P - 3E)(a) - (-8E/3)(3a) = 0 \\ & P - 3E + 8E = 0; \quad E = -P/5 \end{aligned}$$

SUBSTITUTE  $E = -P/5$  INTO Eqs. (1), (2), AND (3).

$$H = -2E = -2(-P/5); \quad H = +2P/5 \quad H = \frac{2P}{5} \uparrow$$

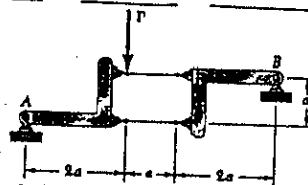
$$G = +4E = 4(-P/5); \quad G = -4P/5 \quad G = \frac{4P}{5} \downarrow$$

$$F = P - 3E = P - 3(-P/5); \quad F = +8P/5 \quad F = \frac{8P}{5} \uparrow$$

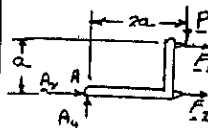


6.119

FOR EACH FRAME SHOWN FIND THE  
REACTIONS AND WHETHER FRAME IS RIGID

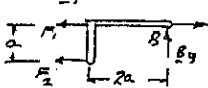


(a)



FREE BODY: LEFT PORTION

$$\begin{aligned} +\sum F_y = 0: & A_y - P = 0; \quad A_y = P; \quad A_y = P \uparrow \\ +\sum M_A = 0: & -P(2a) - F_1(a) = 0 \quad F_1 = -2P \\ +\sum F_x = 0: & A_x + F_1 + F_2 = 0; \quad A_x - 2P + F_2 = 0 \end{aligned} \quad (1)$$



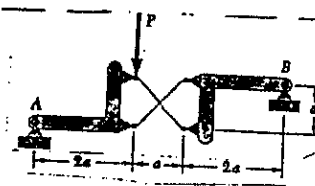
FREE BODY: RIGHT PORTION

$$\begin{aligned} +\sum F_y = 0: & B_y = 0 \quad B_y = 0 \\ +\sum M_B = 0: & F_2(a) = 0 \quad F_2 = 0 \\ +\sum F_x = 0: & B_x - F_1 - F_2 = 0; \quad B_x - (-2P) = 0 \\ & B_x = -2P \quad B_x = 2P \leftarrow \end{aligned}$$

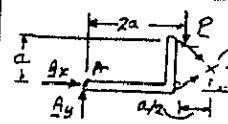
FROM (1) WITH  $F_2 = 0$ ,  $A_x = 2P$

$$A = 2, 2P \angle 26.6^\circ; \quad B = 2P \rightarrow$$

FRAME IS RIGID

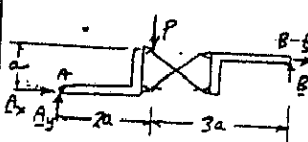


(b)



FREE BODY: LEFT PORTION

$$\begin{aligned} +\sum M_C = 0: & A_x(\frac{a}{2}) - A_y(\frac{5a}{2}) + P(\frac{a}{2}) = 0 \\ & P - A_x + 5A_y = 0 \end{aligned} \quad (1)$$

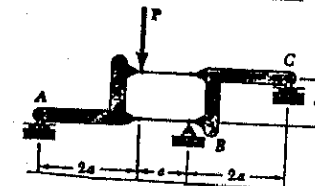


FREE BODY: ENTIRE FRAME

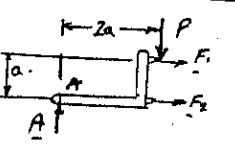
$$\begin{aligned} +\sum M_B = 0: & A_x(a) - A_y(5a) + P(3a) = 0 \\ & 3P - A_x + 5A_y = 0 \end{aligned} \quad (2)$$

$$\text{EQ (2) - EQ (1): } 3P - P = 0; \quad 2P = 0 \quad P = 0$$

FOR  $P \neq 0$ , EQUILIBRIUM IS NOT MAINTAINED AND  
FRAME IS NOT RIGID

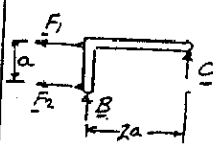


(c)



FREE BODY: LEFT PORTION

$$\begin{aligned} +\sum F_y = 0: & A - P = 0; \quad A = P; \quad A = P \uparrow \\ +\sum M_A = 0: & -P(2a) - F_1(a) = 0 \\ & F_1 = -2P \end{aligned}$$



FREE BODY: RIGHT PORTION

$$\begin{aligned} +\sum M_B = 0: & F_1(a) + C(2a) = 0 \\ & C = -\frac{1}{2}F_1 = -\frac{1}{2}(-2P) \\ & C = P \quad C = P \uparrow \end{aligned}$$

$$+\sum F_y = 0: \quad B + C = 0$$

$$B + P = 0$$

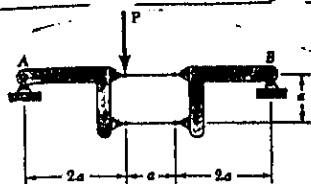
$$B = -P$$

FRAME IS RIGID

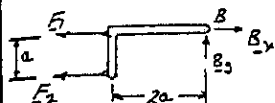


6.120

FOR EACH FRAME FIND THE REACTIONS AND WHETHER THE FRAME IS RIGID.

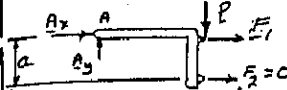


(a)



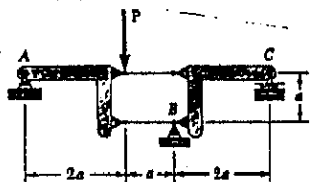
FREE BODY: RIGHT PORTION

$$+\circlearrowleft \Sigma M_B = 0: F_2(a) = 0 \\ F_2 = 0$$

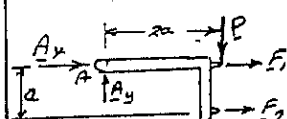


FREE BODY: LEFT PORTION

$$+\circlearrowleft \Sigma M_A = 0: P(2a) = 0 \\ P = 0$$

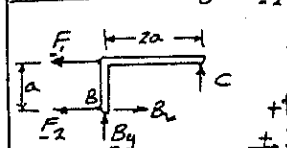
FOR  $P \neq 0$ , EQUILIBRIUM IS NOT MAINTAINED. FRAME IS NOT RIGID

(b) THIS FRAME IS INDETERMINATE, 2 FREE BODIES = 6 FES 5 REACTION COMPONENTS PLUS 2 LINK FORCES = 7 UNES



FREE BODY: LEFT PORTION

$$+\circlearrowleft \Sigma M_A = 0: F_2(a) - P(2a) = 0 \\ F_2 = 2P$$



FREE BODY: RIGHT PORTION

$$+\circlearrowleft \Sigma M_C = 0: -F_2(a) + B_x(a) - B_y(2a) = 0 \\ B_y = \frac{1}{2}(B_x - P)$$

$$+\uparrow \Sigma F_y = 0: C + B_y = 0; C = -\frac{1}{2}(B_x - P)$$

$$\pm \Sigma F_x = 0: -F_1 - F_2 + B_x = 0$$

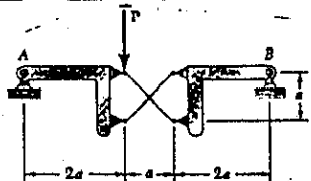
$$F_1 = B_x - F_2 = B_x - 2P$$

RETURN TO FREE BODY OF LEFT PORTION

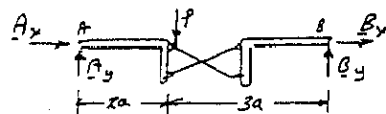
$$+\uparrow \Sigma F_y = 0: A_y - P = 0 \quad A_y = P$$

$$\pm \Sigma F_x = 0: A_x + F_1 + F_2 = 0; A_x = -F_1 - F_2$$

$$A_x = -(B_x - 2P) - 2P: A_x = -B_x$$

WE NOTE THAT REACTIONS CAN BE FOUND FOR AN ARBITRARY VALUE OF  $B_x$ . FRAME IS RIGID

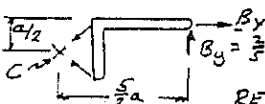
(c)



FREE BODY: ENTIRE FRAME

$$+\circlearrowleft \Sigma M_A = 0: B_y(3a) - P(2a) = 0 \\ B_y = \frac{2}{3}P \quad B_y = \frac{2}{3}P \uparrow$$

$$+\uparrow \Sigma F_y = 0: A_y - P + \frac{2}{3}P = 0; A_y = \frac{1}{3}P \quad A_y = \frac{1}{3}P \uparrow$$



FREE BODY: RIGHT PORTION

$$+\circlearrowleft \Sigma M_C = 0: \frac{2}{3}P(\frac{1}{2}a) - B_x(\frac{1}{2}a) = 0 \\ B_x = 2P \quad B_x = 2P \rightarrow$$

RETURN TO FREE BODY OF ENTIRE FRAME

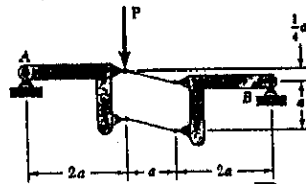
$$\pm \Sigma F_x = 0: A_x + 2P = 0, A_x = -2P; A_x = 2P \rightarrow$$

$$A = 2.09P \angle 16.7^\circ$$

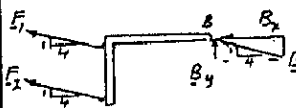
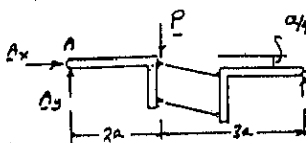
$$B = 2.04P \angle 11.3^\circ$$

6.121

FOR EACH FRAME FIND THE REACTIONS AND WHETHER THE FRAME IS RIGID.



(a)

FREE BODY: RIGHT PORTION FOR EQUILIBRIUM, B MUST BE PARALLEL TO LINKS, THAT IS  $B_x = 4B_y$ 

FREE BODY: ENTIRE FRAME

$$+\circlearrowleft \Sigma M_A = 0: B_y(3a) - (4B_y)(\frac{3}{4}a) - P(2a) = 0$$

$$B_y(5a) - 3B_y a - P(2a) = 0$$

$$B_y = \frac{2}{5}P \quad B_y = \frac{2}{5}P \uparrow$$

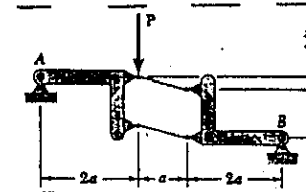
$$B_x = 4B_y \quad B_x = \frac{8}{5}P \leftarrow$$

$$A_y = 2P \rightarrow$$

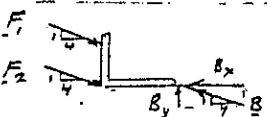
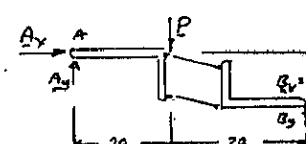
$$A_x = \frac{8}{5}P \uparrow$$

$$A = 2.06P \angle 14.0^\circ$$

$$B = 2.06P \angle 14.0^\circ$$



(b)

FREE BODY: RIGHT PORTION FOR EQUILIBRIUM, B MUST BE PARALLEL TO LINKS, THAT IS  $B_x = 4B_y$ 

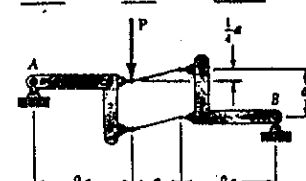
FREE BODY: ENTIRE FRAME

$$+\circlearrowleft \Sigma M_A = 0: B_y(3a) - 4B_y(\frac{3}{4}a) - P(2a) = 0$$

$$B_y(5a) - 3B_y a - P(2a) = 0$$

$$5B_y - 3B_y - 2P = 0$$

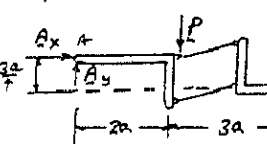
$$P = 0$$

FOR  $P \neq 0$ , EQUILIBRIUM IS NOT MAINTAINED. FRAME IS NOT RIGID.

(c)



FREE BODY: RIGHT PORTION

FOR EQUILIBRIUM, B MUST BE PARALLEL TO LINKS, THAT IS  $B_x = 4B_y$ 

FREE BODY: ENTIRE FRAME

$$+\circlearrowleft \Sigma M_A = 0: B_y(3a) - (4B_y)(\frac{3}{4}a) - P(2a) = 0$$

$$B_y(5a) - 3B_y a - P(2a) = 0$$

$$B_y = \frac{2}{5}P \quad B_y = \frac{2}{5}P \uparrow$$

$$B_x = 4(\frac{2}{5}P) \quad B_x = \frac{8}{5}P \leftarrow$$

$$A_y = 2P \rightarrow$$

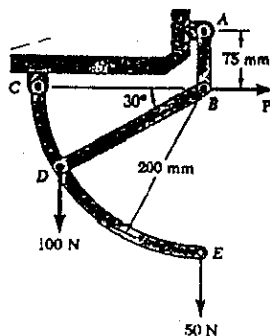
$$A_x = \frac{8}{5}P \uparrow$$

$$A = 2.06P \angle 14.0^\circ$$

$$B = 1.03P \angle 14.0^\circ$$

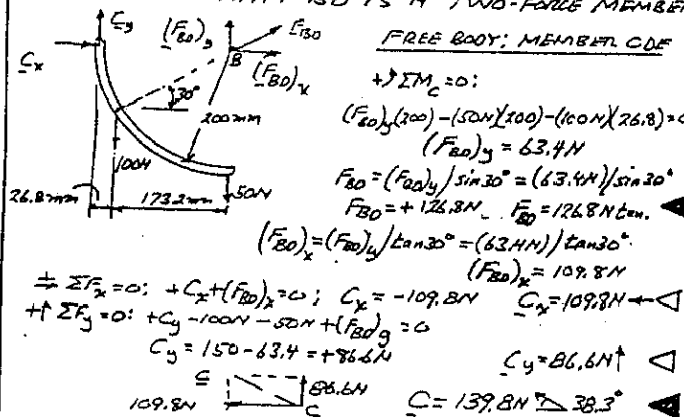
FRAME IS RIGID:  $A = 2.25P \angle 36.9^\circ; B = 1.03P \angle 14.0^\circ$

6.122

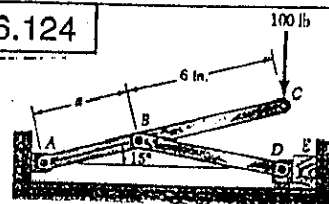


FIND:  
(a) Force  $P$  for equilibrium,  
(b) Force in  $BD$ ,  
(c) Reaction at  $C$ .

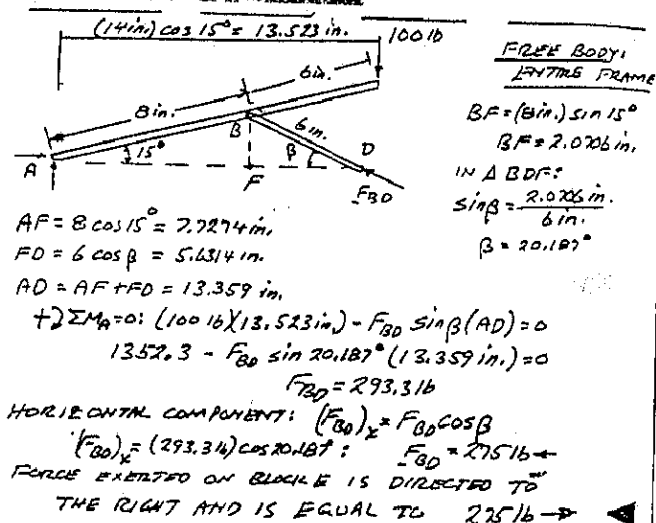
WE NOTE THAT  $BD$  IS A TWO-FORCE MEMBER.



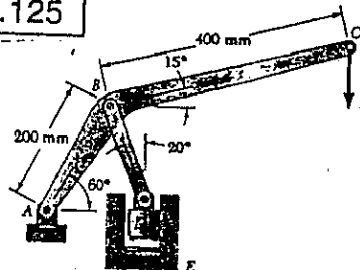
6.124



GIVEN:  $a = 8 \text{ in.}$   
 $BD = 6 \text{ in.}$   
FIND: HORIZONTAL  
FORCE EXERTED  
ON BLOCK E

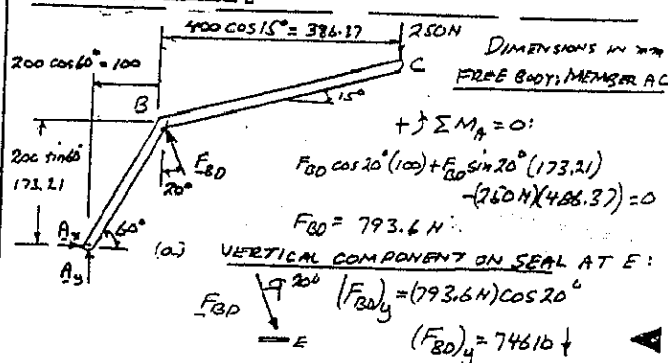


6.125

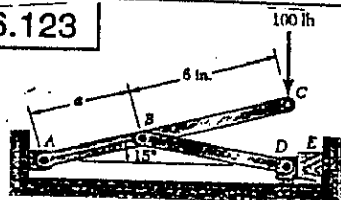


GIVEN:  $P = 250 \text{ N}$

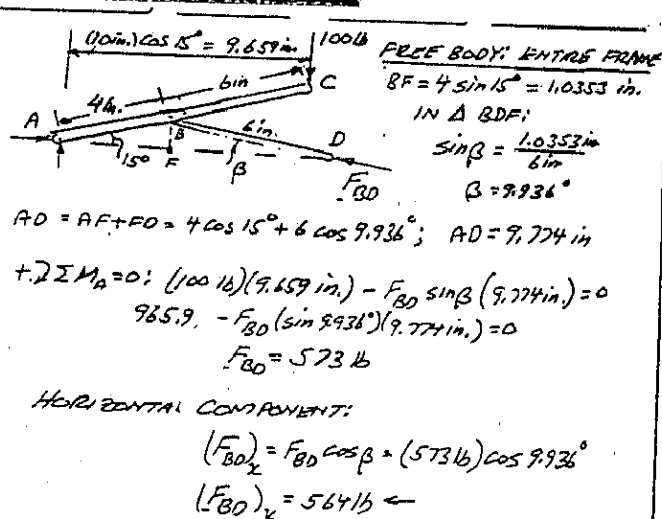
FIND: (a) VERTICAL  
COMPONENT OF FORCE  
EXERTED ON SEAL.  
(b) REACTION AT A.



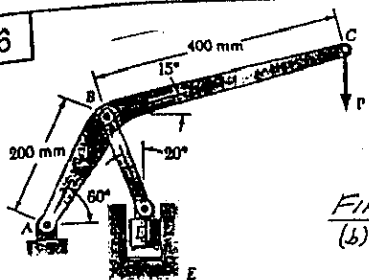
6.123



GIVEN:  $a = 4 \text{ in.}$   
 $BD = 6 \text{ in.}$   
FIND: HORIZONTAL  
FORCE EXERTED ON  
BLOCK E.



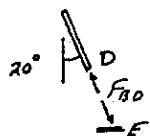
6.126



GIVEN: VERTICAL COMPONENT OF FORCE EXERTED ON SEAL E IS 900 N

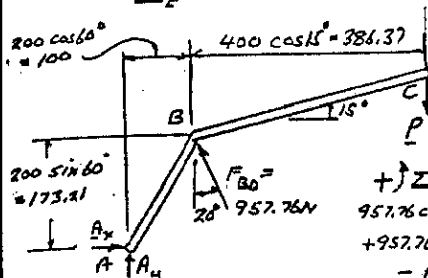
FIND: (a) FORCE P.  
(b) REACTION AT A

WE NOTE THAT BD IS A TWO-FORCE MEMBER



$$(F_{80})_y = F_{80} \cos 20^\circ = 900 \text{ N}$$

$$F_{80} = 957.76 \text{ N comp.}$$



FREE BODY: MEMBER ABC  
DIMENSIONS IN mm

$$+\circlearrowleft \Sigma M_A = 0:$$

$$957.76 \cos 20^\circ (100) + 957.76 \sin 20^\circ (173.21) - P(406.37) = 0$$

$$P = 301.7 \text{ N} \quad P = 302 \text{ N} \downarrow$$

$$+\rightarrow \Sigma F_x = 0: A_x - (957.76 \text{ N}) \sin 20^\circ = 0; A_x = 327.6 \text{ N}; A_x = 322.6 \text{ N} \rightarrow$$

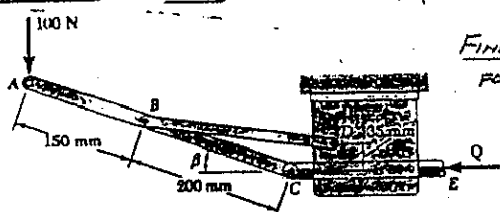
$$+\uparrow \Sigma F_y = 0: A_y + (957.76 \text{ N}) \cos 20^\circ - 301.7 \text{ N} = 0$$

$$A_y = 598.3 \text{ N} \quad A_y = 598.3 \text{ N} \downarrow$$

$$A = 682 \text{ N} \angle 61.3^\circ$$

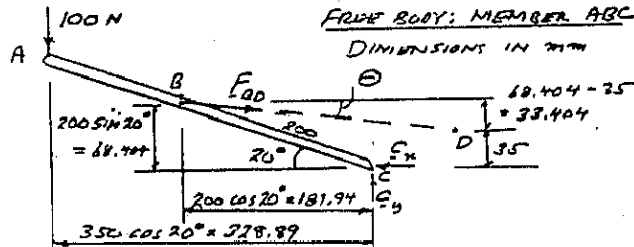
6.127

GIVEN:  $BD = 250 \text{ mm}$ ,  $\beta = 20^\circ$ .



FIND: FORCE Q FOR EQUILIBRIUM

WE NOTE THAT BD IS A TWO-FORCE MEMBER  
FREE BODY: MEMBER ABC  
DIMENSIONS IN mm



$$\text{SINCE } BD = 250, \theta = \sin^{-1} \frac{33.404}{250}; \theta = 7.679^\circ$$

$$+\circlearrowleft \Sigma M_A = 0: (F_{80} \sin \theta) 187.94 - (F_{80} \cos \theta) 68.404 - (100 \text{ N}) 328.89 = 0$$

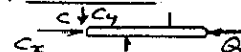
$$F_{80} [187.94 \sin 7.679^\circ - 68.404 \cos 7.679^\circ] = 32889$$

$$F_{80} = 770.6 \text{ N}$$

$$+\rightarrow \Sigma F_x = 0: (770.6 \text{ N}) \cos 7.679^\circ - C_x = 0$$

$$C_x = +763.7 \text{ N}$$

MEMBER CD

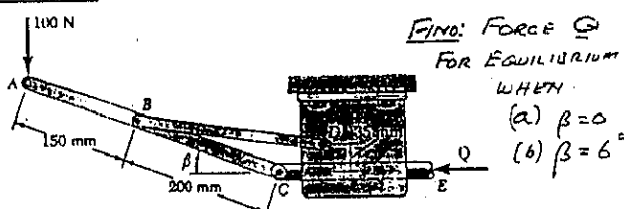


$$+\rightarrow \Sigma F_x = 0: Q = C_x = 763.7 \text{ N}$$

$$Q = 764 \text{ N} \leftarrow$$

6.128

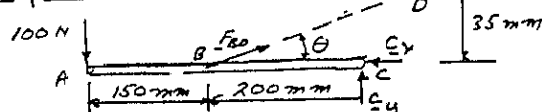
GIVEN:  $BD = 250 \text{ mm}$



FIND: FORCE Q FOR EQUILIBRIUM WHEN

- (a)  $\beta = 0$   
(b)  $\beta = 6^\circ$

WE NOTE THAT BD IS A TWO-FORCE MEMBER  
(a)  $\beta = 0$ : FREE BODY: MEMBER ABC



$$\text{SINCE } BD = 250 \text{ mm}, \sin \theta = \frac{35 \text{ mm}}{250 \text{ mm}}; \theta = 8.048^\circ$$

$$+\circlearrowleft \Sigma M_C = 0: (100 \text{ N}) (350 \text{ mm}) - F_{80} \sin \theta (200 \text{ mm}) = 0$$

$$F_{80} = 1250 \text{ N}$$

$$+\rightarrow \Sigma F_x = 0: F_{80} \cos \theta - C_x = 0$$

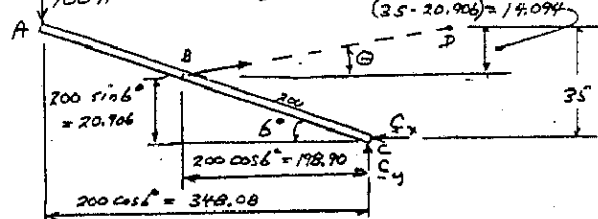
$$(1250 \text{ N}) (\cos 8.048^\circ) - C_x = 0 \quad C_x = 1237.7 \text{ N}$$

$$\text{MEMBER CE} \quad +\rightarrow \Sigma F_x = 0: (1237.7 \text{ N}) - Q = 0$$

$$Q = 1237.7 \text{ N}$$

$$Q = 1238 \text{ N} \leftarrow$$

(b)  $\beta = 6^\circ$ : FREE BODY: MEMBER ABC  
DIMENSIONS IN mm



$$\text{SINCE } BD = 250 \text{ mm}, \theta = \sin^{-1} \frac{14.094}{250 \text{ mm}}$$

$$\theta = 3.232^\circ$$

$$+\circlearrowleft \Sigma M_A = 0: (F_{80} \sin \theta) 198.90 + (F_{80} \cos \theta) 20.906 - (100 \text{ N}) 348.08 = 0$$

$$F_{80} [198.90 \sin 3.232^\circ + 20.906 \cos 3.232^\circ] = 34808$$

$$F_{80} = 1084.8 \text{ N}$$

$$+\rightarrow \Sigma F_x = 0: F_{80} \cos \theta - C_x = 0$$

$$(1084.8 \text{ N}) \cos 3.232^\circ - C_x = 0$$

$$C_x = +1083.1 \text{ N}$$

MEMBER DE:

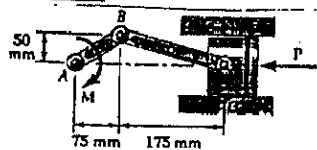


$$+\rightarrow \Sigma F_x = 0: Q = C_x$$

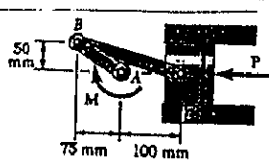
$$Q = 1083.1 \text{ N}$$

$$Q = 1083 \text{ N} \leftarrow$$

6.129

GIVEN:  $M = 1.5 \text{ kN}\cdot\text{m}$ . FIND: Force  $P$ 

(a)



(b)

FREE BODY: PISTON

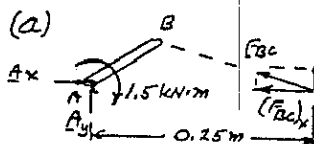
$$\begin{aligned} \sum F_x = 0: (F_{BC})_x - P &= 0 \\ P &= (F_{BC})_x \quad (1) \end{aligned}$$

SINCE BC IS A TWO-FORCE MEMBER

$$\frac{(F_{BC})_y}{50} = \frac{(F_{BC})_x}{175}; \quad (F_{BC})_y = \frac{50}{175} (F_{BC})_x$$

$$\text{USE EQ(1): } (F_{BC})_y = \frac{50}{175} P; \quad (F_{BC})_y = \frac{2}{7} P$$

(a)



FREE BODY: CRANK AB

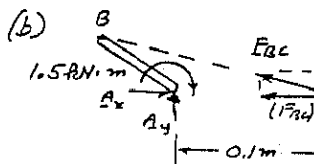
$$+\circlearrowleft \sum M_A = 0:$$

$$(F_{BC})_y(0.25 \text{ m}) - 1.5 \text{ kN}\cdot\text{m} = 0$$

$$\frac{2}{7} P(0.25) = 1.5$$

$$P = 21 \text{ kN} \quad P = 21 \text{ kN} \leftarrow$$

(b)



FREE BODY: CRANK AB

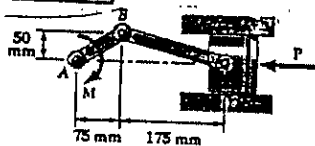
$$+\circlearrowleft \sum M_A = 0:$$

$$(F_{BC})_y(0.1 \text{ m}) - 1.5 \text{ kN}\cdot\text{m} = 0$$

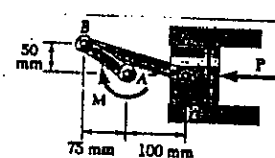
$$\frac{2}{7} P(0.1) = 1.5$$

$$P = 52.5 \text{ kN} \quad P = 52.5 \text{ kN} \leftarrow$$

6.130

GIVEN:  $P = 16 \text{ kN}$ . FIND: Couple  $M$ 

(a)



(b)

FREE BODY: PISTON

$$\sum F_x = 0: (F_{BC})_x = 16 \text{ kN}$$

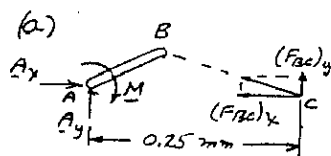
SINCE BC IS A

TWO-FORCE MEMBER

$$\frac{(F_{BC})_y}{50} = \frac{(F_{BC})_x}{175}; \quad (F_{BC})_y = \frac{50}{175} (F_{BC})_x$$

$$(F_{BC})_y = \frac{16 \text{ kN}}{175} \times 50; \quad (F_{BC})_y = 4.571 \text{ kN}$$

(a)



FREE BODY: CRANK AB

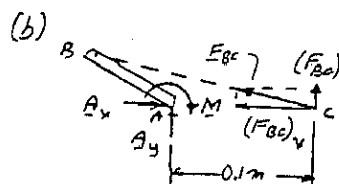
$$+\circlearrowleft \sum M_A = 0:$$

$$M - (F_{BC})_y(0.25 \text{ m})$$

$$M = (4.571 \text{ kN})(0.25 \text{ m})$$

$$M = 1.143 \text{ kN}\cdot\text{m} \quad M = 1.143 \text{ kN}\cdot\text{m} \leftarrow$$

(b)



FREE BODY: CRANK AB

$$+\circlearrowleft \sum M_A = 0:$$

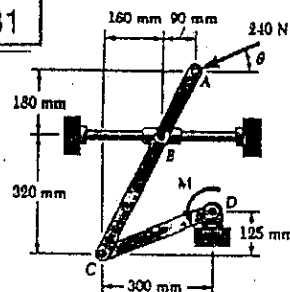
$$M - (F_{BC})_y(0.1 \text{ m}) = 0$$

$$M = (4.571 \text{ kN})(0.1 \text{ m})$$

$$M = 0.4571 \text{ kN}\cdot\text{m}$$

$$M = 457 \text{ N}\cdot\text{m} \leftarrow$$

6.131

GIVEN:  $\theta = 0$ FIND: Couple  $M$  FOR EQUILIBRIUM

FREE BODY: MEMBER ABC

$$+\circlearrowleft \sum F_x = 0: C_x - 240 \text{ N} = 0$$

$$C_x = +240 \text{ N}$$

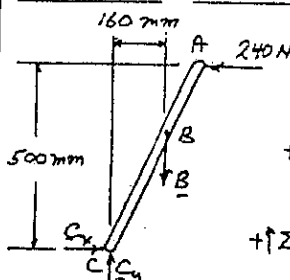
$$+\circlearrowleft \sum M_C = 0:$$

$$(240 \text{ N})(500 \text{ mm}) - B(40 \text{ mm}) = 0$$

$$B = +750 \text{ N}$$

$$+\uparrow \sum F_y = 0: C_y - 750 \text{ N} = 0$$

$$C_y = +750 \text{ N}$$



FREE BODY: MEMBER CD

$$+\circlearrowleft \sum M_D = 0:$$

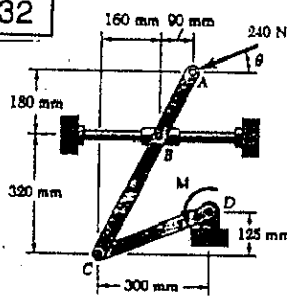
$$M + (750 \text{ N})(300 \text{ mm})$$

$$- (240 \text{ N})(125 \text{ mm}) = 0$$

$$M = -195 \times 10^3 \text{ N}\cdot\text{mm}$$

$$M = 195 \text{ kN}\cdot\text{m} \leftarrow$$

6.132

GIVEN:  $\theta = 90^\circ$ FIND: Couple  $M$  FOR EQUILIBRIUM

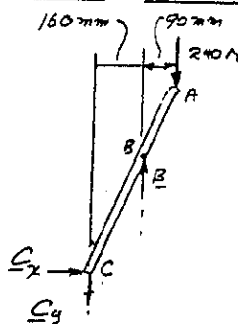
FREE BODY: MEMBER ABC

$$+\circlearrowleft \sum F_x = 0: C_x = 0$$

$$+\circlearrowleft \sum M_B = 0:$$

$$C_y(160 \text{ mm}) - (240 \text{ N})(90 \text{ mm}) = 0$$

$$C_y = +135 \text{ N}$$



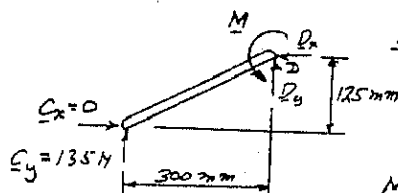
FREE BODY: MEMBER CD

$$+\circlearrowleft \sum M_D = 0:$$

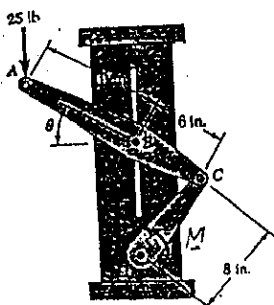
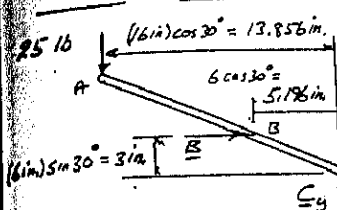
$$M - (135 \text{ N})(300 \text{ mm}) = 0$$

$$M = +40.5 \times 10^3 \text{ N}\cdot\text{mm}$$

$$M = 40.5 \text{ kN}\cdot\text{m} \leftarrow$$



6.133

GIVEN:  $\theta = 30^\circ$ FIND: COUPLE  $M$   
FOR EQUILIBRIUM

FREE BODY: MEMBER ABC

$$+\circlearrowleft \Sigma M_C = 0$$

$$(25 \text{ lb})(5.196 \text{ in}) - B(3 \text{ in}) = 0$$

$$B = +115.47 \text{ N}$$

$$+\uparrow \Sigma F_y = 0$$

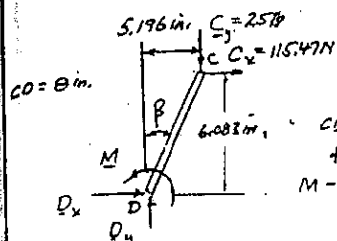
$$-25 \text{ lb} + C_y = 0$$

$$C_y = +25 \text{ lb}$$

$$+\rightarrow \Sigma F_x = 0: 115.47 \text{ N} - C_x = 0$$

$$C_x = +115.47 \text{ lb}$$

FREE BODY: MEMBER CD



$$\beta = \sin^{-1} \frac{5.196}{8}; \beta = 40.505^\circ$$

$$CD \cos \beta = (8 \text{ in}) \cos 40.505^\circ = 6.083 \text{ in}$$

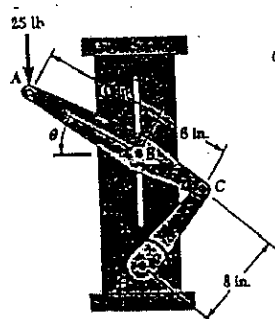
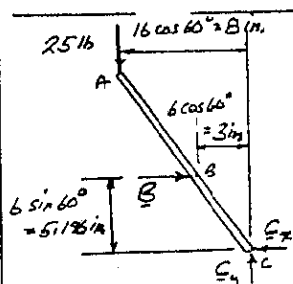
$$+\circlearrowleft \Sigma M_D = 0$$

$$M - (25 \text{ lb})(5.196 \text{ in}) - (115.47 \text{ lb})(6.083 \text{ in}) = 0$$

$$M = +832.3 \text{ lb}\cdot\text{in.}$$

$$M = 832 \text{ lb}\cdot\text{in.}$$

6.134

GIVEN:  $\theta = 60^\circ$ FIND: COUPLE  $M$   
FOR EQUILIBRIUM

FREE BODY: MEMBER ABC

$$+\circlearrowleft \Sigma M_C = 0: (25 \text{ lb})(3 \text{ in}) - B(5.196 \text{ in}) = 0$$

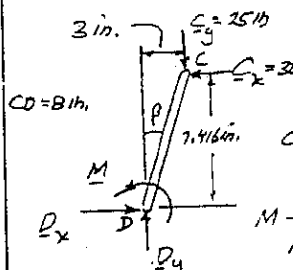
$$B = +38.49 \text{ lb}$$

$$+\rightarrow \Sigma F_x = 0: 38.49 \text{ lb} - C_x = 0$$

$$C_x = +38.49 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: -25 \text{ lb} + C_y = 0$$

$$C_y = +25 \text{ lb}$$



FREE BODY: MEMBER CD

$$\beta = \sin^{-1} \frac{3}{8}; \beta = 22.024^\circ$$

$$CD \cos \beta = (8 \text{ in}) \cos 22.024^\circ = 7.416 \text{ in.}$$

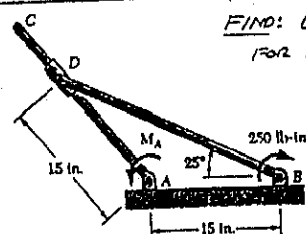
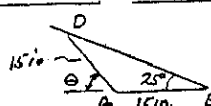
$$+\circlearrowleft \Sigma M_D = 0$$

$$M - (25 \text{ lb})(3 \text{ in}) - (38.49 \text{ lb})(7.416 \text{ in}) = 0$$

$$M = +360.4 \text{ lb}\cdot\text{in.}$$

$$M = 360 \text{ lb}\cdot\text{in.}$$

6.135

FIND: COUPLE  $M_A$   
FOR EQUILIBRIUMGEOMETRY:  $\triangle ABD$  IS ISOSCELES  
 $\therefore \theta = \angle ABD = 50^\circ$ 

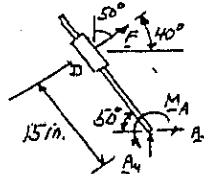
$$BD = 2[(15 \text{ in.}) \cos 25^\circ] = 27.189 \text{ in.}$$

FREE BODY: MEMBER AC

 $F$  IS  $\perp$  TO  $AD$ ,  $\angle 40^\circ$ 

$$+\circlearrowleft \Sigma M_A = 0: M_A - F(15 \text{ in.}) = 0$$

$$M_A = (15 \text{ in.})F \quad (1)$$



FREE BODY: MEMBER BD

$$+\circlearrowleft \Sigma M_B = 0$$

$$(F \sin 65^\circ)(27.189 \text{ in.}) - 250 \text{ lb}\cdot\text{in.} = 0$$

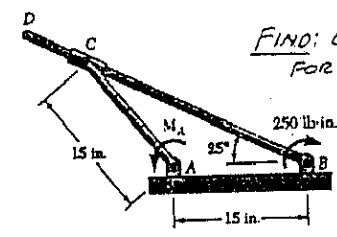
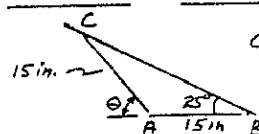
$$F = +10.145 \text{ lb}$$

FROM EQ (1):

$$M_A = (15 \text{ in.})(10.145 \text{ lb})$$

$$M_A = 152.2 \text{ lb}\cdot\text{in.}$$

6.136

FIND: COUPLE  $M_A$   
FOR EQUILIBRIUMGEOMETRY:  $\triangle ABC$  IS ISOSCELES

$$\therefore \theta = 50^\circ$$

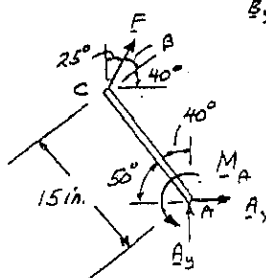
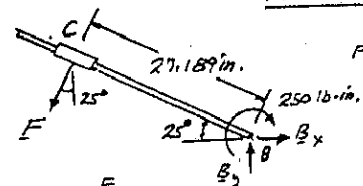
$$BC = 2[(15 \text{ in.}) \cos 25^\circ] = 27.189 \text{ in.}$$

FREE BODY: MEMBER BC

$$+\circlearrowleft \Sigma M_B = 0$$

$$F(27.189 \text{ in.}) - 250 \text{ lb}\cdot\text{in.} = 0$$

$$F = +9.195 \text{ lb}$$



FREE BODY: MEMBER AC

$$\beta = 90^\circ - 25^\circ - 40^\circ = 25^\circ$$

$$+\circlearrowleft \Sigma M_A = 0$$

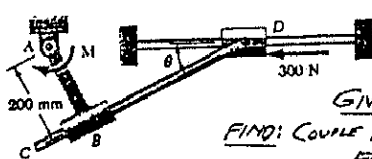
$$M_A - (F \cos 25^\circ)(15 \text{ in.}) = 0$$

$$M_A = (9.195 \text{ lb}) \cos 25^\circ (15 \text{ in.})$$

$$M_A = +125.0 \text{ lb}\cdot\text{in.}$$

$$M_A = 125 \text{ lb}\cdot\text{in.}$$

6.137



GIVEN:  $\theta = 30^\circ$   
FIND: COUPLE  $M$  FOR  
EQUILIBRIUM

FREE BODY: ROD CD

IN  $\triangle ABD$ :

$$AD = (200 \text{ mm}) / \sin 30^\circ$$

$$AD = 400 \text{ mm} = 0.4 \text{ m}$$

$$+\uparrow \Sigma F = 0$$

$$D \sin 30^\circ - (300 \text{ N}) \cos 30^\circ = 0$$

$$D = +519.62 \text{ N} \quad P = 379.62 \text{ N} \uparrow$$

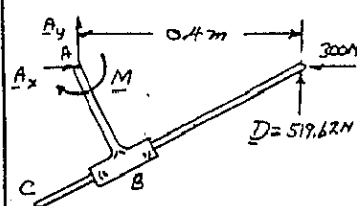
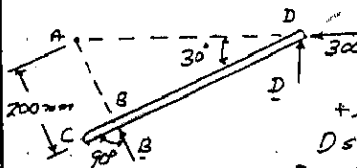
FREE BODY: ENTIRE FRAME

$$+\circlearrowleft \Sigma M_A = 0$$

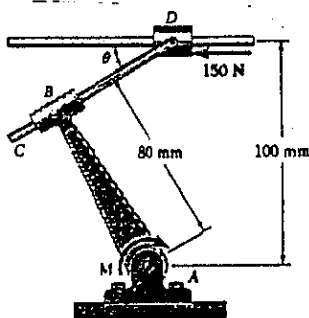
$$(519.62 \text{ N})(0.4 \text{ m}) - M = 0$$

$$M = +207.84 \text{ N}\cdot\text{m}$$

$$M = 208 \text{ N}\cdot\text{m}$$



6.138

GIVEN:  $\theta = 30^\circ$ 

FIND:  
COUPLE  $M$   
FOR EQUILIBRIUM

FREE BODY: ENTIRE FRAME

GEOMETRY

$$a = 80 \cos 60^\circ = 40 \text{ mm}$$

$$b = 80 \sin 60^\circ = 69.282 \text{ mm}$$

$$c = 100 - b = 30.718 \text{ mm}$$

$$d = c / \tan 30^\circ = 53.205 \text{ mm}$$

$$e = d - a = 53.205 - 40$$

$$e = 13.205 \text{ mm}$$

$$+\circlearrowleft \Sigma M_A = 0$$

$$(150 \text{ N})(100 \text{ mm}) + D(13.205 \text{ mm}) - M = 0$$

$$M = 15 \text{ N}\cdot\text{m} + D(13.205 \text{ mm}) \quad (1)$$

FREE BODY: ROD CD

$$+\circlearrowleft \Sigma F = 0$$

$$D \sin 30^\circ - (150 \text{ N}) \cos 30^\circ = 0$$

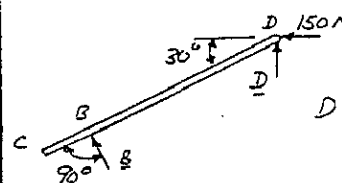
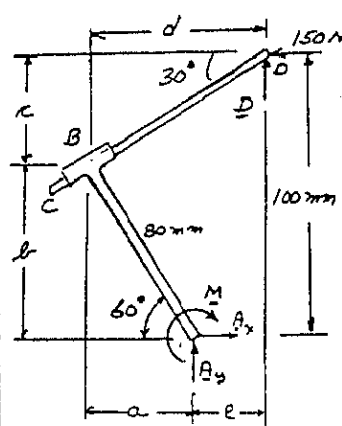
$$D = 259.81 \text{ N}$$

RETURN TO EQ(1):

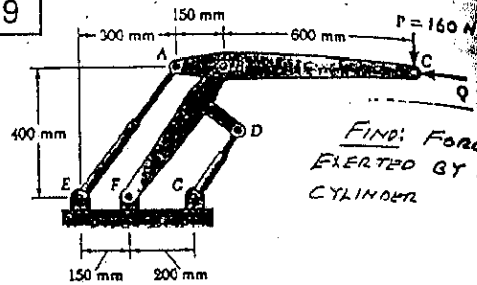
$$M = 15 \text{ N}\cdot\text{m} + (259.81 \text{ N})(0.013205 \text{ m})$$

$$M = +18.431 \text{ N}\cdot\text{m}$$

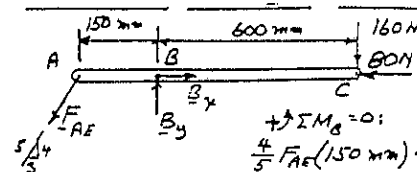
$$M = 18.43 \text{ N}\cdot\text{m}$$



6.139



FIND: FORCE  
EXERTED BY PIN  
CYLINDER



FREE BODY: MEMBER ABC

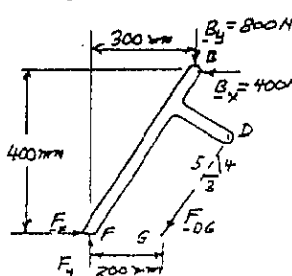
$$+\circlearrowleft \Sigma M_B = 0$$

$$\frac{4}{5} F_{AE}(150 \text{ mm}) - (160 \text{ N})(600 \text{ mm}) = 0$$

$$F_{AE} = +800 \text{ N} \quad F_{AE} = 800 \text{ N T}$$

$$+\circlearrowleft \Sigma F_x = 0: -\frac{3}{5}(800 \text{ N}) + B_x - 80 \text{ N} = 0 \quad B_x = +560 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: -\frac{4}{5}(800 \text{ N}) + B_y - 160 \text{ N} = 0 \quad B_y = +800 \text{ N}$$



FREE BODY: MEMBER BDE

$$+\circlearrowleft \Sigma M_D = 0$$

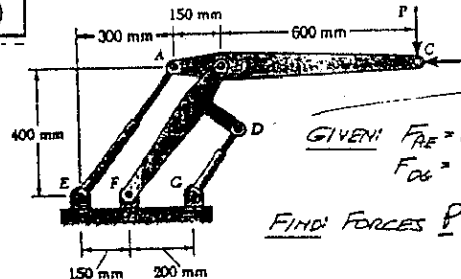
$$(560 \text{ N})(400 \text{ mm}) - (800 \text{ N})(300 \text{ mm})$$

$$- \frac{4}{5} F_{DE}(200 \text{ mm}) = 0$$

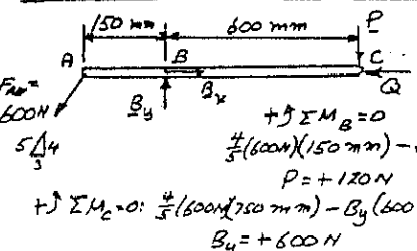
$$F_{DE} = -100 \text{ N}$$

$$F_{DE} = 100 \text{ N C}$$

6.140



GIVEN:  $F_{AE} = 600 \text{ N T}$   
 $F_{DE} = 50 \text{ N T}$

FIND: FORCES  $P$  AND  $Q$ 

FREE BODY: MEMBER ABC

$$+\circlearrowleft \Sigma M_B = 0$$

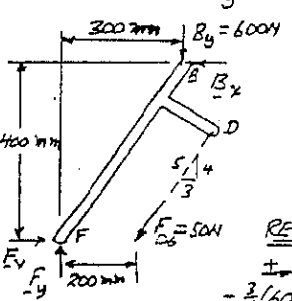
$$\frac{4}{5}(600 \text{ N})(150 \text{ mm}) - P(600 \text{ mm}) = 0$$

$$P = +120 \text{ N}$$

$$P = 120 \text{ N} \uparrow$$

$$+\circlearrowleft \Sigma M_C = 0: \frac{4}{5}(600 \text{ N})(750 \text{ mm}) - B_y(600 \text{ mm}) = 0$$

$$B_y = +600 \text{ N}$$



FREE BODY: MEMBER BDE

$$+\circlearrowleft \Sigma M_D = 0$$

$$B_x(400 \text{ mm}) - (100 \text{ N})(300 \text{ mm})$$

$$- \frac{4}{5}(50 \text{ N})(200 \text{ mm}) = 0$$

$$B_x = +40 \text{ N}$$

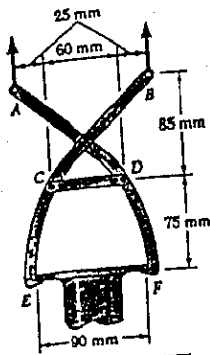
RETURN TO FREE BODY: MEMBER ABC

$$+\circlearrowleft \Sigma F_x = 0$$

$$-\frac{3}{5}(600 \text{ N}) + 40 \text{ N} - Q = 0$$

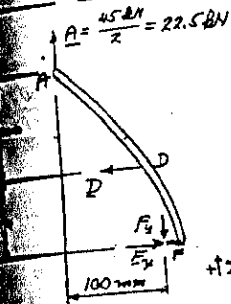
$$Q = +110 \text{ N}$$

$$Q = 110 \text{ N} \leftarrow$$

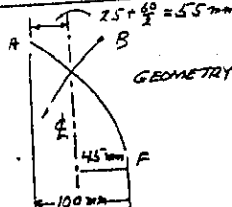


**GIVEN:** TONGS  
EXERT UPWARD  
FORCE OF 45 kN  
ON PIPE CAP.

**FIND:** FORCES EXERTED  
AT D AND F ON  
TONG ADF.



**FREE BODY: TONG ADF**



**GEOMETRY**

$$A = \frac{45 \text{ kN}}{2} = 22.5 \text{ kN}$$

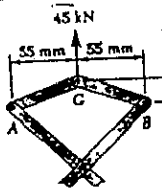
$$\uparrow \sum F_y = 0: 22.5 \text{ kN} - F_y = 0 \quad F_y = +22.5 \text{ kN}$$

$$\sum M_D = 0: D(75 \text{ mm}) - (22.5 \text{ kN})(100 \text{ mm}) = 0 \quad D = +30 \text{ kN}$$

$$\uparrow \sum F_x = 0: -30 \text{ kN} + F_x = 0 \quad F_x = +30 \text{ kN}$$

$$F = 37.5 \text{ kN} \angle 36.9^\circ$$

142



**GIVEN:** TOGGLE SHOWN  
IS ADDED TO TONGS  
OF PROB. 6.141  
**FIND:** FORCES EXERTED  
AT D AND E ON  
TONG ADF

**FREE BODY: TOGGLE**

BY SYMMETRY:  $A_y = \frac{1}{2}(45 \text{ kN}) = 22.5 \text{ kN}$

AG IS A TWO-FORCE MEMBER.

$$\frac{22.5 \text{ kN}}{22 \text{ mm}} = \frac{A_x}{55 \text{ mm}} \quad A_x = 56.25 \text{ kN}$$

**FREE BODY: TONG ADF**

$$\uparrow \sum F_y = 0: 22.5 \text{ kN} - F_y = 0 \quad F_y = +22.5 \text{ kN}$$

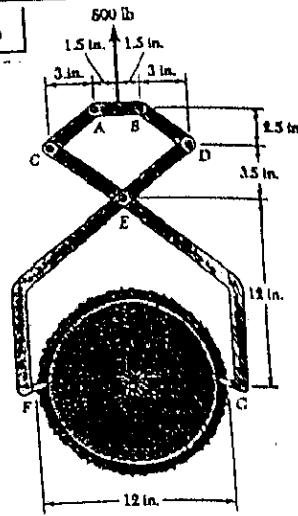
$$\uparrow \sum M_D = 0: D(75 \text{ mm}) - (22.5 \text{ kN})(100 \text{ mm}) - (56.25 \text{ kN})(60 \text{ mm}) = 0$$

$$D = +150 \text{ kN} \quad D = 150 \text{ kN}$$

$$\uparrow \sum F_x = 0: 56.25 \text{ kN} - 150 \text{ kN} + F_x = 0 \quad F_x = 93.75 \text{ kN}$$

$$F = 96.4 \text{ kN} \angle 13.5^\circ$$

6.143



**GIVEN:** WEIGHT  
OF LOG IS  
800 lb

**FIND:** FORCES  
EXERTED AT  
E AND F ON  
TONG DEF.

**FREE BODY:**  
**MEMBERS CA, AB, BD**

BY SYMMETRY

$$D_y = \frac{1}{2}(800 \text{ lb}) = 400 \text{ lb}$$

SINCE BD IS A TWO-FORCE MEMBER

$$\frac{D_y}{2.5 \text{ in}} = \frac{D_x}{3 \text{ in}}; \frac{400 \text{ lb}}{2.5 \text{ in}} = \frac{D_x}{3 \text{ in}}; \quad D_x = 480 \text{ lb}$$

**FREE BODY: TONG DEF**

$$\uparrow \sum F_y = 0: 400 \text{ lb} + E_y - 400 \text{ lb} = 0 \quad E_y = 0$$

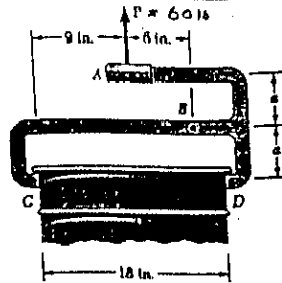
$$\uparrow \sum M_E = 0: (400 \text{ lb})(10.5 \text{ in}) + (480 \text{ lb})(15 \text{ in}) - E_x(12 \text{ in}) = 0$$

$$E_x = 970 \text{ lb} \quad E = 970 \text{ lb}$$

$$\uparrow \sum F_x = 0: 970 \text{ lb} - 480 \text{ lb} - F_x = 0 \quad F_x = 490 \text{ lb}$$

$$F = 633 \text{ lb} \angle 39.2^\circ$$

6.144



**GIVEN:**  $\alpha = 5^\circ$   
WEIGHT OF  
BARREL = 60 lb

**FIND:** FORCES  
EXERTED AT B AND  
D ON TONG ABD

NOTE THAT BC IS A TWO-FORCE MEMBER.

**FREE BODY: TONG ABD**

$$\frac{B_y}{15} = \frac{B_x}{5} \quad B_x = 3B_y$$

$$\uparrow \sum M_D = 0: B_y(3 \text{ in}) + 3B_y(5 \text{ in}) - (60 \text{ lb})(9 \text{ in}) = 0$$

$$B_y = 30 \text{ lb} \quad B_x = 90 \text{ lb}$$

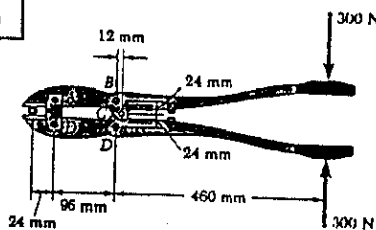
$$\uparrow \sum F_x = 0: -90 \text{ lb} + D_x = 0 \quad D_x = 90 \text{ lb}$$

$$\uparrow \sum F_y = 0: 60 \text{ lb} - 30 \text{ lb} - D_y = 0 \quad D_y = 30 \text{ lb}$$

$$B = 94.9 \text{ lb} \angle 18.4^\circ$$

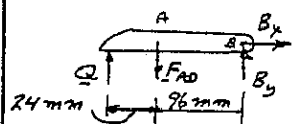
$$D = 94.9 \text{ lb} \angle 18.4^\circ$$

6.145



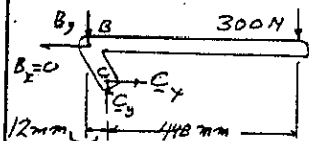
FIND: MAGNITUDE  
OF FORCES  
EXERTED  
ON BOLT

WE NOTE THAT AD IS A TWO-FORCE MEMBER



FREE BODY: JAW AB

$$\begin{aligned} \pm \Sigma F_x = 0: & B_x = 0 \\ + \Sigma M_A = 0: & B_y(96 \text{ mm}) - Q(24 \text{ mm}) = 0 \\ & Q = 4 B_y \quad (1) \end{aligned}$$

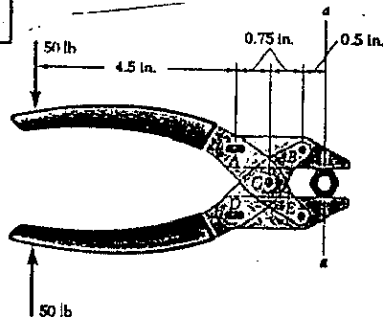


FREE BODY: HANDLE

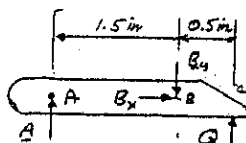
$$\begin{aligned} + \Sigma M_C = 0: & B_y(12 \text{ mm}) - (300 \text{ N})(448 \text{ mm}) = 0 \\ & B_y = 11.2 \times 10^3 \text{ N} = 11.2 \text{ kN} \end{aligned}$$

$$\text{EQ(1): } Q = 4 B_y = 4(11.2 \text{ kN}) = 44.8 \text{ kN} \quad Q = 44.8 \text{ kN}$$

6.146

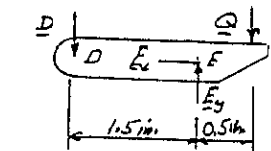


FIND: GRIPPING  
FORCES  
EXERTED ALONG  
LINE a-a ON  
THE NUT



FREE BODY: UPPER JAW

$$\begin{aligned} \pm \Sigma F_x = 0: & B_x = 0 \\ + \Sigma M_A = 0: & Q(0.5 \text{ in}) - B_y(1.5 \text{ in}) = 0 \\ & B_y = Q/3 \end{aligned}$$

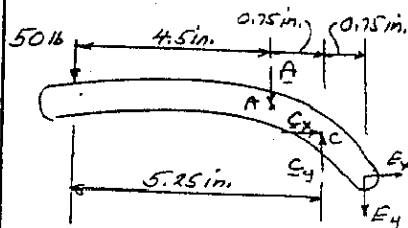


FREE BODY: LOWER JAW

$$\begin{aligned} \pm \Sigma F_x = 0: & E_x = 0 \\ + \Sigma M_D = 0: & E_y(1.5 \text{ in}) - Q(2 \text{ in}) = 0 \\ & E_y = 4Q/3 \end{aligned}$$

FREE BODY:  
HANDLE

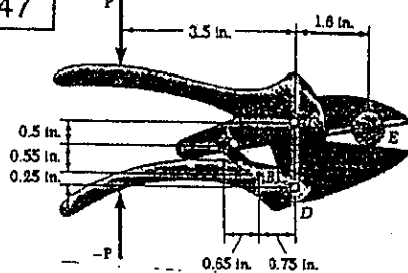
$$\begin{aligned} \text{FROM ABOVE:} \\ A = Q/3 \\ E_y = 4Q/3 \end{aligned}$$



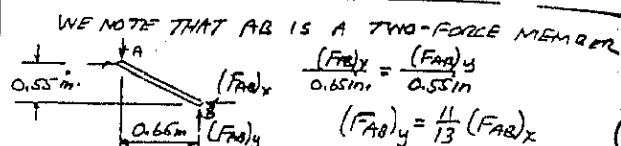
$$\begin{aligned} + \Sigma M_C = 0: & (50 \text{ lb})(5.25 \text{ in}) + A(0.75 \text{ in}) - E_y(0.75 \text{ in}) = 0 \\ & 262.5 \text{ lb} \cdot \text{in} + (Q/3)(0.75 \text{ in}) - (4Q/3)(0.75 \text{ in}) = 0 \\ & 262.5 \text{ lb} \cdot \text{in} - Q(0.75 \text{ in}) = 0 \\ & Q = 350 \text{ lb} \end{aligned}$$

$$Q = 350 \text{ lb}$$

6.147

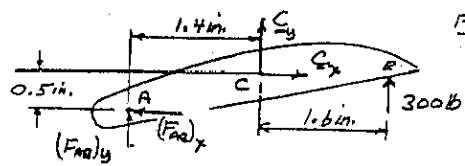


FIND:  
MAGNITUDE  
FOR 300-LB  
VERTICAL  
CUTTING FORCE  
ON BRANCH  
AT E.



WE NOTE THAT AB IS A TWO-FORCE MEMBER

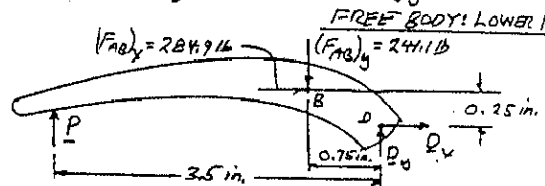
$$\begin{aligned} \frac{(F_{AB})_x}{0.65 \text{ in}} &= \frac{(F_{AB})_y}{0.55 \text{ in}} \\ (F_{AB})_y &= \frac{11}{13} (F_{AB})_x \quad (1) \end{aligned}$$



FREE BODY:  
BLADE AC

$$\begin{aligned} + \Sigma M_E = 0: & (300 \text{ lb})(1.6 \text{ in}) - (F_{AB})_x(0.5 \text{ in}) - (F_{AB})_y(1.4 \text{ in}) = 0 \\ \text{USE EQ(1):} & (F_{AB})_x(0.5 \text{ in}) + \frac{11}{13} (F_{AB})_x(1.4 \text{ in}) = 480 \text{ lb} \cdot \text{in} \\ & 1.6846 (F_{AB})_x = 480 \quad (F_{AB})_x = 284.9 \text{ lb} \end{aligned}$$

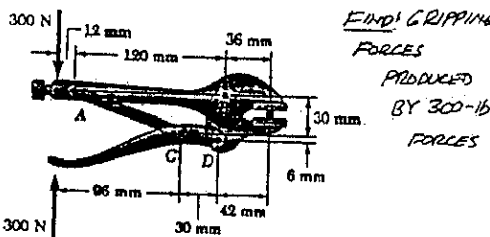
$$(F_{AB})_y = \frac{11}{13} (284.9 \text{ lb}) \quad (F_{AB})_y = 241.1 \text{ lb}$$



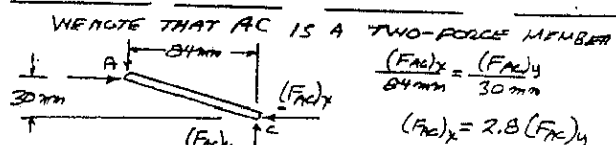
FREE BODY: LOWER HANDLE

$$\begin{aligned} + \Sigma M_D = 0: & (241.1 \text{ lb})(0.75 \text{ in}) - (284.9 \text{ lb})(0.25 \text{ in}) - P(3.5 \text{ in}) = 0 \\ & P = 31.3 \text{ lb} \end{aligned}$$

6.148

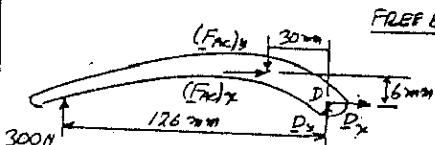


FIND: GRIPPING  
FORCES  
PRODUCED  
BY 300-N  
FORCES



WE NOTE THAT AC IS A TWO-FORCE MEMBER

$$\begin{aligned} \frac{(F_{AC})_x}{84 \text{ mm}} &= \frac{(F_{AC})_y}{30 \text{ mm}} \\ (F_{AC})_x &= 2.8 (F_{AC})_y \end{aligned}$$



FREE BODY: LOWER HANDLE

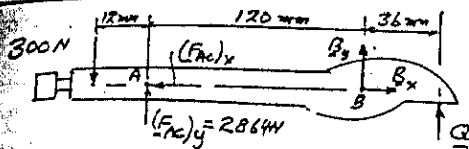
$$\begin{aligned} + \Sigma M_D = 0: & (F_{AC})_y(30 \text{ mm}) - (F_{AC})_x(6 \text{ mm}) - (300 \text{ N})(126 \text{ mm}) = 0 \\ & (F_{AC})_y(30) - 2.8 (F_{AC})_y(6) - (300)(126) = 0 \\ & (F_{AC})_y = 2884 \text{ N} \end{aligned}$$

(CONTINUED)



# 148 CONTINUED

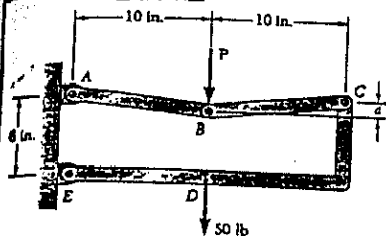
FREE BODY: UPPER HANDLE



$$+\circlearrowleft \Sigma M_B = 0: (300N)(132mm) - (2864N)(120mm) + Q(36mm) = 0$$

$$Q = 8450N \quad Q = 8.45kN$$

## 6.149 and 6.150

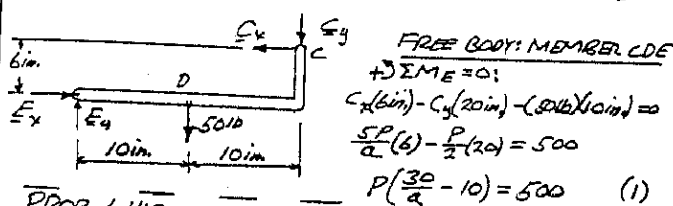


FIND FORCE  $P$   
REQUIRED FOR  
EQUILIBRIUM  
PROB. 6.149  
WHEN  $\alpha = 1 \text{ in.}$   
PROB. 6.150  
WHEN  $\alpha = 0.5 \text{ in.}$

WE NOTE THAT AB AND BC ARE TWO-FORCE MEMBERS

FREE BODY: DOBBLE  
BY SYMMETRY:  
 $C_y = P/2$

$$\frac{C_x}{10 \text{ in.}} = \frac{C_y}{\alpha}; C_x = \frac{10}{\alpha} C_y = \frac{10}{\alpha} \cdot \frac{P}{2} = \frac{5P}{\alpha}$$



FREE BODY: MEMBER CDE  
 $+\circlearrowleft \Sigma M_E = 0:$   
 $C_x(6 \text{ in.}) - C_y(20 \text{ in.}) - (50 \text{ lb})(10 \text{ in.}) = 0$   
 $\frac{5P}{\alpha}(6) - \frac{P}{2}(20) = 500$   
 $P(\frac{30}{\alpha} - 10) = 500 \quad (1)$

PROB. 6.149:  $\alpha = 1 \text{ in.}$

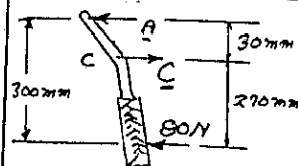
$$F_{D1}: P(\frac{30}{1} - 10) = 500; 20P = 500 \quad P = 25 \text{ lb}$$

PROB. 6.150:  $\alpha = 0.5 \text{ in.}$

$$F_{D2}: P(\frac{30}{0.5} - 10) = 500; 50P = 500 \quad P = 10 \text{ lb}$$

## 6.151 CONTINUED

FREE BODY: RIGHT HANDLE



$$+\circlearrowleft \Sigma M_C = 0:$$

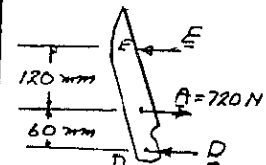
$$A(30 \text{ mm}) - (80N)(270 \text{ mm}) = 0$$

$$A = +720N$$

$$+\circlearrowright \Sigma F_x = 0: C - 720N - 80N = 0$$

$$C = +800N$$

FREE BODY: LEFT BLADE

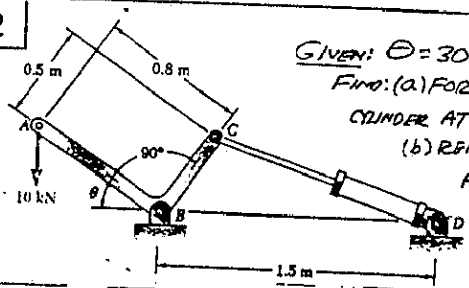


$$+\circlearrowleft \Sigma M_D = 0$$

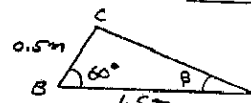
$$E(180 \text{ mm}) - (720N)(60 \text{ mm}) = 0$$

$$E = 240N$$

## 6.152



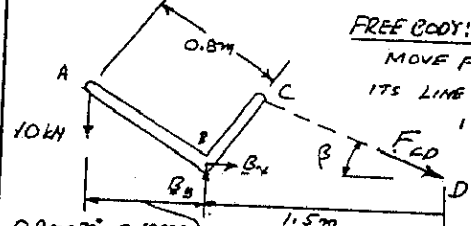
GIVEN:  $\theta = 30^\circ$   
FIND: (a) FORCE OF  
CABLE AT C.  
(b) REACTION  
AT B.



GEOMETRY: IN  $\triangle ABC$   
LAW OF COSINES  
 $(CD)^2 = (0.5)^2 + (1.5)^2 - 2(0.5)(1.5)\cos 60^\circ$   
 $CD = 1.3229 \text{ m}$

LAW OF SINES

$$\frac{\sin \beta}{0.5 \text{ m}} = \frac{\sin 60^\circ}{1.3229 \text{ m}}; \sin \beta = 0.3223; \beta = 19.107^\circ$$



FREE BODY: ENTIRE SYSTEM  
MOVE FORCE  $F_{CD}$  ALONG  
ITS LINE OF ACTION SO  
IT ACTS AT D.

$$0.8 \cos 30^\circ = 0.69282 \text{ m}$$

$$+\circlearrowleft \Sigma M_B = 0: (10 \text{ kN})(0.69282 \text{ m}) - F_{CD} \sin \beta (1.5 \text{ m}) = 0$$

$$6.9282 \text{ kN} \cdot \text{m} - F_{CD} \sin 19.107^\circ (1.5 \text{ m}) = 0$$

$$F_{CD} = 14.111 \text{ kN}$$

$$+\circlearrowright \Sigma F_x = 0: B_x + F_{CD} \cos \beta = 0$$

$$B_x + (14.111 \text{ kN}) \cos 19.107^\circ = 0$$

$$B_x = -13.333 \text{ kN} \quad B_x = 13.333 \text{ kN}$$

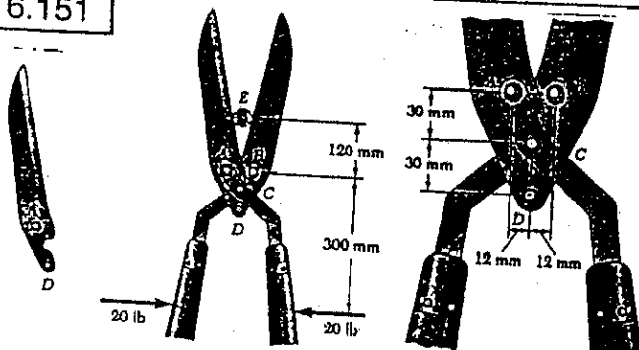
$$+\uparrow \Sigma F_y = 0: B_y - 10 \text{ kN} - F_{CD} \sin \beta = 0$$

$$B_y - 10 \text{ kN} - (14.111 \text{ kN}) \sin 19.107^\circ = 0$$

$$B_y = +14.619 \text{ kN} \quad B_y = 14.619 \text{ kN}$$

$$B = 19.79 \text{ kN} \angle 47.6^\circ$$

## 6.151

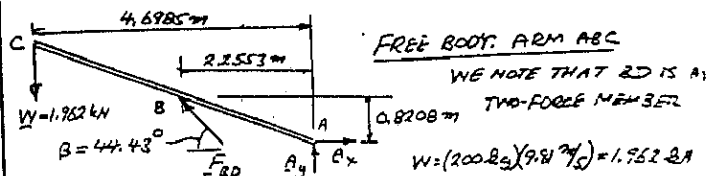
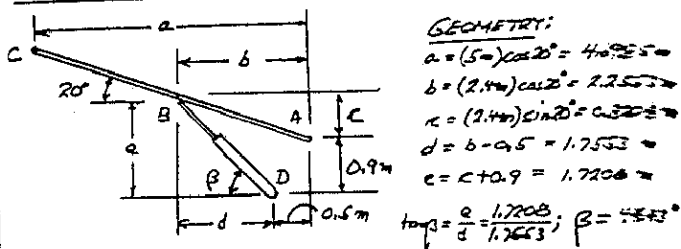
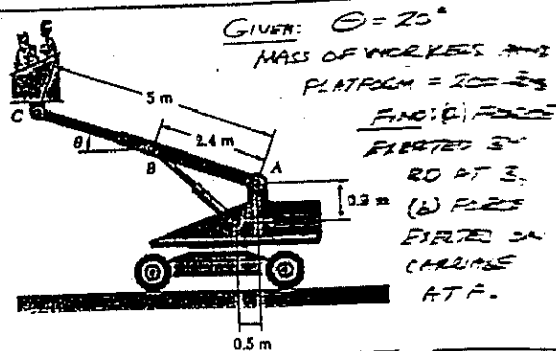


FIND: MAGNITUDE OF FORCES EXERTED AT E.

BY SYMMETRY VERTICAL COMPONENTS  $C_y, D_y, E_y$  ARE 0.  
THEN BY CONSIDERING  $\Sigma F_y = 0$  ON THE  
BLADES OR HANDLES, WE FIND THAT  $A_y$  AND  $B_y$  ARE 0.  
THUS FORCES AT A, B, C, D, AND E ARE  
HORIZONTAL

(CONTINUED)

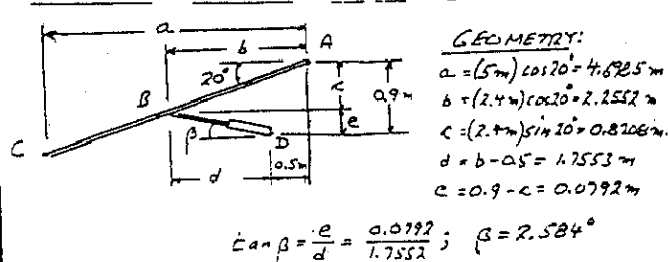
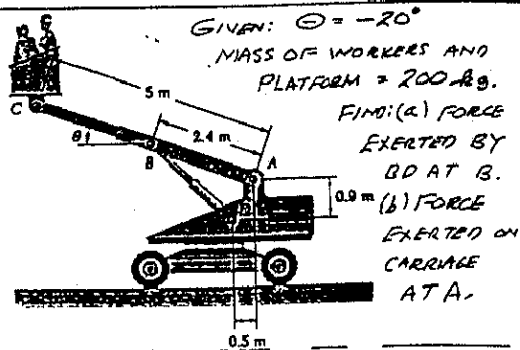
6.153



(a)  $\sum M_A = 0$ :  $(1962\text{ N})(4.6985\text{ m}) - F_{BD} \sin 44.43^\circ (2.2553\text{ m}) + F_{BD} \cos 44.43^\circ (0.8208\text{ m}) = 0$   
 $9285 - F_{BD}(0.9927) = 0$ ;  $F_{BD} = 9287\text{ N}$   
 $F_{BD} = 9.29\text{ kN} \nearrow 44.4^\circ$

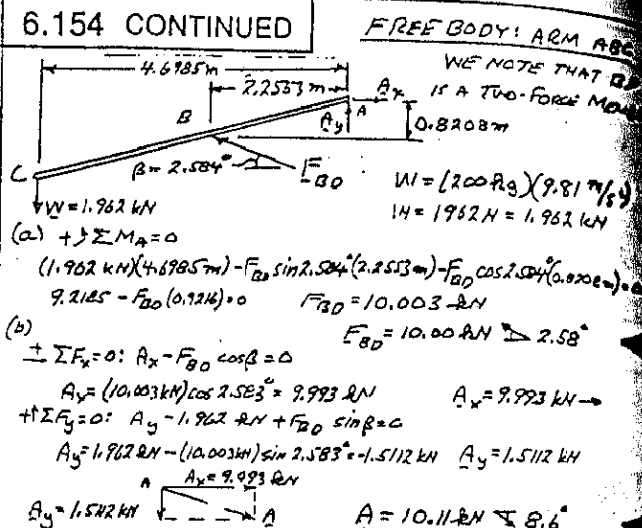
(b)  $\sum F_x = 0$ :  $A_x - F_{BD} \cos \beta = 0$   
 $A_x = (9287\text{ N}) \cos 44.43^\circ = 6.632\text{ kN}$   $A_x = 6.632\text{ kN}$   
 $\sum F_y = 0$ :  $A_y - 1962\text{ N} + F_{BD} \sin \beta = 0$   
 $A_y = 1962\text{ N} - (9287\text{ N}) \sin 44.43^\circ = 4539\text{ N}$   
 $A_y = 4.539\text{ kN}$   
 $A = 8.04\text{ kN} \searrow 34.4^\circ$

6.154

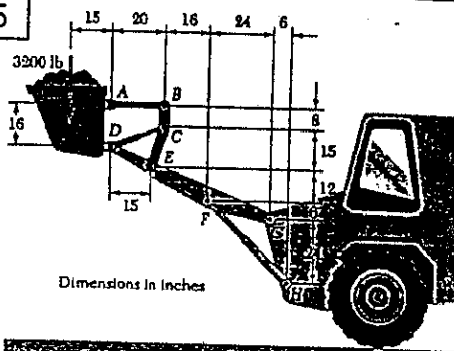


(CONTINUED)

6.154 CONTINUED

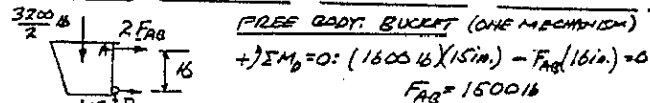


6.155

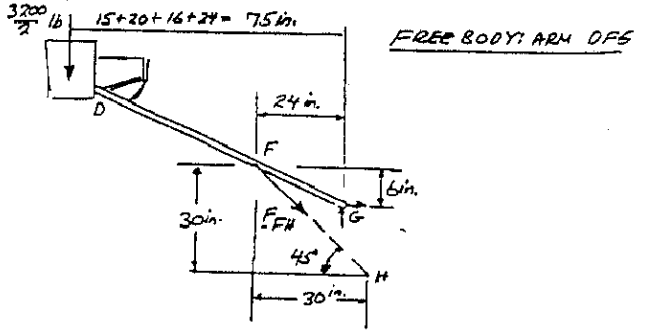
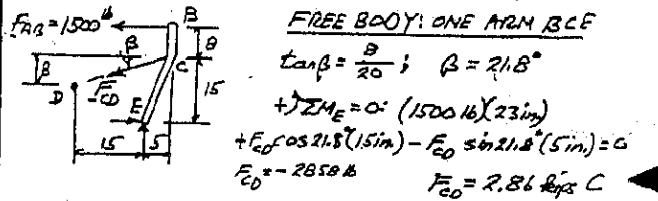


Dimensions in inches

FIND: FORCE EXERTED BY CD AND FH.



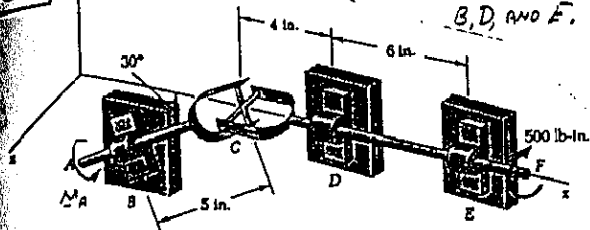
NOTE: THERE ARE 2 IDENTICAL SUPPORT MECHANISMS





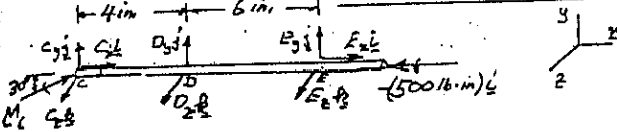
61

FIND: (a) MAGNITUDE  $M_A$ .  
(b) REACTIONS AT B, D, AND E.



WE RECALL FROM FIG. 4.10, PAGE 187, THAT A UNIVERSAL JOINT EXERTS ON MEMBERS IT CONNECTS A FORCE OF UNKNOWN DIRECTION AND A COUPLE ABOUT AN AXIS PERPENDICULAR TO THE CROSS DISC.

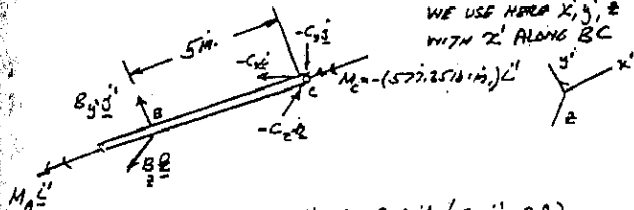
FREE BODY: SHAFT DE



$$\Sigma M_y = 0: M_C \cos 30^\circ - 500(6) = 0 \quad M_C = 577.35 \text{ lb}\cdot\text{in}$$

FREE BODY: SHAFT BC

WE USE HERE  $x', y', z'$  WITH  $x'$  ALONG BC



$$\Sigma M_C = 0: -M_A L' - (577.35 \text{ lb}\cdot\text{in}) L' + (-5 \text{ in}) L' x (B_y j' + B_z k') = 0$$

EQUATE COEFFICIENTS OF UNIT VECTORS TO ZERO:

$$\textcircled{1} M_A - 577.35 \text{ lb}\cdot\text{in} = 0 \quad M_A = 577.35 \text{ lb}\cdot\text{in}$$

$$\textcircled{2} B_z = 0$$

$$\textcircled{3} B_y = 0$$

$$\Sigma F = 0: B + C = 0, \text{ SINCE } B = 0, \quad C = 0$$

RETURN TO FREE BODY OF SHAFT DE

$$\Sigma M_D = 0 \quad (\text{NOTE THAT } C = 0 \text{ AND } M_C = 577.35 \text{ lb}\cdot\text{in})$$

$$(577.35 \text{ lb}\cdot\text{in})(\cos 30^\circ i + \sin 30^\circ j) - (500(6) \text{ lb}\cdot\text{in}) i + (6 \text{ in}) i x (E_x i + E_y j + E_z k) = 0$$

$$(500(6) \text{ lb}\cdot\text{in}) i + (250(6) \text{ lb}\cdot\text{in}) j - (500(6) \text{ lb}\cdot\text{in}) i + (6 \text{ in}) E_y j - (6 \text{ in}) E_z k = 0$$

EQUATE COEFFICIENTS OF UNIT VECTORS TO ZERO:

$$\textcircled{1} 250(6) \text{ lb}\cdot\text{in} - (6 \text{ in}) E_z = 0 \quad E_z = 48.1 \text{ lb}$$

$$\textcircled{2} E_y = 0$$

$$\Sigma F = 0: C + D + E = 0$$

$$0 + D_y j + D_z k + E_x i + (48.1 \text{ lb}) k = 0$$

$$\textcircled{1} E_x = 0$$

$$\textcircled{2} D_y = 0$$

$$\textcircled{3} D_z + 48.1 \text{ lb} = 0 \quad D_z = -48.1 \text{ lb}$$

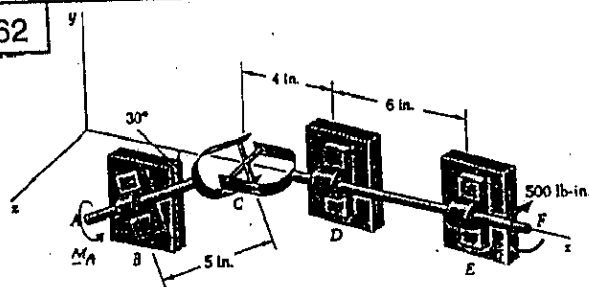
REACTIONS ARE:

$$B = 0$$

$$D = -(48.1 \text{ lb}) k$$

$$E = (48.1 \text{ lb}) k$$

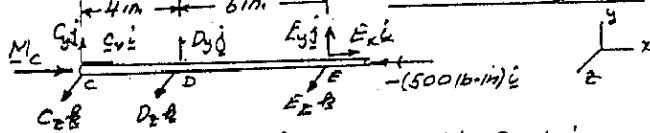
\*6.162



GIVEN: ROTATE SHAFT UNTIL CROSSPIECE ATTACHED TO SHAFT CF IS VERTICAL, THEN

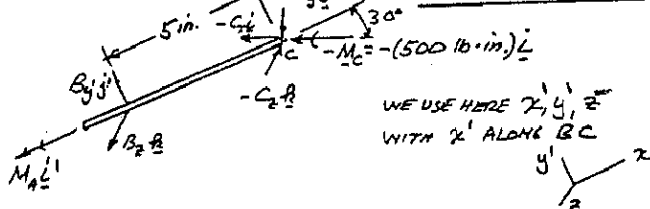
FIND: (a) MAGNITUDE  $M_A$ . (b) REACTIONS AT B, D, AND E.

FREE BODY: SHAFT DE



$$\Sigma M_x = 0: M_C - 500(6) = 0 \quad M_C = 500 \text{ lb}\cdot\text{in}$$

FREE BODY: SHAFT BC



WE USE HERE  $x', y', z'$  WITH  $x'$  ALONG BC

WE RESOLVE  $-(500 \text{ lb}\cdot\text{in}) i$  INTO COMPONENTS ALONG  $x'$  AND  $y'$  AXES:

$$\Sigma M_C = 0: M_A L' - (500 \text{ lb}\cdot\text{in})(\cos 30^\circ i + \sin 30^\circ j) + (5 \text{ in}) L' x (B_y j' + B_z k') = 0$$

$$M_A L' - (433 \text{ lb}\cdot\text{in}) i - (250 \text{ lb}\cdot\text{in}) j + (5 \text{ in}) B_y j - (5 \text{ in}) B_z k = 0$$

EQUATE TO ZERO COEFFICIENTS OF UNIT VECTORS:

$$\textcircled{1} M_A - 433 \text{ lb}\cdot\text{in} = 0 \quad M_A = 433 \text{ lb}\cdot\text{in}$$

$$\textcircled{2} -250 \text{ lb}\cdot\text{in} - (5 \text{ in}) B_z = 0 \quad B_z = -50 \text{ lb}$$

$$\textcircled{3} B_y = 0$$

$$\text{REACTION AT B: } B = -(50 \text{ lb}) k$$

$$\Sigma F = 0: B + C = 0$$

$$-(50 \text{ lb}) k - C = 0 \quad C = -(50 \text{ lb}) k$$

RETURN TO FREE BODY OF SHAFT DE:

$$\Sigma M_D = 0: (6 \text{ in}) i x (E_x i + E_y j + E_z k) - (4 \text{ in}) i x (-(50 \text{ lb}) k) - (500(6) \text{ lb}\cdot\text{in}) i + (500(6) \text{ lb}\cdot\text{in}) i = 0$$

$$(6 \text{ in}) E_y j - (6 \text{ in}) E_z k - (200 \text{ lb}\cdot\text{in}) j = 0$$

$$\textcircled{1} E_y = 0$$

$$\textcircled{2} -6 \text{ in} E_z - 200 \text{ lb}\cdot\text{in} = 0 \quad E_z = -33.3 \text{ lb}$$

$$\Sigma F = 0: C + D + E = 0$$

$$-(50 \text{ lb}) k + D_y j + D_z k + E_x i - (33.3 \text{ lb}) k = 0$$

$$\textcircled{1} E_x = 0$$

$$\textcircled{2} -50 \text{ lb} - 33.3 \text{ lb} + D_z = 0$$

$$D_z = 83.3 \text{ lb}$$

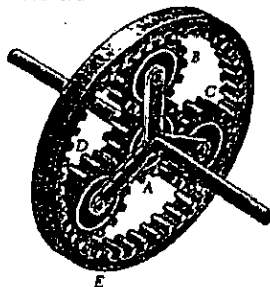
REACTIONS ARE:

$$B = -(50 \text{ lb}) k$$

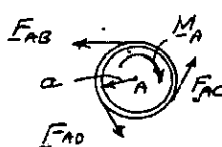
$$D = (83.3 \text{ lb}) k$$

$$E = -(33.3 \text{ lb}) k$$

6.159



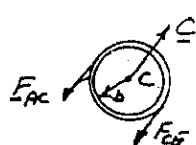
GIVEN: RADIUS OF GEAR A IS  $a = 18 \text{ mm}$ , GEAR B IS  $b$ ,  $M_A = 10 \text{ N}\cdot\text{m}$  APPLIED TO GEAR A,  $M_S = 50 \text{ N}\cdot\text{m}$  APPLIED TO GEAR C. FIND: (a) VALUE OF  $b$ , (b) COUPLE  $M_E$  APPLIED TO GEAR E.



GEAR A: BY SYMMETRY

$$F_{AB} = F_{AC} = F_{AD} \quad +\sum M_A = 0: -M_A + 3F_{AC}a = 0$$

$$F_{AC} = \frac{M_A}{3a}$$



GEAR C:  $+\sum M_C = 0: F_{AC}b - F_{EC}c = 0$

$$F_{EC} = F_{AC} = \frac{M_A}{3a}$$

SPIDER BCD:

BY SYMMETRY:  $B = C = D$

$$+\sum M_B = 0$$

$$-M_S + 3C(a+b) = 0$$

$$-M_S + 3\left(\frac{M_A}{3a}\right)(a+b) = 0$$

$$\frac{M_S}{M_A} = 2\frac{a+b}{a} = 2\left(1 + \frac{b}{a}\right)$$

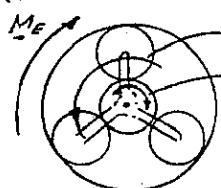
(a)

GIVEN:  $M_S = 50 \text{ N}\cdot\text{m}$  AND  $M_A = 10 \text{ N}\cdot\text{m}$

$$\frac{50 \text{ N}\cdot\text{m}}{10 \text{ N}\cdot\text{m}} = 2\left(1 + \frac{b}{a}\right)$$

$$\frac{b}{a} = 1.5 \quad \text{FOR } a = 18 \text{ mm}, \quad b = 1.5(18 \text{ mm}); \quad b = 27 \text{ mm}$$

(b)



$$M_S = 50 \text{ N}\cdot\text{m}$$

$$M_A = 10 \text{ N}\cdot\text{m}$$

FREE BODY:

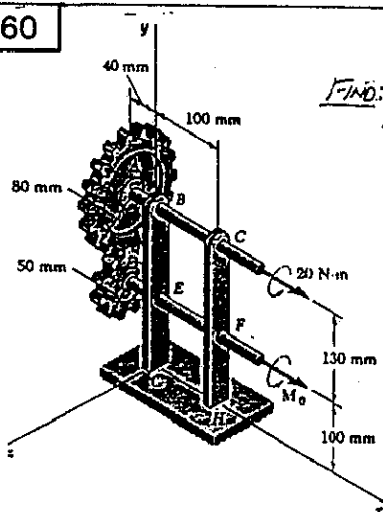
ENTIRE SYSTEM

$$+\sum M = 0$$

$$10 \text{ N}\cdot\text{m} - 50 \text{ N}\cdot\text{m} + M_E = 0$$

$$M_E = 40 \text{ N}\cdot\text{m}$$

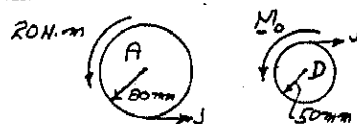
6.160



FIND: (a) COUPLE  $M_0$  FOR EQUILIBRIUM  
(b) REACTIONS AT G AND H.

(CONTINUED)

6.160 CONTINUED

PROJECTIONS ON  $yz$  PLANE

$$\text{GEAR A: } +\sum M_A = 0: 20 \text{ N}\cdot\text{m} - J(0.08 \text{ m}) = 0$$

$$J = 250 \text{ N}$$

$$\text{GEAR D: } +\sum M_D = 0: M_0 - J(0.05 \text{ m}) = 0$$

$$M_0 = (250 \text{ N})(0.05 \text{ m}) = 0$$

$$M_0 = 12.5 \text{ N}\cdot\text{m} \quad M_0 = (12.5 \text{ N}\cdot\text{m}) \hat{i}$$

(a)

(b) PROJECTIONS ON  $xz$  PLANE GEAR A AND AXLE AC:

$$+\sum M_B = 0: (250 \text{ N})(40 \text{ mm}) - C(100 \text{ mm}) = 0$$

$$C = 100 \text{ N}$$

$$+\sum M_C = 0:$$

$$(250 \text{ N})(140 \text{ mm}) - B(100 \text{ mm}) = 0$$

$$B = 350 \text{ N}$$

GEAR D AND AXLE DF:

$$+\sum M_E = 0: - (250 \text{ N})(40 \text{ mm}) + F(100 \text{ mm}) = 0$$

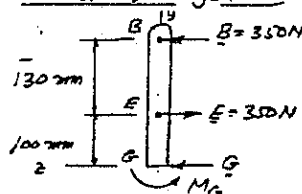
$$F = 100 \text{ N}$$

$$+\sum M_F = 0:$$

$$-(250 \text{ N})(140 \text{ mm}) + E(100 \text{ mm}) = 0$$

$$E = 350 \text{ N}$$

PROJECTIONS ON  $yz$  PLANE BRACKET BG:



$$\sum F_x = 0:$$

$$350 \text{ N} - 350 \text{ N} + G = 0$$

$$G = 0$$

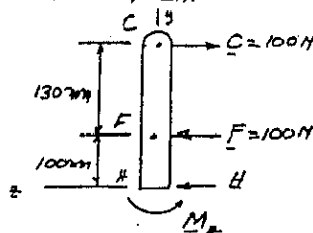
$$\sum M_x = 0:$$

$$(350 \text{ N})(130 \text{ mm}) + M_G = 0$$

$$M_G = -45.500 \text{ N}\cdot\text{m}$$

$$M_G = -(45.5 \text{ N}\cdot\text{m}) \hat{i}$$

BRACKET CH:



$$\sum F_z = 0:$$

$$100 \text{ N} - 100 \text{ N} + H = 0$$

$$H = 0$$

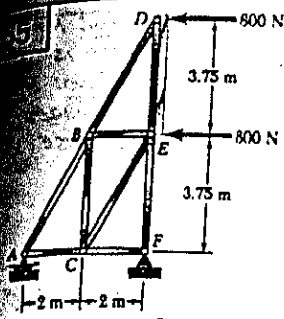
$$\sum M_z = 0:$$

$$-(100 \text{ N})(130 \text{ mm}) + M_H = 0$$

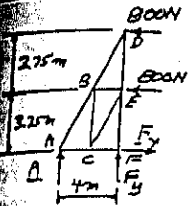
$$M_H = 13.000 \text{ N}\cdot\text{m}$$

$$M_H = (13.3 \text{ N}\cdot\text{m}) \hat{i}$$





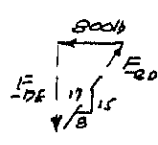
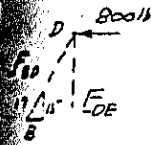
FIND: FORCE IN EACH MEMBER



FREE BODY ENTIRE TRUSS

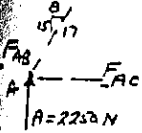
$$\begin{aligned} +\sum M_A &= 0 \\ (800\text{ N})(7.5\text{ m}) + (800\text{ N})(3.75\text{ m}) - A(2\text{ m}) &= 0 \\ A &= +2250\text{ N} \quad A = 2250\text{ N} \uparrow \\ +\sum F_y &= 0: 2250\text{ N} + F_y = 0 \\ F_y &= -2250\text{ N} \quad F_y = 2250\text{ N} \downarrow \\ +\sum F_x &= 0: -800\text{ N} - 800\text{ N} + F_x = 0 \\ F_x &= +1600\text{ N} \quad F_x = 1600\text{ N} \rightarrow \end{aligned}$$

JOINT D:



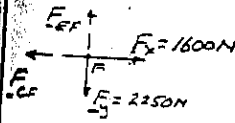
$$\begin{aligned} \frac{800\text{ N}}{8} &= \frac{F_{DE}}{15} = \frac{F_{ED}}{17} \\ F_{BD} &= 1700\text{ N C} \\ F_{DE} &= 1500\text{ N T} \end{aligned}$$

JOINT A:



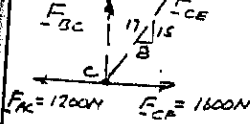
$$\begin{aligned} \frac{2250\text{ N}}{15} &= \frac{F_{AB}}{17} = \frac{F_{AC}}{8} \\ F_{AB} &= 2550\text{ N C} \\ F_{AC} &= 1200\text{ N T} \end{aligned}$$

JOINT F:



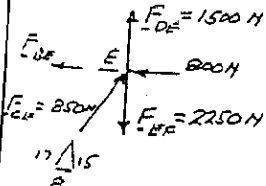
$$\begin{aligned} +\sum F_x &= 0: 1600\text{ N} - F_{CF} = 0 \\ F_{CF} &= +1600\text{ N} \quad F_{CF} = 1600\text{ N T} \\ +\sum F_y &= 0: F_{FE} - 2250\text{ N} = 0 \\ F_{FE} &= +2250\text{ N} \quad F_{FE} = 2250\text{ N T} \end{aligned}$$

JOINT C:



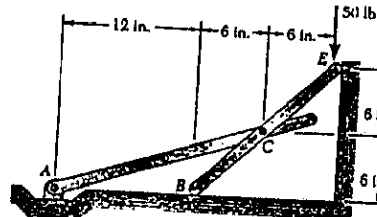
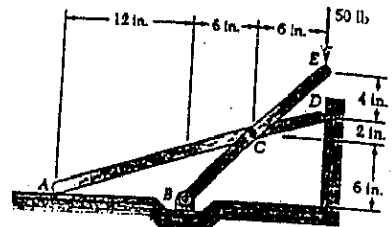
$$\begin{aligned} +\sum F_x &= 0: \frac{8}{17} F_{CD} - 1200\text{ N} + 1600\text{ N} = 0 \\ F_{CD} &= -850\text{ N} \quad F_{CD} = 850\text{ N C} \\ +\sum F_y &= 0: F_{BC} + \frac{15}{17} F_{CD} = 0 \\ F_{BC} &= -\frac{15}{17} F_{CD} = -\frac{15}{17}(-850\text{ N}) \\ F_{BC} &= +750\text{ N} \quad F_{BC} = 750\text{ N T} \end{aligned}$$

JOINT E:



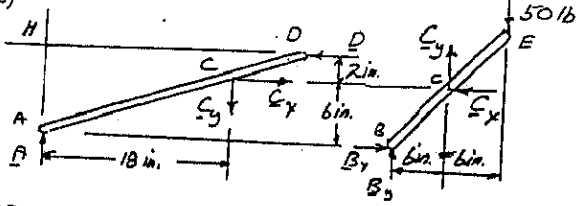
$$\begin{aligned} +\sum F_x &= 0: -F_{BE} - 800\text{ N} + \frac{8}{17}(850\text{ N}) = 0 \\ F_{BE} &= -400\text{ N} \quad F_{BE} = 400\text{ N C} \\ +\sum F_y &= 0: 1500\text{ N} - 2250\text{ N} + \frac{15}{17}(850\text{ N}) = 0 \\ 0 &= 0 \quad (\text{check}) \end{aligned}$$

6.166



FIND: FOR EACH FRAME, THE FORCES EXERTED AT B AND C ON MEMBER BCE

(a)



FREE BODY OF MEMBER ACD

$$+\sum M_A = 0: C_x(2\text{ in}) - C_y(18\text{ in}) = 0 \quad C_x = 9C_y \quad (1)$$

FREE BODY OF MEMBER BCE

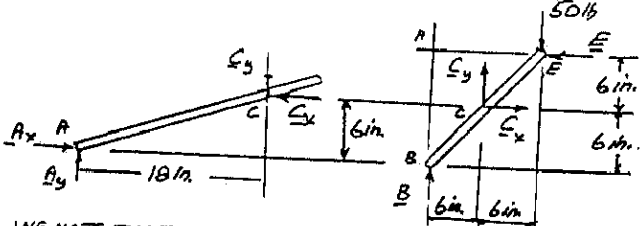
$$+\sum M_B = 0: C_x(6\text{ in}) + C_y(6\text{ in}) - (50\text{ lb})(12\text{ in}) = 0$$

$$\begin{aligned} \text{SUBSTITUTE FROM (1): } 9C_y(6) + C_y(6) - 600 &= 0 \\ C_y &= +10\text{ lb}; \quad C_x = 9C_y = 90\text{ lb} = +90\text{ lb} \\ C &= 90.6\text{ lb} \angle 6.3^\circ \end{aligned}$$

$$\begin{aligned} +\sum F_x &= 0: B_x - 90\text{ lb} = 0 \quad B_x = 90\text{ lb} \\ +\sum F_y &= 0: B_y + 10\text{ lb} - 50\text{ lb} = 0 \quad B_y = 40\text{ lb} \end{aligned}$$

(b)

$$B = 98.5\text{ lb} \angle 24.0^\circ$$



NOTE THAT AC IS A TWO-FORCE MEMBER

$$\frac{C_x}{18\text{ in}} = \frac{C_y}{6\text{ in}} \quad C_x = 3C_y \quad (1)$$

ON FREE BODY OF MEMBER BCE

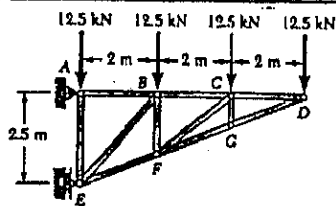
$$+\sum M_B = 0: C_x(6\text{ in}) + C_y(6\text{ in}) - (50\text{ lb})(12\text{ in}) = 0$$

$$\begin{aligned} \text{SUBSTITUTE FROM (1): } 3C_y(6) + C_y(6) - 600 &= 0 \\ C_y &= +25\text{ lb}; \quad C_x = 3C_y = 75\text{ lb} \\ C &= 79.1\text{ lb} \angle 18.4^\circ \end{aligned}$$

$$\begin{aligned} +\sum F_y &= 0: B + C_y - 50\text{ lb} = 0 \\ B + 25\text{ lb} - 50\text{ lb} &= 0 \\ B &= +25\text{ lb} \end{aligned}$$

$$B = 25\text{ lb} \uparrow$$

6.167



FIND: THE  
FORCE IN EACH  
MEMBER

JOINT D:

$$\begin{aligned} \sum F_x = 0: & F_{CD} - F_{DG} = 0 \\ \sum F_y = 0: & 12.5 \text{ kN} - F_{CD} \sin 39.81^\circ - F_{DG} \cos 39.81^\circ = 0 \end{aligned}$$

JOINT G:

$$\begin{aligned} \sum F_x = 0: & F_{CG} = 0 \\ \sum F_y = 0: & F_{FG} = 32.5 \text{ kN C} \end{aligned}$$

JOINT C:

$$\begin{aligned} \sum F_x = 0: & 12.5 \text{ kN} - F_{BC} \cos 39.81^\circ - F_{CG} \sin 39.81^\circ = 0 \\ \sum F_y = 0: & F_{BC} \sin 39.81^\circ - F_{CG} \cos 39.81^\circ = 0 \end{aligned}$$

$$\begin{aligned} \sum F_x = 0: & 30 \text{ kN} - F_{BC} - F_{CF} \cos 39.81^\circ = 0 \\ \sum F_y = 0: & F_{BC} \sin 39.81^\circ - F_{CF} \sin 39.81^\circ = 0 \end{aligned}$$

JOINT F:

$$\begin{aligned} \sum F_x = 0: & -F_{BF} - F_{CF} \cos 39.81^\circ = 0 \\ \sum F_y = 0: & F_{BF} \sin 39.81^\circ - F_{CF} \sin 39.81^\circ = 0 \end{aligned}$$

$$\begin{aligned} \sum F_x = 0: & F_{BF} - F_{CF} \cos 39.81^\circ = 0 \\ \sum F_y = 0: & F_{BF} \sin 39.81^\circ - F_{CF} \sin 39.81^\circ = 0 \end{aligned}$$

JOINT B:

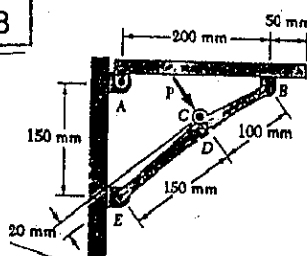
$$\begin{aligned} \sum F_x = 0: & F_{AB} - F_{BC} \cos 57.34^\circ = 0 \\ \sum F_y = 0: & F_{AB} \sin 57.34^\circ - F_{BC} \sin 57.34^\circ = 0 \end{aligned}$$

$$\begin{aligned} \sum F_x = 0: & 45.0 \text{ kN} - F_{AB} \cos 57.34^\circ = 0 \\ \sum F_y = 0: & F_{AB} \sin 57.34^\circ - 30.0 \text{ kN} = 0 \end{aligned}$$

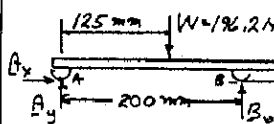
JOINT E:

$$\begin{aligned} \sum F_x = 0: & F_{AE} - F_{BE} \cos 57.34^\circ = 0 \\ \sum F_y = 0: & F_{AE} \sin 57.34^\circ - F_{BE} \sin 57.34^\circ = 0 \end{aligned}$$

6.168

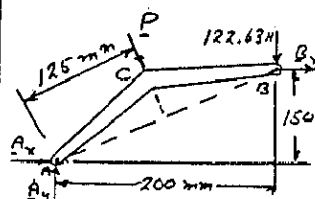


GIVEN: MASS OF  
SHELF IS 20 kg.  
FIND: FORCE P  
REQUIRED TO  
RELEASE BRACE



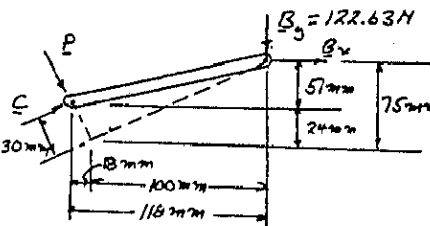
FREE BODY: SHELF

$$\begin{aligned} W &= (20 \text{ kg})(9.81 \text{ m/s}^2) = 196.2 \text{ N} \\ \sum M_A = 0: & B_y(200 \text{ mm}) - (196.2 \text{ N})(125 \text{ mm}) = 0 \\ B_y &= 122.63 \text{ N} \end{aligned}$$



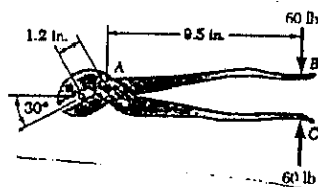
FREE BODY: PORTION ACB

$$\begin{aligned} \sum M_A = 0: & -B_y(150 \text{ mm}) - P(125 \text{ mm}) - (122.63 \text{ N})(200 \text{ mm}) = 0 \\ B_y &= -163.5 - 0.8333P \quad (1) \end{aligned}$$

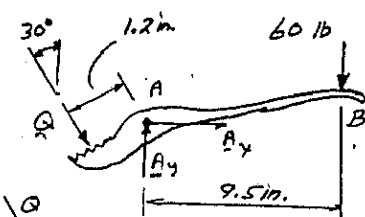


$$\begin{aligned} \sum M_C = 0: & (122.63 \text{ N})(110 \text{ mm}) + B_y(57 \text{ mm}) = 0 \\ & + (122.63 \text{ N})(118 \text{ mm}) + (-163.5 - 0.8333P)(51 \text{ mm}) = 0 \\ P &= 144.28 \text{ N} \end{aligned}$$

6.169



FIND: (a) MAGNITUDE  
OF FORCES EXERTED  
ON ROD.  
(b) FORCE EXERTED  
AT A ON PORTION  
AB OF PLIERS



FREE BODY: PORTION AB

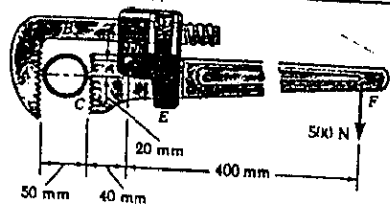
$$\begin{aligned} \sum M_A = 0: & Q(1.2 \text{ in}) - (60 \text{ lb})(9.5 \text{ in}) = 0 \\ Q &= 475 \text{ lb} \end{aligned}$$

$$\begin{aligned} \sum F_x = 0: & Q(\sin 30^\circ) + A_x = 0 \\ & (475 \text{ lb})(\sin 30^\circ) + A_x = 0 \\ A_x &= -237.5 \text{ lb} \quad A_x = 237.5 \text{ lb} \leftarrow \\ \sum F_y = 0: & -Q(\cos 30^\circ) + A_y - 60 \text{ lb} = 0 \\ & -(475 \text{ lb})(\cos 30^\circ) + A_y - 60 \text{ lb} = 0 \\ A_y &= +471.4 \text{ lb} \quad A_y = 471.4 \text{ lb} \uparrow \\ A &= 528 \text{ lb} \angle 63.3^\circ \end{aligned}$$

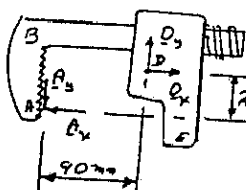




6.173



FIND: COMPONENTS OF FORCES ON PIPE AT A AND AT C.

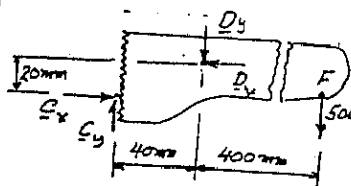


FREE BODY: PORTION ABDE

THIS IS A TWO-FORCE MEMBER?

$$\frac{A_y}{20\text{ mm}} = \frac{A_x}{90\text{ mm}}; A_x = 4.5 A_y$$

$$D_y = A_y; D_x = A_x = 4.5 D_y \quad (1)$$



FREE BODY: PORTION CF

$$+\uparrow \Sigma M_C = 0$$

$$D_x(20\text{ mm}) - D_y(40\text{ mm}) - (500\text{ N})(440\text{ mm}) = 0$$

SUBSTITUTE FROM (1)

$$4.5 D_y(20) - D_y(40) - 220 \times 10^3 = 0$$

$$D_y = 4400\text{ N} = +4.4\text{ kN}$$

$$D_x = 4.5(D_y) = +19.8\text{ kN}$$

$$+\uparrow \Sigma F_y = 0: C_y - 4.4\text{ kN} - 0.5\text{ kN} = 0; C_y = +4.9\text{ kN}$$

$$+\rightarrow \Sigma F_x = 0: C_x - 19.8\text{ kN} = 0; C_x = +19.8\text{ kN}$$

FROM PORTION ABDE:

$$A_x = D_x = +19.8\text{ kN}$$

$$A_y = D_y = +4.4\text{ kN}$$

WE NOTE THAT ALL COMPONENTS FOUND ABOVE ACT IN DIRECTIONS DRAWN. COMPONENTS ON THE PIPE ARE EQUAL AND OPPOSITE TO THOSE ON WRENCH.

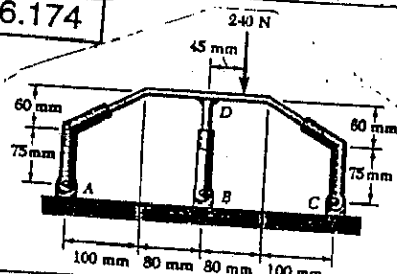


$$A_x = 19.8\text{ kN} \rightarrow A_y = 4.4\text{ kN} \uparrow$$

$$C_x = 19.8\text{ kN} \leftarrow C_y = 4.9\text{ kN} \uparrow$$

NOTE: FREE BODY OF PIPE ALSO INCLUDES REACTIONS, P AND M, EXERTED BY GROUND.

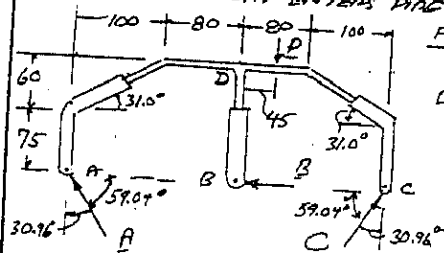
6.174



FIND: REACTIONS AT A, B, AND C.

NOTE:

REAR REACTION IS  $\perp$  TO SLOPE OF PIPE WHERE WELDMENT ENTERS PIPE.



FREE BODY: ENTIRE FRAME

$$\text{DIMENSIONS IN mm}$$

$$P = 240\text{ N}$$

$$\tan^{-1} \frac{60}{100} = 30.96^\circ$$

(CONTINUED)

6.174 CONTINUED

$$+\uparrow \Sigma M_A = 0: C \cos 30.96^\circ (360\text{ mm}) - (240\text{ N})(225\text{ mm}) = 0$$

$$C = +174.92\text{ N} \quad C = 174.9\text{ N} \angle 59.0^\circ$$

$$+\uparrow \Sigma M_C = 0: -A \cos 30.96^\circ (360\text{ mm}) + (240\text{ N})(135\text{ mm}) = 0$$

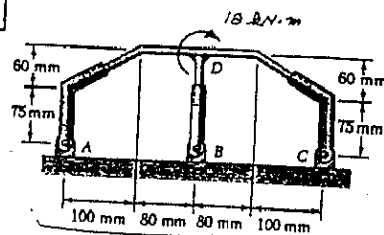
$$A = 104.95\text{ N} \quad A = 105.0\text{ N} \angle 59.0^\circ$$

$$+\rightarrow \Sigma F_x = 0: -A \sin 30.96^\circ + C \sin 30.96^\circ - B = 0$$

$$B = (C - A) \sin 30.96^\circ = (174.92\text{ N} - 104.95\text{ N}) \sin 30.96^\circ$$

$$B = +36.0\text{ N} \quad B = 36.0\text{ N} \leftarrow$$

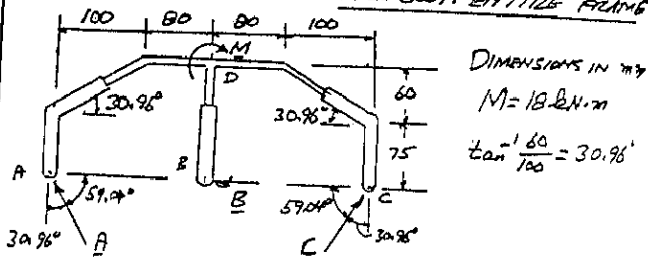
6.175



FIND: THE REACTIONS AT A, B, AND C.

NOTE: EACH REACTION IS  $\perp$  TO SLOPE OF PIPE WHERE WELDMENT ENTERS PIPE.

FREE BODY: ENTIRE FRAME



DIMENSIONS IN mm

$$M = 18\text{ kN} \cdot \text{m}$$

$$\tan^{-1} \frac{60}{100} = 30.96^\circ$$

$$+\uparrow \Sigma M_A = 0: M - C \cos 30.96^\circ (360\text{ mm}) = 0$$

$$18\text{ kN} \cdot \text{m} - C \cos 30.96^\circ (0.36\text{ m}) = 0$$

$$C = +58.31\text{ kN} \quad C = 58.3\text{ kN} \angle 59.0^\circ$$

$$+\uparrow \Sigma M_C = 0: M + A \cos 30.96^\circ (360\text{ mm}) = 0$$

$$18\text{ kN} \cdot \text{m} + A \cos 30.96^\circ (0.36\text{ m}) = 0$$

$$A = -58.31\text{ kN} \quad A = 58.3\text{ kN} \angle 59.0^\circ$$

$$+\rightarrow \Sigma F_x = 0: -A \sin 30.96^\circ - B + C \sin 30.96^\circ = 0$$

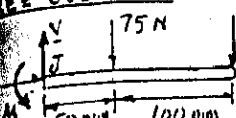
$$B = (C - A) \sin 30.96^\circ$$

$$B = (58.31\text{ kN} - (-58.31\text{ kN})) \sin 30.96^\circ$$

$$B = +60.0\text{ kN} \quad B = 60\text{ kN} \leftarrow$$

**GIVEN:** FRAME AND LOADING OF PROB. 6.75.  
**FIND:** INTERNAL FORCES AT POINT J.

CUT MEMBER BCD AT POINT J AND CONSIDER THE FREE BODY JD:

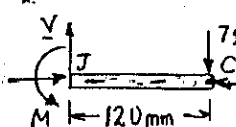


$$\begin{aligned} \sum F_x = 0: & \quad H = 0 \\ \sum F_y = 0: & \quad V - 75\text{ N} - 75\text{ N} = 0 \\ & \quad V = +150.0\text{ N} \quad V = 150.0\text{ N} \uparrow \\ \sum M_J = 0: & \quad M - (75\text{ N})(0.05\text{ m}) - (75\text{ N})(0.15\text{ m}) = 0 \\ & \quad M = +15.00\text{ N}\cdot\text{m} \quad M = 15.00\text{ N}\cdot\text{m} \end{aligned}$$

**7.2**

**GIVEN:** FRAME AND LOADING OF PROB. 6.76.  
**FIND:** INTERNAL FORCES AT POINT J.

CUT MEMBER ABC AT POINT J AND CONSIDER THE FREE BODY JC. WE RECALL FROM THE SOLUTION OF PROB. 6.76 THAT THE REACTION AT C IS  $C_x = 125\text{ N} \angle 36.9^\circ$  OR  $C_x = 100\text{ N} \rightarrow$ ,  $C_y = 75\text{ N} \uparrow$ .

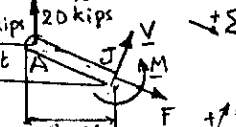


$$\begin{aligned} \sum F_x = 0: & \quad H = 100.0\text{ N} \rightarrow \\ \sum F_y = 0: & \quad V - 75\text{ N} = 0 \\ & \quad V = 75.0\text{ N} \uparrow \\ \sum M_J = 0: & \quad M - (75\text{ N})(0.120\text{ m}) = 0 \\ & \quad M = 9.00\text{ N}\cdot\text{m} \end{aligned}$$

**7.3**

**GIVEN:** FRAME AND LOADING OF PROB. 6.B1.  
**FIND:** INTERNAL FORCES AT POINT J LOCATED HALFWAY BETWEEN A AND B.

WE CUT MEMBER ABC AT POINT J AND CONSIDER THE FREE BODY AJ. WE RECALL FROM THE SOLUTION OF PROB. 6.B1 THAT THE COMPONENTS OF THE REACTION AT A ARE  $A_x = 25\text{ kips} \leftarrow$  AND  $A_y = 20\text{ kips} \uparrow$ .




$$\begin{aligned} \sum F_x = 0: & \quad F - \frac{2.5}{\sqrt{1^2 + 2.5^2}}(25) - \frac{1}{\sqrt{1^2 + 2.5^2}}(20) = 0 \\ & \quad F = +30.6\text{ kips}, \quad F = 30.6\text{ kips} \rightarrow \\ \sum F_y = 0: & \quad V - \frac{1}{\sqrt{1^2 + 2.5^2}}(25) + \frac{2.5}{\sqrt{1^2 + 2.5^2}}(20) = 0 \\ & \quad V = -9.28\text{ kips}, \quad V = 9.28\text{ kips} \downarrow \\ \sum M_J = 0: & \quad M + (25\text{ kips})(1\text{ ft}) - (20\text{ kips})(2.5\text{ ft}) = 0 \\ & \quad M = +25.0\text{ kips}\cdot\text{ft}, \quad M = 25.0\text{ kips}\cdot\text{ft} \end{aligned}$$

**7.4**

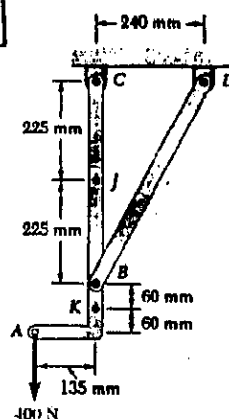
**GIVEN:** FRAME AND LOADING OF PROB. 6.B1.  
**FIND:** INTERNAL FORCES AT POINT K LOCATED HALFWAY BETWEEN B AND C.

WE DISCONNECT MEMBER ABC AND CUT IT AT POINT K. WE CONSIDER THE FREE BODY KC. WE RECALL FROM THE SOLUTION OF PROB. 6.B1 THAT THE COMPONENTS OF THE FORCE EXERTED AT C ON KC ARE  $C_x = 50\text{ kips} \rightarrow$ ,  $C_y = 10\text{ kips} \uparrow$ .



$$\begin{aligned} \sum F_x = 0: & \quad -F + \frac{2.5}{\sqrt{1^2 + 2.5^2}}(50) + \frac{1}{\sqrt{1^2 + 2.5^2}}(10) = 0 \\ & \quad F = +50.14\text{ kips}, \quad F = 50.1\text{ kips} \leftarrow \\ \sum F_y = 0: & \quad -V + \frac{1}{\sqrt{1^2 + 2.5^2}}(50) - \frac{2.5}{\sqrt{1^2 + 2.5^2}}(10) = 0 \\ & \quad V = +9.285\text{ kips}, \quad V = 9.28\text{ kips} \uparrow \\ \sum M_K = 0: & \quad -M + (50\text{ kips})(1\text{ ft}) - (10\text{ kips})(2.5\text{ ft}) = 0 \\ & \quad M = +25.0\text{ kip}\cdot\text{ft}, \quad M = 25.0\text{ kip}\cdot\text{ft} \end{aligned}$$

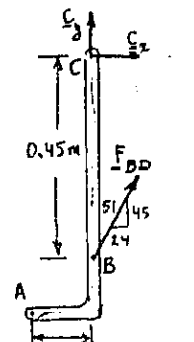
**7.5**



**GIVEN:** STRUCTURE AND LOADING SHOWN.

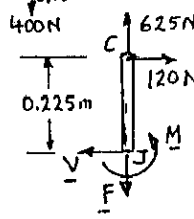
**FIND:** INTERNAL FORCES AT POINT J.

**FREE BODY: MEMBER ABC**



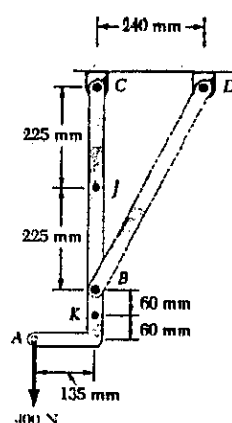
$$\begin{aligned} \sum M_C = 0: & \quad (400\text{ N})(0.135\text{ m}) + \left(\frac{24}{51}F_{BD}\right)(0.45\text{ m}) = 0 \\ & \quad F_{BD} = -255\text{ N}, \quad F_{BD} = 255\text{ N} \leftarrow \\ \sum F_x = 0: & \quad C_x + \frac{24}{51}(-255\text{ N}) = 0 \\ & \quad C_x = +120\text{ N}, \quad C_x = 120\text{ N} \rightarrow \\ \sum F_y = 0: & \quad C_y - 400\text{ N} + \frac{45}{51}(-255\text{ N}) = 0 \\ & \quad C_y = +625\text{ N}, \quad C_y = 625\text{ N} \uparrow \end{aligned}$$

**FREE BODY: CJ**



$$\begin{aligned} \sum F_x = 0: & \quad -F + 120\text{ N} = 0 \\ & \quad F = 120\text{ N} \leftarrow \\ \sum F_y = 0: & \quad -V + 625\text{ N} = 0 \\ & \quad V = 625\text{ N} \uparrow \\ \sum M_J = 0: & \quad M - (120\text{ N})(0.225\text{ m}) = 0 \\ & \quad M = +27.0\text{ N}\cdot\text{m}, \quad M = 27.0\text{ N}\cdot\text{m} \end{aligned}$$

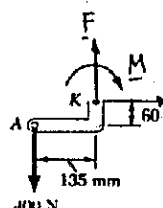
**7.6**



**GIVEN:** STRUCTURE AND LOADING SHOWN.

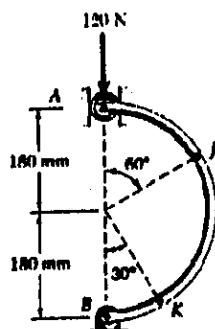
**FIND:** INTERNAL FORCES AT POINT K.

**FREE BODY: AK**



$$\begin{aligned} \sum F_x = 0: & \quad F - 400\text{ N} = 0 \\ & \quad F = 400\text{ N} \leftarrow \\ \sum F_y = 0: & \quad V = 0 \\ & \quad V = 0 \\ \sum M_K = 0: & \quad (400\text{ N})(0.135\text{ m}) - M = 0 \\ & \quad M = +54.0\text{ N}\cdot\text{m}, \quad M = 54.0\text{ N}\cdot\text{m} \end{aligned}$$

7.7



GIVEN:

SEMICIRCULAR ROD  
LOADED AS SHOWN

FIND:

INTERNAL FORCES  
AT POINT J

FREE BODY: ROD AB

$$\sum M_B = 0: -A(360 \text{ mm}) = 0$$

$$A = 0$$

$$\sum F_x = 0: B_x + A = 0$$

$$B_x = -A = 0$$

$$\sum F_y = 0: B_y - 120 \text{ N} = 0$$

$$B_y = 120 \text{ N}, B = 120 \text{ N}$$

FREE BODY: AJ

$$\sum F_y = 0: F - (120 \text{ N}) \sin 60^\circ = 0$$

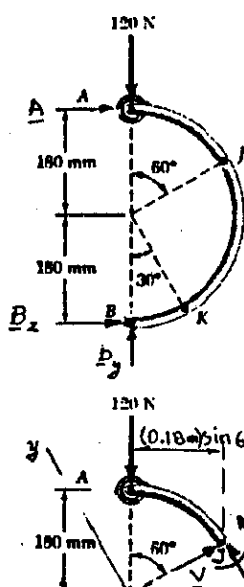
$$F = +103.9 \text{ N}, F = 103.9 \text{ N}$$

$$\sum F_x = 0: V - (120 \text{ N}) \cos 60^\circ = 0$$

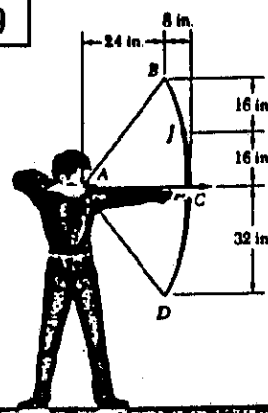
$$V = +60.0 \text{ N}, V = 60.0 \text{ N}$$

$$\sum M_J = 0: M - (120 \text{ N})(0.18 \text{ m}) \sin 60^\circ = 0$$

$$M = +18.71 \text{ N}\cdot\text{m}, M = 18.71 \text{ N}\cdot\text{m}$$



7.9



GIVEN:

ARCHER PULLING WITH  
A 45-lb FORCE ON THE  
BOWSTRING

FIND:

INTERNAL FORCES AT  
POINT J.  
(ASSUME THAT THE SHAPE  
OF THE BOW IS A PARABOLA)

FREE BODY: POINT A

$$\sum F_x = 0: 2\left(\frac{3}{5}T\right) - 45 \text{ lb} = 0 \quad T = 37.5 \text{ lb}$$

FREE BODY: PORTION OF BOW BC

$$\sum F_y = 0: F_c - 30 \text{ lb} = 0$$

$$F_c = 30 \text{ lb}$$

$$\sum F_x = 0: V_c - 22.5 \text{ lb} = 0$$

$$V_c = 22.5 \text{ lb}$$

$$\sum M_c = 0: (22.5 \text{ lb})(32 \text{ in}) + (30 \text{ lb})(8 \text{ in}) - M_c = 0$$

$$M_c = 960 \text{ lb}\cdot\text{in.}$$

EQUATION OF PARABOLA

$$z = ky^2$$

$$\text{AT B: } 0 = k(32)^2 \quad k = \frac{1}{128}$$

THEREFORE, EQUATION IS

$$z = \frac{y^2}{128}$$

(1)

THE SLOPE AT J IS OBTAINED BY DIFFERENTIATING (1):

$$dz = \frac{2y}{128} dy, \tan \theta = \frac{dz}{dy} = \frac{y}{64}$$

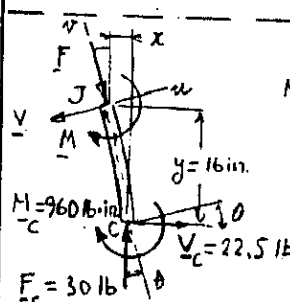
(2)

FREE BODY: PORTION OF BOW CJ

MAKING  $y = 16 \text{ in.}$  IN EQS (1) AND (2):

$$x = \frac{(16)^2}{128} = 2.00 \text{ in.}$$

$$\tan \theta = \frac{16}{64} = 0.25 \quad \theta = 14.04^\circ$$



$$\sum F_x = 0: -F + (30 \text{ lb}) \cos 14.04^\circ - (22.5 \text{ lb}) \sin 14.04^\circ = 0$$

$$F = +23.6 \text{ lb}$$

$$F = 23.6 \text{ lb}$$

$$\sum F_y = 0: -V + (30 \text{ lb}) \sin 14.04^\circ + (22.5 \text{ lb}) \cos 14.04^\circ = 0$$

$$V = +29.1 \text{ lb}$$

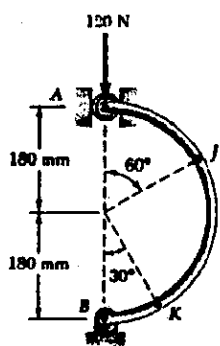
$$V = 29.1 \text{ lb}$$

$$\sum M_J = 0: -M - 960 \text{ lb}\cdot\text{in.} + (30 \text{ lb})(2 \text{ in.}) + (22.5 \text{ lb})(16 \text{ in.}) = 0$$

$$M = -540 \text{ lb}\cdot\text{in.}$$

$$M = 540 \text{ lb}\cdot\text{in.}$$

7.8



GIVEN:

SEMICIRCULAR ROD  
LOADED AS SHOWN

FIND:

INTERNAL FORCES  
AT POINT K.REACTION AT B: (SEE SOLUTION OF PROB. 7.7)  $B = 120 \text{ N}$ 

FREE BODY: BK

$$\sum F_x = 0: -F + (120 \text{ N}) \sin 30^\circ = 0$$

$$F = +60.0 \text{ N}, F = 60.0 \text{ N}$$

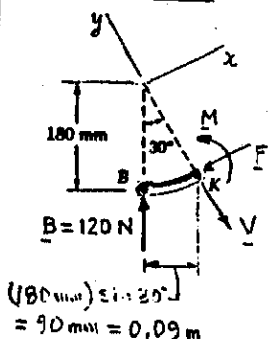
$$\sum F_y = 0: -V + (120 \text{ N}) \cos 30^\circ = 0$$

$$V = +103.9 \text{ N}, V = 103.9 \text{ N}$$

$$\sum M_K = 0: M - (120 \text{ N})(0.09 \text{ m}) = 0$$

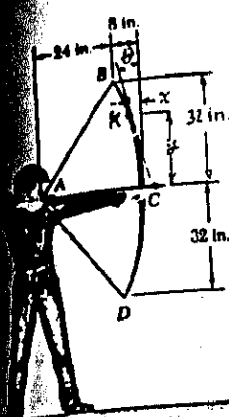
$$M = +10.80 \text{ N}\cdot\text{m}$$

$$M = 10.80 \text{ N}\cdot\text{m}$$



$$(180 \text{ mm}) \sin 30^\circ = 90 \text{ mm} = 0.09 \text{ m}$$

10. GIVEN: ARCHER AND BOW OF PROB. 7.9, WITH ARCHER PULLING WITH A 45-lb FORCE ON BOWSTRING. MAGNITUDE AND LOCATION IN THE BOW OF THE MAXIMUM AXIAL FORCE, (b) SHEARING FORCE, (c) BENDING MOMENT. FOLLOWING RESULTS WERE OBTAINED IN THE FIRST OF THE SOLUTION OF PROB. 7.9:



INTERNAL FORCES AT C (ON BC)  
 $F_c = 30 \text{ lb}$ ,  $V_c = 22.5 \text{ lb}$ ,  $M_c = 960 \text{ lb}\cdot\text{in.}$

EQUATION OF PARABOLA (BC)

$$x = \frac{y^2}{128} \quad (1)$$

SLOPE (ANGLE  $\theta$ )

$$\tan \theta = \frac{dy}{dx} = \frac{y}{64} \quad (2)$$

FREE BODY: PORTION OF BOW CK

(a) MAXIMUM AXIAL FORCE

$$\sum F_x = 0: -F + (30 \text{ lb}) \cos \theta - (22.5 \text{ lb}) \sin \theta = 0$$

$$F = 30 \cos \theta - 22.5 \sin \theta$$

F IS LARGEST AT C ( $\theta = 0$ )

$$F_m = 30.0 \text{ lb AT C}$$

(b) MAXIMUM SHEARING FORCE

$$\sum F_y = 0: -V + (30 \text{ lb}) \sin \theta + (22.5 \text{ lb}) \cos \theta = 0$$

$$V = 30 \sin \theta + 22.5 \cos \theta$$

V IS LARGEST AT B (AT D)

WHERE  $\theta = \theta_{\max} = \tan^{-1}(1/2) = 26.56^\circ$

$$V_m = 30 \sin 26.56^\circ + 22.5 \cos 26.56^\circ$$

$$V_m = 33.5 \text{ lb AT B AND D}$$

MAXIMUM BENDING MOMENT

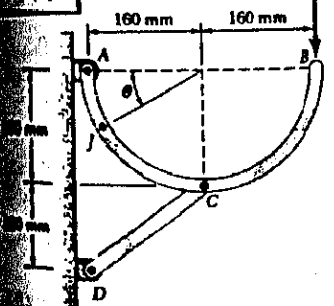
$$\sum M_A = 0: M - 960 \text{ lb}\cdot\text{in.} + (30 \text{ lb})x + (22.5 \text{ lb})y = 0$$

$$M = 960 - 30x - 22.5y$$

M IS LARGEST AT C, WHERE  $x = y = 0$ .

$$M_m = 960 \text{ lb}\cdot\text{in. AT C}$$

7.11

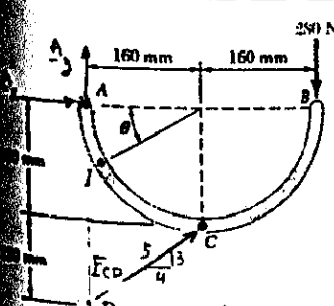


GIVEN:

SEMICIRCULAR ROD LOADED AS SHOWN.

FIND:

INTERNAL FORCES AT POINT J WHERE  $\theta = 30^\circ$



FREE BODY: ROD ACB

$$\sum M_A = 0:$$

$$\left(\frac{4}{5} F_{CD}\right)(0.16 \text{ m}) + \left(\frac{3}{5} F_{CD}\right)(0.16 \text{ m}) - (280 \text{ N})(0.32 \text{ m}) = 0$$

$$F_{CD} = 400 \text{ N}$$

$$\sum F_x = 0: A_x + \frac{4}{5}(400 \text{ N}) = 0$$

$$A_x = -320 \text{ N} \quad A_x = 320 \text{ N}$$

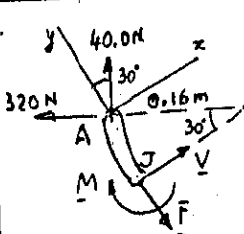
$$\sum F_y = 0: A_y + \frac{3}{5}(400 \text{ N}) - 280 \text{ N} = 0$$

$$A_y = +40.0 \text{ N} \quad A_y = 40.0 \text{ N}$$

(CONTINUED)

7.11 CONTINUED

FREE BODY: AJ



$$\sum F_x = 0: (320 \text{ N}) \sin 30^\circ + (40.0 \text{ N}) \cos 30^\circ - F = 0$$

$$F = +194.6 \text{ N} \quad F = 194.6 \text{ N}$$

$$\sum F_y = 0: (40.0 \text{ N}) \sin 30^\circ - (320 \text{ N}) \cos 30^\circ + V = 0$$

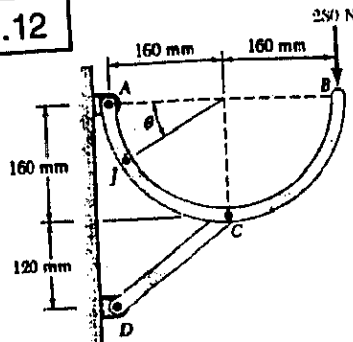
$$V = +257 \text{ N} \quad V = 257 \text{ N}$$

$$\sum M_J = 0: (320 \text{ N})(0.16 \text{ m}) \sin 30^\circ$$

$$- (40.0 \text{ N})(0.16 \text{ m})(1 - \cos 30^\circ) - M = 0$$

$$M = +24.7 \text{ N}\cdot\text{m} \quad M = 24.7 \text{ N}\cdot\text{m}$$

7.12

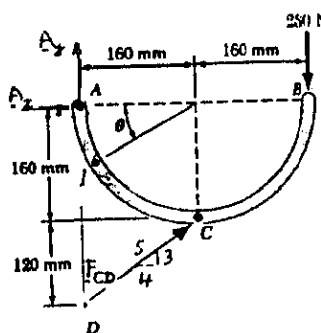


GIVEN:

SEMICIRCULAR ROD LOADED AS SHOWN.

FIND:

MAGNITUDE AND LOCATION OF MAXIMUM BENDING MOMENT IN THE ROD.



FREE BODY: ROD ACB

$$\sum M_A = 0:$$

$$\left(\frac{4}{5} F_{CD}\right)(0.16 \text{ m}) + \left(\frac{3}{5} F_{CD}\right)(0.16 \text{ m}) - (280 \text{ N})(0.32 \text{ m}) = 0$$

$$F_{CD} = 400 \text{ N}$$

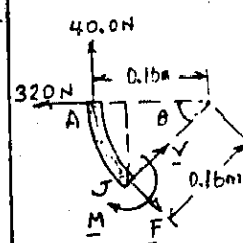
$$\sum F_x = 0: A_x + \frac{4}{5}(400 \text{ N}) = 0$$

$$A_x = -320 \text{ N} \quad A_x = 320 \text{ N}$$

$$\sum F_y = 0: A_y + \frac{3}{5}(400 \text{ N}) - 280 \text{ N} = 0$$

$$A_y = +40.0 \text{ N} \quad A_y = 40.0 \text{ N}$$

FREE BODY: AJ (FOR  $\theta < 90^\circ$ )



$$\sum M_J = 0: (320 \text{ N})(0.16 \text{ m}) \sin \theta - (40.0 \text{ N})(0.16 \text{ m})(1 - \cos \theta) - M = 0$$

$$M = 51.2 \sin \theta + 6.4 \cos \theta - 6.4 \quad (1)$$

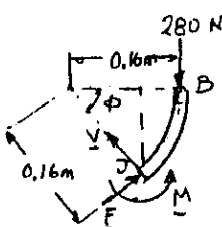
FOR MAXIMUM VALUE BETWEEN A AND C

$$\frac{dM}{d\theta} = 0: 51.2 \cos \theta - 6.4 \sin \theta = 0$$

$$\tan \theta = \frac{51.2}{6.4} = 8 \quad \theta = 82.87^\circ$$

CARRYING INTO (1):

$$M = 51.2 \sin 82.87^\circ + 6.4 \cos 82.87^\circ - 6.4 = +45.20 \text{ N}\cdot\text{m}$$



FREE BODY: BJ (FOR  $\theta > 90^\circ$ )

$$\sum M_J = 0:$$

$$M - (280 \text{ N})(0.16 \text{ m})(1 - \cos \theta) = 0$$

$$M = (44.8 \text{ N}\cdot\text{m})(1 - \cos \theta)$$

LARGEST VALUE OCCURS FOR  $\theta = 90^\circ$

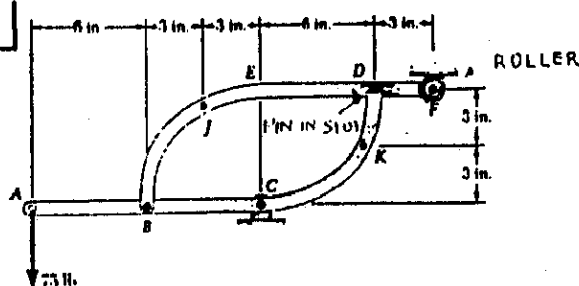
THAT IS, AT C, AND IS

$$M_c = 44.8 \text{ N}\cdot\text{m}$$

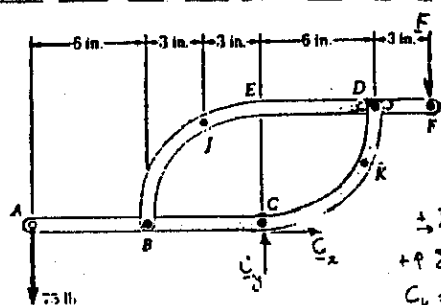
WE CONCLUDE THAT

$$M_{\max} = 45.2 \text{ N}\cdot\text{m FOR } \theta = 82.9^\circ$$

7.13



**GIVEN:** TWO MEMBERS, CONSISTING EACH OF A STRAIGHT AND A QUARTER-CIRCULAR ROD, SUPPORT A 75-lb LOAD.  
**FIND:** INTERNAL FORCES AT POINT J.



FREE BODY:

ENTIRE FRAME

$$+\circlearrowleft \sum M_C = 0;$$

$$(75 \text{ lb})(12 \text{ in}) - F(9 \text{ in}) = 0$$

$$F = 100 \text{ lb} \quad \leftarrow$$

$$+\rightarrow \sum F_x = 0; C_x = 0$$

$$+\uparrow \sum F_y = 0; C_y - 75 \text{ lb} - 100 \text{ lb} = 0$$

$$C_y = +175 \text{ lb}, C = 175 \text{ lb} \quad \leftarrow$$

FREE BODY: MEMBER CBEDF

$$+\circlearrowleft \sum M_B = 0;$$

$$D(12 \text{ in}) - (100 \text{ lb})(15 \text{ in}) = 0$$

$$D = 125 \text{ lb} \quad \leftarrow$$

$$+\rightarrow \sum F_x = 0; B_x = 0$$

$$+\uparrow \sum F_y = 0; B_y + 125 \text{ lb} - 100 \text{ lb} = 0$$

$$B_y = -25 \text{ lb}, B = 25 \text{ lb} \quad \leftarrow$$

FREE BODY: BJ

$$+\rightarrow \sum F_x = 0; F - (25 \text{ lb}) \sin 30^\circ = 0$$

$$F = 12.50 \text{ lb} \angle 30^\circ \quad \leftarrow$$

$$+\uparrow \sum F_y = 0; V - (25 \text{ lb}) \cos 30^\circ = 0$$

$$V = 21.7 \text{ lb} \angle 60^\circ \quad \leftarrow$$

$$+\circlearrowleft \sum M_J = 0; -M + (25 \text{ lb})(3 \text{ in}) = 0$$

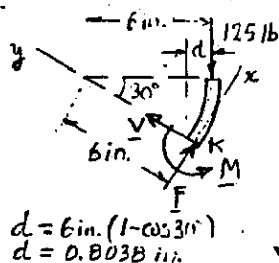
$$M = 75.0 \text{ lb-in.} \quad \leftarrow$$

7.14

(SEE FIGURE OF PROB. 7.13)

**GIVEN:** TWO MEMBERS, CONSISTING EACH OF A STRAIGHT AND A QUARTER-CIRCULAR ROD, SUPPORT A 75-lb LOAD.  
**FIND:** INTERNAL FORCES AT POINT K.

SEE SOLUTION OF PROB. 7.13 UP TO DASHED LINE.



FREE BODY: DK

WE FOUND IN PROB. 7.13 THAT  
 $D = 125 \text{ lb} \uparrow$  ON BEDF, THUS  
 $D = 125 \text{ lb} \downarrow$  ON DK.

$$+\rightarrow \sum F_x = 0; F - (125 \text{ lb}) \cos 30^\circ = 0$$

$$F = 108.3 \text{ lb} \angle 60^\circ \quad \leftarrow$$

$$+\uparrow \sum F_y = 0; V - (125 \text{ lb}) \sin 30^\circ = 0$$

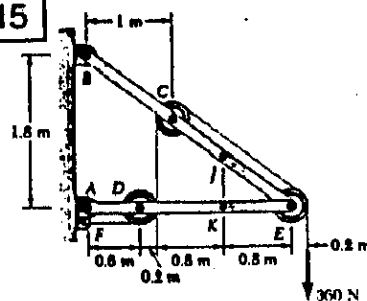
$$V = 62.5 \text{ lb} \angle 30^\circ \quad \leftarrow$$

$$+\circlearrowleft \sum M_K = 0; M - (125 \text{ lb})d = 0$$

$$M = (125 \text{ lb})d = (125 \text{ lb})(0.8038 \text{ m}) = 100.5 \text{ lb-in.}$$

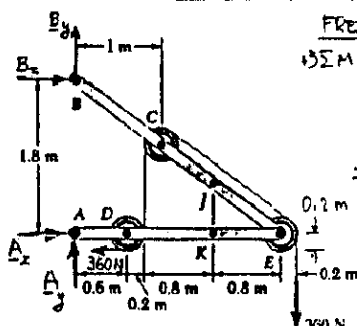
$$M = 100.5 \text{ lb-in.} \quad \leftarrow$$

7.15



GIVEN:

FRAME AND LOADING  
 SHOWN. FOR EACH  
 PULLEY  $R = 200 \text{ mm}$   
**FIND:**  
 INTERNAL FORCES  
 AT POINT J.



FREE BODY: FRAME AND PULLEY

$$+\circlearrowleft \sum M_A = 0; -B_x(1.8 \text{ m}) - (360 \text{ N})(0.2 \text{ m}) - (360 \text{ N})(2.6 \text{ m}) = 0$$

$$B_x = -560 \text{ N}, B_x = 560 \text{ N} \quad \leftarrow$$

$$+\rightarrow \sum F_x = 0; A_x - 560 \text{ N} - 360 \text{ N} = 0$$

$$A_x = +920 \text{ N}, A_x = 920 \text{ N} \quad \leftarrow$$

$$+\uparrow \sum F_y = 0; A_y + B_y - 360 \text{ N} = 0$$

$$A_y + B_y = 360 \text{ N} \quad (1)$$

FREE BODY: MEMBER AE

WE RECALL FROM PROB. 6.40 THAT THE FORCES APPLIED TO A PULLEY MAY BE APPLIED DIRECTLY TO THE AXES OF THE PULLEY.

$$+\circlearrowleft \sum M_E = 0; -A_y(2.4 \text{ m}) - (360 \text{ N})(1.8 \text{ m}) = 0$$

$$A_y = -270 \text{ N}, A_y = 270 \text{ N} \quad \leftarrow$$

$$\text{FROM (1): } B_y = 360 \text{ N} + 270 \text{ N}$$

$$B_y = 630 \text{ N}, B_y = 630 \text{ N} \quad \leftarrow$$

FREE BODY: BJ

$$+\uparrow \sum F_y = 0; \frac{3}{5}(630 \text{ N}) + \frac{4}{5}(560 \text{ N}) - 360 \text{ N} - \frac{3}{5}(360 \text{ N}) - F = 0$$

$$F = +250 \text{ N}, F = 250 \text{ N} \quad \leftarrow$$

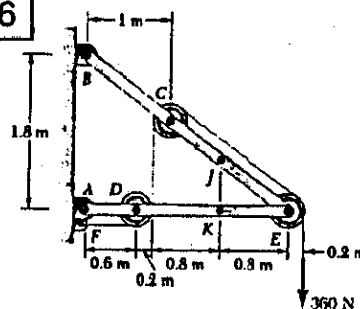
$$+\rightarrow \sum F_x = 0; \frac{4}{5}(630 \text{ N}) - \frac{3}{5}(560 \text{ N}) - \frac{4}{5}(360 \text{ N}) + V = 0$$

$$V = 120.0 \text{ N}, V = 120.0 \text{ N} \quad \leftarrow$$

$$+\circlearrowleft \sum M_J = 0; (560 \text{ N})(1.2 \text{ m}) - (630 \text{ N})(1.6 \text{ m}) + (360 \text{ N})(0.6 \text{ m}) + M = 0$$

$$M = +120.0 \text{ N-m}, M = 120.0 \text{ N-m} \quad \leftarrow$$

7.16



GIVEN:

FRAME AND LOADING  
 SHOWN. FOR EACH  
 PULLEY  $R = 200 \text{ mm}$   
**FIND:**  
 INTERNAL FORCES  
 AT POINT K.

SEE SOLUTION OF PROB. 7.15 UP TO DASHED LINE.  
 WE FOUND

$$A_x = 920 \text{ N} \rightarrow, A_y = 270 \text{ N} \downarrow$$

FREE BODY: AK

$$+\rightarrow \sum F_x = 0; 920 \text{ N} - 360 \text{ N} - F = 0$$

$$F = +560 \text{ N}, F = 560 \text{ N} \quad \leftarrow$$

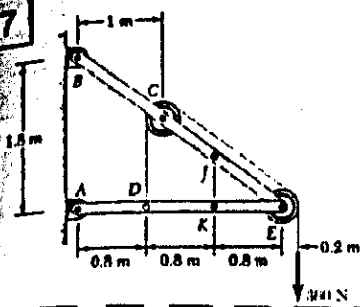
$$+\uparrow \sum F_y = 0; 360 \text{ N} - 270 \text{ N} - V = 0$$

$$V = +90.0 \text{ N}, V = 90.0 \text{ N} \quad \leftarrow$$

$$+\circlearrowleft \sum M_K = 0; (270 \text{ N})(1.6 \text{ m}) - (360 \text{ N})(1 \text{ m}) - M = 0$$

$$M = +72.0 \text{ N-m}, M = 72.0 \text{ N-m} \quad \leftarrow$$

17



**GIVEN:**  
FRAME AND LOADING SHOWN. FOR EACH PULLEY  $R = 200\text{mm}$

**FIND:**  
INTERNAL FORCES AT POINT J.

**FREE BODY: FRAME AND PULLEYS**

$$+\circlearrowleft \Sigma M_A = 0: -B_2(1.8\text{m}) - (360\text{N})(2.6\text{m}) = 0$$

$$B_2 = -520\text{N} \quad B_2 = 520\text{N} \leftarrow$$

$$+\circlearrowleft \Sigma F_x = 0: A_2 - 520\text{N} = 0$$

$$A_2 = +520\text{N} \quad A_2 = 520\text{N} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: A_y + B_2 - 360\text{N} = 0$$

$$A_y + B_2 = 360\text{N} \quad (1)$$

**FREE BODY: MEMBER AE**

$$+\circlearrowleft \Sigma M_E = 0:$$

$$-A_y(2.4\text{m}) - (360\text{N})(1.6\text{m}) = 0$$

$$A_y = -240\text{N} \quad A_y = 240\text{N} \uparrow$$

$$\text{FROM (1): } B_y = 360\text{N} + 240\text{N}$$

$$B_y = +600\text{N} \quad B_y = 600\text{N} \uparrow$$

**FREE BODY: BJ**

WE RECALL FROM PROB. 6.10 THAT THE FORCES APPLIED TO A PULLEY MAY BE APPLIED DIRECTLY TO ITS AXLE.

$$+\circlearrowleft \Sigma F_y = 0: \frac{2}{3}(600\text{N}) + \frac{1}{3}(520\text{N}) - 360\text{N} - \frac{2}{3}(360\text{N}) - F = 0$$

$$F = +200\text{N} \quad F = 200\text{N} \leftarrow$$

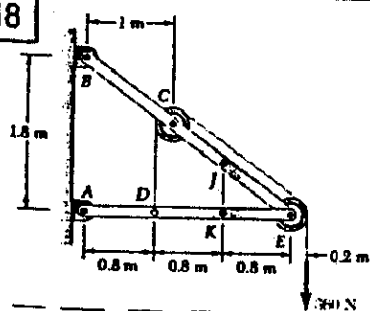
$$+\circlearrowleft \Sigma F_x = 0: \frac{1}{3}(600\text{N}) - \frac{2}{3}(520\text{N}) - \frac{1}{3}(360\text{N}) + V = 0$$

$$V = +120.0\text{N} \quad V = 120.0\text{N} \uparrow$$

$$+\circlearrowleft \Sigma M_J = 0: (520\text{N})(1.2\text{m}) - (600\text{N})(1.6\text{m}) + (360\text{N})(0.6\text{m}) + M = 0$$

$$M = +120.0\text{N}\cdot\text{m} \quad M = 120.0\text{N}\cdot\text{m}$$

7.18



**GIVEN:**  
FRAME AND LOADING SHOWN. FOR EACH PULLEY  $R = 200\text{mm}$

**FIND:**  
INTERNAL FORCES AT POINT K.

SEE SOLUTION OF PROB. 7.17 UP TO DASHED LINE. WE FOUND

$$A_2 = 520\text{N} \rightarrow, \quad A_y = 240\text{N} \uparrow$$

**FREE BODY: AK**

$$+\circlearrowleft \Sigma F_x = 0: 520\text{N} - F = 0$$

$$F = +520\text{N} \quad F = 520\text{N} \leftarrow$$

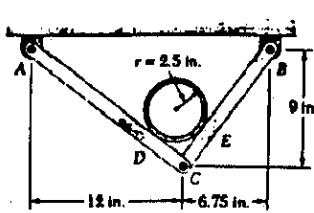
$$+\circlearrowleft \Sigma F_y = 0: 360\text{N} - 240\text{N} - V = 0$$

$$V = +120.0\text{N} \quad V = 120.0\text{N} \uparrow$$

$$+\circlearrowleft \Sigma M_K = 0: (240\text{N})(1.6\text{m}) - (360\text{N})(0.8\text{m}) - M = 0$$

$$M = +96.0\text{N}\cdot\text{m} \quad M = 96.0\text{N}\cdot\text{m}$$

7.19



**GIVEN:**  
PIPE SUPPORTED EVERY 9 ft BY FRAME SHOWN. PIPE AND CONTENTS WEIGH 10 lb/ft.

**FIND:** MAGNITUDE AND LOCATION OF  $M_{\text{max}}$  IN AC

**FREE BODY: 10-ft SECTION OF PIPE**

$$+\circlearrowleft \Sigma F_x = 0: D - \frac{4}{5}(90\text{lb}) = 0 \quad D = 72\text{lb} \leftarrow$$

$$+\circlearrowleft \Sigma F_y = 0: E - \frac{3}{5}(90\text{lb}) = 0 \quad E = 54\text{lb} \leftarrow$$

**FREE BODY: FRAME**

$$+\circlearrowleft \Sigma M_B = 0: -A_2(18.75\text{m})$$

$$+ (72\text{lb})(2.5\text{in}) + (54\text{lb})(8.75\text{in}) = 0$$

$$A_2 = +34.8\text{lb} \quad A_2 = 34.8\text{lb} \leftarrow$$

$$+\circlearrowleft \Sigma F_y = 0:$$

$$B_y + 34.8\text{lb} - \frac{4}{5}(72\text{lb}) - \frac{3}{5}(54\text{lb}) = 0$$

$$B_y = +55.2\text{lb} \quad B_y = 55.2\text{lb} \leftarrow$$

$$+\circlearrowleft \Sigma F_x = 0: A_2 + B_x - \frac{3}{5}(72\text{lb}) + \frac{4}{5}(54\text{lb}) = 0 \quad A_2 + B_x = 0 \quad (1)$$

**FREE BODY: MEMBER AC**

$$+\circlearrowleft \Sigma M_C = 0:$$

$$(72\text{lb})(2.5\text{in}) - (34.8\text{lb})(12\text{in})$$

$$- A_2(9\text{in}) = 0$$

$$A_2 = -26.4\text{lb} \quad A_2 = 26.4\text{lb} \leftarrow$$

$$\text{FROM (1): } B_x = -A_2 = +26.4\text{lb}$$

$$B_x = 26.4\text{lb} \rightarrow$$

**FREE BODY: PORTION AJ**

FOR  $x \leq 12.5\text{ in. (AJ} \leq \text{AD):}$

$$+\circlearrowleft \Sigma M_J = 0: (26.4\text{lb})\frac{2}{3}x - (34.8\text{lb})\frac{4}{3}x + M = 0$$

$$M = 12x, \quad M_{\text{max}} = 150\text{ lb}\cdot\text{in. for } x = 12.5\text{ in.}$$

$$M_{\text{max}} = 150.0\text{ lb}\cdot\text{in. at D}$$

FOR  $x > 12.5\text{ in. (AJ} > \text{AD):}$

$$+\circlearrowleft \Sigma M_J = 0: (26.4\text{lb})\frac{2}{3}x - (34.8\text{lb})\frac{4}{3}x$$

$$+ (72\text{lb})(x - 12.5) + M = 0$$

$$M = 900 - 60x, \quad M_{\text{max}} = 150\text{ lb}\cdot\text{in. for } x = 12.5\text{ in.}$$

$$\text{THUS: } M_{\text{max}} = 150.0\text{ lb}\cdot\text{in. at D.}$$

7.20

**GIVEN:** FRAME OF PROB. 7.19.

**FIND:** MAGNITUDE AND LOCATION OF  $M_{\text{max}}$  IN BC.

SEE SOLUTION OF PROB. 7.19 UP TO DASHED LINE. WE FOUND

$$B_x = 26.4\text{lb} \rightarrow, \quad B_y = 55.2\text{lb} \uparrow$$

**FREE BODY: PORTION BK**

FOR  $x \leq 8.75\text{ in. (BK} \leq \text{BE):}$

$$+\circlearrowleft \Sigma M_K = 0: (55.2\text{lb})\frac{2}{3}x - (26.4\text{lb})\frac{4}{3}x - M = 0$$

$$M = 12x, \quad M_{\text{max}} = 105.0\text{ lb}\cdot\text{in. for } x = 8.75\text{ in.}$$

$$M_{\text{max}} = 105.0\text{ lb}\cdot\text{in. at E}$$

FOR  $x > 8.75\text{ in. (BK} > \text{BE):}$

$$+\circlearrowleft \Sigma M_K = 0: (55.2\text{lb})\frac{2}{3}x - (26.4\text{lb})\frac{4}{3}x$$

$$- (54\text{lb})(x - 8.75\text{ in.}) - M = 0$$

$$M = 472.5 - 42x, \quad M_{\text{max}} = 105.0\text{ lb}\cdot\text{in. for } x = 8.75\text{ in.}$$

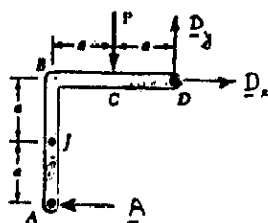
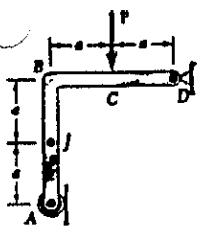
$$\text{THUS: } M_{\text{max}} = 105.0\text{ lb}\cdot\text{in. at E}$$

7.21

GIVEN: BENT ROD SUPPORTED AND LOADED AS SHOWN.  
FIND: FOR EACH CASE, THE INTERNAL FORCES AT J.

CASE (A)

FREE-BODY DIAGRAM



$$+\circlearrowleft \Sigma M_D = 0: Pa - A_y(2a) = 0$$

$$A_y = \frac{P}{2} \leftarrow$$

FREE BODY: AJ

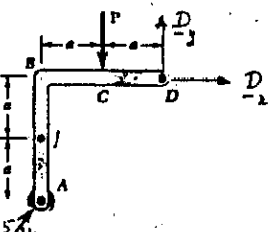
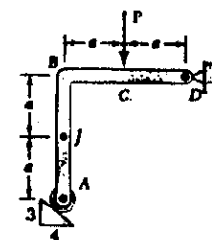
$$+\uparrow \Sigma F_y = 0: F - \frac{P}{2} = 0 \quad F = \frac{P}{2}$$

$$+\rightarrow \Sigma F_x = 0: V - \frac{P}{2} = 0 \quad V = \frac{P}{2}$$

$$+\circlearrowleft \Sigma M_J = 0: M - \frac{P}{2}a = 0 \quad M = \frac{1}{2}Pa$$

CASE (B)

FREE-BODY DIAGRAM



$$+\circlearrowleft \Sigma M_D = 0: Pa - \left(\frac{4}{5}A_y\right)(2a) + \left(\frac{3}{5}A_x\right)(2a) = 0 \quad A_y = \frac{5}{2}P$$

FREE BODY: AJ

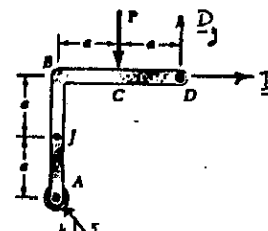
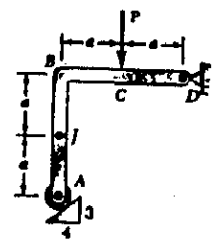
$$+\uparrow \Sigma F_y = 0: -F + \frac{4}{5}\left(\frac{5}{2}P\right) = 0 \quad F = 2P$$

$$+\rightarrow \Sigma F_x = 0: -V + \frac{3}{5}\left(\frac{5}{2}P\right) = 0 \quad V = \frac{3}{2}P$$

$$+\circlearrowleft \Sigma M_J = 0: -M + \frac{3}{5}\left(\frac{5}{2}P\right)a = 0 \quad M = \frac{3}{2}Pa$$

CASE (C)

FREE-BODY DIAGRAM



$$+\circlearrowleft \Sigma M_D = 0: Pa - \left(\frac{4}{5}D_y\right)(2a) - \left(\frac{3}{5}D_x\right)(2a) = 0 \quad D_y = \frac{5}{2}P$$

$$A_y = \frac{5}{2}P$$

FREE BODY: AJ

$$+\uparrow \Sigma F_y = 0: -F + \frac{4}{5}\left(\frac{5}{2}P\right) = 0 \quad F = \frac{2}{7}P$$

$$+\rightarrow \Sigma F_x = 0: V - \frac{3}{5}\left(\frac{5}{2}P\right) = 0 \quad V = \frac{3}{2}P$$

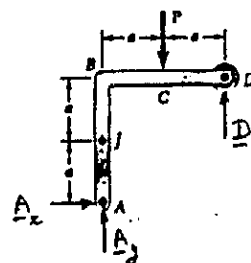
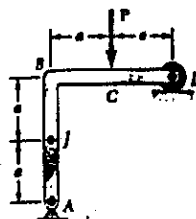
$$+\circlearrowleft \Sigma M_J = 0: M - \frac{3}{5}\left(\frac{5}{2}P\right)a = 0 \quad M = \frac{3}{2}Pa$$

7.22

GIVEN: BENT ROD SUPPORTED AND LOADED AS SHOWN.  
FIND: FOR EACH CASE, THE INTERNAL FORCES AT J.

CASE (A)

FREE-BODY DIAGRAM



$$+\circlearrowleft \Sigma M_D = 0: D_x(2a) - Pa = 0$$

$$+\rightarrow \Sigma F_x = 0: A_x = 0$$

$$+\uparrow \Sigma F_y = 0: A_y - P + \frac{P}{2} = 0 \quad A_y = +\frac{P}{2}$$

$$D_x = \frac{P}{2} \uparrow$$

$$A_x = \frac{P}{2} \uparrow$$

FREE BODY: AJ

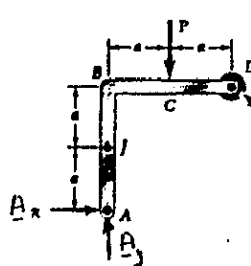
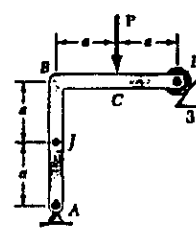
$$+\uparrow \Sigma F_y = 0: \frac{P}{2} - F = 0 \quad F = \frac{P}{2}$$

$$+\rightarrow \Sigma F_x = 0: V = 0$$

$$+\circlearrowleft \Sigma M_J = 0: M = 0$$

CASE (B)

FREE-BODY DIAGRAM



$$+\circlearrowleft \Sigma M_D = 0: \left(\frac{4}{5}D_x\right)(2a) + \left(\frac{3}{5}D_y\right)(2a) - Pa = 0$$

$$+\rightarrow \Sigma F_x = 0: A_x - \frac{4}{5}\left(\frac{5}{2}P\right) = 0 \quad A_x = +2P$$

$$+\uparrow \Sigma F_y = 0: A_y + \frac{3}{5}\left(\frac{5}{2}P\right) - P = 0, \quad A_y = +\frac{11}{14}P$$

$$D_x = \frac{5}{2}P$$

$$A_x = \frac{2}{7}P$$

$$A_y = \frac{11}{14}P$$

FREE BODY: AJ

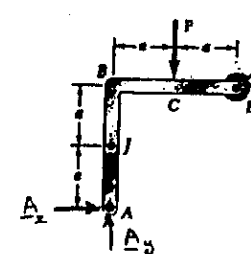
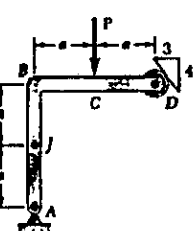
$$+\uparrow \Sigma F_y = 0: -F + \frac{11}{14}P = 0 \quad F = \frac{11}{14}P$$

$$+\rightarrow \Sigma F_x = 0: -V + \frac{2}{7}P = 0 \quad V = \frac{2}{7}P$$

$$+\circlearrowleft \Sigma M_J = 0: -M + \left(\frac{2}{7}P\right)a = 0 \quad M = \frac{2}{7}Pa$$

CASE (C)

FREE-BODY DIAGRAM



$$+\circlearrowleft \Sigma M_D = 0: \left(\frac{4}{5}D_x\right)(2a) - \left(\frac{3}{5}D_y\right)(2a) - Pa = 0$$

$$+\rightarrow \Sigma F_x = 0: A_x - \frac{4}{5}\left(\frac{5}{2}P\right) = 0 \quad A_x = +2P$$

$$+\uparrow \Sigma F_y = 0: A_y - \frac{3}{5}\left(\frac{5}{2}P\right) - P = 0, \quad A_y = +\frac{5}{2}P$$

$$D_x = \frac{5}{2}P$$

$$A_x = 2P$$

$$A_y = \frac{5}{2}P$$

FREE BODY: AJ

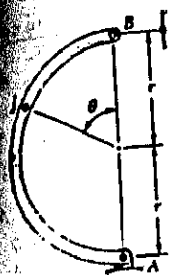
$$+\uparrow \Sigma F_y = 0: -F + \frac{5}{2}P = 0 \quad F = \frac{5}{2}P$$

$$+\rightarrow \Sigma F_x = 0: -V + 2P = 0 \quad V = 2P$$

$$+\circlearrowleft \Sigma M_J = 0: -M + (2P)a = 0 \quad M = 2Pa$$



7.23 AND 7.24



**GIVEN:**  
SEMICIRCULAR ROD OF WEIGHT  $W$  AND UNIFORM CROSS SECTION SUPPORTED AS SHOWN.

**FIND:**  
BENDING MOMENT AT J WHEN  
 $\theta = 60^\circ$  (PROB. 7.23)  
 $\theta = 150^\circ$  (PROB. 7.24)

**FREE BODY: ROD**

$$+\circlearrowleft \sum M_A = 0: W\left(\frac{2r}{\pi}\right) - B(2r) = 0$$

$$B = \frac{W}{\pi}$$

$$+\rightarrow \sum F_x = 0: \frac{W}{\pi} - A_2 = 0 \quad A_2 = \frac{W}{\pi}$$

$$+\uparrow \sum F_y = 0: A_3 - W = 0 \quad A_3 = W$$

**FREE BODY: PORTION BJ**

$$+\circlearrowleft \sum M_J = 0:$$

$$M - \frac{W}{\pi} r (1 - \cos \theta) - \frac{W\theta}{\pi} d = 0$$

$$M = \frac{W}{\pi} r (1 - \cos \theta) + \frac{W\theta}{\pi} d$$

$$\text{BUT } d = r \sin \theta - \frac{r}{2} \sin \frac{\theta}{2}$$

$$= r \sin \theta - \frac{r}{2} \sin \frac{\theta}{2}$$

$$= r \sin \theta - \frac{r}{2} \sin \frac{\theta}{2}$$

$$= r \sin \theta - \frac{r}{2} \sin \frac{\theta}{2}$$

$$\text{THUS: } M = \frac{W}{\pi} r (1 - \cos \theta) + \frac{W}{\pi} r \theta \sin \theta - \frac{W}{\pi} r (1 - \cos \theta)$$

$$M = \frac{W r}{\pi} \theta \sin \theta \quad (1)$$

7.23

MAKING  $\theta = 60^\circ = \frac{\pi}{3}$  IN EQ. (1):

$$M = \frac{W r}{\pi} \frac{\pi}{3} \sin 60^\circ = W r \frac{\sin 60^\circ}{3} \quad M = 0.289 W r$$

7.24

MAKING  $\theta = 150^\circ = \frac{5\pi}{6}$  IN EQ. (1):

$$M = \frac{W r}{\pi} \frac{5\pi}{6} \sin 150^\circ = \frac{5}{12} W r \quad M = 0.417 W r \quad (\text{ON BJ})$$

ALTERNATIVE SOLUTION TO PROB. 7.24:

**FREE BODY: AJ**

$$+\circlearrowleft \sum M_J = 0:$$

$$-M + W r \sin \phi - \frac{W}{\pi} r (1 - \cos \phi) - \frac{W\phi}{\pi} d = 0$$

$$M = W r \sin \phi - \frac{W}{\pi} r (1 - \cos \phi) - \frac{W\phi}{\pi} d$$

$$\text{BUT } d = r \sin \phi - \frac{r}{2} \sin \frac{\phi}{2}$$

$$= r \sin \phi - \frac{r}{2} \sin \frac{\phi}{2}$$

$$= r \sin \phi - \frac{r}{2} \sin \frac{\phi}{2}$$

$$= r \sin \phi - \frac{r}{2} \sin \frac{\phi}{2}$$

$$= r \sin \phi - \frac{r}{2} \sin \frac{\phi}{2}$$

THUS:

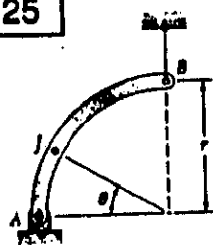
$$M = W r \sin \phi - \frac{W}{\pi} r (1 - \cos \phi) - \frac{W}{\pi} r \phi \sin \phi + \frac{W}{\pi} r (1 - \cos \phi)$$

$$M = W r \left(1 - \frac{\phi}{\pi}\right) \sin \phi \quad (2)$$

MAKING  $\phi = 180^\circ - 150^\circ = 30^\circ = \frac{\pi}{6}$  IN EQ. (2):

$$M = W r \left(1 - \frac{1}{6}\right) \sin 30^\circ = \frac{5}{12} W r \quad M = 0.417 W r \quad (\text{ON AJ})$$

7.25



**GIVEN:**

QUARTER CIRCULAR ROD OF WEIGHT  $W$  AND UNIFORM CROSS SECTION SUPPORTED AS SHOWN

**FIND:**

BENDING MOMENT AT J WHEN  $\theta = 30^\circ$ .

**FREE BODY: ROD**

$$+\rightarrow \sum F_x = 0: A_2 = 0$$

$$+\circlearrowleft \sum M_B = 0:$$

$$W\left(\frac{2r}{\pi}\right) - A_3 r = 0$$

$$A_3 = \frac{2W}{\pi} \quad A = \frac{2W}{\pi}$$

**FREE BODY: PORTION AJ**

$$+\circlearrowleft \sum M_J = 0:$$

$$M + W' d - \frac{2W}{\pi} r (1 - \cos \theta) = 0$$

$$M = \frac{2W}{\pi} r (1 - \cos \theta) - W' d \quad (1)$$

$$\text{BUT } W' = W \frac{\theta}{\pi/2} = \frac{2W\theta}{\pi} \quad (2)$$

AND

$$d = \frac{r}{2} \cos \frac{\theta}{2} - r \cos \theta$$

$$= \frac{r}{2} \sin \frac{\theta}{2} \cos \frac{\theta}{2} - r \cos \theta$$

$$= \frac{r}{2} \sin \frac{\theta}{2} \cos \frac{\theta}{2} - r \cos \theta$$

$$d = \frac{r}{2} (\sin \theta - \cos \theta) \quad (3)$$

SUBSTITUTING FROM (2) AND (3) INTO (1):

$$M = \frac{2W}{\pi} r (1 - \cos \theta) - \frac{2W\theta}{\pi} \left( \frac{\sin \theta}{2} - \cos \theta \right)$$

$$M = \frac{2W r}{\pi} (1 - \cos \theta - \sin \theta + \theta \cos \theta) \quad (4)$$

MAKING  $\theta = 30^\circ = \frac{\pi}{6}$  IN EQ. (4):

$$M = \frac{2W r}{\pi} \left[ \frac{1 - \cos 30^\circ - \sin 30^\circ + \frac{1}{6} \cos 30^\circ}{1} \right] \quad M = 0.0557 W r$$

THE SOLUTIONS OF PROBS. 7.26 AND 7.27 ARE GIVEN ON THE NEXT PAGE

7.28

**GIVEN:** ROD OF PROB. 7.25.

**FIND:** MAGNITUDE AND LOCATION OF MAXIMUM BENDING MOMENT.

WE RECALL EQ. (4) OF PROB. 7.25:

$$M = \frac{2W r}{\pi} (1 - \cos \theta - \sin \theta + \theta \cos \theta) \quad (4)$$

$$\frac{dM}{d\theta} = \frac{2W r}{\pi} (\sin \theta - \cos \theta + \cos \theta - \theta \sin \theta)$$

$$\text{SETTING } \frac{dM}{d\theta} = 0:$$

$$\sin \theta (1 - \theta) = 0$$

THE ROOTS OF THIS EQUATION FOR  $0 \leq \theta \leq \frac{\pi}{2}$  ARE

$$\theta = 0 \quad \text{AND} \quad \theta = 1 \text{ RAD} = 57.3^\circ$$

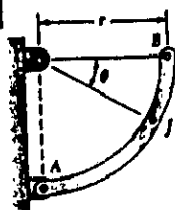
FOR  $\theta = 0$ ,  $M = 0$ . FOR  $\theta = 1 \text{ RAD} = 57.3^\circ$ , EQ. (4) YIELDS

$$M = \frac{2W r}{\pi} (1 - \cos 57.3^\circ - \sin 57.3^\circ + 1 \times \cos 57.3^\circ)$$

$$= \frac{2W r}{\pi} (1 - \sin 57.3^\circ) = 0.1009 W r$$

THUS:  $M_{\max} = 0.1009 W r$  for  $\theta = 57.3^\circ$

7.26



GIVEN:

QUARTER CIRCULAR ROD OF WEIGHT  $W$  AND UNIFORM CROSS SECTION SUPPORTED AS SHOWN.

FIND:

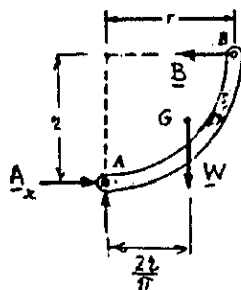
BENDING MOMENT AT  $J$  WHEN  $\theta = 30^\circ$

FREE BODY: ROD

$$+\circlearrowleft \Sigma M_A = 0:$$

$$B\ell - W\left(\frac{2\ell}{\pi}\right) = 0$$

$$B = \frac{2W}{\pi}$$



FREE BODY: PORTION BJ

$$+\circlearrowleft \Sigma M_J = 0:$$

$$\frac{2W}{\pi} \ell \sin \theta - W'd - M = 0$$

$$M = \frac{2W}{\pi} \ell \sin \theta - W'd \quad (1)$$

$$\text{BUT } W' = W \frac{\theta}{\pi/2} = \frac{2W\theta}{\pi} \quad (2)$$

AND

$$d = \frac{2}{\pi} \cos \frac{\theta}{2} - \ell \cos \theta$$

$$= \frac{2}{\pi} \frac{\sin \theta/2}{\theta/2} \cos \frac{\theta}{2} - \ell \cos \theta$$

$$= \frac{2}{\pi} \frac{\sin \theta/2 \cos \theta/2}{\theta/2} - \ell \cos \theta$$

$$d = \frac{2}{\pi} \left( \frac{\sin \theta}{\theta} - \cos \theta \right) \quad (3)$$

SUBSTITUTING FROM (2) AND (3) INTO (1):

$$M = \frac{2W}{\pi} \ell \sin \theta - \frac{2W\theta}{\pi} \frac{2}{\pi} \left( \frac{\sin \theta}{\theta} - \cos \theta \right)$$

$$M = \frac{2W\ell}{\pi} \theta \cos \theta \quad (4)$$

MAKING  $\theta = 30^\circ = \frac{\pi}{6}$  IN EQ. (4):

$$M = \frac{2W\ell}{\pi} \left( \frac{\pi}{6} \right) \cos 30^\circ = \frac{W\ell}{3} \cos 30^\circ \quad M = 0.289 W\ell$$

7.27

GIVEN: ROD OF PROB. 7.26.

FIND: MAGNITUDE AND LOCATION OF MAXIMUM BENDING MOMENT.

WE RECALL EQ. (4) OF PROB. 7.26:

$$M = \frac{2W\ell}{\pi} \theta \cos \theta \quad (4)$$

$$\frac{dM}{d\theta} = 0:$$

$$\cos \theta - \theta \sin \theta = 0$$

$$\tan \theta = \frac{1}{\theta}$$

SOLVING BY SUCCESSIVE APPROXIMATIONS:

$$\theta = 49.293^\circ = 0.86033 \text{ RAD}$$

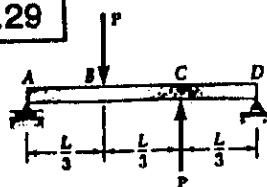
SUBSTITUTING INTO EQ. (4):

$$M = \frac{2W\ell}{\pi} (0.86033 \text{ RAD}) \cos 49.293^\circ = 0.3572 W\ell$$

THUS:  $M_{\max} = 0.357 W\ell$  for  $\theta = 49.3^\circ$ 

THE SOLUTION OF PROB. 7.28 IS GIVEN ON THE PRECEDING PAGE

7.29



GIVEN:

BEAM AND LOADING

(a) DRAW V AND M DIAG.

(b) DETERMINE  $|V|_{\max}$  $|M|_{\max}$ .

FREE BODY: ENTIRE BEAM

$$+\circlearrowleft \Sigma M_D = 0:$$

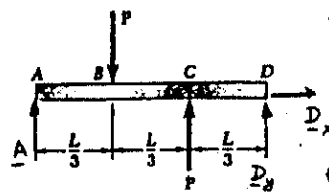
$$P\left(\frac{2L}{3}\right) - P\left(\frac{L}{3}\right) - A\ell = 0$$

$$A = P/3$$

$$+\rightarrow \Sigma F_x = 0: D_x = 0$$

$$+\uparrow \Sigma F_y = 0: \frac{P}{3} - P + P + D_y = 0$$

$$D_y = -P/3 \quad D = P/3$$



(a) SHEAR AND BENDING MOMENT. SINCE THE LOADING CONSISTS OF CONCENTRATED LOADS, THE SHEAR DIAGRAM IS MADE OF HORIZONTAL STRAIGHT-LINE SEGMENTS AND THE B.M. DIAGRAM IS MADE OF OBLIQUE STRAIGHT-LINE SEGMENTS. WE SHALL DETERMINE  $V$  AND  $M$  JUST TO THE RIGHT OF A, B, AND C.

$$+\uparrow \Sigma F_y = 0: -V_1 + \frac{P}{3} = 0 \quad V_1 = +P/3$$

$$+\circlearrowleft \Sigma M_1 = 0: M_1 - \frac{P}{3}(0) = 0 \quad M_1 = 0$$

$$+\uparrow \Sigma F_y = 0: -V_2 + \frac{P}{3} - P = 0, \quad V_2 = -2P/3$$

$$+\circlearrowleft \Sigma M_2 = 0: M_2 - \frac{P}{3}\left(\frac{L}{3}\right) + P(0) = 0$$

$$M_2 = +PL/9$$

$$+\uparrow \Sigma F_y = 0: \frac{P}{3} - P + P - V_3 = 0$$

$$V_3 = +P/3$$

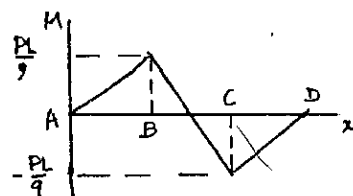
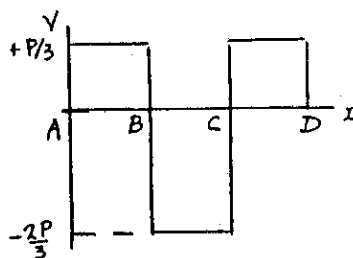
$$+\circlearrowleft \Sigma M_3 = 0: M_3 - \frac{P}{3}\left(\frac{2L}{3}\right) + P\left(\frac{L}{3}\right) - P(0) = 0$$

$$M_3 = -PL/9$$

$$\text{JUST TO THE LEFT OF D:}$$

$$+\uparrow \Sigma F_y = 0: V_4 - \frac{P}{3} = 0 \quad V_4 = +P/3$$

$$+\circlearrowleft \Sigma M_4 = 0: -M_4 - \frac{P}{3}(0) = 0 \quad M_4 = 0$$



$$(b) |V|_{\max} = 2P/3; |M|_{\max} = PL/9$$

GIVEN:  
BEAM AND LOADING SHOWN  
(a) DRAW V AND M DIAGRAM  
(b) DETERMINE  $|V|_{\max}$  AND  $|M|_{\max}$ .

PARTS. BECAUSE OF SYMMETRY OF

LOADING!  $A = D = \frac{1}{4} wL$   $\triangleleft$   
 $(D) = \frac{1}{2} (w \frac{L}{2})$

(a) SHEAR AND BENDING MOMENT

JUST TO THE RIGHT OF A:

$V_1 = +\frac{1}{4} wL$   $\triangleleft$   
 $M_1 = 0$   $\triangleleft$

AT B:  $+\uparrow \Sigma F_y = 0: \frac{1}{4} wL - V_2 = 0, V_2 = +\frac{1}{4} wL$   $\triangleleft$   
 $+\circlearrowleft \Sigma M_B = 0: M_2 - (\frac{1}{4} wL)(\frac{L}{4}) = 0$   $\triangleleft$   
 $M_2 = +wL^2/16$   $\triangleleft$

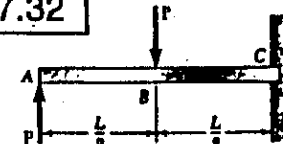
AT CENTER LINE:  
 $+\uparrow \Sigma F_y = 0: \frac{1}{4} wL - \frac{1}{4} wL - V_3 = 0, V_3 = 0$   $\triangleleft$   
 $+\circlearrowleft \Sigma M_3 = 0:$   $\triangleleft$   
 $M_3 - (\frac{1}{4} wL)(\frac{L}{2}) + (\frac{wL}{4})(\frac{L}{8}) = 0, M_3 = \frac{3}{32} wL^2$   $\triangleleft$

THE REMAINDERS OF THE DIAGRAMS ARE OBTAINED FROM SYMMETRY.

(b)  $|V|_{\max} = wL/4$   $\triangleleft$   
 $|M|_{\max} = 3wL^2/32$   $\triangleleft$

PARABOLA  $\triangleleft$   
 STRAIGHT LINE  $\triangleleft$

7.32



GIVEN:  
BEAM AND LOADING SHOWN.  
(a) DRAW V AND M DIAGRAMS.  
(b) DETERMINE  $|V|_{\max}$  AND  $|M|_{\max}$ .

(a) SHEAR AND BENDING MOMENT

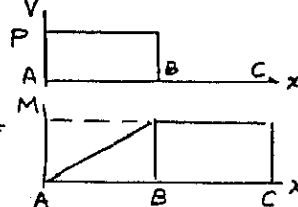
JUST TO THE RIGHT OF A:  $V_1 = +P, M_1 = 0$   $\triangleleft$

JUST TO THE RIGHT OF B:

$+\uparrow \Sigma F_y = 0: P - P - V_2 = 0, V_2 = 0$   $\triangleleft$   
 $+\circlearrowleft \Sigma M_2 = 0: M_2 - P(\frac{L}{2}) = 0, M_2 = +PL/2$   $\triangleleft$

JUST TO THE LEFT OF C:

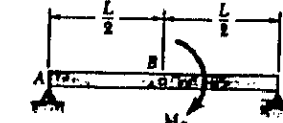
$+\uparrow \Sigma F_y = 0: P - P - V_3 = 0, V_3 = 0$   $\triangleleft$   
 $+\circlearrowleft \Sigma M_3 = 0:$   $\triangleleft$   
 $M_3 + P(\frac{L}{2}) - PL = 0, M_3 = +PL/2$   $\triangleleft$



(b)  $|V|_{\max} = P$   $\triangleleft$

$|M|_{\max} = PL/2$   $\triangleleft$

7.33



GIVEN:  
BEAM AND LOADING SHOWN.  
(a) DRAW V AND M DIAGRAMS  
(b) DETERMINE  $|V|_{\max}$  AND  $|M|_{\max}$ .

FREE BODY: ENTIRE BEAM

$+\circlearrowleft \Sigma M_A = 0: CL - M_1 = 0$   $\triangleleft$   
 $C = M_0/L$   $\triangleleft$

$+\uparrow \Sigma F_x = 0: A_x = 0$   $\triangleleft$   
 $+\uparrow \Sigma F_y = 0: A_y + \frac{M_0}{L} = 0, A_y = -\frac{M_0}{L}$   $\triangleleft$   
 $A = M_0/L$   $\triangleleft$

(a) SHEAR AND BENDING MOMENT:

JUST TO THE RIGHT OF A:

$V_1 = -M_0/L, M_1 = 0$   $\triangleleft$

JUST TO THE LEFT OF B:

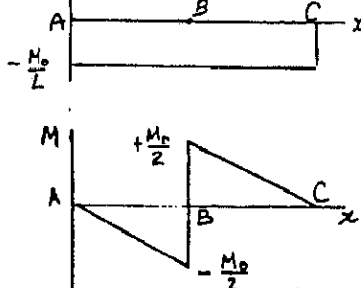
$V_2 = -M_0/L, M_2 = -\frac{M_0}{L} \frac{L}{2} = -\frac{M_0}{2}$   $\triangleleft$

JUST TO THE RIGHT OF B:

$V_3 = -M_0/L, M_3 = +\frac{M_0}{L} \frac{L}{2} = +\frac{M_0}{2}$   $\triangleleft$

JUST TO THE LEFT OF C:

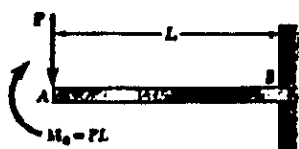
$V_4 = -M_0/L, M_4 = 0$   $\triangleleft$



(b)  $|V|_{\max} = M_0/L$   $\triangleleft$

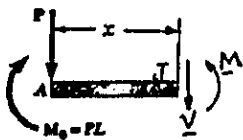
$|M|_{\max} = M_0/2$   $\triangleleft$

7.34



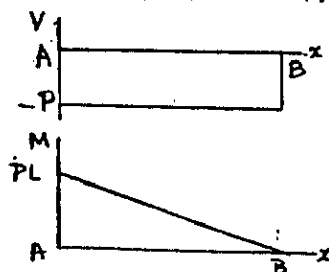
GIVEN:  
BEAM AND LOADING SHOWN.  
(a) DRAW V AND M DIAGRAMS  
(b) DETERMINE  $|V|_{\max}$  AND  $|M|_{\max}$ .

FREE BODY: PORTION AJ



$$\begin{aligned} +\uparrow \Sigma F_y = 0: -P - V &= 0, \quad V = -P \\ +\circlearrowleft \Sigma M_J = 0: M + Px - PL &= 0 \\ M &= P(L-x) \end{aligned}$$

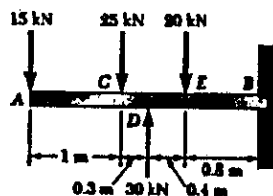
(a) THE V AND M DIAGRAMS ARE OBTAINED BY PLOTTING THE FUNCTIONS V AND M.



(b)  $|V|_{\max} = P$

$|M|_{\max} = PL$

7.35



GIVEN:  
BEAM AND LOADING SHOWN.  
(a) DRAW V AND M DIAGRAMS  
(b) DETERMINE  $|V|_{\max}$  AND  $|M|_{\max}$ .

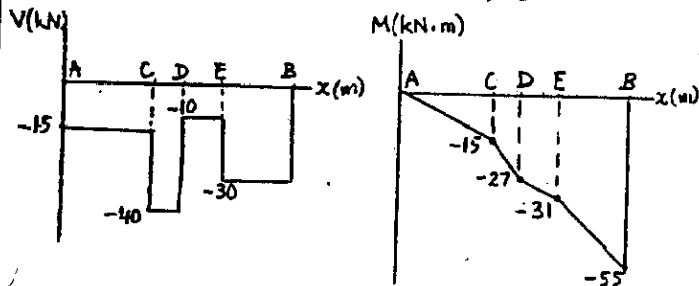
JUST TO THE RIGHT OF A:  
 $+\uparrow \Sigma F_y = 0: V_1 = -15 \text{ kN} \quad M_1 = 0$

JUST TO THE RIGHT OF C:  
 $V_2 = -40 \text{ kN} \quad M_2 = -15 \text{ kN}\cdot\text{m}$

JUST TO THE RIGHT OF D:  
 $V_3 = -10 \text{ kN}, M_3 = -27 \text{ kN}\cdot\text{m}$

JUST TO THE RIGHT OF E:  
 $V_4 = 30 \text{ kN} - 60 \text{ kN}, V_4 = -30 \text{ kN}$   
 $M_4 = 30 \times 0.4 - 15 \times 1.7 - 25 \times 0.7$   
 $M_4 = -31 \text{ kN}\cdot\text{m}$

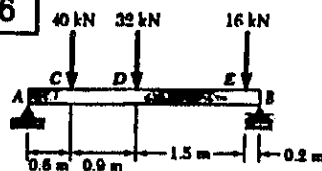
AT B:  $M_B = 30 \times 1.2 - 15 \times 2.5 - 25 \times 1.5 - 20 \times 0.8, M_B = -55 \text{ kN}\cdot\text{m}$



(b)

$|V|_{\max} = 40.0 \text{ kN}; |M|_{\max} = 55.0 \text{ kN}\cdot\text{m}$

7.36



GIVEN:  
BEAM AND LOADING SHOWN.  
(a) DRAW V AND M DIAGRAMS  
(b) DETERMINE  $|V|_{\max}$  AND  $|M|_{\max}$ .

FREE BODY: ENTIRE BEAM

$$\begin{aligned} +\circlearrowleft \Sigma M_A &= 0 \\ B(3.2 \text{ m}) - (40 \text{ kN})(0.6 \text{ m}) - (32 \text{ kN})(1.5 \text{ m}) - (16 \text{ kN})(3.0 \text{ m}) &= 0 \\ B &= 37.5 \text{ kN} \end{aligned}$$

$\Sigma F_x = 0: A_x = 0$

$+\uparrow \Sigma F_y = 0: A_y + 37.5 \text{ kN} - 40 \text{ kN} - 32 \text{ kN} - 16 \text{ kN} = 0 \quad A_y = +50.5 \text{ kN}$   
 $A = 50.5 \text{ kN} \uparrow$

(a) SHEAR AND BENDING MOMENT.

JUST TO THE RIGHT OF A:

$V_1 = 50.5 \text{ kN} \quad M_1 = 0$

JUST TO THE RIGHT OF C:

$+\uparrow \Sigma F_y = 0: 50.5 \text{ kN} - 40 \text{ kN} - V_2 = 0$

$V_2 = +10.5 \text{ kN}$

$+\circlearrowleft \Sigma M_C = 0: M_2 - (50.5 \text{ kN})(0.6 \text{ m}) = 0$

$M_2 = +30.3 \text{ kN}\cdot\text{m}$

JUST TO THE RIGHT OF D:

$+\uparrow \Sigma F_y = 0: 50.5 - 40 - 32 - V_3 = 0$

$V_3 = -21.5 \text{ kN}$

$+\circlearrowleft \Sigma M_D = 0: M_3 - (50.5)(1.5) + (40)(0.9) = 0$

$M_3 = +39.8 \text{ kN}\cdot\text{m}$

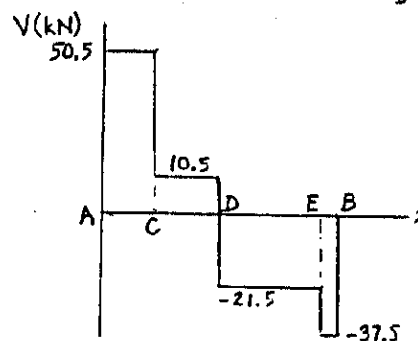
JUST TO THE RIGHT OF E:

$+\uparrow \Sigma F_y = 0: V_4 + 37.5 = 0 \quad V_4 = -37.5 \text{ kN}$

$+\circlearrowleft \Sigma M_E = 0: -M_4 + (37.5)(0.2) = 0$

$M_4 = +7.50 \text{ kN}\cdot\text{m}$

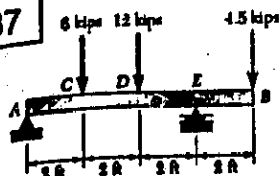
AT B:  $V_B = M_B = 0$



(b)  $|V|_{\max} = 50.5 \text{ kN}$

$|M|_{\max} = 39.8 \text{ kN}\cdot\text{m}$

7.37



GIVEN:

BEAM AND LOADING SHOWN  
(a) DRAW V AND M DIAGRAMS(b) DETERMINE  $|V|_{\max}$  AND  $|M|_{\max}$ 

FREE BODY: ENTIRE BEAM

$$\begin{aligned}
 +\sum M_A = 0: & E(6\text{ ft}) - (6\text{ kips})(2\text{ ft}) - (12\text{ kips})(4\text{ ft}) - (4.5\text{ kips})(8\text{ ft}) = 0 \\
 & E = +16\text{ kips} \quad E = 16\text{ kips} \uparrow \\
 +\sum F_y = 0: & A_2 = 0
 \end{aligned}$$

$$\begin{aligned}
 +\sum F_y = 0: & A_2 + 16\text{ kips} - 6\text{ kips} - 12\text{ kips} - 4.5\text{ kips} = 0 \\
 & A_2 = +6.50\text{ kips} \quad A = 6.50\text{ kips} \uparrow
 \end{aligned}$$

(a) SHEAR AND BENDING MOMENT

JUST TO THE RIGHT OF A:

$$V_1 = +6.50\text{ kips}, M_1 = 0$$

JUST TO THE RIGHT OF C:

$$+\sum F_y = 0: 6.50\text{ kips} - 6\text{ kips} - V_2 = 0$$

$$V_2 = +0.50\text{ kips}$$

$$+\sum M_C = 0:$$

$$M_2 - (6.50\text{ kips})(2\text{ ft}) = 0, M_2 = +13\text{ kip}\cdot\text{ft}$$

JUST TO THE RIGHT OF D:

$$+\sum F_y = 0: 6.50 - 6 - 12 - V_3 = 0$$

$$V_3 = +11.5\text{ kips}$$

$$+\sum M_D = 0:$$

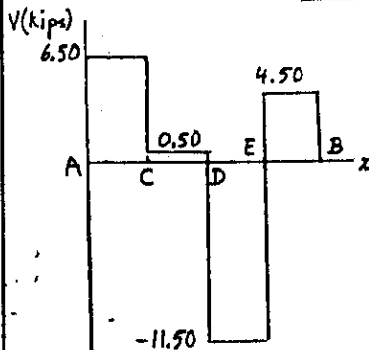
$$M_3 - (6.50)(4) - (6)(2) = 0, M_3 = +14\text{ kip}\cdot\text{ft}$$

JUST TO THE RIGHT OF E:

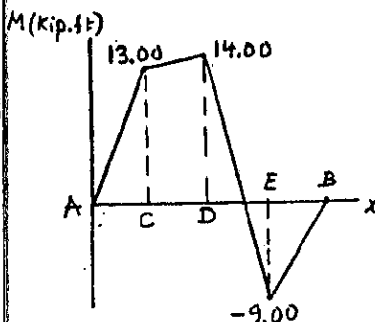
$$+\sum F_y = 0: V_4 - 4.5 = 0, V_4 = +4.5\text{ kips}$$

$$+\sum M_E = 0: -M_4 - (4.5)(2) = 0, M_4 = -9\text{ kip}\cdot\text{ft}$$

$$\text{AT B: } V_B = M_B = 0$$

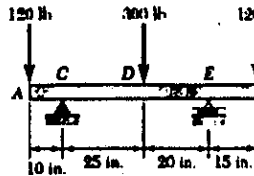


$$(b) |V|_{\max} = 11.50\text{ kips}$$



$$|M|_{\max} = 14.00\text{ kip}\cdot\text{ft}$$

7.38



GIVEN:

BEAM AND LOADING SHOWN  
(a) DRAW V AND M DIAGRAMS(b) DETERMINE  $|V|_{\max}$  AND  $|M|_{\max}$ 

FREE BODY: ENTIRE BEAM

$$\begin{aligned}
 +\sum M_C = 0: & (120\text{ lb})(10\text{ in.}) - (300\text{ lb})(25\text{ in.}) + E(45\text{ in.}) - (120\text{ lb})(60\text{ in.}) = 0 \\
 & E = +300\text{ lb} \quad E = 300\text{ lb} \uparrow \\
 \sum F_x = 0: & C_x = 0
 \end{aligned}$$

$$+\sum F_y = 0: C_y + 300\text{ lb} - 120\text{ lb} - 300\text{ lb} - 120\text{ lb} = 0 \quad C_y = +240\text{ lb}$$

$$C = 240\text{ lb} \uparrow$$

(a) SHEAR AND BENDING MOMENT

JUST TO THE RIGHT OF A:

$$+\sum F_y = 0: -120\text{ lb} - V_1 = 0, V_1 = -120\text{ lb}, M_1 = 0$$

JUST TO THE RIGHT OF C:

$$+\sum F_y = 0: 240\text{ lb} - 120\text{ lb} - V_2 = 0, V_2 = +120\text{ lb}$$

$$+\sum M_C = 0: M_2 + (120\text{ lb})(10\text{ in.}) = 0, M_2 = -1200\text{ lb}\cdot\text{in.}$$

JUST TO THE RIGHT OF D:

$$+\sum F_y = 0: 240 - 120 - 300 - V_3 = 0$$

$$V_3 = -180\text{ lb}$$

$$+\sum M_D = 0:$$

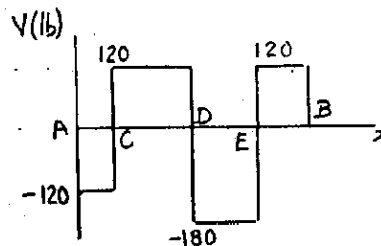
$$M_3 + (120)(35) - (240)(25) = 0, M_3 = +1800\text{ lb}\cdot\text{in.}$$

JUST TO THE RIGHT OF E:

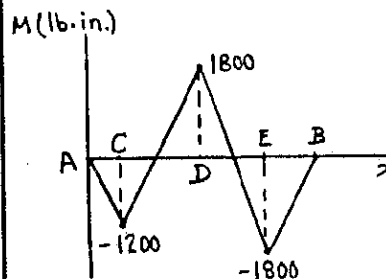
$$+\sum F_y = 0: V_4 - 120\text{ lb} = 0, V_4 = +120\text{ lb}$$

$$+\sum M_E = 0: -M_4 - (120)(15\text{ in.}) = 0, M_4 = -1800\text{ lb}\cdot\text{in.}$$

$$\text{AT B: } V_B = M_B = 0$$

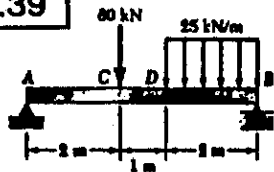


$$(b) |V|_{\max} = 180.0\text{ lb}$$



$$|M|_{\max} = 1800\text{ lb}\cdot\text{in.}$$

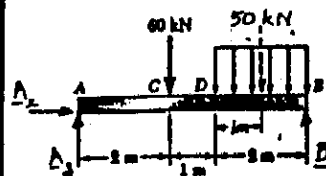
7.39



GIVEN:

BEAM AND LOADING SHOWN.

(a) DRAW V AND M DIAGRAMS

(b) DETERMINE  $|V|_{max}$  AND  $|M|_{max}$ .

FREE BODY: ENTIRE BEAM

$$+\uparrow \Sigma M_A = 0:$$

$$B(5m) - (60kN)(2m) - (50kN)(4m) = 0$$

$$B = +64.0kN, B = 64.0kN \uparrow$$

$$\Sigma F_x = 0: A_x = 0$$

$$+\uparrow \Sigma F_y = 0: A_y + 64.0kN - 60kN - 50kN = 0, A_y = +46.0kN$$

$$A = 46.0kN \uparrow$$

(a) SHEAR AND BENDING-MOMENT DIAGRAMS.

FROM A TO C:

$$+\uparrow \Sigma F_y = 0: 46 - V = 0, V = +46kN$$

$$+\uparrow \Sigma M_J = 0: M - 46x = 0, M = (46x)kN \cdot m$$

FROM C TO D:

$$+\uparrow \Sigma F_y = 0: 46 - 60 - V = 0, V = -14kN$$

$$+\uparrow \Sigma M_J = 0: M - 46x + 60(x-2) = 0$$

$$M = (120 - 14x)kN \cdot m$$

$$\text{FOR } x = 2m: M_C = +92.0kN \cdot m$$

$$\text{FOR } x = 3m: M_D = +78.0kN \cdot m$$

FROM D TO B:

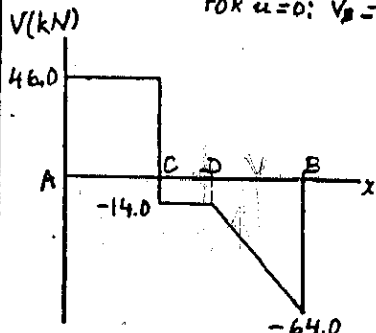
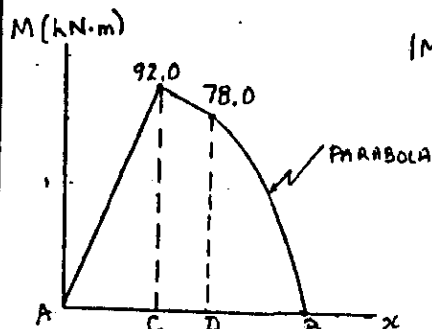
$$+\uparrow \Sigma F_y = 0: V + 64 - 25u = 0$$

$$V = (25u - 64)kN$$

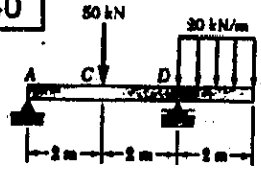
$$+\uparrow \Sigma M_J = 0: 64u - (25u)(\frac{u}{2}) - M = 0$$

$$M = (64u - 12.5u^2)kN \cdot m$$

$$\text{FOR } u = 0: V_B = -64kN, M_B = 0$$

(b)  $|V|_{max} = 64.0kN$  $|M|_{max} = 92.0kN \cdot m$ 

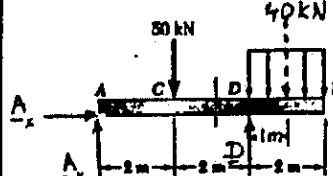
7.40



GIVEN:

BEAM AND LOADING SHOWN.

(a) DRAW V AND M DIAGRAMS

(b) DETERMINE  $|V|_{max}$  AND  $|M|_{max}$ .

FREE BODY: ENTIRE BEAM

$$+\uparrow \Sigma M_A = 0:$$

$$D(4m) - (50kN)(2m) - (40kN)(5m) = 0$$

$$D = +75.0kN, D = 75.0kN \uparrow$$

$$\Sigma F_x = 0: A_x = 0$$

$$+\uparrow \Sigma F_y = 0: A_y + 75.0kN - 50kN - 40kN = 0, A_y = +15.0kN$$

$$A = 15.0kN \uparrow$$

(a) SHEAR AND BENDING-MOMENT DIAGRAMS

FROM A TO C:

$$+\uparrow \Sigma F_y = 0: 15 - V = 0, V = +15.0kN$$

$$+\uparrow \Sigma M_J = 0: M - 15x = 0, M = (15x)kN \cdot m$$

FROM C TO D:

$$+\uparrow \Sigma F_y = 0: 15 - 50 - V = 0, V = -35.0kN$$

$$+\uparrow \Sigma M_J = 0: M - 15x + 50(x-2) = 0, M = (100 - 35x)kN \cdot m$$

$$\text{FOR } x = 2m: M_C = +30.0kN \cdot m$$

$$\text{FOR } x = 4m: M_D = -40.0kN \cdot m$$

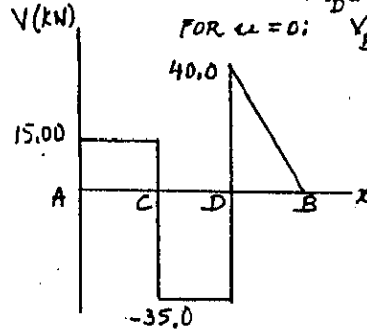
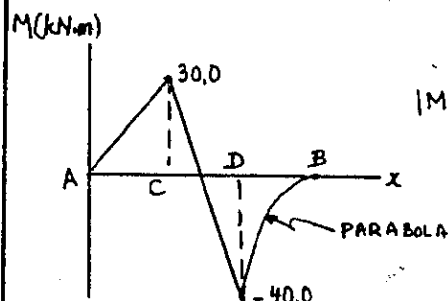
FROM D TO B:

$$+\uparrow \Sigma F_y = 0: V - 20u = 0, V = (20u)kN$$

$$+\uparrow \Sigma M_J = 0: -M - (20u)(\frac{u}{2}) = 0, M = (-10u^2)kN \cdot m$$

$$\text{FOR } u = 2m: V_D = 40kN, M_D = -40kN \cdot m$$

$$\text{FOR } u = 0: V_B = M_B = 0$$

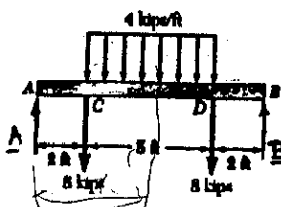
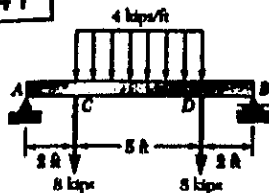
(b)  $|V|_{max} = 40.0kN$  $|M|_{max} = 40.0kN \cdot m$

7.41

GIVEN:

BEAM AND LOADING SHOWN.

(a) DRAW V AND M DIAGRAMS.

(b) DETERMINE  $|V|_{\max}$  AND  $|M|_{\max}$ .FREE BODY: ENTIRE BEAM  
BECAUSE OF SYMMETRY OF  
LOADING:

$$A = B = \frac{1}{2} (\text{TOTAL LOAD})$$

$$= \frac{1}{2} (8 + 8 + 4 \times 4) \text{ kips}$$

$$A = B = 18 \text{ kips} \uparrow$$

(a) SHEAR AND BENDING-MOMENT DIAGRAMS



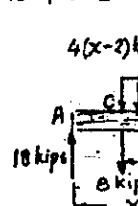
FROM A TO C:

$$+\uparrow \Sigma F_y = 0: 18 \text{ kips} - V = 0$$

$$V = +18 \text{ kips}$$

$$+\circlearrowleft \Sigma M_j = 0: M - 18x = 0$$

$$M = + (18x) \text{ kip}\cdot\text{ft}$$



FROM C TO D:

$$+\uparrow \Sigma F_y = 0: 18 - 4(x-2) - V = 0$$

$$V = 18 - 4x$$

$$+\circlearrowleft \Sigma M_j = 0:$$

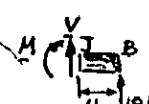
$$M + 8(x-2) + 4(x-2)\frac{x-2}{2} - 18x = 0$$

$$M = 18x - 8(x-2) - 2(x-2)^2$$

$$\text{FOR } x = 2: V_C = +10 \text{ kips}, M_C = +36 \text{ kip}\cdot\text{ft}$$

$$\text{FOR } x = 4.5: V_E = 0, M_E = +48.5 \text{ kip}\cdot\text{ft}$$

$$\text{FOR } x = 7: V_D = -10 \text{ kips}, M_D = +36 \text{ kip}\cdot\text{ft}$$



FROM D TO B:

$$+\uparrow \Sigma F_y = 0: V + 18 = 0$$

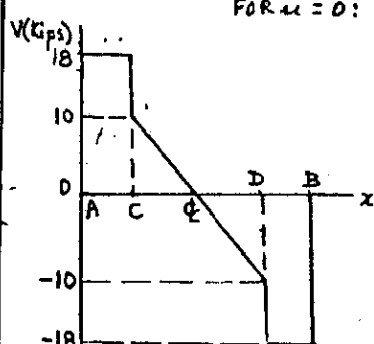
$$V = -18 \text{ kips}$$

$$+\circlearrowleft \Sigma M_j = 0: 18u - M = 0$$

$$M = (18u) \text{ kip}\cdot\text{ft}$$

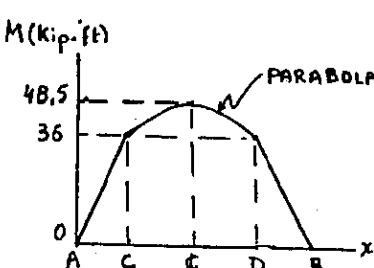
$$\text{FOR } u = 2 \text{ ft: } M_D = +36 \text{ kip}\cdot\text{ft}$$

$$\text{FOR } u = 0: M_B = 0$$



(b)

$$|V|_{\max} = 18.00 \text{ kips}$$



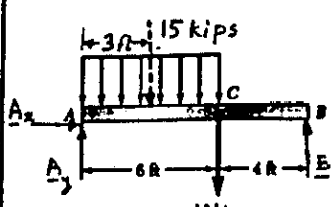
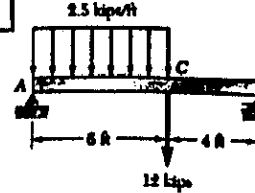
$$|M|_{\max} = 48.5 \text{ kip}\cdot\text{ft}$$

7.42

GIVEN:

BEAM AND LOADING SHOWN.

(a) DRAW V AND M DIAGRAMS.

(b) DETERMINE  $|V|_{\max}$  AND  $|M|_{\max}$ .

FREE BODY: ENTIRE BEAM

$$+\uparrow \Sigma M_A = 0:$$

$$B(10 \text{ ft}) - (15 \text{ kips})(3 \text{ ft}) - (12 \text{ kips})(6 \text{ ft}) = 0$$

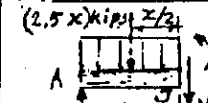
$$B = +11.70 \text{ kips}, B = 11.70 \text{ kips} \uparrow$$

$$\Sigma F_x = 0: A_x = 0$$

$$+\uparrow \Sigma F_y = 0: A_y - 15 - 12 + 11.70 = 0$$

$$A_y = +15.30 \text{ kips}, A = 15.30 \text{ kips} \uparrow$$

(a) SHEAR AND BENDING-MOMENT DIAGRAMS



FROM A TO C:

$$+\uparrow \Sigma F_y = 0: 15.30 - 2.5x - V = 0$$

$$V = (15.30 - 2.5x) \text{ kips}$$

$$+\circlearrowleft \Sigma M_j = 0: M + (2.5x)\left(\frac{x}{2}\right) - 15.30x = 0$$

$$M = 15.30x - 1.25x^2$$

$$\text{FOR } x = 0: V_A = +15.30 \text{ kips}, M_A = 0$$

$$\text{FOR } x = 6 \text{ ft: } V_C = +0.300 \text{ kip}, M_C = +46.8 \text{ kip}\cdot\text{ft}$$

FROM C TO B:

$$+\uparrow \Sigma F_y = 0: V + 11.70 = 0$$

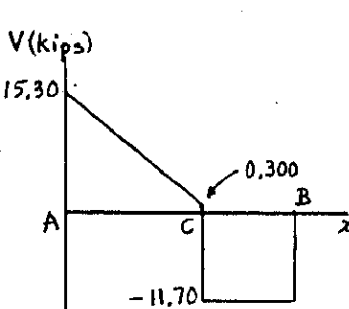
$$V = -11.70 \text{ kips}$$

$$+\circlearrowleft \Sigma M_j = 0: 11.70u - M = 0$$

$$M = (11.70u) \text{ kip}\cdot\text{ft}$$

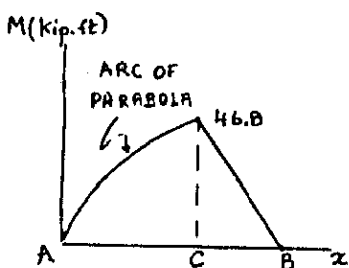
$$\text{FOR } u = 4 \text{ ft: } M_C = +46.8 \text{ kip}\cdot\text{ft}$$

$$\text{FOR } u = 0: M_B = 0$$



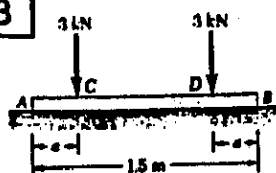
(b)

$$|V|_{\max} = 15.30 \text{ kips}$$



$$|M|_{\max} = 46.8 \text{ kip}\cdot\text{ft}$$

7.43



GIVEN:

BEAM RESTING ON GROUND AND LOADED AS SHOWN ( $a = 0.3 \text{ m}$ ).

- (a) DRAW V AND M DIAGRAMS.  
(b) DETERMINE  $|V|_{\max}$  AND  $|M|_{\max}$ .

FREE BODY: ENTIRE BEAM

$$+\uparrow \Sigma F_y = 0:$$

$$w_y(1.5 \text{ m}) - 3 \text{ kN} - 3 \text{ kN} = 0$$

$$w_y = 4 \text{ kN/m}$$

(a) SHEAR AND BENDING-MOMENT

FROM A TO C:

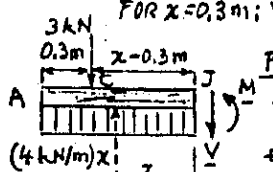
$$+\uparrow \Sigma F_y = 0: 4x - V = 0 \quad V = (4x) \text{ kN}$$

$$+\circlearrowleft \Sigma M_J = 0: M - (4x)\frac{x}{2} = 0$$

$$M = (2x^2) \text{ kN}\cdot\text{m}$$

$$\text{FOR } x = 0: V_A = M_A = 0$$

$$\text{FOR } x = 0.3 \text{ m}: V_C = 1.2 \text{ kN}, M_C = 0.18 \text{ kN}\cdot\text{m}$$



FROM C TO D:

$$+\uparrow \Sigma F_y = 0: 4x - 3 \text{ kN} - V = 0$$

$$V = (4x - 3) \text{ kN}$$

$$+\circlearrowleft \Sigma M_J = 0:$$

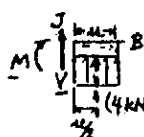
$$M + (3 \text{ kN})(x - 0.3) - (4x)\frac{x}{2} = 0$$

$$M = (2x^2 - 3x + 0.9) \text{ kN}\cdot\text{m}$$

$$\text{FOR } x = 0.3 \text{ m}: V_C = -1.8 \text{ kN}, M_C = +0.18 \text{ kN}\cdot\text{m}$$

$$\text{FOR } x = 0.75 \text{ m}: V_E = 0, M_E = -0.225 \text{ kN}\cdot\text{m}$$

$$\text{FOR } x = 1.2 \text{ m}: V_D = +1.8 \text{ kN}, M_D = +0.18 \text{ kN}\cdot\text{m}$$



FROM D TO B:

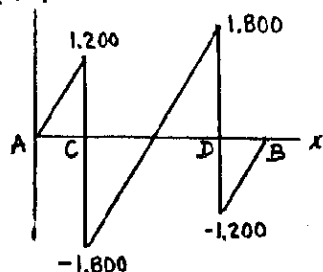
$$+\uparrow \Sigma F_y = 0: V + 4u = 0 \quad V = -(4u) \text{ kN}$$

$$+\circlearrowleft \Sigma M_J = 0: (4u)\frac{u}{2} - M = 0, M = 2u^2$$

$$\text{FOR } u = 0: V_B = M_B = 0$$

$$\text{FOR } u = 0.3 \text{ m}: V_D = -1.2 \text{ kN}, M_D = +0.18 \text{ kN}\cdot\text{m}$$

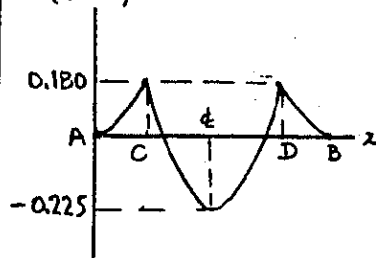
V (kN)



(b)

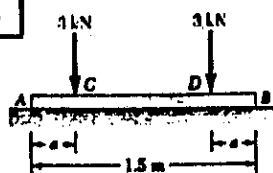
$$|V|_{\max} = 1.800 \text{ kN}$$

M (kN·m)



$$|M|_{\max} = 0.225 \text{ kN}\cdot\text{m}$$

7.44



GIVEN:

BEAM RESTING ON GROUND AND LOADED AS SHOWN ( $a = 0.5 \text{ m}$ ).

- (a) DRAW V AND M DIAGRAMS.  
(b) DETERMINE  $|V|_{\max}$  AND  $|M|_{\max}$ .

FREE BODY: ENTIRE BEAM

$$+\uparrow \Sigma F_y = 0:$$

$$w_y(1.5 \text{ m}) - 3 \text{ kN} - 1 \text{ kN} = 0$$

$$w_y = 4 \text{ kN/m}$$

(a) SHEAR AND BENDING-MOMENT

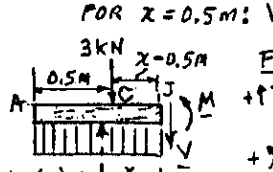
FROM A TO C:

$$+\uparrow \Sigma F_y = 0: 4x - V = 0 \quad V = (4x) \text{ kN}$$

$$+\circlearrowleft \Sigma M_J = 0: M - (4x)\frac{x}{2} = 0, M = (2x^2) \text{ kN}\cdot\text{m}$$

$$\text{FOR } x = 0: V_A = M_A = 0$$

$$\text{FOR } x = 0.5 \text{ m}: V_C = 2 \text{ kN}, M_C = 0.500 \text{ kN}\cdot\text{m}$$



FROM C TO D:

$$+\uparrow \Sigma F_y = 0: 4x - 3 \text{ kN} - V = 0$$

$$V = (4x - 3) \text{ kN}$$

$$+\circlearrowleft \Sigma M_J = 0:$$

$$M + (3 \text{ kN})(x - 0.5) - (4x)\frac{x}{2} = 0$$

$$M = (2x^2 - 3x + 1.5) \text{ kN}\cdot\text{m}$$

$$\text{FOR } x = 0.5 \text{ m}: V_C = -1.00 \text{ kN}, M_C = 0.500 \text{ kN}\cdot\text{m}$$

$$\text{FOR } x = 0.75 \text{ m}: V_E = 0, M_E = 0.375 \text{ kN}\cdot\text{m}$$

$$\text{FOR } x = 1.0 \text{ m}: V_D = 1.00 \text{ kN}, M_D = 0.500 \text{ kN}\cdot\text{m}$$



FROM D TO B:

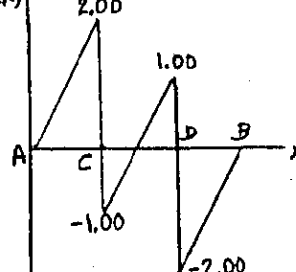
$$+\uparrow \Sigma F_y = 0: V + 4u = 0 \quad V = -(4u) \text{ kN}$$

$$+\circlearrowleft \Sigma M_J = 0: (4u)\frac{u}{2} - M = 0, M = 2u^2$$

$$\text{FOR } u = 0: V_B = M_B = 0$$

$$\text{FOR } u = 0.5 \text{ m}: V_D = -2 \text{ kN}, M_D = 0.500 \text{ kN}\cdot\text{m}$$

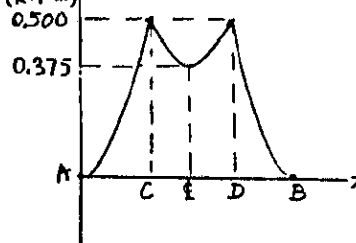
V (kN)



(b)

$$|V|_{\max} = 2.00 \text{ kN}$$

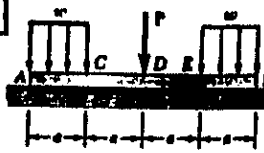
M (kN·m)



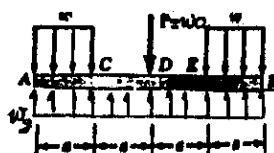
$$|M|_{\max} = 0.500 \text{ kN}\cdot\text{m}$$



7.47



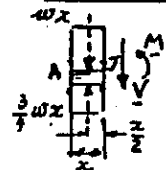
GIVEN:  
BEAM AND LOADING SHOWN  
WITH  $P = wa$ .  
(a) DRAW V AND M DIAGRAMS.  
(b) DETERMINE  $|V|_{\max}$  AND  $|M|_{\max}$ .



FREE BODY: ENTIRE BEAM

$$\begin{aligned} +\uparrow \Sigma F_y &= 0: \\ V_b(4a) - 2wa - wa &= 0 \\ V_b &= \frac{3}{4}w \end{aligned}$$

(a) SHEAR AND BENDING-MOMENT DIAGRAMS

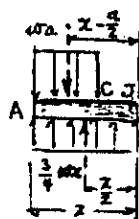


FROM A TO C:

$$\begin{aligned} +\uparrow \Sigma F_y &= 0: \frac{3}{4}wx - wx - V = 0, \quad V = -\frac{1}{4}wx \\ +\uparrow \Sigma M_x &= 0: M + (wx)\frac{x}{2} - (\frac{3}{4}wx)\frac{x}{2} = 0 \\ M &= -\frac{1}{8}wx^2 \end{aligned}$$

FOR  $x=0$ :  $V_A = M_A = 0$

FOR  $x=a$ :  $V_C = -\frac{1}{4}wa$ ,  $M_C = -\frac{1}{8}wa^2$



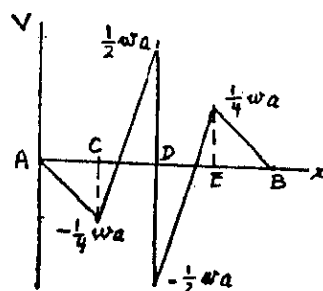
FROM C TO D:

$$\begin{aligned} +\uparrow \Sigma F_y &= 0: \frac{3}{4}wx - wa - V = 0 \\ V &= (\frac{3}{4}x - a)w \\ +\uparrow \Sigma M_x &= 0: \\ M + wa(x - \frac{a}{2}) - \frac{3}{4}wx(\frac{x}{2}) &= 0 \\ M &= \frac{3}{8}wx^2 - wa(x - \frac{a}{2}) \quad (1) \end{aligned}$$

FOR  $x=a$ :  $V_C = -\frac{1}{4}wa$ ,  $M_C = -\frac{1}{8}wa^2$

FOR  $x=2a$ :  $V_D = +\frac{1}{2}wa$ ,  $M_D = 0$

BECAUSE OF THE SYMMETRY OF THE LOADING, WE CAN REDUCE THE VALUES OF V AND M FOR THE RIGHT-HAND HALF OF THE BEAM FROM THE VALUES OBTAINED FOR ITS LEFT-HAND HALF.

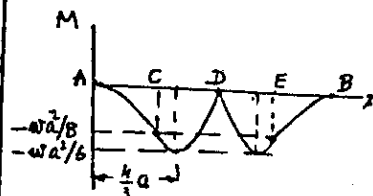


(b)  $|V|_{\max} = \frac{1}{2}wa$

TO FIND  $|M|_{\max}$ , WE DIFFERENTIATE EQ.(1) AND SET  $\frac{dM}{dx} = 0$ :

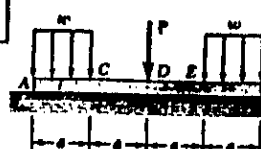
$$\frac{dM}{dx} = \frac{3}{4}wx - wa = 0, \quad x = \frac{4}{3}a, \quad M = \frac{3}{8}w(\frac{4}{3}a)^2 - wa(\frac{4}{3}a - \frac{1}{2}) = -\frac{wa^2}{6}$$

$$|M|_{\max} = \frac{1}{6}wa^2$$

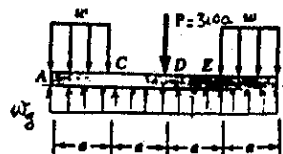


B.M. DIAGRAM CONSISTS OF FOUR DISTINCT ARCS OF PARABOLA.

7.48



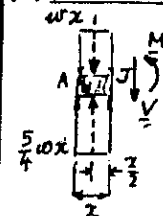
GIVEN:  
BEAM AND LOADING SHOWN  
WITH  $P = 3wa$ .  
(a) DRAW V AND M DIAGRAMS.  
(b) DETERMINE  $|V|_{\max}$  AND  $|M|_{\max}$ .



FREE BODY: ENTIRE BEAM

$$\begin{aligned} +\uparrow \Sigma F_y &= 0: \\ V_b(4a) - 2wa - 3wa &= 0 \\ V_b &= \frac{5}{4}w \end{aligned}$$

(a) SHEAR AND BENDING-MOMENT DIAGRAMS

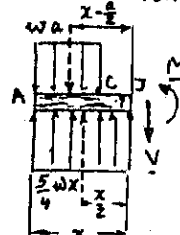


FROM A TO C:

$$\begin{aligned} +\uparrow \Sigma F_y &= 0: \frac{5}{4}wx - wx - V = 0, \quad V = +\frac{1}{4}wx \\ +\uparrow \Sigma M_x &= 0: M + (wx)\frac{x}{2} - (\frac{5}{4}wx)\frac{x}{2} = 0 \\ M &= +\frac{1}{8}wx^2 \end{aligned}$$

FOR  $x=0$ :  $V_A = M_A = 0$

FOR  $x=a$ :  $V_C = +\frac{1}{4}wa$ ,  $M_C = +\frac{1}{8}wa^2$



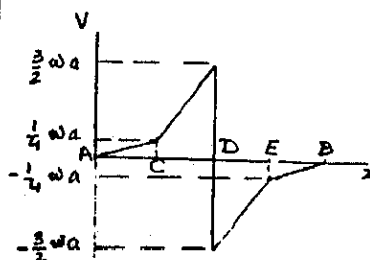
FROM C TO D:

$$\begin{aligned} +\uparrow \Sigma F_y &= 0: \frac{5}{4}wx - wa - V = 0 \\ V &= (\frac{5}{4}x - a)w \\ +\uparrow \Sigma M_x &= 0: \\ M + wa(x - \frac{a}{2}) - \frac{5}{4}wx(\frac{x}{2}) &= 0 \\ M &= \frac{5}{8}wx^2 - wa(x - \frac{a}{2}) \quad (1) \end{aligned}$$

FOR  $x=a$ :  $V_C = +\frac{1}{4}wa$ ,  $M_C = +\frac{1}{8}wa^2$

FOR  $x=2a$ :  $V_D = -\frac{3}{2}wa$ ,  $M_D = +wa^2$

BECAUSE OF THE SYMMETRY OF THE LOADING, WE CAN REDUCE THE VALUES OF V AND M FOR THE RIGHT-HAND HALF OF THE BEAM FROM THE VALUES OBTAINED FOR ITS LEFT-HAND HALF.



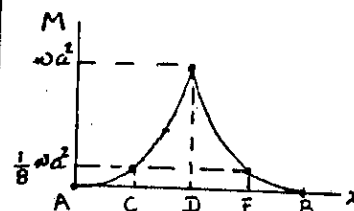
(b)  $|V|_{\max} = \frac{3}{2}wa$

TO FIND  $|M|_{\max}$ , WE DIFFERENTIATE EQ.(1) AND SET  $\frac{dM}{dx} = 0$ :

$$\frac{dM}{dx} = \frac{5}{4}wx - wa = 0, \quad x = \frac{4}{5}a < a \text{ (OUTSIDE PORTION CD)}$$

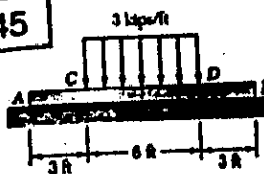
THE MAX. VALUE OF  $|M|$  OCCURS AT D:

$$|M|_{\max} = wa^2$$



B.M. DIAGRAM CONSISTS OF FOUR DISTINCT ARCS OF PARABOLA.

7.45



GIVEN:  
BEAM RESTING ON GROUND  
AND LOADED AS SHOWN.  
(a) DRAW V AND M DIAGRAMS.  
(b) DETERMINE  $|V|_{\max}$  AND  $|M|_{\max}$ .

FREE BODY: ENTIRE BEAM

$$+\uparrow \Sigma F_y = 0:$$

$$\frac{4}{3}(12 \text{ ft}) - (3 \text{ kips/ft})(6 \text{ ft}) = 0$$

$$w_f = 1.5 \text{ kips/ft} \quad \triangleleft$$

(a) SHEAR AND BM DIAGRAMS

FROM A TO C:

$$+\uparrow \Sigma F_y = 0: 1.5x - V = 0 \quad V = (1.5x) \text{ kips}$$

$$+\circlearrowleft \Sigma M_J = 0: M - (1.5x)\frac{x}{2} = 0 \quad M = (0.75x^2) \text{ kip}\cdot\text{ft}$$

$$\text{FOR } x = 0: V_A = M_A = 0$$

$$\text{FOR } x = 3 \text{ ft: } V_C = 4.5 \text{ kips, } M_C = 6.75 \text{ kip}\cdot\text{ft} \quad \triangleleft$$

FROM C TO D:

$$+\uparrow \Sigma F_y = 0: 1.5x - 3(x-3) - V = 0$$

$$V = (9 - 1.5x) \text{ kips}$$

$$+\circlearrowleft \Sigma M_J = 0:$$

$$M + 3(x-3)\frac{x-3}{2} - (1.5x)\frac{x}{2} = 0$$

$$M = [0.75x^2 - 1.5(x-3)^2] \text{ kip}\cdot\text{ft}$$

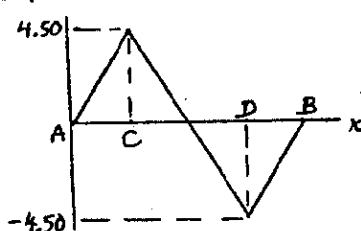
$$\text{FOR } x = 3 \text{ ft: } V_C = 4.5 \text{ kips, } M_C = 6.75 \text{ kip}\cdot\text{ft}$$

$$\text{FOR } x = 6 \text{ ft: } V_D = 0, M_D = 13.50 \text{ kip}\cdot\text{ft}$$

$$\text{FOR } x = 9 \text{ ft: } V_D = -4.5 \text{ kips, } M_D = 6.75 \text{ kip}\cdot\text{ft}$$

$$\text{AT B: } V_B = M_B = 0$$

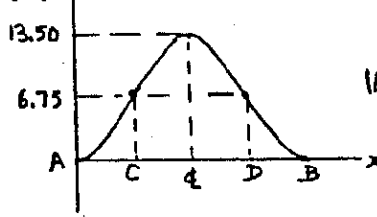
V (kips)



(b)

$$|V|_{\max} = 4.50 \text{ kips} \quad \triangleleft$$

M (kip·ft)

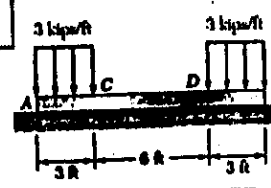


$$|M|_{\max} = 13.50 \text{ kip}\cdot\text{ft} \quad \triangleleft$$

B.M. DIAGRAM CONSISTS OF  
THREE DISTINCT ARCS OF  
PARABOLA, ALL LOCATED  
ABOVE THE X AXIS.

THUS:  $M \geq 0$  EVERYWHERE

7.46



GIVEN:  
BEAM RESTING ON GROUND  
AND LOADED AS SHOWN.  
(a) DRAW V AND M DIAGRAMS.  
(b) DETERMINE  $|V|_{\max}$  AND  $|M|_{\max}$ .

FREE BODY: ENTIRE BEAM

$$+\uparrow \Sigma F_y = 0:$$

$$\frac{4}{3}(12 \text{ ft}) - (3 \text{ kips/ft})(6 \text{ ft}) = 0$$

$$w_f = 1.5 \text{ kips/ft} \quad \triangleleft$$

(a) SHEAR AND BENDING-MOMENT DIAGRAMS:

FROM A TO C:

$$+\uparrow \Sigma F_y = 0: 1.5x - 3x - V = 0 \quad V = (-1.5x) \text{ kips}$$

$$+\circlearrowleft \Sigma M_J = 0: M + (3x)\frac{x}{2} - (1.5x)\frac{x}{2} = 0$$

$$M = (-0.75x^2) \text{ kip}\cdot\text{ft}$$

$$\text{FOR } x = 0: V_A = M_A = 0$$

$$\text{FOR } x = 3 \text{ ft: } V_C = -4.5 \text{ kips, } M_C = -6.75 \text{ kip}\cdot\text{ft} \quad \triangleleft$$

FROM C TO D:

$$+\uparrow \Sigma F_y = 0: 1.5x - 9 - V = 0, \quad V = (1.5x - 9) \text{ kips}$$

$$+\circlearrowleft \Sigma M_J = 0:$$

$$M + 9(x-3) - (1.5x)\frac{x}{2} = 0$$

$$M = 0.75x^2 - 9x + 13.5$$

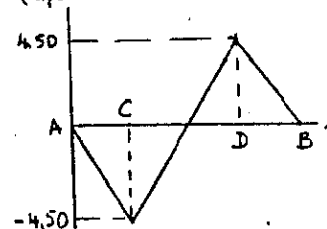
$$\text{FOR } x = 3 \text{ ft: } V_C = -4.5 \text{ kips, } M_C = -6.75 \text{ kip}\cdot\text{ft}$$

$$\text{FOR } x = 6 \text{ ft: } V_D = 0, M_D = -13.50 \text{ kip}\cdot\text{ft}$$

$$\text{FOR } x = 9 \text{ ft: } V_D = 4.5 \text{ kips, } M_D = -6.75 \text{ kip}\cdot\text{ft}$$

$$\text{AT B: } V_B = M_B = 0$$

V (kips)

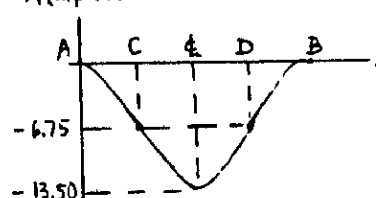


(b)

$$|V|_{\max} = 4.50 \text{ kips} \quad \triangleleft$$

B.M. DIAGRAM CONSISTS OF THREE DISTINCT ARCS OF  
PARABOLA.

M (kip·ft)

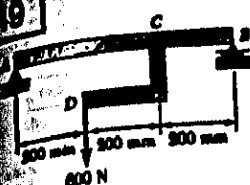


$$|M|_{\max} = 13.50 \text{ kip}\cdot\text{ft} \quad \triangleleft$$

SINCE ENTIRE DIAGRAM IS BELOW THE X AXIS:

 $M \leq 0$  EVERYWHERE

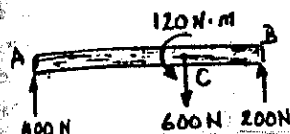
**GIVEN:**  
BEAM AND LOADING SHOWN.  
DRAW V AND M DIAGRAMS AND  
DETERMINE V AND M  
(a) JUST TO THE LEFT OF C.  
(b) JUST TO THE RIGHT OF C



**FREE BODY: ENTIRE BEAM**

$$\begin{aligned} +\sum M_A = 0: & B(0.6\text{ m}) - (600\text{ N})(0.2\text{ m}) = 0 \\ & B = 200\text{ N} \quad B = 200\text{ N} \uparrow \\ \sum F_x = 0: & A_x = 0 \\ +\sum F_y = 0: & A_y - 600\text{ N} + 200\text{ N} = 0 \\ & A_y = 400\text{ N} \quad A = 400\text{ N} \uparrow \end{aligned}$$

WE REPLACE THE 600-N LOAD BY AN EQUIVALENT FORCE-COUPLE SYSTEM AT C



**JUST TO THE RIGHT OF A:**

$$V_1 = +400\text{ N}, M_1 = 0$$

(a) **JUST TO THE LEFT OF C:**

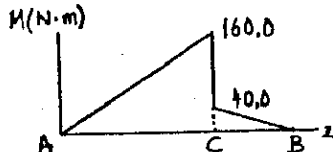
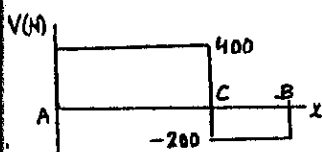
$$\begin{aligned} V_2 &= +400\text{ N} \\ M_2 &= (400\text{ N})(0.4\text{ m}) = 160.0\text{ N}\cdot\text{m} \end{aligned}$$

(b) **JUST TO THE RIGHT OF C:**

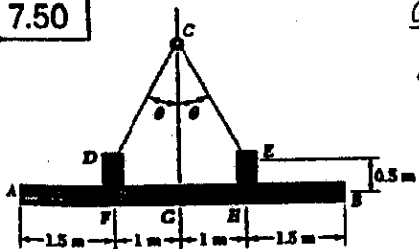
$$\begin{aligned} V_3 &= -200\text{ N} \\ M_3 &= (200\text{ N})(0.2\text{ m}) = 40.0\text{ N}\cdot\text{m} \end{aligned}$$

**JUST TO THE LEFT OF B:**

$$V_4 = -200\text{ N}, M_4 = 0$$



**7.50**



**GIVEN:**  
STRUCTURAL MEMBER  
CONSISTING OF 3-kN BEAM  
AB AND TWO CHANNELS  
OF NEGLIGIBLE WEIGHT  
IS LIFTED WITH  $\theta = 30^\circ$ .

(a) **DRAW V AND M DIAGRAMS**

(b) **DETERMINE  $|V|_{\max}$  AND  $|M|_{\max}$**

**FREE BODY:**

**BEAM AND CHANNELS**

FROM SYMMETRY:

$$F_3 = D_3$$

THUS:

$$F_x = D_x = D_y \tan \theta \quad (1)$$

$$D_y = F_y = 1.5\text{ kN}$$

$$+\sum F_y = 0: D_y + F_y - 3\text{ kN} = 0$$

$$\text{FROM (1): } D_x = (1.5\text{ kN}) \tan \theta, \quad F_x = (1.5\text{ kN}) \tan \theta$$

(CONTINUED)

**7.50 CONTINUED**

WE REPLACE THE FORCES AT D AND E BY EQUIVALENT FORCE-COUPLE SYSTEMS AT F AND H, WHERE

$$\begin{aligned} M_0 &= (1.5\text{ kN} \tan \theta)(0.5\text{ m}) \\ &= (750\text{ N}) \tan \theta \quad (2) \end{aligned}$$

WE NOTE THAT THE WEIGHT OF THE BEAM PER UNIT LENGTH IS

$$w = \frac{W}{L} = \frac{3\text{ kN}}{5\text{ m}} = 0.6\text{ kN/m} = 600\text{ N/m}$$

(a) **SHEAR AND BENDING-MOMENT DIAGRAMS**

FROM A TO F:

$$\begin{aligned} +\sum F_y = 0: & -V - 600x = 0 \quad V = (-600x)\text{ N} \\ +\sum M_x = 0: & M + (600x)\frac{x}{2} = 0, \quad M = (-300x^2)\text{ N}\cdot\text{m} \\ \text{FOR } x = 0: & V_A = M_A = 0 \\ \text{FOR } x = 1.5\text{ m}: & V_F = -900\text{ N}, M_F = -675\text{ N}\cdot\text{m} \end{aligned}$$

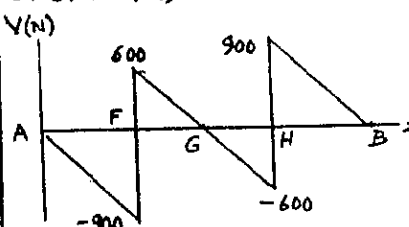
FROM F TO H:

$$\begin{aligned} +\sum F_y = 0: & 1500 - 600x - V = 0 \quad V = (1500 - 600x)\text{ N} \\ +\sum M_x = 0: & M + (600x)\frac{x}{2} - 1500(x - 1.5) - M_0 = 0 \\ & M = M_0 - 300x^2 + 1500(x - 1.5)\text{ N}\cdot\text{m} \end{aligned}$$

$$\text{FOR } x = 1.5\text{ m}: V_F = +600\text{ N}, M_F = (M_0 - 675)\text{ N}\cdot\text{m}$$

$$\text{FOR } x = 2.5\text{ m}: V_H = 0, M_H = (M_0 - 375)\text{ N}\cdot\text{m}$$

FROM G TO B, THE V AND M DIAGRAMS WILL BE OBTAINED BY SYMMETRY.

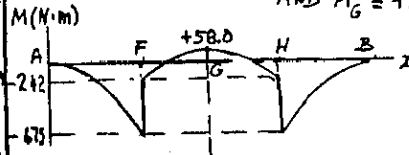


$$(b) |V|_{\max} = 900\text{ N}$$

RECALLING THAT  $\theta = 30^\circ$ , EQ. (2) YIELDS  $M_0 = 433\text{ N}\cdot\text{m}$

THUS, JUST TO THE RIGHT OF F:  $M = 433 - 675 = -242\text{ N}\cdot\text{m}$

$$\text{AND } M_G = 433 - 375 = +58.0\text{ N}\cdot\text{m}$$



$$|M|_{\max} = 675\text{ N}\cdot\text{m}$$

**7.51**

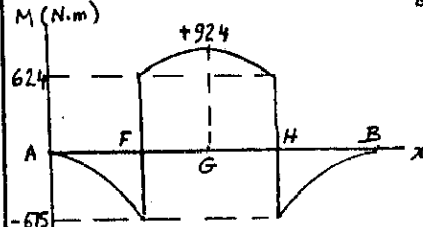
**SOLVE PROB. 7.50 WHEN  $\theta = 60^\circ$ .**

SEE SOLUTION OF PROB. 7.50 UP TO DASHED LINE (INCLUDING SHEAR DIAGRAM).

MAKING  $\theta = 60^\circ$  IN EQ. (2):  $M_0 = 750 \tan 60^\circ = 1299\text{ N}\cdot\text{m}$

THUS, JUST TO THE RIGHT OF F:  $M = 1299 - 675 = 624\text{ N}\cdot\text{m}$

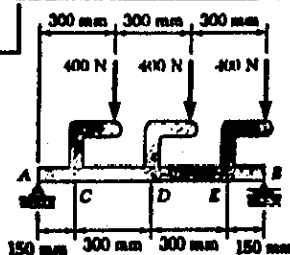
$$\text{AND } M_G = 1299 - 375 = 924\text{ N}\cdot\text{m}$$



$$(b) |V|_{\max} = 900\text{ N}$$

$$|M|_{\max} = 924\text{ N}\cdot\text{m}$$

7.52

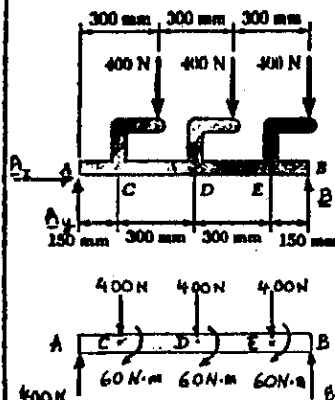


GIVEN:  
BEAM AND LOADING SHOWN  
DRAW V AND M DIAGRAMS  
DETERMINE  $|V|_{\max}$  AND  $|M|_{\max}$

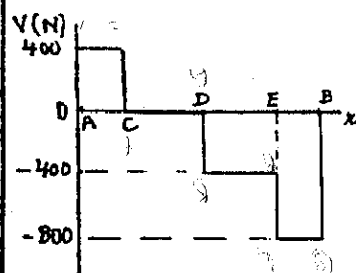
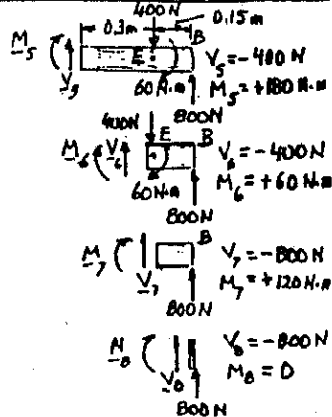
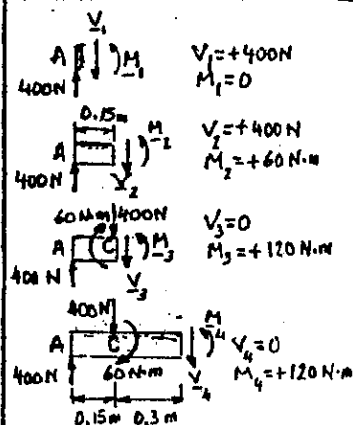
FREE BODY: ENTIRE BEAM

$$\begin{aligned} +\uparrow \Sigma M_A &= 0: B(0.9\text{ m}) - (400\text{ N})(0.3\text{ m}) - (400\text{ N})(0.6\text{ m}) - (400\text{ N})(0.9\text{ m}) = 0 \\ B &= +800\text{ N} \quad B = 800\text{ N} \uparrow \\ \Sigma F_x &= 0: A_x = 0 \\ +\uparrow \Sigma F_y &= 0: A_y + 800\text{ N} - 3(400\text{ N}) = 0 \\ A_y &= +400\text{ N} \quad A = 400\text{ N} \uparrow \end{aligned}$$

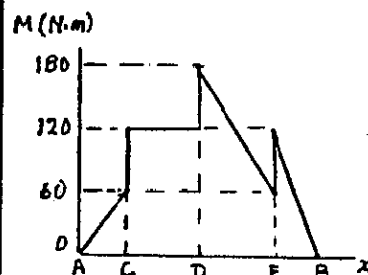
WE REPLACE THE LOADS BY EQUIVALENT FORCE-COUPLE SYSTEMS AT C, D, AND E.



WE CONSIDER SUCCESSIVELY THE FOLLOWING F-B DIAGRAM

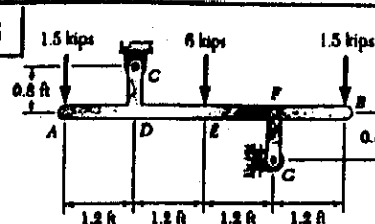


(b)  $|V|_{\max} = 800\text{ N}$

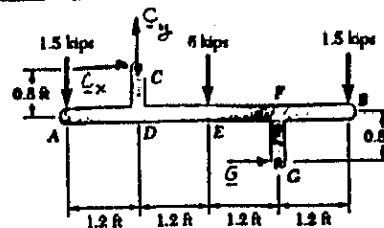


$|M|_{\max} = 180.0\text{ N}\cdot\text{m}$

7.53



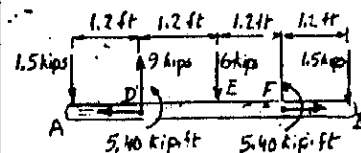
GIVEN: BEAM AND LOADING SHOWN  
DRAW V AND M DIAGRAM  
DETERMINE  $|V|_{\max}$  AND  $|M|_{\max}$



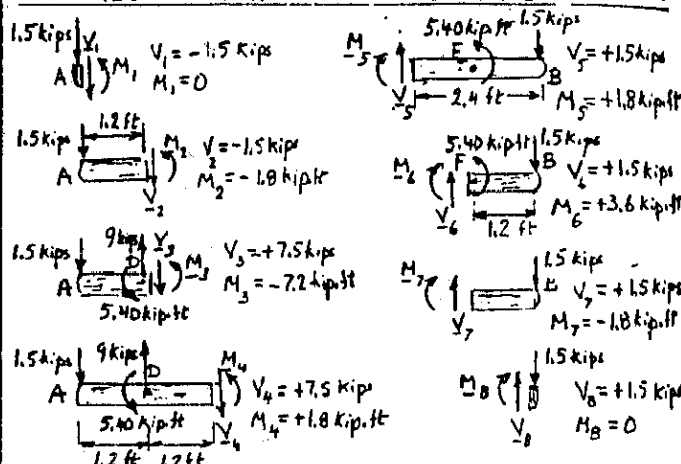
FREE BODY: ENTIRE BEAM

$$\begin{aligned} +\uparrow \Sigma M_C &= 0: (1.5\text{ kips})(1.2\text{ ft}) - (6\text{ kips})(1.2\text{ ft}) + G(1.6\text{ ft}) - (1.5\text{ kips})(3.6\text{ ft}) = 0 \\ G &= 6.75\text{ kips} \rightarrow \\ \pm \Sigma F_x &= 0: C_x + 6.75\text{ kips} = 0 \\ C_x &= -6.75\text{ kips} \leftarrow \\ +\uparrow \Sigma F_y &= 0: C_y - 1.5 - 6 - 1.5 = 0 \\ C_y &= 9.00\text{ kips} \uparrow \end{aligned}$$

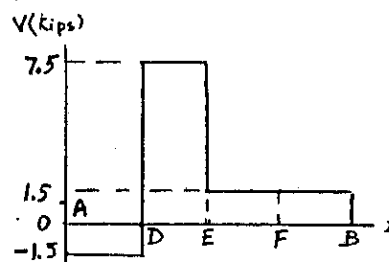
WE REPLACE THE REACTIONS BY EQUIVALENT FORCE-COUPLE SYSTEMS AT D AND F.



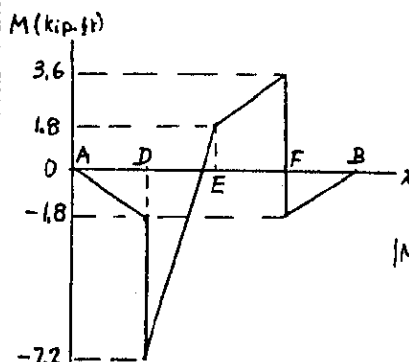
WE CONSIDER SUCCESSIVELY THE FOLLOWING F-B DIAGRAMS



(AXIAL FORCES HAVE BEEN OMITTED FROM F-B DIAGRAM)

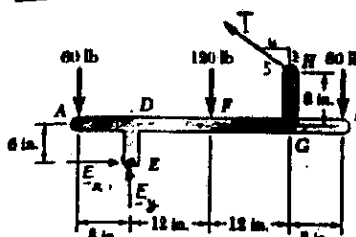
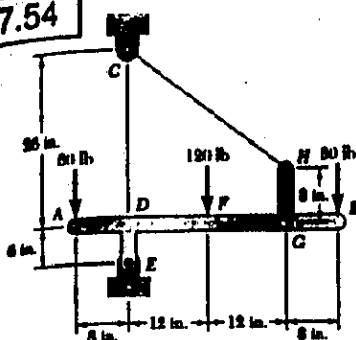


$|V|_{\max} = 7.50\text{ kips}$



$|M|_{\max} = 7.20\text{ kip}\cdot\text{ft}$

7.54



GIVEN:

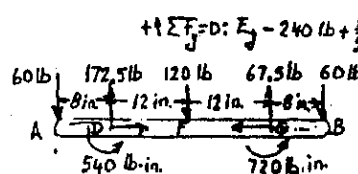
BEAM AND LOADING SHOWN

DRAW V AND M DIAGRAMS

DETERMINE  $|V|_{\max}$  AND  $|M|_{\max}$ 

FREE BODY: ENTIRE BEAM

$$\begin{aligned} +\sum M_E &= 0: (60 \text{ lb})(8 \text{ in.}) - (120 \text{ lb})(12 \text{ in.}) - (60 \text{ lb})(32 \text{ in.}) \\ &+ \left(\frac{4}{3}T\right)(14 \text{ in.}) + \left(\frac{2}{3}T\right)(24 \text{ in.}) = 0 \\ T &= 112.5 \text{ lb} \\ +\sum F_x &= 0: E_x - \frac{4}{3}(112.5 \text{ lb}) = 0 \\ E_x &= 90.0 \text{ lb} \end{aligned}$$



WE REPLACE THE REACTIONS AT E AND H BY EQUIVALENT FORCE-COUPLE SYSTEMS AT D AND G, RESPECTIVELY.

WE CONSIDER SUCCESSIVELY THE FOLLOWING F-B DIAGRAM

$$\begin{aligned} 60 \text{ lb} \quad V_1 &= -60 \text{ lb} \\ A \quad M_1 &= 0 \end{aligned}$$

$$\begin{aligned} 60 \text{ lb} \quad V_2 &= -60 \text{ lb} \\ A \quad M_2 &= -480 \text{ lb-in} \end{aligned}$$

$$\begin{aligned} 60 \text{ lb} \quad V_3 &= +112.5 \text{ lb} \\ A \quad M_3 &= -1020 \text{ lb-in} \end{aligned}$$

$$\begin{aligned} 60 \text{ lb} \quad V_4 &= +112.5 \text{ lb} \\ A \quad M_4 &= +330 \text{ lb-in} \end{aligned}$$

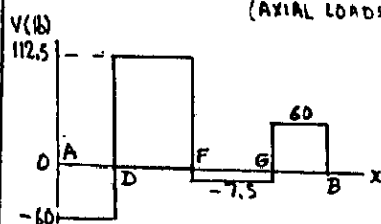
(AXIAL LOADS HAVE BEEN OMITTED)

$$\begin{aligned} 67.5 \text{ lb} \quad V_5 &= -7.5 \text{ lb} \\ Y_5 \quad M_5 &= +330 \text{ lb-in} \end{aligned}$$

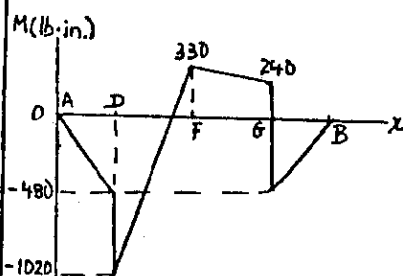
$$\begin{aligned} 67.5 \text{ lb} \quad V_6 &= -7.5 \text{ lb} \\ Y_6 \quad M_6 &= +240 \text{ lb-in} \end{aligned}$$

$$\begin{aligned} 60 \text{ lb} \quad V_7 &= +60 \text{ lb} \\ Y_7 \quad M_7 &= -480 \text{ lb-in} \end{aligned}$$

$$\begin{aligned} 60 \text{ lb} \quad V_8 &= +60 \text{ lb} \\ Y_8 \quad M_8 &= 0 \end{aligned}$$



$$|V|_{\max} = 112.5 \text{ lb}$$



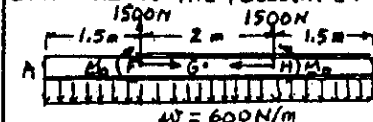
$$|M|_{\max} = 1020 \text{ lb-in}$$

7.55

GIVEN: STRUCTURAL MEMBER OF PROB. 7.50.

FIND: (a) ANGLE  $\theta$  FOR WHICH  $|M|_{\max}$  IS AS SMALL AS POSSIBLE(b) CORRESPONDING VALUE OF  $|M|_{\max}$ 

SEE SOLUTION OF PROB. 7.50 FOR REDUCTION OF LOADING ON BEAM AB TO THE FOLLOWING:



WHERE

$$M_0 = (750 \text{ N-m}) \tan \theta$$

[EQUATION (2)]

$$\begin{aligned} \text{THE LARGEST NEGATIVE B.M. OCCURS JUST TO THE LEFT OF F:} \\ +\sum M_1 = 0: M_1 + (400 \text{ N})(1.5 \text{ m}) = 0, M_1 = -675 \text{ N-m} \end{aligned}$$

$$\begin{aligned} \text{THE LARGEST POSITIVE B.M. OCCURS AT G:} \\ +\sum M_2 = 0: M_2 - M_0 + (1500 \text{ N})(1.25 \text{ m} - 1 \text{ m}) = 0 \\ M_2 = M_0 - 375 \text{ N-m} \end{aligned}$$

$$\text{EQUATING } M_2 \text{ AND } -M_1: M_0 - 375 = +675$$

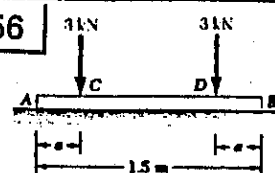
$$M_0 = 1050 \text{ N-m}$$

$$\text{FROM EQ. (2): } \tan \theta = \frac{1050}{750} = 1.400$$

$$(a) \theta = 54.5^\circ$$

$$(b) |M|_{\max} = 675 \text{ N-m}$$

7.56



GIVEN: BEAM RESTING ON GROUND AND LOADED AS SHOWN.

FIND: (a) DISTANCE  $a$  FOR WHICH  $|M|_{\max}$  IS AS SMALL AS POSSIBLE(b) CORRESPONDING VALUE OF  $|M|_{\max}$ 

FORCE PER UNIT LENGTH EXERTED BY GROUND:

$$w_g = \frac{6 \text{ kN}}{1.5 \text{ m}} = 4 \text{ kN/m}$$

$$\begin{aligned} \text{THE LARGEST POSITIVE B.M. OCCURS JUST TO THE LEFT OF C:} \\ +\sum M_1 = 0: M_1 + (4a) \frac{a}{2} = 0, M_1 = -2a^2 \end{aligned}$$

$$\begin{aligned} \text{THE LARGEST NEGATIVE B.M. OCCURS AT THE CENTER LINE:} \\ +\sum M_2 = 0: M_2 + 3(0.75-a) - 3(0.375) = 0 \\ M_2 = -(1.125-3a) \end{aligned}$$

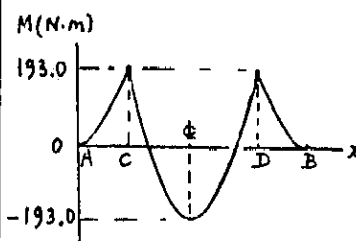
$$\text{EQUATING } M_1 \text{ AND } -M_2:$$

$$2a^2 = 1.125 - 3a$$

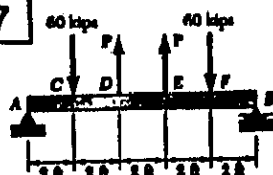
$$a^2 + 1.5a - 0.5625 = 0$$

$$(a) \text{ SOLVING THE QUADRATIC EQ.: } a = 0.31066, a = 0.311 \text{ m}$$

$$(b) \text{ SUBSTITUTING: } |M|_{\max} = M_1 = 2(0.31066)^2, |M|_{\max} = 193.0 \text{ N-m}$$



7.57



GIVEN:

BEAM AND LOADING SHOWN

FIND:

- (a) VALUE OF P FOR WHICH  $|M|_{\max}$  IS AS SMALL AS POSSIBLE  
(b) CORRESPONDING VALUE OF  $|M|_{\max}$

FREE BODY: ENTIRE BEAM

BECAUSE OF SYMMETRY OF BEAM AND LOADING, WE HAVE

$$A = B = (60 \text{ kips} - P) \uparrow$$

LARGEST POSITIVE B.M. OCCURS AT C:

$$+\sum M_C = 0: M_C - (60 - P)(2a) = 0 \quad (1)$$

$$M_C = 120 - 2P$$

LARGEST NEGATIVE B.M. OCCURS AT CENTERLINE:

$$+\sum M_E = 0: M_E + (60)(3) - (60 - P)(5) - P(1) = 0$$

$$M_E = 120 - 4P$$

$$(a) \text{ EQUATING } M_C \text{ AND } -M_E:$$

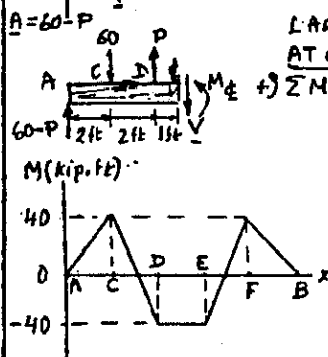
$$120 - 2P = -(120 - 4P)$$

$$6P = 240, P = 40.0 \text{ kips}$$

$$(b) \text{ SUBSTITUTING INTO (1):}$$

$$|M|_{\max} = M_C = 120 - 2(40)$$

$$|M|_{\max} = 40 \text{ kip}\cdot\text{ft}$$



7.58 CONTINUED

MAXIMUM VALUE OF B.M. OCCURS AT D

$$+\sum M_D = 0: M_D + wa(\frac{3a}{2}) - (2\alpha wa)a = 0$$

$$M_{\max} = M_D = wa^2(2\alpha - \frac{3}{2}) \quad (6)$$

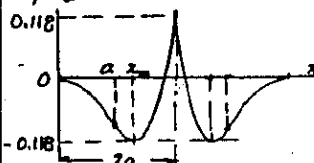
EQUATING  $-M_{\min}$  AND  $M_{\max}$ :

$$wa^2 \frac{1-\alpha}{2\alpha} = wa^2(2\alpha - \frac{3}{2})$$

$$4\alpha^2 - 2\alpha - 1 = 0$$

$$\alpha = \frac{2 + \sqrt{20}}{8} \quad \alpha = \frac{1 + \sqrt{5}}{4} = 0.809$$

$$(a) \text{ SUBSTITUTE IN (2): } k = 4(0.809) - 2 \quad k = 1.236$$

 $M/wa^2$ 

$$(b) \text{ SUBSTITUTE FOR } \alpha \text{ IN (5):}$$

$$|M|_{\max} = -M_{\min} = wa^2 \frac{1-0.809}{2(0.809)}$$

$$|M|_{\max} = 0.1180wa^2$$

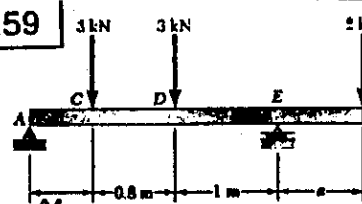
SUBSTITUTE FOR  $x$  IN (4):

$$x_{\min} = \frac{a}{0.809} = 1.236a$$

B.M. DIAGRAM CONSISTS OF 4 ARCS OF PARABOLA.

COMPARE THIS DIAGRAM WITH THOSE OF PROB. 7.47 AND 7.48

7.59



GIVEN: BEAM AND LOADING SHOWN.

FIND:

- (a) DISTANCE  $a$  FOR WHICH  $|M|_{\max}$  IS AS SMALL AS POSSIBLE  
(b) CORRESPONDING VALUE OF  $|M|_{\max}$

FREE BODY: ENTIRE BEAM

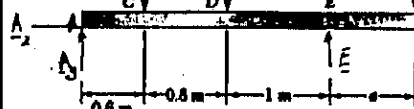
$$\sum F_y = 0: A_y = 0$$

$$+\sum M_E = 0: -A_y(L) +$$

$$+ (3)(1.8) + 3(1) - (2)a = 0$$

$$A_y = 3.5 \text{ kN} - \frac{5}{6}a$$

$$A = 3.5 \text{ kN} - \frac{5}{6}a \uparrow$$



FREE BODY: AC

$$+\sum M_C = 0: M_C - (3.5 - \frac{5}{6}a)(0.6) = 0, M_C = 2.1 - \frac{a}{2}$$

FREE BODY: AD

$$+\sum M_D = 0: M_D - (3.5 - \frac{5}{6}a)(1.4) + (3 \text{ kN})(0.8) = 0$$

$$M_D = 2.5 - \frac{7}{6}a$$

FREE BODY: EB

$$+\sum M_E = 0: -M_E - (2 \text{ kN})a = 0$$

$$M_E = -2a$$

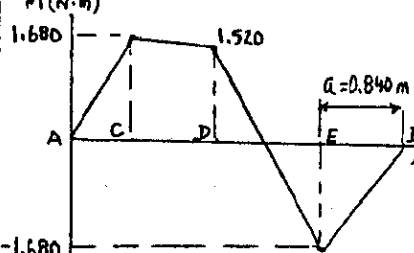
WE SHALL ASSUME THAT  $M_C > M_D$  AND, THUS, THAT  $M_{\max} = M_C$ .WE SET  $M_{\max} = |M_{\min}|$  OR  $M_C = |M_E|$ :

$$2.1 - \frac{a}{2} = 2a$$

$$a = 0.840 \text{ m}$$

$$|M|_{\max} = M_C = |M_E| = 2a = 2(0.840)$$

$$|M|_{\max} = 1.680 \text{ N}\cdot\text{m}$$

 $M(\text{N}\cdot\text{m})$ 

WE MUST CHECK OUR ASSUMPTION.

$$M_D = 2.5 - \frac{7}{6}(0.840)$$

$$= 1.520 \text{ N}\cdot\text{m}$$

THUS,  $M_C > M_D$ , O.K.

THE ANSWERS ARE

$$(a) a = 0.840 \text{ m}$$

$$(b) |M|_{\max} = 1.680 \text{ N}\cdot\text{m}$$

7.58



GIVEN:

BEAM AND LOADING SHOWN (SAME AS FOR PROBS 7.47 &amp; 7.48)

FIND:

- (a) RATIO  $k = P/wa$  FOR WHICH  $|M|_{\max}$  IS AS SMALL AS POSSIBLE  
(b) CORRESPONDING VALUE OF  $|M|_{\max}$

FREE BODY: ENTIRE BEAM

$$+\sum F_y = 0:$$

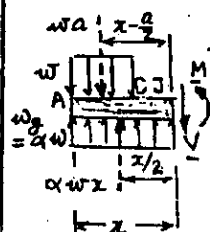
$$w_a(4a) - 2wa - kwa = 0$$

$$w_a = \frac{w}{4}(2 + k)$$

$$\text{SETTING } w_a/w = \alpha \quad (1)$$

$$\text{WE HAVE } k = 4\alpha - 2 \quad (2)$$

MINIMUM VALUE OF B.M. FOR  $M$  TO HAVE NEGATIVE VALUES, WE MUST HAVE  $w_a < w$ . WE VERIFY THAT  $M$  WILL THEN BE NEGATIVE AND KEEP DECREASING IN THE PORTION AC OF THE BEAM. THEREFORE,  $M_{\min}$  WILL OCCUR BETWEEN C AND D.



FROM C TO D:

$$+\sum M_x = 0: M + wa(x - \frac{a}{2}) - \alpha wa(\frac{x}{2}) = 0$$

$$M = \frac{1}{2}wa(\alpha x^2 - 2ax + a^2) \quad (3)$$

WE DIFFERENTIATE AND SET  $\frac{dM}{dx} = 0$ :

$$\alpha x - a = 0 \quad x_{\min} = \frac{a}{\alpha} \quad (4)$$

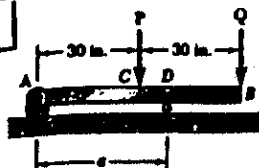
SUBSTITUTE IN (3):

$$M_{\min} = \frac{1}{2}wa^2(\frac{1}{\alpha} - \frac{2}{\alpha} + 1)$$

$$M_{\min} = -\frac{1}{2}wa^2 \frac{1-\alpha}{2\alpha} \quad (5)$$

(CONTINUED)

7.60



GIVEN: BEAM SHOWN WITH  $P = Q = 150 \text{ lb}$ .

FIND: (a) DISTANCE  $a$  FOR WHICH  $|M|_{\max}$  IS AS SMALL AS POSSIBLE  
(b) CORRESPONDING VALUE OF  $|M|_{\max}$ .

FREE BODY: ENTIRE BEAM

$$+\circlearrowleft \sum M_A = 0; D a - (150)(30) - (150)(60) = 0$$

$$D = \frac{13500}{a}$$

FREE BODY: C-B

$$+\circlearrowleft \sum M_C = 0; -M_C - (150)(30) + \frac{13500}{a}(a-30) = 0$$

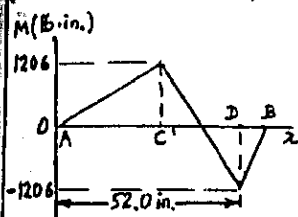
$$M_C = 9000(1 - \frac{30}{a})$$

FREE BODY: D-B

$$+\circlearrowleft \sum M_D = 0; -M_D - (150)(60-a) = 0$$

$$M_D = -150(60-a)$$

(a) WE SET  $M_{\max} = |M_{\min}|$  OR  $M_C = -M_D; 9000(1 - \frac{30}{a}) = 150(60-a)$



$$60 - \frac{2700}{a} = 60 - a$$

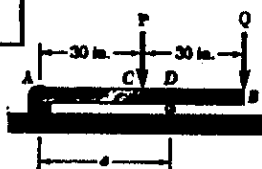
$$a^2 = 2700 \quad a = 51.96 \text{ in.}$$

$$a = 52.0 \text{ in.}$$

$$(b) |M|_{\max} = -M_D = 150(60 - 51.96)$$

$$|M|_{\max} = 1206 \text{ lb-in.}$$

7.61



GIVEN: BEAM SHOWN WITH  $P = 300 \text{ lb}$  AND  $Q = 150 \text{ lb}$ .

FIND: DISTANCE  $a$  FOR WHICH  $|M|_{\max}$  IS AS SMALL AS POSSIBLE  
(b) CORRESPONDING VALUE OF  $|M|_{\max}$ .

FREE BODY: ENTIRE BEAM

$$+\circlearrowleft \sum M_A = 0; D a - (300)(30) - (150)(60) = 0$$

$$D = \frac{18000}{a}$$

FREE BODY: C-B

$$+\circlearrowleft \sum M_C = 0; -M_C - (150)(30) + \frac{18000}{a}(a-30) = 0$$

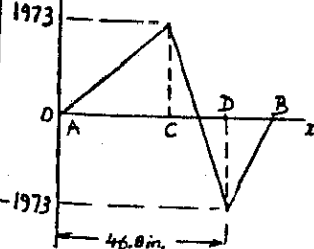
$$M_C = 13,500(1 - \frac{30}{a})$$

FREE BODY: D-B

$$+\circlearrowleft \sum M_D = 0; -M_D - (150)(60-a) = 0$$

$$M_D = -150(60-a)$$

(a) WE SET  $M_{\max} = |M_{\min}|$  OR  $M_C = -M_D; 13,500(1 - \frac{30}{a}) = 150(60-a)$



$$90 - \frac{2600}{a} = 60 - a$$

$$a^2 + 30a - 3600 = 0$$

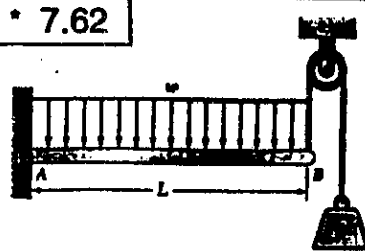
$$a = \frac{-30 + \sqrt{15.150}}{2} = 46.847$$

$$a = 46.8 \text{ in.}$$

$$(b) |M|_{\max} = -M_D = 150(60 - 46.847)$$

$$|M|_{\max} = 1973 \text{ lb-in.}$$

7.62



GIVEN:

BEAM WITH COUNTERWEIGHT  
FIND:  $W$  FOR WHICH  $|M|_{\max}$  IS AS SMALL AS POSSIBLE AND CORRESPONDING  $|M|_{\max}$ .  
CONSIDER FOLLOWING CASES:  
(a) LOAD  $w$  PERMANENTLY APPLIED TO BEAM  
(b) LOAD  $w$  MAY BE APPLIED OR REMOVED

FREE BODY: PORTION JB

$$+\circlearrowleft \sum M_J = 0; W u - (w u) \frac{u}{2} - M = 0$$

$$M = W u - \frac{1}{2} w u^2 \quad (1)$$

MAXIMUM (POSITIVE) VALUE OF  $M$ : DIFFERENTIATE (1) AND SET  $dM/du = 0$ :

$$\frac{dM}{du} = W - w u = 0 \quad u_m = \frac{W}{w} \quad (2)$$

SUBSTITUTE INTO EQ. (1):

$$M_{\max} = W \left( \frac{W}{w} \right) - \frac{1}{2} w \left( \frac{W}{w} \right)^2 \quad M_{\max} = \frac{1}{2} \frac{W^2}{w} \quad (3)$$

LARGEST NEGATIVE VALUE OF  $M$  OCCURS AT A

$$\text{SETTING } u = L \text{ IN EQ. (1): } M_B = W L - \frac{1}{2} w L^2 \quad (4)$$

SETTING  $M_{\max} = |M_{\min}|$ , OR  $M_{\max} = -M_B$ :

$$\frac{1}{2} \frac{W^2}{w} = -W L + \frac{1}{2} w L^2$$

$$W^2 + 2 w L W - w^2 L^2 = 0 \quad W = \frac{-2 w L \pm \sqrt{4 w^2 L^2 + 4 w^2 L^2}}{2} = (\sqrt{2} - 1) w L$$

$$W = 0.4142 w L, \quad W = 0.414 w L$$

CARRYING INTO (2) AND (4):  $u_m = 0.414 L$  (FROM B)

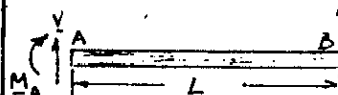
$$|M|_{\max} = -M_B = -(0.4142 w L^2 - 0.5 w L^2)$$

$$|M|_{\max} = 0.0858 w L^2$$

(b) LOAD  $w$  MAY BE APPLIED OR REMOVED

WITH NO LOAD  $w$ :  $M_{\max}$  OCCURS AT A:

$$M_{\max} = M_A = W L \quad (5)$$



WE MUST CONSIDER THE FOLLOWING POSSIBILITIES:  
(SUB "W" MEANS THAT LOAD  $w$  IS APPLIED; SUB "NL", THAT IT IS NOT)

$$(1) (M_{\max})_{NL} = (M_{\max})_w \text{ OR } W L = \frac{1}{2} \frac{W^2}{w} \quad W = 2 w L$$

WITH THIS VALUE OF  $W$ , WE HAVE

$$|M|_{\max} = W L = (2 w L) L = 2 w L^2$$

$$(2) (M_{\max})_{NL} = |M_{\min}|_w \text{ OR } (M_A)_{NL} = -(M_B)_w$$

$$W L = -(W L - \frac{1}{2} w L^2)$$

$$2 W L = \frac{1}{2} w L^2$$

$$W = 0.250 w L$$

WITH THIS VALUE OF  $W$ , WE HAVE

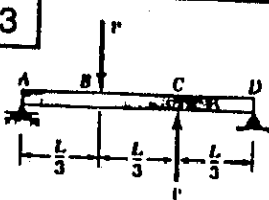
$$|M|_{\max} = W L = (0.250 w L) L = 0.250 w L^2$$

THE COUNTERWEIGHT, THEREFORE, SHOULD BE

$$W = 0.250 w L$$

$$\text{WITH } |M|_{\max} = 0.250 w L^2$$

7.63



GIVEN:  
BEAM AND LOADING SHOWN.  
(1) DRAW V AND M DIAGRAMS  
(2) DETERMINE  $|V|_{\max}$  AND  $|M|_{\max}$ .

FREE BODY: ENTIRE BEAM

$$\begin{aligned} \sum F_y = 0: & P\left(\frac{L}{3}\right) - A\left(\frac{L}{3}\right) - AL = 0 \\ A &= P/3 \end{aligned}$$

SHEAR DIAGRAM

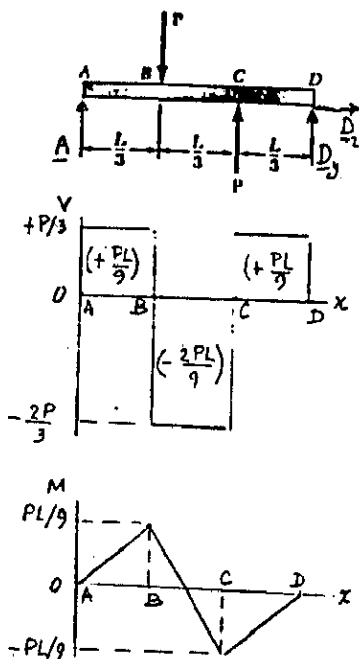
WE NOTE THAT  $V_A = A = +P/3$

$$|V|_{\max} = 2P/3$$

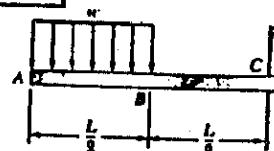
B.M. DIAGRAM

WE NOTE THAT  $M_A = 0$

$$|M|_{\max} = PL/9$$



7.65



GIVEN:  
BEAM AND LOADING SHOWN.  
(1) DRAW V AND M DIAGRAMS  
(2) DETERMINE  $|V|_{\max}$  AND  $|M|_{\max}$ .

FREE BODY: ENTIRE BEAM

$$\begin{aligned} \sum F_y = 0: & C - w\left(\frac{L}{2}\right) = 0 \\ C &= \frac{1}{2}wL \end{aligned}$$

$$\begin{aligned} \sum M_C = 0: & \left(\frac{1}{2}wL\right)\left(\frac{3L}{4}\right) - M_C = 0 \\ M_C &= \frac{3}{8}wL^2 \end{aligned}$$

SHEAR DIAGRAM

AT A:  $V_A = 0$

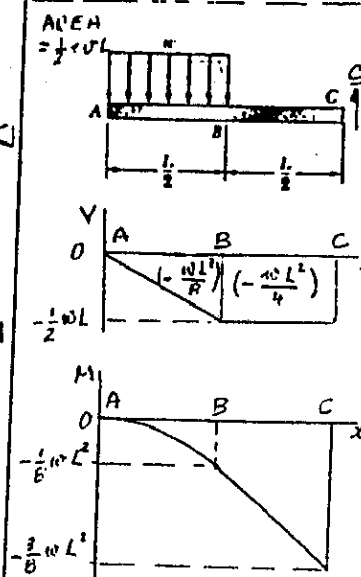
$$|V|_{\max} = \frac{1}{2}wL$$

B.M. DIAGRAM

AT A:  $M = 0, \frac{dM}{dx} = V = 0$

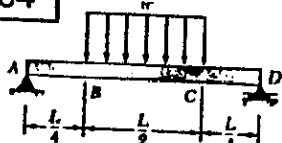
$$|M|_{\max} = \frac{3}{8}wL^2$$

FROM A TO B: ARC OF PARABOLA



SINCE V HAS NO DISCONTINUITY AT B, THE SLOPE OF THE PARABOLA AT B IS EQUAL TO THE SLOPE OF THE STRAIGHT-LINE SEGMENT.

7.64



GIVEN:  
BEAM AND LOADING SHOWN.  
(1) DRAW V AND M DIAGRAMS  
(2) DETERMINE  $|V|_{\max}$  AND  $|M|_{\max}$ .

REACTIONS AT A AND D  
BECAUSE OF THE SYMMETRY  
OF THE SUPPORTS AND LOADING,

$$\begin{aligned} A &= D = \frac{1}{2}\left(w\left(\frac{L}{2}\right)\right) = \frac{1}{4}wL \\ A &= D = \frac{1}{4}wL \end{aligned}$$

SHEAR DIAGRAM

AT A:  $V_A = +\frac{1}{4}wL$

FROM B TO C:  
SLOPE STRAIGHT  
LINE

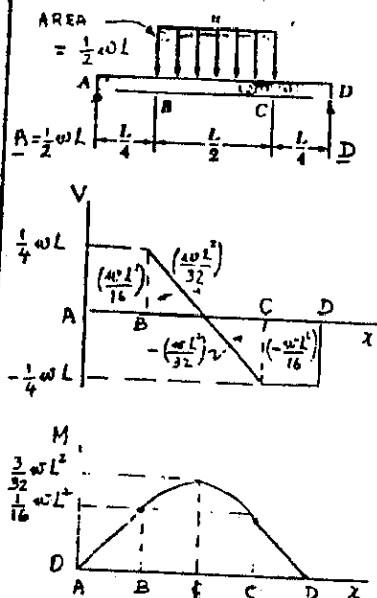
$$|V|_{\max} = \frac{1}{4}wL$$

B.M. DIAGRAM

AT A:  $M_A = 0$

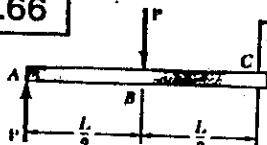
FROM B TO C:  
ARC OF PARABOLA

$$|M|_{\max} = \frac{3}{32}wL^2$$



SINCE V HAS A DISCONTINUITY AT B AND C, THE SLOPE OF THE PARABOLA AT THESE POINTS IS THE SAME AS THE SLOPE OF THE ADJOINING STRAIGHT-LINE SEGMENT.

7.66



GIVEN:  
BEAM AND LOADING SHOWN.  
(1) DRAW V AND M DIAGRAMS  
(2) DETERMINE  $|V|_{\max}$  AND  $|M|_{\max}$ .

FREE BODY: ENTIRE BEAM

$$\begin{aligned} \sum F_y = 0: & C = 0 \\ \sum M_C = 0: & M_C = \frac{1}{2}PL \end{aligned}$$

SHEAR DIAGRAM

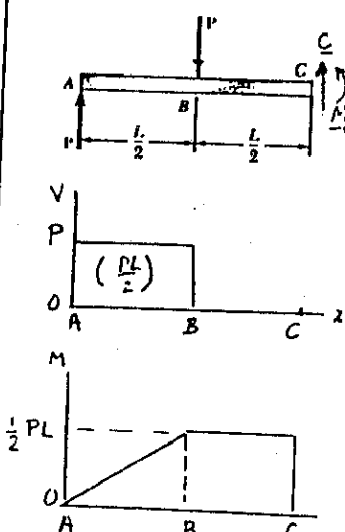
AT A:  $V_A = +P$

$$|V|_{\max} = P$$

B.M. DIAGRAM

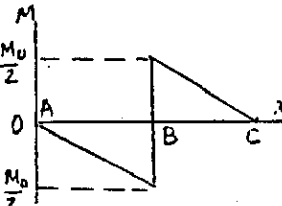
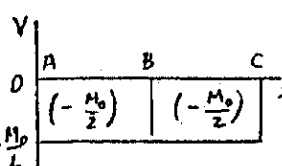
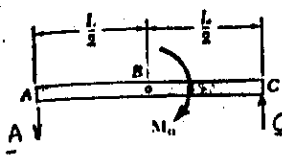
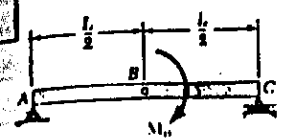
AT A:  $M_A = 0$

$$|M|_{\max} = \frac{1}{2}PL$$





67



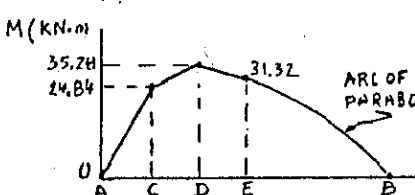
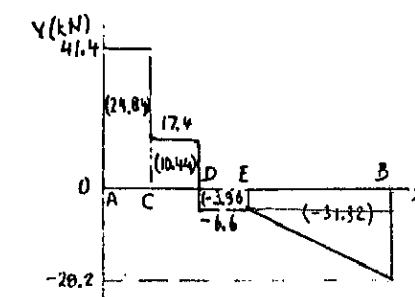
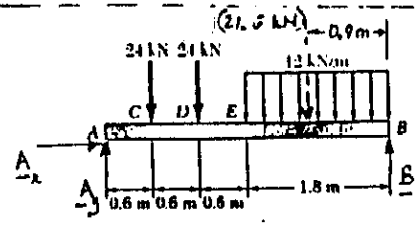
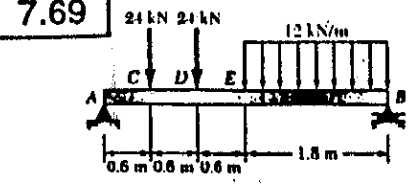
**GIVEN:**  
BEAM AND LOADING SHOWN  
(a) DRAW V AND M DIAGRAMS.  
(b) DETERMINE  $|V|_{max}$  AND  $|M|_{max}$

**FREE BODY: ENTIRE BEAM**  
 $\sum F_y = 0: A = C$   
 $\sum M_C = 0: AL - M_0 = 0$   
 $A = C = \frac{M_0}{L}$

**SHEAR DIAGRAM**  
ATA:  $V_A = -\frac{M_0}{L}$   
 $|V|_{max} = \frac{M_0}{L}$

**B.M. DIAGRAM**  
ATA:  $M_A = 0$   
AT B, M INCREASES BY  $M_0$  ON ACCOUNT OF APPLIED COUPLE.  
 $|M|_{max} = M_0/2$

7.69



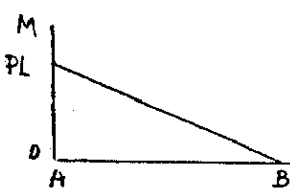
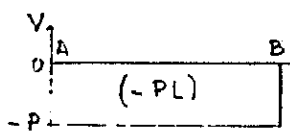
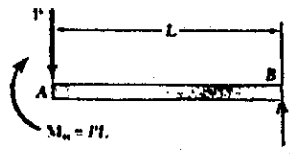
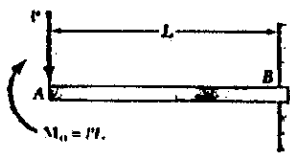
**GIVEN:**  
BEAM AND LOADING SHOWN  
(a) DRAW V AND M DIAGRAMS  
(b) DETERMINE  $|V|_{max}$  AND  $|M|_{max}$

**FREE BODY: ENTIRE BEAM**  
 $\sum M_B = 0: (24 \text{ kN})(3 \text{ m}) + (24 \text{ kN})(2.4 \text{ m}) + (12 \text{ kN/m})(0.9 \text{ m})(1.8 \text{ m}) - A_y(3.6 \text{ m}) = 0$   
 $A_y = 41.4 \text{ kN}$   
 $\sum F_x = 0: A_x = 0$

**SHEAR DIAGRAM**  
ATA:  $V_A = A_y = 41.4 \text{ kN}$   
 $|V|_{max} = 41.4 \text{ kN}$

**B.M. DIAGRAM**  
ATA:  $M_A = 0$   
 $|M|_{max} = 35.3 \text{ kN-m}$   
THE SLOPE OF THE PARABOLA AT E IS THE SAME AS THAT OF THE SEGMENT DE

7.68



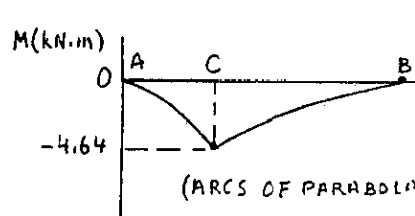
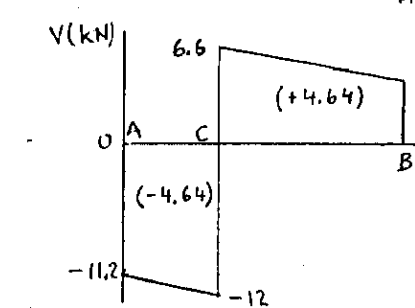
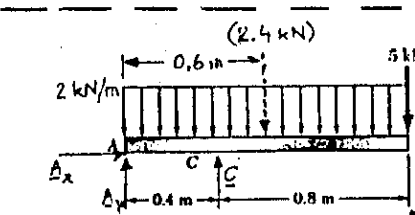
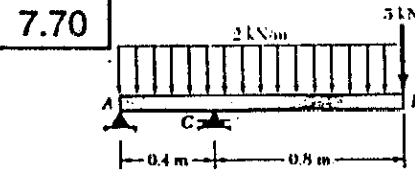
**GIVEN:**  
BEAM AND LOADING SHOWN.  
(a) DRAW V AND M DIAGRAMS.  
(b) DETERMINE  $|V|_{max}$  AND  $|M|_{max}$

**FREE BODY: ENTIRE BEAM**  
 $\sum F_y = 0: B - P = 0$   
 $B = P$   
 $\sum M_B = 0: M_B - PL + PL = 0$   
 $M_B = 0$

**SHEAR DIAGRAM**  
ATA:  $V_A = -P$   
 $|V|_{max} = P$

**B.M. DIAGRAM**  
AT A:  $M_A = M_B = PL$   
 $|M|_{max} = PL$

7.70



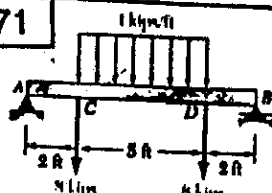
**GIVEN:**  
BEAM AND LOADING SHOWN  
(a) DRAW V AND M DIAGRAMS  
(b) DETERMINE  $|V|_{max}$  AND  $|M|_{max}$

**FREE BODY: BEAM**  
 $\sum M_A = 0: C(0.4 \text{ m}) - (2.4 \text{ kN})(0.6 \text{ m}) - (5 \text{ kN})(1.2 \text{ m}) = 0$   
 $C = 18.6 \text{ kN}$   
 $\sum F_x = 0: A_x = 0$   
 $\sum F_y = 0: A_y + 18.6 - 2.4 - 5 = 0$   
 $A_y = -11.2 \text{ kN}$

**SHEAR DIAGRAM**  
ATA:  $V_A = A_y = -11.2 \text{ kN}$   
 $|V|_{max} = 12.00 \text{ kN}$

**B.M. DIAGRAM**  
AT A:  $M_A = 0$   
 $|M|_{max} = 4.64 \text{ kN-m}$

7.71



GIVEN:

BEAM AND LOADING SHOWN

(a) DRAW V AND M DIAGRAMS

(b) DETERMINE  $|V|_{max}$  AND  $|M|_{max}$ 

REACTIONS AT SUPPORTS

BECAUSE OF THE SYMMETRY:

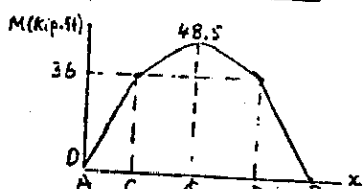
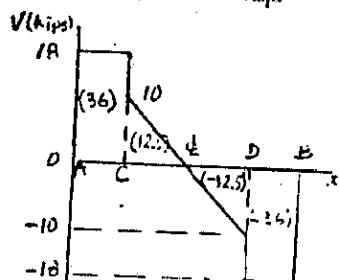
$$A = B = \frac{1}{2}(8R + 4 \times 5) \text{ kips}$$

$$A = B = 18 \text{ kips} \uparrow$$

SHEAR DIAGRAM

AT A:  $V_A = +18 \text{ kips}$ 

$$|V|_{max} = 18 \text{ kips}$$



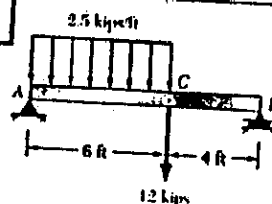
B.M. DIAGRAM

AT A:  $M_A = 0$ 

$$|M|_{max} = 48.5 \text{ kip-ft}$$

DISCONTINUITIES IN SLOPE AT C AND D, DUE TO THE DISCONTINUITIES OF V.

7.72



GIVEN:

BEAM AND LOADING SHOWN

(a) DRAW V AND M DIAGRAMS

(b) DETERMINE  $|V|_{max}$  AND  $|M|_{max}$ 

FREE BODY: BEAM

$$\sum F_x = 0: A_x = 0$$

$$+\circlearrowleft \sum M_B = 0: (12 \text{ kips})(4 \text{ ft}) + (15 \text{ kips})(7 \text{ ft}) - A_y(10 \text{ ft}) = 0$$

$$A_y = +15.3 \text{ kips} \triangleleft$$

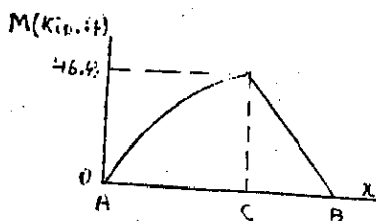
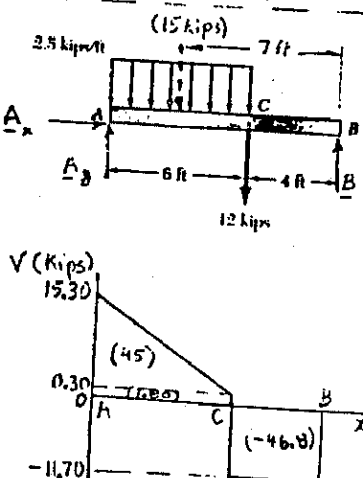
$$+\uparrow \sum F_y = 0: B + 15.3 - 15 - 12 = 0$$

$$B = +11.7 \text{ kips} \triangleleft$$

SHEAR DIAGRAM

AT A:  $V_A = A_y = 15.3 \text{ kips}$ 

$$|V|_{max} = 15.3 \text{ kips}$$

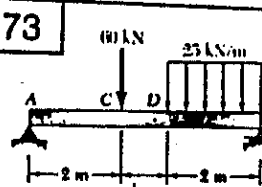


B.M. DIAGRAM

AT A:  $M_A = 0$ 

$$|M|_{max} = 46.4 \text{ kip-ft}$$

7.73



GIVEN:

BEAM AND LOADING SHOWN

(a) DRAW V AND M DIAGRAMS

(b) DETERMINE  $|V|_{max}$  AND  $|M|_{max}$ 

FREE BODY: BEAM

$$\sum F_x = 0: A_x = 0$$

$$+\circlearrowleft \sum M_B = 0:$$

$$(60 \text{ kN})(3 \text{ m}) + (50 \text{ kN})(1 \text{ m}) - A_y(5 \text{ m}) = 0$$

$$A_y = +46.0 \text{ kN}$$

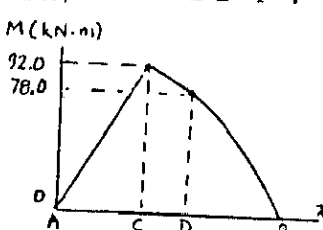
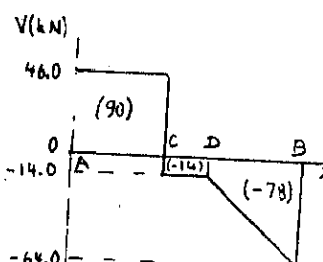
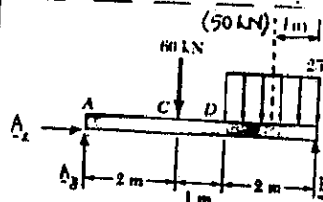
$$+\uparrow \sum F_y = 0: B + 46.0 \text{ kN} - 60 \text{ kN} - 50 \text{ kN} = 0$$

$$B = +64.0 \text{ kN}$$

SHEAR DIAGRAM

AT A:  $V_A = A_y = +46.0 \text{ kN}$ 

$$|V|_{max} = 64.0 \text{ kN}$$



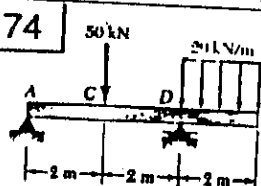
B.M. DIAGRAM

AT A:  $M_A = 0$ 

$$|M|_{max} = 92.0 \text{ kN-m}$$

PARABOLA FROM D TO B, ITS SLOPE AT D IS SAME THAT OF STRAIGHT-LINE SEGMENT CD SINCE V HAS NO DISCONTINUITY AT D.

7.74



GIVEN:

BEAM AND LOADING SHOWN

(a) DRAW V AND M DIAGRAMS

(b) DETERMINE  $|V|_{max}$  AND  $|M|_{max}$ 

FREE BODY: BEAM

$$\sum F_x = 0: A_x = 0$$

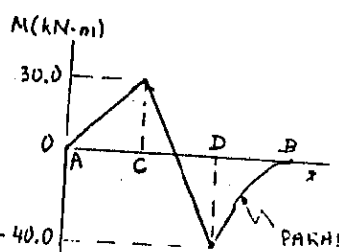
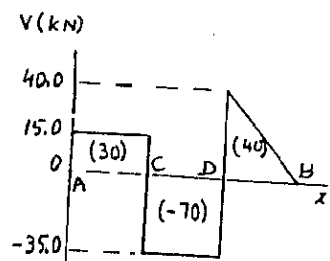
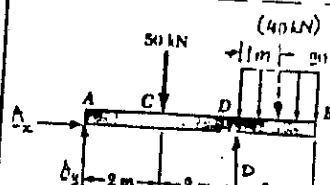
$$+\circlearrowleft \sum M_B = 0:$$

$$(50 \text{ kN})(2 \text{ m}) - (40 \text{ kN})(1 \text{ m}) - A_y(4 \text{ m}) = 0$$

$$A_y = +15.00 \text{ kN} \triangleleft$$

$$+\uparrow \sum F_y = 0: D + 15.00 - 50 - 40 = 0$$

$$D = +75.0 \text{ kN} \triangleleft$$



B.M. DIAGRAM

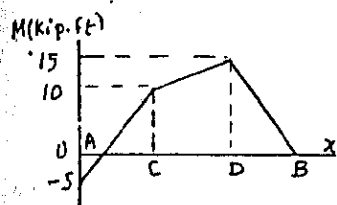
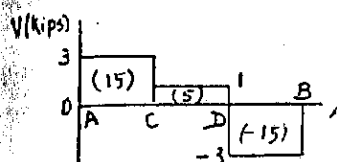
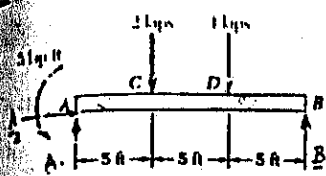
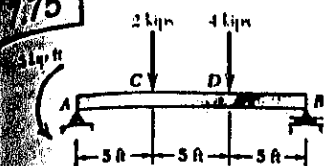
AT A:  $M_A = 0$ 

$$|M|_{max} = 40.0 \text{ kN-m}$$

AT B: SLOPE  $= \frac{dM}{dx} = V_B = 0$ 

PARABOLA

7.75



GIVEN:

BEAM AND LOADING SHOWN

(a) DRAW V AND M DIAGRAMS

(b) DETERMINE  $|V|_{\max}$  AND  $|M|_{\max}$ 

FREE BODY: BEAM

$$+\circlearrowleft \sum M_B = 0:$$

$$5 \text{ kip} \cdot \text{ft} + (2 \text{ kips})(10 \text{ ft}) + (4 \text{ kips})(5 \text{ ft}) - A_y(15 \text{ ft}) = 0$$

$$A_y = +3.00 \text{ kips} \quad \triangleleft$$

$$\sum F_x = 0: A_x = 0$$

SHEAR DIAGRAM

$$\text{AT A: } V_A = A_y = +3.00 \text{ kips}$$

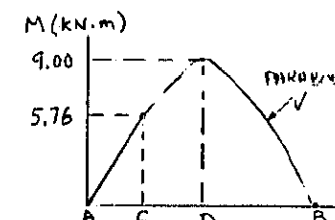
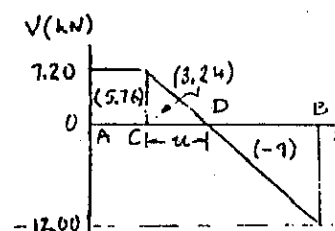
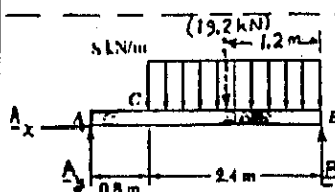
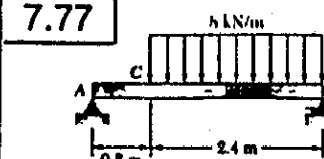
$$|V|_{\max} = 3.00 \text{ kips} \quad \triangleleft$$

B.M. DIAGRAM

$$\text{AT A: } M_A = -5 \text{ kip} \cdot \text{ft}$$

$$|M|_{\max} = 15.00 \text{ kip} \cdot \text{ft} \quad \triangleleft$$

7.77



GIVEN:

BEAM AND LOADING SHOWN

(a) DRAW V AND M DIAGRAMS

(b) DETERMINE MAGNITUDE AND LOCATION OF  $|M|_{\max}$ 

FREE BODY: BEAM

$$\sum F_x = 0: A_x = 0$$

$$+\circlearrowleft \sum M_B = 0: (19.2 \text{ kN})(1.2 \text{ m}) - A_y(3.2 \text{ m}) = 0$$

$$A_y = +7.20 \text{ kN} \quad \triangleleft$$

SHEAR DIAGRAM

$$V_A = V_C = A_y = +7.20 \text{ kN}$$

TO DETERMINE POINT D WHERE  $V=0$ , WE WRITE

$$V_D - V_C = -w \cdot u$$

$$0 - 7.20 \text{ kN} = -(8 \text{ kN/m})u$$

$$u = 0.9 \text{ m} \quad \triangleleft$$

WE NEXT COMPUTE ALL AREAS

B.M. DIAGRAM

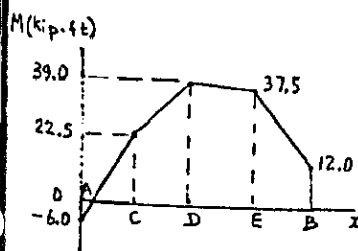
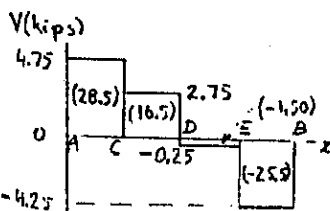
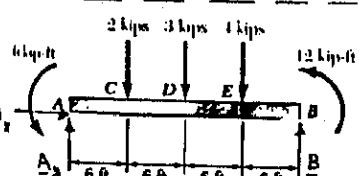
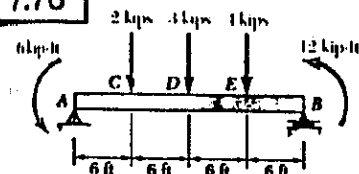
$$\text{AT A: } M_A = 0$$

LARGEST VALUE OCCURS AT D, WITH  $AD = 0.8 + 0.9 = 1.700 \text{ m}$ 

$$|M|_{\max} = 9.00 \text{ kN} \cdot \text{m},$$

$$1.700 \text{ m FROM A} \quad \triangleleft$$

7.76



GIVEN:

BEAM AND LOADING SHOWN

(a) DRAW V AND M DIAGRAMS

(b) DETERMINE  $|V|_{\max}$  AND  $|M|_{\max}$ 

FREE BODY: BEAM

$$+\circlearrowleft \sum M_B = 0:$$

$$6 \text{ kip} \cdot \text{ft} + 12 \text{ kip} \cdot \text{ft} + (2 \text{ kips})(18 \text{ ft}) + (3 \text{ kips})(12 \text{ ft}) + (4 \text{ kips})(6 \text{ ft}) - A_y(24 \text{ ft}) = 0$$

$$A_y = +4.75 \text{ kips} \quad \triangleleft$$

$$\sum F_x = 0: A_x = 0$$

SHEAR DIAGRAM

$$\text{AT A: } V_A = A_y = +4.75 \text{ kips}$$

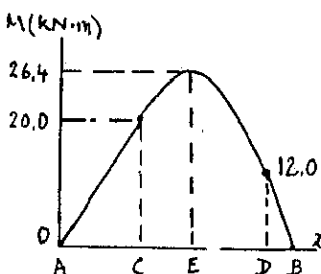
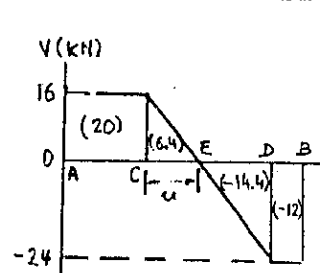
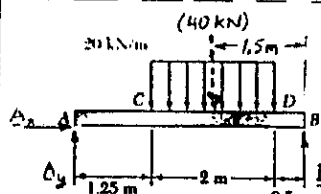
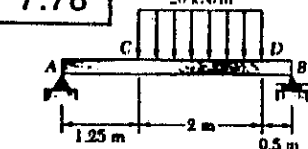
$$|V|_{\max} = 4.75 \text{ kips} \quad \triangleleft$$

B.M. DIAGRAM

$$\text{AT A: } M_A = -6 \text{ kip} \cdot \text{ft}$$

$$|M|_{\max} = 39.0 \text{ kip} \cdot \text{ft} \quad \triangleleft$$

7.78



GIVEN:

BEAM AND LOADING SHOWN

(a) DRAW V AND M DIAGRAMS

(b) DETERMINE MAGNITUDE AND LOCATION OF  $|M|_{\max}$ 

FREE BODY: BEAM

$$\sum F_x = 0: A_x = 0$$

$$+\circlearrowleft \sum M_B = 0: (40 \text{ kN})(1.5 \text{ m}) - A_y(3.75 \text{ m}) = 0$$

$$A_y = +16.00 \text{ kN} \quad \triangleleft$$

SHEAR DIAGRAM

$$V_A = V_C = A_y = +16.00 \text{ kN}$$

TO DETERMINE POINT E WHERE  $V=0$ , WE WRITE

$$V_E - V_C = -w \cdot u$$

$$0 - 16 \text{ kN} = -(20 \text{ kN/m})u$$

$$u = 0.800 \text{ m} \quad \triangleleft$$

WE NEXT COMPUTE ALL AREAS

B.M. DIAGRAM

$$\text{AT A: } M_A = 0$$

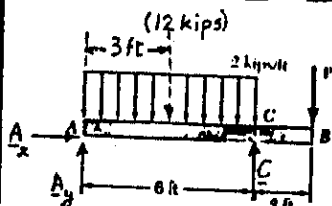
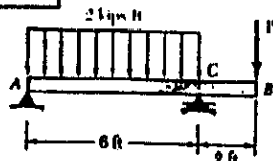
LARGEST VALUE OCCURS AT E WITH  $AE = 1.25 + 0.8 = 2.05 \text{ m}$ 

$$|M|_{\max} = 26.4 \text{ kN} \cdot \text{m},$$

$$2.05 \text{ m FROM A} \quad \triangleleft$$

FROM A TO C AND D TO B: STRAIGHT-LINE SEGMENTS  
FROM C TO D: PARABOLA

7.79



GIVEN

BEAM AND LOADING SHOWN  
DRAW V AND M DIAGRAMS AND  
DETERMINE MAGNITUDE AND  
LOCATION OF  $|M|_{max}$  FOR  
(a)  $P = 6$  kips, (b)  $P = 3$  kips.

FREE BODY: BEAM

$$\sum F_x = 0: A_x = 0$$

$$+\uparrow \sum M_A = 0:$$

$$C(6R) - (12 \text{ kips})(3R) - P(8R) = 0$$

$$C = 6 \text{ kips} + \frac{4}{3}P \quad \triangleleft (1)$$

$$\sum F_y = 0:$$

$$A_y + (6 + \frac{4}{3}P) - 12 - P = 0$$

$$A_y = 6 \text{ kips} - \frac{1}{3}P \quad \triangleleft (2)$$

LOAD DIAGRAM

SUBSTITUTING FOR  $P$  IN  
EQU. (2) AND (1):

$$A_y = 6 - \frac{1}{3}(6) = 4 \text{ kips}$$

$$C = 6 + \frac{4}{3}(6) = 14 \text{ kips}$$

SHEAR DIAGRAM

$$V_A = A_y = +4 \text{ kips}$$

TO DETERMINE POINT D  
WHERE  $V = 0$ ;

$$V_D - V_A = -wx$$

$$0 - 4 \text{ kips} = -(2 \text{ kips/ft})x$$

$$x = 2 \text{ ft}$$

WE COMPUTE ALL AREAS

B.M. DIAGRAM

$$\text{AT A: } M_A = 0$$

$$|M|_{max} = 12.00 \text{ kip}\cdot\text{ft, AT C}$$

PARABOLA FROM A TO C

LOAD DIAGRAM

SUBSTITUTING FOR  $P$  IN  
EQS. (2) AND (1):

$$A = 6 - \frac{1}{3}(3) = 5 \text{ kips}$$

$$C = 6 + \frac{4}{3}(3) = 10 \text{ kips}$$

SHEAR DIAGRAM

$$V_A = A_y = +5 \text{ kips}$$

TO DETERMINE D WHERE  $V = 0$ :

$$V_D - V_A = -wx$$

$$0 - (5 \text{ kips}) = -(2 \text{ kips/ft})x$$

$$x = 2.5 \text{ ft}$$

WE COMPUTE ALL AREAS

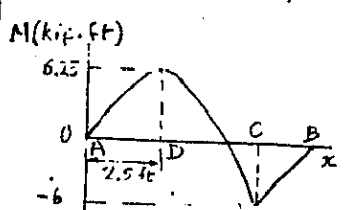
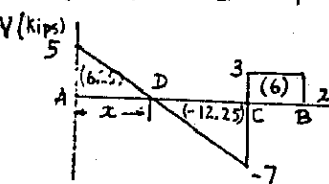
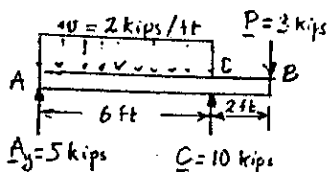
B.M. DIAGRAM

$$\text{AT A: } M_A = 0$$

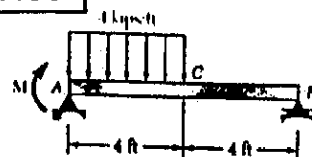
$$|M|_{max} = 6.25 \text{ kip}\cdot\text{ft, AT D}$$

$$2.50 \text{ ft FROM A}$$

PARABOLA FROM A TO C

(b)  $P = 3$  kips

7.80



GIVEN:

BEAM AND LOADING  
DRAW V AND M DIAGRAMS  
DETERMINE MAGNITUDE  
LOCATION OF  $|M|_{max}$   
(a)  $M = 0$ , (b)  $M = 24$  kip-ft

FREE BODY: BEAM

$$\sum F_x = 0: A_x = 0$$

$$+\uparrow \sum M_B = 0:$$

$$(16 \text{ kips})(6 \text{ ft}) - A_y(8 \text{ ft})$$

$$A_y = 12 \text{ kips} - \frac{1}{8}M$$

$$+\uparrow \sum F_y = 0: B + 12 - \frac{1}{8}M$$

$$B = 4 \text{ kips} + \frac{1}{8}M$$

LOAD DIAGRAM

MAKING  $M = 0$  IN (1)

$$A_y = +12 \text{ kips}, B = 4 \text{ kips}$$

SHEAR DIAGRAM

$$V_A = A_y = +12 \text{ kips}$$

TO DETERMINE POINT D  
WHERE  $V = 0$ :

$$V_D - V_A = -wx$$

$$0 - 12 \text{ kips} = -(4 \text{ kips/ft})x$$

$$x = 3 \text{ ft}$$

WE COMPUTE ALL AREAS

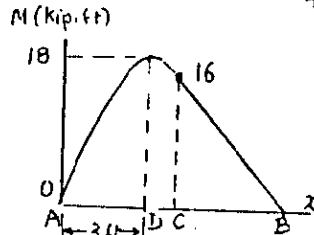
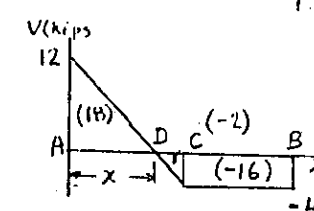
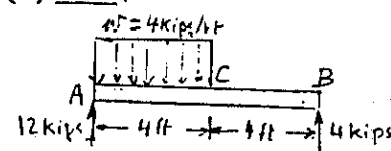
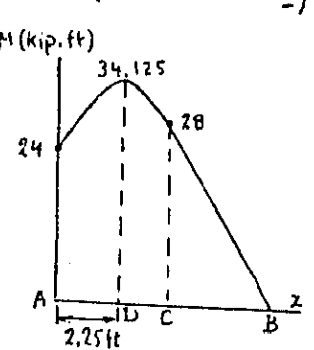
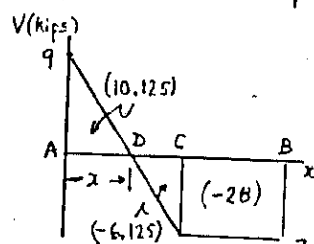
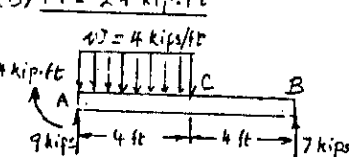
B.M. DIAGRAM

$$\text{AT A: } M_A = 0$$

$$|M|_{max} = 18.00 \text{ kip}\cdot\text{ft, AT D}$$

$$3 \text{ ft FROM A}$$

PARABOLA FROM A TO C

(a)  $M = 0$ (b)  $M = 24 \text{ kip}\cdot\text{ft}$ 

LOAD DIAGRAM

MAKING  $M = 24 \text{ kip}\cdot\text{ft}$  IN (1)

$$A_y = 12 - \frac{1}{8}(24) = 9 \text{ kips}$$

$$B = 4 + \frac{1}{8}(24) = 7 \text{ kips}$$

SHEAR DIAGRAM

$$V_A = A_y = +9 \text{ kips}$$

TO DETERMINE POINT D  
WHERE  $V = 0$ :

$$V_D - V_A = -wx$$

$$0 - 9 \text{ kips} = -(4 \text{ kips/ft})x$$

$$x = 2.25 \text{ ft}$$

B.M. DIAGRAM

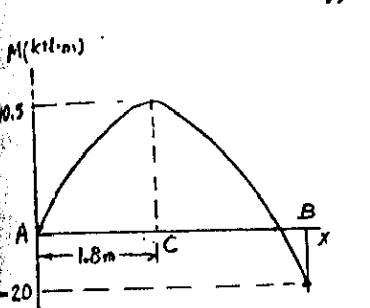
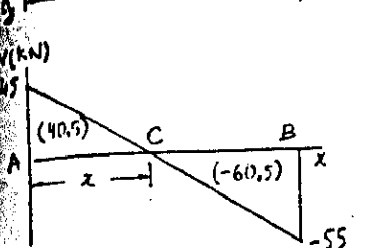
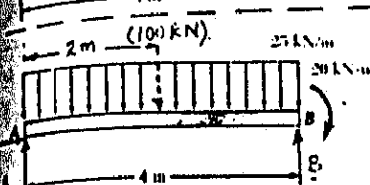
$$\text{AT A: } M_A = +24 \text{ kip}\cdot\text{ft}$$

$$|M|_{max} = 34.1 \text{ kip}\cdot\text{ft, AT D}$$

$$2.25 \text{ ft FROM A}$$

PARABOLA FROM A TO C

81



GIVEN:

BEAM AND LOADING SHOWN.

(a) DRAW V AND M DIAGRAMS

(b) DETERMINE MAGNITUDE AND LOCATION OF  $|M|_{max}$ 

FREE BODY: BEAM

$$\uparrow \Sigma M_A = 0: B(4m) - (100kN)(2m) - 20kN \cdot m = 0$$

$$B = +55 kN$$

$$\Sigma F_x = 0: A_x = 0$$

$$\uparrow \Sigma F_y = 0: A_y + 55 - 100 = 0$$

$$A_y = +45 kN$$

SHEAR DIAGRAM

AT A:  $V_A = A_y = +45 kN$ TO DETERMINE POINT C WHERE  $V = 0$ :

$$V_C - V_A = -wx$$

$$0 - 45 kN = -(25 kN/m)x$$

$$x = 1.8 m$$

WE COMPUTE ALL AREAS

B.M. DIAGRAM

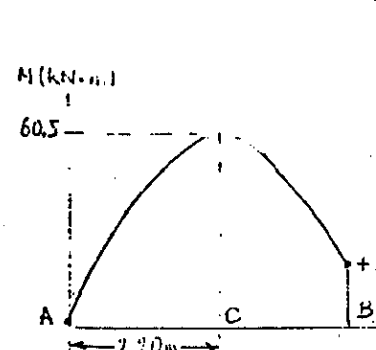
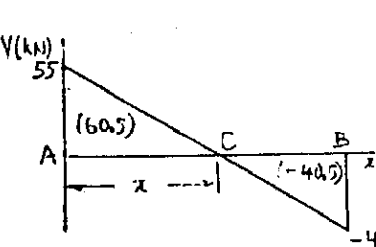
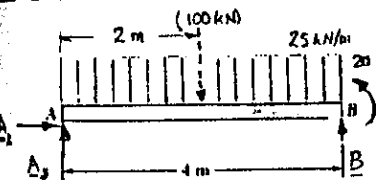
AT A:  $M_A = 0$ AT B:  $M_B = -20 kN \cdot m$  $|M|_{max} = 40.5 kN \cdot m$ 

1.800 m FROM A.

SINGLE ARC OF PARABOLA

7.82

SOLVE PROB. 7.81, ASSUMING THAT 20-KN-m COUPLE AT B IS COUNTERCLOCKWISE.



FREE BODY: BEAM

$$\uparrow \Sigma M_A = 0: B(4m) - (100kN)(2m) + 20kN \cdot m = 0$$

$$B = +45 kN$$

$$\Sigma F_x = 0: A_x = 0$$

$$\uparrow \Sigma F_y = 0: A_y + 45 - 100 = 0$$

$$A_y = +55 kN$$

SHEAR DIAGRAM

AT A:  $V_A = A_y = +55 kN$ TO DETERMINE POINT C WHERE  $V = 0$ :

$$V_C - V_A = -wx$$

$$0 - 55 kN = -(25 kN/m)x$$

$$x = 2.20 m$$

WE COMPUTE ALL AREAS

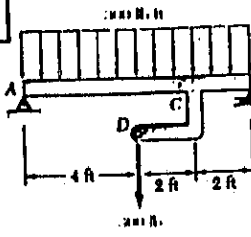
B.M. DIAGRAM

AT A:  $M_A = 0$ AT B:  $M_B = +20 kN \cdot m$  $|M|_{max} = 60.5 kN \cdot m$ 

2.20 m FROM A

SINGLE ARC OF PARABOLA

7.83



GIVEN:

STRUCTURE AND LOADING SHOWN

(a) DRAW V AND M DIAGRAMS FOR BEAM AB.

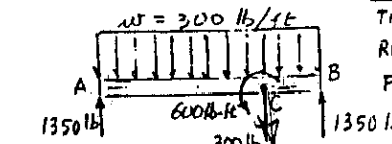
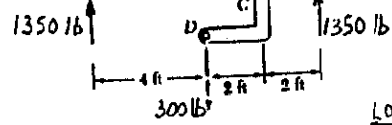
(b) DETERMINE MAGNITUDE AND LOCATION OF  $|M|_{max}$ 

REACTIONS AT SUPPORTS

BECAUSE OF SYMMETRY OF LOAD:

$$A = B = \frac{1}{2}(300 \times 8 + 300)$$

$$A = B = 1350 lb \uparrow$$



LOAD DIAGRAM FOR AB

THE 300-LB FORCE AT D IS

REPLACED BY AN EQUIVALENT

FORCE-COUPLE SYSTEM AT C.

SHEAR DIAGRAM

AT A:  $V_A = A = 1350 lb$ 

TO DETERMINE POINT E

WHERE  $V = 0$ :

$$V_E - V_A = -wx$$

$$0 - 1350 lb = -(300 lb/ft)x$$

$$x = 4.50 ft$$

WE COMPUTE ALL AREAS

B.M. DIAGRAM

AT A:  $M_A = 0$ 

NOTE 600-LB-ft DROP AT C

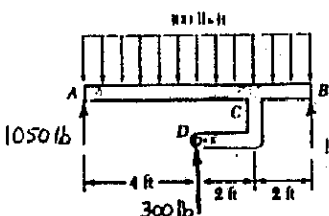
DUE TO COUPLE

 $|M|_{max} = 3040 lb \cdot ft$ 

4.50 ft FROM A.

7.84

SOLVE PROB. 7.83, ASSUMING THAT 300-LB FORCE APPLIED AT D IS DIRECTED UPWARD.

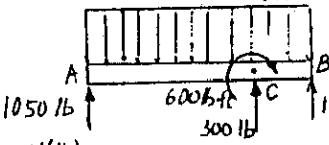
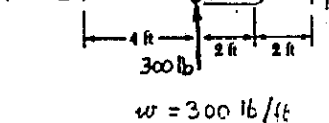


REACTIONS AT SUPPORTS

BECAUSE OF SYMMETRY OF LOAD:

$$A = B = \frac{1}{2}(300 \times 8 - 300)$$

$$A = B = 1050 lb \uparrow$$



LOAD DIAGRAM

THE 300-LB FORCE AT D IS

REPLACED BY AN EQUIVALENT

FORCE-COUPLE SYSTEM AT C

SHEAR DIAGRAM

AT A:  $V_A = A = 1050 lb$ 

TO DETERMINE POINT E

WHERE  $V = 0$ :

$$V_E - V_A = -wx$$

$$0 - 1050 lb = -(300 lb/ft)x$$

$$x = 3.50 ft$$

WE COMPUTE ALL AREAS

B.M. DIAGRAM

AT A:  $M_A = 0$ 

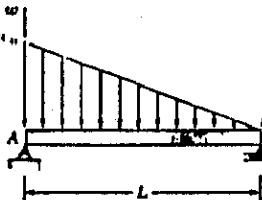
NOTE 600-LB INCREASE AT

C DUE TO COUPLE

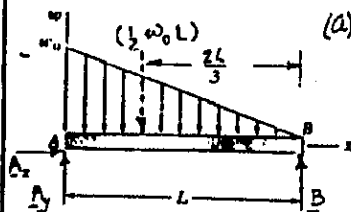
 $|M|_{max} = 1838 lb \cdot ft$ 

3.50 ft FROM A

7.85



GIVEN: BEAM AND  
LOADING SHOWN.  
(a) WRITE EQUATIONS  
FOR  $V(x)$  AND  $M(x)$ .  
(b) DETERMINE MAGNITUDE  
AND LOCATION OF  $M_{max}$ .



(a) FREE BODY: BEAM

$$\sum F_x = 0: A_x = 0$$

$$+\circlearrowleft \sum M_A = 0:$$

$$\left(\frac{1}{2}w_0L\right)\left(\frac{2L}{3}\right) - A_yL = 0$$

$$A_y = \frac{1}{3}w_0L$$

$$\text{THUS: } V_A = A_y = \frac{1}{3}w_0L, M_A = 0 \quad (1)$$

$$\text{LOAD: } w(x) = w_0\left(1 - \frac{x}{L}\right)$$

$$\text{SHEAR: FROM EQ. (7.2): } V(x) - V_A = -\int_0^x w(x)dx = -w_0 \int_0^x \left(1 - \frac{x}{L}\right)dx$$

INTEGRATING AND RECALLING (1):

$$V(x) - \frac{1}{3}w_0L = -w_0\left(x - \frac{x^2}{2L}\right)$$

$$V(x) = \frac{w_0}{6L}(3x^2 - 6Lx + 2L^2) \quad (2)$$

BENDING MOMENT: FROM EQ. (7.4) AND RECALLING THAT  $M_A = 0$ .

$$M(x) - M_A = \int_0^x V(x)dx \quad M(x) = \frac{w_0}{6L}\left(x^3 - 3Lx^2 + 2L^2x\right) \quad (3)$$

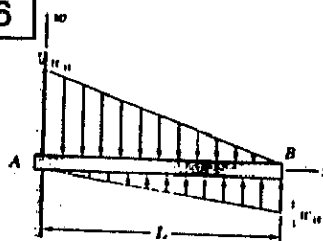
(b) MAXIMUM BENDING MOMENT

$$\frac{dM}{dx} = V = 0. \text{ EQ. (2); } 3x^2 - 6Lx + 2L^2 = 0$$

$$x = \frac{6 \pm \sqrt{36 - 24L^2}}{6} = 0.42265L$$

CARRYING INTO (3):  $M_{max} = 0.0642w_0L^2$  AT  $x = 0.423L$ 

7.86



GIVEN: BEAM AND  
LOADING SHOWN.  
(a) WRITE EQUATIONS  
FOR  $V(x)$  AND  $M(x)$ .  
(b) DETERMINE MAGNITUDE  
AND LOCATION OF  $M_{max}$ .

(a) WE NOTE THAT AT B ( $x=L$ ):  $V_B = 0$ ,  $M_B = 0$  (1)

$$\text{LOAD: } w(x) = w_0\left(1 - \frac{x}{L}\right) - \frac{1}{3}w_0\left(\frac{x}{L}\right) = w_0\left(1 - \frac{4x}{3L}\right)$$

SHEAR: WE USE EQ. (7.2) BETWEEN C ( $x=x$ ) AND B ( $x=L$ ):

$$V_B - V_C = -\int_x^L w(x)dx \quad 0 - V(x) = -\int_x^L w(x)dx$$

$$V(x) = w_0 \int_x^L \left(1 - \frac{4x}{3L}\right)dx = w_0 \left[x - \frac{2x^2}{3L}\right]_x^L = w_0 \left(L - \frac{2L}{3} - x + \frac{2x^2}{3L}\right)$$

$$V(x) = \frac{w_0}{3L}(2x^2 - 3Lx + L^2) \quad (2)$$

BENDING MOMENT: WE USE EQ. (7.4) BETWEEN C ( $x=x$ ) AND B ( $x=L$ ):

$$M_B - M_C = \int_x^L V(x)dx \quad 0 - M(x) = \frac{w_0}{3L} \int_x^L (2x^2 - 3Lx + L^2)dx$$

$$M(x) = -\frac{w_0}{3L} \left[\frac{2}{3}x^3 - \frac{3}{2}Lx^2 + L^2x\right]_x^L = -\frac{w_0}{18L} [4L^3 - 9L^2x + 6L^2x^2]$$

$$M(x) = \frac{w_0}{18L}(4x^3 - 9Lx^2 + 6L^2x - L^3) \quad (3)$$

(b) MAXIMUM BENDING MOMENT

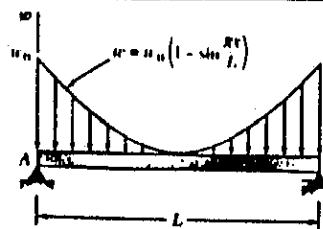
$$\frac{dM}{dx} = V = 0. \text{ EQ. (2); } 2x^2 - 3Lx + L^2 = 0$$

$$x = \frac{3 \pm \sqrt{9 - 8L^2}}{4} L = \frac{L}{2}$$

CARRYING INTO (3):

$$M_{max} = \frac{w_0L^3}{72}, \text{ AT } x = L/2$$

7.87



GIVEN: BEAM  
LOADING SHOWN.  
(a) WRITE EQUATIONS  
FOR  $V(x)$  AND  $M(x)$ .  
(b) DETERMINE  
MAGNITUDE  
AND LOCATION OF  $M_{max}$ .

(a) REACTIONS AT SUPPORTS:  $A = B = \frac{1}{2}W$ , WHERE  $W = \int_0^L w dx$ 

$$W = \int_0^L w dx = w_0 \int_0^L \left(1 - \sin \frac{\pi x}{L}\right) dx = w_0 \left[x + \frac{L}{\pi} \cos \frac{\pi x}{L}\right]_0^L = w_0L$$

$$\text{THUS } V_A = A = \frac{1}{2}W = \frac{1}{2}w_0L \left(1 - \frac{2}{\pi}\right), \quad M_A = 0$$

$$\text{LOAD: } w(x) = w_0\left(1 - \sin \frac{\pi x}{L}\right)$$

SHEAR: THEN EQ. (7.2):  $V(x) - V_A = -\int_0^x w(x)dx = -w_0 \int_0^x \left(1 - \sin \frac{\pi x}{L}\right)dx$ 

INTEGRATING AND RECALLING FIRST OF EQS. (1),

$$V(x) - \frac{1}{2}w_0L \left(1 - \frac{2}{\pi}\right) = -w_0 \left[x + \frac{L}{\pi} \cos \frac{\pi x}{L}\right]_0^x$$

$$V(x) = \frac{1}{2}w_0L \left(1 - \frac{2}{\pi}\right) - w_0 \left(x + \frac{L}{\pi} \cos \frac{\pi x}{L}\right) + w_0 \frac{L}{\pi}$$

$$V(x) = w_0 \left(\frac{L}{2} - x - \frac{L}{\pi} \cos \frac{\pi x}{L}\right) \quad (2)$$

BENDING MOMENT: FROM EQ. (7.4) AND RECALLING THAT  $M_A = 0$ 

$$M(x) - M_A = \int_0^x V(x)dx = w_0 \left[\frac{L}{2}x - \frac{1}{2}x^2 - \left(\frac{L}{\pi}\right) \sin \frac{\pi x}{L}\right]_0^x$$

$$M(x) = \frac{1}{2}w_0 \left(Lx - x^2 - \frac{2L^2}{\pi^2} \sin \frac{\pi x}{L}\right) \quad (3)$$

(b) MAXIMUM BENDING MOMENT

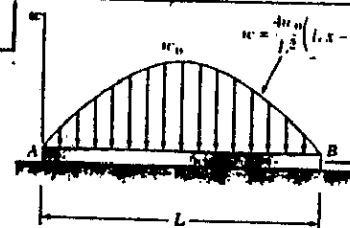
 $\frac{dM}{dx} = V = 0$ . THIS OCCURS I-T  $x = \frac{L}{2}$  AS WE MAY CHECK FROM

$$V\left(\frac{L}{2}\right) = w_0 \left(\frac{L}{2} - \frac{L}{2} - \frac{L}{\pi} \cos \frac{\pi}{2}\right) = 0$$

$$\text{FROM (3): } M\left(\frac{L}{2}\right) = \frac{1}{2}w_0 \left(\frac{L^2}{2} - \frac{L^2}{4} - \frac{2L^2}{\pi^2} \sin \frac{\pi}{2}\right) = \frac{1}{8}w_0L^2 \left(1 - \frac{8}{\pi^2}\right) = 0.0237W$$

$$M_{max} = 0.0237w_0L^2, \text{ AT } x = L/2$$

7.88



GIVEN: BEAM  
LOADING SHOWN.  
(a) WRITE EQUATIONS  
FOR  $V(x)$  AND  $M(x)$ .  
(b) DETERMINE  
MAGNITUDE  
AND LOCATION OF  $M_{max}$ .

(a) FROM FIG. 5.8A: TOTAL LOAD  $= W = \frac{2}{3}w_0L$ 

$$\text{GROUND PRESSURE} = w_g = \frac{W}{L} = \frac{2}{3}w_0.$$

WE ALSO NOTE THAT  $V_A = M_A = 0$  (1)

$$\text{NET LOAD: } w_N(x) = w - w_g = \frac{4w_0}{3L} \left(Lx - x^2 - \frac{L^2}{6}\right)$$

SHEAR: FROM EQ. (7.2):  $V(x) - V_A = -\int_0^x w_N(x)dx$ RECALLING (1):  $V(x) = -\frac{4w_0}{3L} \int_0^x \left(-\frac{L^2}{6} + Lx - x^2\right)dx$ 

$$V(x) = -\frac{4w_0}{3L} \left(-\frac{L^2}{6}x + \frac{L}{2}x^2 - \frac{1}{3}x^3\right), \quad V(x) = \frac{2w_0}{3L} \left(L^2x - 3Lx^2 + 2x^3\right) \quad (2)$$

BENDING MOMENT: FROM EQ. (7.4), WITH  $M_A = 0$ 

$$M(x) - 0 = \int_0^x V(x)dx = \frac{2w_0}{3L} \left(\frac{L^2}{2}x^2 - Lx^3 + \frac{2}{4}x^4\right)$$

$$M(x) = \frac{w_0}{3L} \left(L^2x^2 - 2Lx^3 + x^4\right) \quad (3)$$

(b) MAXIMUM BENDING MOMENT

 $\frac{dM}{dx} = V = 0$ . THIS OCCURS AT  $x = \frac{L}{2}$  AS WE MAY CHECK FROM (1)

$$V\left(\frac{L}{2}\right) = \frac{2w_0}{3L} \left(\frac{L^2}{2} - 3\frac{L}{2} + 2\frac{L^3}{8}\right) = 0$$

$$\text{FROM (3): } M\left(\frac{L}{2}\right) = \frac{w_0}{3L} \left(\frac{L^4}{4} - 2\frac{L^4}{8} + \frac{L^4}{16}\right) = \frac{w_0L^4}{48}$$

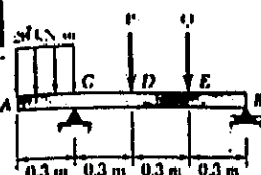
$$M_{max} = w_0L^4/48, \text{ AT } x = L/2$$

CHECK

$$+\circlearrowleft \sum M_C = 0: M_{max} + \frac{w_0L}{3} \left(\frac{2L}{3}\right) - \frac{w_0L}{2} \left(\frac{L}{3}\right) = 0$$

RECALLING THAT  $w_g = \frac{2}{3}w_0$ .

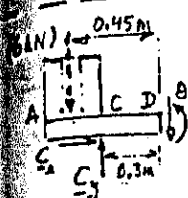
$$M_{max} = \left(\frac{1}{3}w_0L\right)\left(\frac{L}{3}\right) - \left(\frac{w_0L}{3}\right)\left(\frac{2L}{6}\right) = \frac{w_0L^2}{48} \quad (OK)$$



GIVEN:

BEAM AND LOADING SHOWN.  
WE KNOW THAT  $M_D = +800 \text{ N}\cdot\text{m}$   
AND  $M_E = +1300 \text{ N}\cdot\text{m}$ .

- (a) FIND  $P$  AND  $Q$   
(b) DRAW  $V$  AND  $M$  DIAGRAMS



(a) FREE BODY: PORTION AD

$$\sum F_x = 0: C_2 = 0$$

$$+\circlearrowleft \sum M_D = 0:$$

$$-C_2(0.3\text{m}) + 0.800\text{ kN}\cdot\text{m} + (6\text{ kN})(0.45\text{m}) = 0$$

$$C_2 = +11.667 \text{ kN} \quad C = 11.667 \text{ kN} \uparrow$$

FREE BODY: PORTION EB

$$+\circlearrowleft \sum M_E = 0: B(0.3\text{m}) - 1.300\text{ kN}\cdot\text{m} = 0$$

$$B = 4.333 \text{ kN} \uparrow$$

FREE BODY: ENTIRE BEAM

$$+\circlearrowleft \sum M_D = 0: (6\text{ kN})(0.45\text{m})$$

$$- (11.667\text{ kN})(0.3\text{m}) - Q(0.3\text{m})$$

$$+ (4.333\text{ kN})(0.6\text{m}) = 0$$

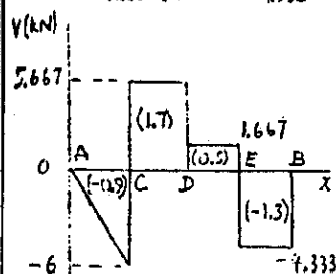
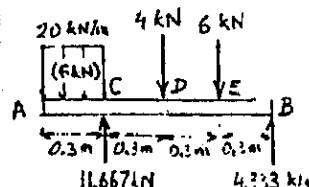
$$Q = 6.00 \text{ kN} \downarrow$$

$$+\uparrow \sum F_y = 0: 11.667\text{ kN} + 4.333\text{ kN}$$

$$- 6\text{ kN} - P - 6\text{ kN} = 0$$

$$P = 4.00 \text{ kN} \downarrow$$

LOAD DIAGRAM



SHEAR FORCE DIAGRAM

$$\text{AT H: } V_A = 0$$

$$|V|_{\text{max}} = 6 \text{ kN}$$

B.M. DIAGRAM

$$\text{AT H: } M_A = 0$$

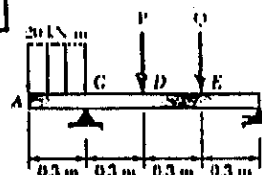
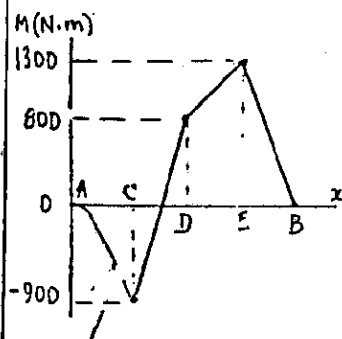
$$|M|_{\text{max}} = 1300 \text{ N}\cdot\text{m}$$

WE CHECK THAT  
 $M_D = +800 \text{ N}\cdot\text{m}$  AND  
 $M_E = +1300 \text{ N}\cdot\text{m}$   
AS GIVEN.

AT C:

$$M_C = -900 \text{ N}\cdot\text{m}$$

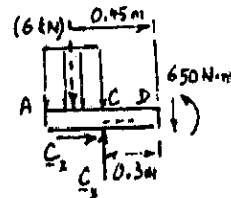
PARABOLA WITH  
HORIZONTAL TANGENT  
AT A.



GIVEN:

BEAM AND LOADING SHOWN.  
WE KNOW THAT  $M_D = +650 \text{ N}\cdot\text{m}$   
AND  $M_E = +1450 \text{ N}\cdot\text{m}$ .

- (a) FIND  $P$  AND  $Q$   
(b) DRAW  $V$  AND  $M$  DIAGRAMS



(a) FREE BODY: PORTION AD

$$\sum F_x = 0: C_2 = 0$$

$$+\circlearrowleft \sum M_D = 0:$$

$$-C_2(0.3\text{m}) + 0.650\text{ kN}\cdot\text{m} + (6\text{ kN})(0.45\text{m}) = 0$$

$$C_2 = +11.167 \text{ kN} \quad C = 11.167 \text{ kN} \uparrow$$

FREE BODY: PORTION EB

$$+\circlearrowleft \sum M_E = 0: B(0.3\text{m}) - 1.450\text{ kN}\cdot\text{m} = 0$$

$$B = 4.833 \text{ kN} \uparrow$$

FREE BODY: ENTIRE BEAM

$$+\circlearrowleft \sum M_D = 0: (6\text{ kN})(0.45\text{m})$$

$$- (11.167\text{ kN})(0.3\text{m}) - Q(0.3\text{m})$$

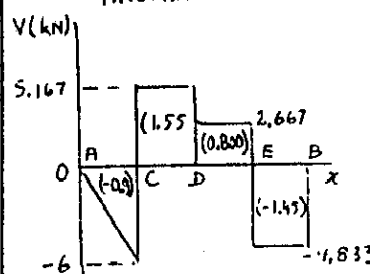
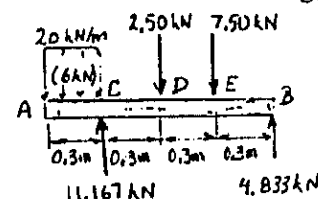
$$+ (4.833\text{ kN})(0.6\text{m}) = 0$$

$$+\uparrow \sum F_y = 0: 11.167\text{ kN} + 4.833\text{ kN}$$

$$- 6\text{ kN} - P - 7.50\text{ kN} = 0$$

$$P = 2.50 \text{ kN} \downarrow$$

LOAD DIAGRAM



SHEAR DIAGRAM

$$\text{AT H: } V_A = 0$$

$$|V|_{\text{max}} = 6 \text{ kN}$$

B.M. DIAGRAM

$$\text{AT H: } M_A = 0$$

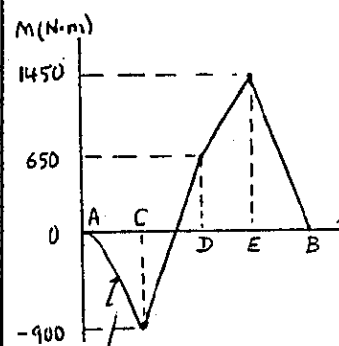
$$|M|_{\text{max}} = 1450 \text{ N}\cdot\text{m}$$

WE CHECK THAT  
 $M_D = +650 \text{ N}\cdot\text{m}$  AND  
 $M_E = +1450 \text{ N}\cdot\text{m}$   
AS GIVEN.

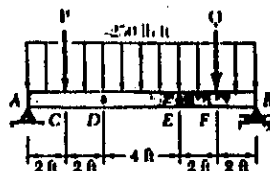
AT C:

$$M_C = -900 \text{ N}\cdot\text{m}$$

PARABOLA WITH  
HORIZONTAL TANGENT  
AT A.

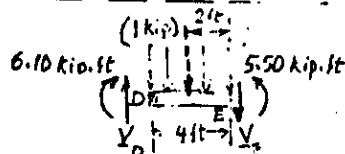


7.91



GIVEN:

BEAM AND LOADING SHOWN.  
WE KNOW THAT  $M_D = +6.10 \text{ kip}\cdot\text{ft}$   
AND  $M_E = +5.50 \text{ kip}\cdot\text{ft}$   
(a) FIND  $P$  AND  $Q$ .  
(b) DRAW  $V$  AND  $M$  DIAGRAMS



(2) FREE BODY: PORTION DE

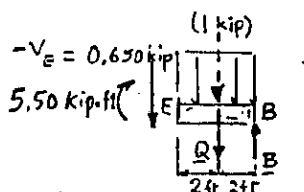
$$\begin{aligned} \uparrow \Sigma M_E = 0: & 5.50 \text{ kip}\cdot\text{ft} - 6.10 \text{ kip}\cdot\text{ft} + (1 \text{ kip})(2 \text{ ft}) - V_D(4 \text{ ft}) = 0 \\ & -V_D(4 \text{ ft}) = 0 \\ & V_D = +0.350 \text{ kip} \end{aligned}$$

$$\begin{aligned} \uparrow \Sigma F_y = 0: & 0.350 \text{ kip} - 1 \text{ kip} - V_E = 0 \\ & V_E = -0.650 \text{ kip} \end{aligned}$$

FREE BODY: PORTION AD

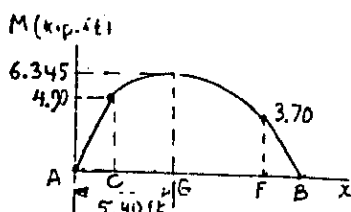
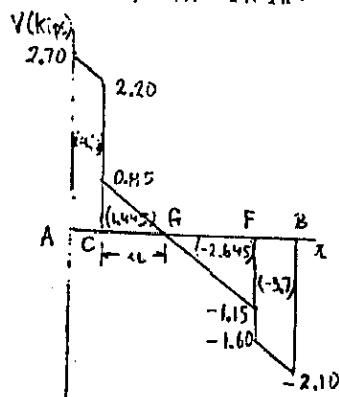
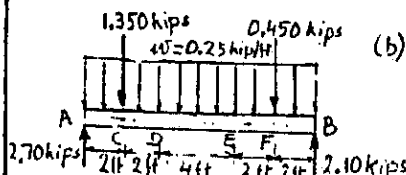
$$\begin{aligned} \uparrow \Sigma M_A = 0: & 6.10 \text{ kip}\cdot\text{ft} - P(2 \text{ ft}) - (1 \text{ kip})(2 \text{ ft}) - (0.350 \text{ kip})(4 \text{ ft}) = 0 \\ & P = 1.350 \text{ kips} \downarrow \end{aligned}$$

$$\begin{aligned} \Sigma F_x = 0: & A_x = 0 \\ \uparrow \Sigma F_y = 0: & A_y - 1 \text{ kip} - 1.350 \text{ kip} - 0.350 \text{ kip} = 0 \\ & A_y = +2.70 \text{ kips} \quad A = 2.70 \text{ kips} \uparrow \end{aligned}$$



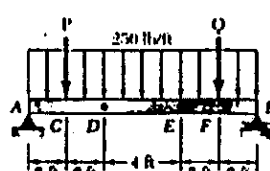
$$\begin{aligned} \uparrow \Sigma M_B = 0: & (0.650 \text{ kip})(4 \text{ ft}) + (1 \text{ kip})(2 \text{ ft}) + Q(2 \text{ ft}) - 5.50 \text{ kip}\cdot\text{ft} = 0 \\ & Q = 0.450 \text{ kip} \uparrow \\ \uparrow \Sigma F_y = 0: & B - 0.450 - 1 - 0.650 = 0 \\ & B = 2.10 \text{ kips} \uparrow \end{aligned}$$

(b) LOAD DIAGRAM



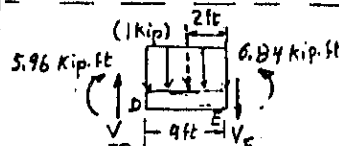
B.M. DIAGRAM CONSISTS OF 3 DISTINCT ARCS OF PARABOLAS.

7.92



GIVEN:

BEAM AND LOADING SHOWN.  
WE KNOW THAT  $M_D = +5.96 \text{ kip}\cdot\text{ft}$   
AND  $M_E = +6.84 \text{ kip}\cdot\text{ft}$   
(a) FIND  $P$  AND  $Q$ .  
(b) DRAW  $V$  AND  $M$  DIAGRAMS



(1) FREE BODY: PORTION DE

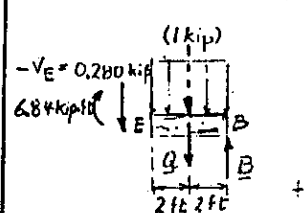
$$\begin{aligned} \uparrow \Sigma M_E = 0: & 6.84 \text{ kip}\cdot\text{ft} - 5.96 \text{ kip}\cdot\text{ft} + (1 \text{ kip})(2 \text{ ft}) - V_D(4 \text{ ft}) = 0 \\ & -V_D(4 \text{ ft}) = 0 \\ & V_D = +0.720 \text{ kip} \end{aligned}$$

$$\begin{aligned} \uparrow \Sigma F_y = 0: & 0.720 \text{ kip} - 1 \text{ kip} - V_E = 0 \\ & V_E = -0.280 \text{ kip} \end{aligned}$$

FREE BODY: PORTION AD

$$\begin{aligned} \uparrow \Sigma M_A = 0: & 5.96 \text{ kip}\cdot\text{ft} - P(2 \text{ ft}) - (1 \text{ kip})(2 \text{ ft}) - (0.720 \text{ kip})(4 \text{ ft}) = 0 \\ & P = 0.540 \text{ kip} \downarrow \end{aligned}$$

$$\begin{aligned} \Sigma F_x = 0: & A_x = 0 \\ \uparrow \Sigma F_y = 0: & A_y - 1 \text{ kip} - 0.540 \text{ kip} - 0.720 \text{ kip} = 0 \\ & A_y = +2.26 \text{ kips} \quad A = 2.26 \text{ kips} \uparrow \end{aligned}$$

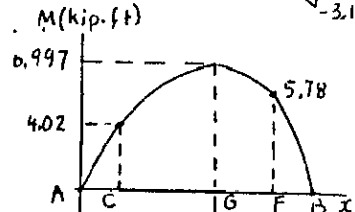
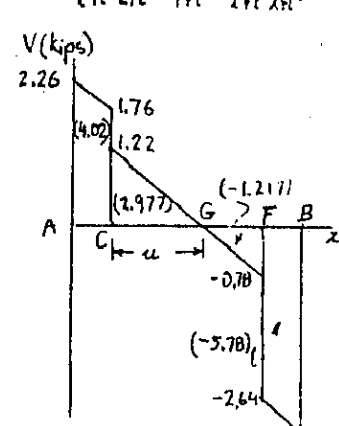
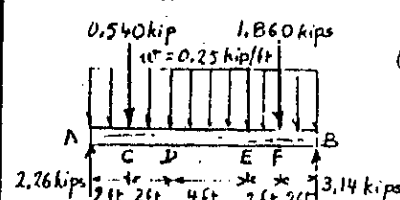


FREE BODY: PORTION EB

$$\begin{aligned} \uparrow \Sigma M_B = 0: & (0.280 \text{ kip})(4 \text{ ft}) + (1 \text{ kip})(2 \text{ ft}) + Q(2 \text{ ft}) - 6.84 \text{ kip}\cdot\text{ft} = 0 \\ & Q = 1.860 \text{ kips} \downarrow \end{aligned}$$

$$\begin{aligned} \uparrow \Sigma F_y = 0: & B - 1.860 - 1 - 0.280 = 0 \\ & B = 3.14 \text{ kips} \uparrow \end{aligned}$$

(b) LOAD DIAGRAM



B.M. DIAGRAM

AT A:  $M_A = 0$

LARGEST VALUE OCCURS AT G WITH  $AG = 2 + 4.88 = 6.88 \text{ ft}$   
 $|M|_{\max} = 6.997 \text{ kip}\cdot\text{ft}$ ,  
6.88 ft FROM A

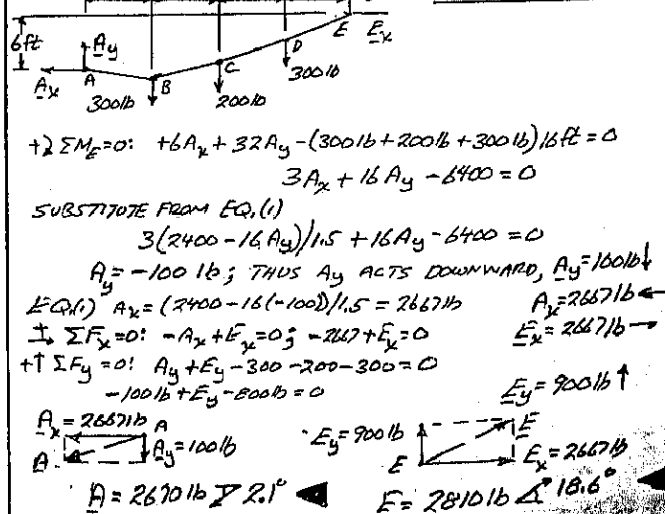
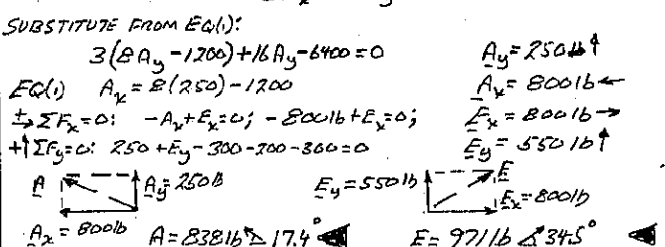
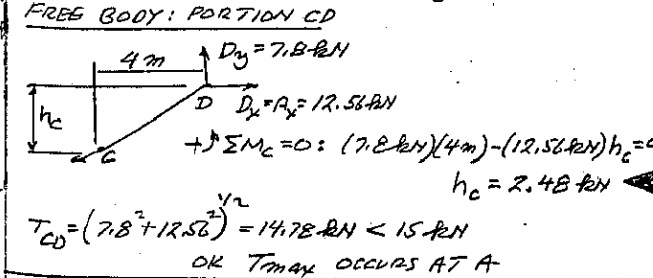
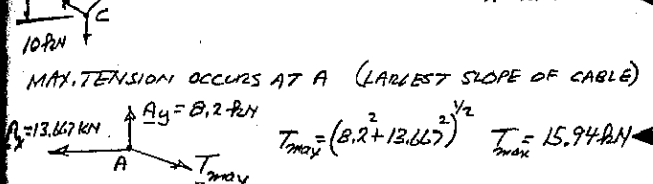
B.M. DIAGRAM CONSISTS OF 3 DISTINCT ARCS OF PARABOLAS.



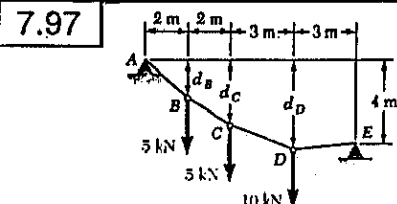
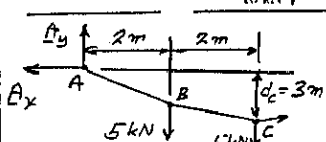


FIND:

- (a) DISTANCE  $h_c$
- (b) COMPONENTS OF REACTION AT D
- (c)  $T_{max}$

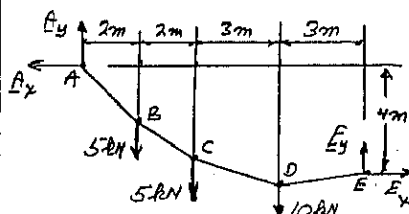


7.97

GIVEN:  $d_C = 3m$ FIND: (a)  $d_B$  and  $d_D$   
(b) REACTION AT E

FREE BODY: PORTION ABC

$$\begin{aligned} \sum M_C = 0 \\ 3A_x - 4A_y + (5kN)(2m) = 0 \\ A_x = \frac{4}{3}A_y - \frac{10}{3} \quad (1) \end{aligned}$$

FREE BODY:  
ENTIRE CABLE

$$\begin{aligned} \sum M_E = 0: 4A_x - 10A_y + (5kN)(8m) + (5kN)(6m) + (10kN)(3m) = 0 \\ 4A_x - 10A_y + 100 = 0 \end{aligned}$$

SUBSTITUTE FROM EQ.(1):  $4(\frac{4}{3}A_y - \frac{10}{3}) - 10A_y + 100 = 0$ 

$$A_y = 18.571 kN; \quad A_y = 18.571 kN \uparrow$$

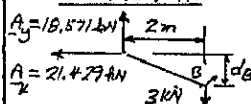
$$EQ.(1) \quad A_x = \frac{4}{3}(18.571) - \frac{10}{3} = 21.429 kN; \quad A_x = 21.429 kN \leftarrow$$

$$\sum F_x = 0: -A_x + E_x = 0; \quad -21.429 + E_x = 0; \quad E_x = 21.429 kN \rightarrow$$

$$\sum F_y = 0: 18.571 kN + E_y + 5kN + 5kN + 10kN = 0; \quad E_y = -14.29 kN \uparrow$$

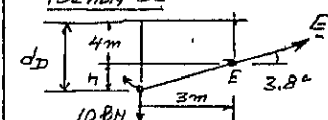
$$E_y = 14.29 kN \uparrow \quad E = 21.5 kN \angle 3.8^\circ$$

PORTION AB



$$\begin{aligned} \sum M_B = 0: \\ (18.571 kN)(2m) - (21.429 kN)d_B = 0 \\ d_B = 1.733 m \end{aligned}$$

PORTION DE



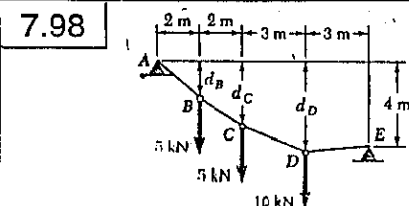
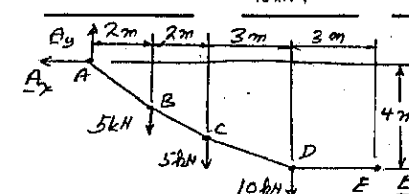
GEOMETRY

$$h = (3m) \tan 3.8^\circ = 0.199 m$$

$$d_D = 4m + 0.199 m$$

$$d_D = 4.20 m$$

7.98

GIVEN: PORTION DE  
IS HORIZONTALFIND: (a)  $d_C$   
(b) REACTION AT  
A AND E

FREE BODY: ENTIRE CABLE

$$\sum F_y = 0: A_y - 5kN - 5kN - 10kN = 0 \quad A_y = 20 kN \uparrow$$

$$\sum M_A = 0: E(4m) - (5kN)(2m) - (5kN)(4m) - (10kN)(7m) = 0$$

$$E = 25 kN$$

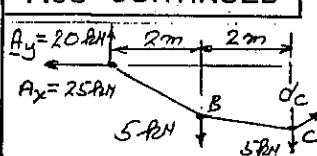
$$\sum F_x = 0: -A_x + 25 kN = 0$$

$$A_x = 25 kN \leftarrow$$

$$A = 32.0 kN \angle 38.7^\circ$$

(CONTINUED)

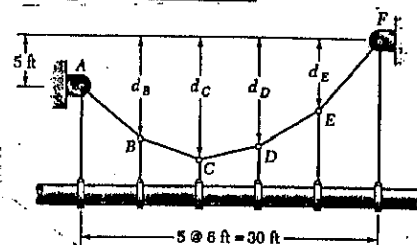
7.98 CONTINUED



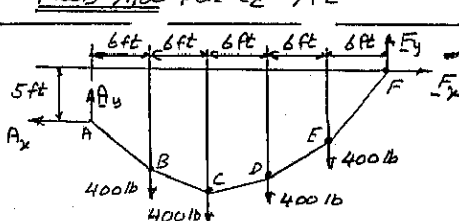
FREE BODY: PORTION ABC

$$\begin{aligned} \sum M_C = 0: (25kN)d_C \\ - (20kN)(4m) + (5kN)(2m) = 0 \\ 25d_C - 70 = 0 \\ d_C = 2.80 m \end{aligned}$$

7.99 and 7.100



GIVEN: TENSION IN EACH HANGER = 400 lb

FIND: (a) MAXIMUM TENSION, (b) DISTANCE  $d_D$ PROB. 7.99 FOR  $d_C = 12 ft$ PROB. 7.100 FOR  $d_C = 9 ft$ FREE BODY:  
ENTIRE CABLE

$$\begin{aligned} \sum M_A = 0: 5F_x - 30F_y + 400 \times 6 + 400 \times 12 + 400 \times 18 + 400 \times 24 = 0 \\ 5F_x - 30F_y + 24000 = 0 \quad (1) \end{aligned}$$

FREE BODY: PORTION CDEF

$$\sum M_C = 0$$

$$d_C F_x - 18F_y + 400 \times 6 + 400 \times 12 = 0$$

$$d_C F_x - 18F_y + 7200 = 0 \quad (2)$$

$$EQ.(1) \times (-0.6): -3F_x + 18F_y - 14400 = 0 \quad (3)$$

$$(2) + (3): d_C F_x - 3F_y - 7200 = 0$$

$$F_y = \frac{7200}{d_C - 3} \quad (4)$$

FREE BODY: PORTION DEF

$$\sum M_D = 0: d_D F_x - 12F_y + 400 \times 6 = 0$$

$$d_D = (12F_y - 2400) / F_x \quad (5)$$

PROB. 7.99 FOR  $d_C = 12 ft$ 

$$EQ.(4): F_y = \frac{7200}{12-3} = 800 lb$$

$$EQ.(1): 5(800) - 30F_y + 24000 = 0; \quad F_y = 933.3$$

$$EQ.(5): d_D = [12(933.3) - 2400] / 800 \quad d_D = 11.00 ft$$

$$F_y = 933.3 \quad F = 1229.3 lb; \quad T_{max} = T_{EF} = F$$

PROB. 7.100 FOR  $d_C = 9 ft$ 

$$EQ.(4): F_y = 7200 / (9-3) = 1200 lb$$

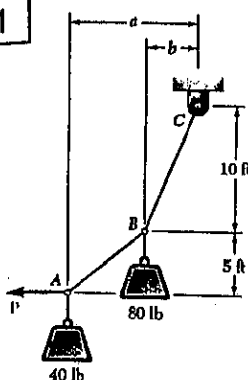
$$EQ.(1): 5(1200) - 30F_y + 24000 = 0; \quad F_y = 1000 lb$$

$$EQ.(5): d_D = [12(1000) - 2400] / 1200; \quad d_D = 8.00 ft$$

$$F_y = 1000 lb \quad F = 1562 lb; \quad T_{max} = T_{EF} = F$$

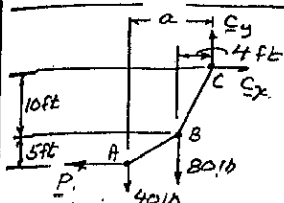
$$T_{max} = 1562 lb$$

7.101



GIVEN:  $b = 4 \text{ ft}$

FIND: (a) Force  $P$ ,  
(b) Distance  $a$ .



FREE BODY: ENTIRE CABLE

$$\begin{aligned} +\sum M_C = 0 \\ (80 \text{ lb})(4 \text{ ft}) + (40 \text{ lb})a - P(15 \text{ ft}) = 0 \\ 15P - 320 - 40a = 0 \quad (1) \end{aligned}$$

FREE BODY: PORTION AB

$$\begin{aligned} +\sum M_B = 0: P(5 \text{ ft}) - (40 \text{ lb})(a - 4 \text{ ft}) = 0 \\ a = 4 + \frac{P}{8} \quad (2) \end{aligned}$$

$$\text{EQ(1): } 15P - 320 - 40\left(4 + \frac{P}{8}\right) = 0$$

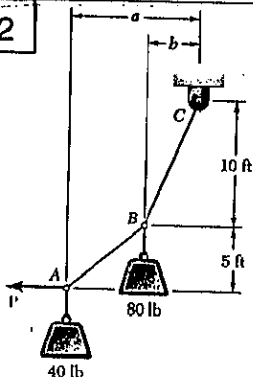
$$10P - 480 = 0$$

$$P = 48 \text{ lb}$$

$$\text{EQ(2): } a = 4 + \frac{48}{8} = 4 + 6 = 10 \text{ ft}$$

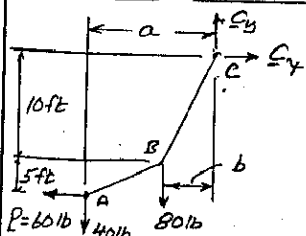
$$a = 10 \text{ ft}$$

7.102



GIVEN:  $P = 60 \text{ lb}$

FIND: DISTANCES  
 $a$  AND  $b$ .



FREE BODY: ENTIRE CABLE

$$\begin{aligned} +\sum M_C = 0: \\ (60 \text{ lb})(15 \text{ ft}) - (40 \text{ lb})a - (80 \text{ lb})b = 0 \\ a = 22.5 - 2b \quad (1) \end{aligned}$$

FREE BODY: PORTION AB

$$\begin{aligned} +\sum M_B = 0: (60 \text{ lb})(5 \text{ ft}) - (40 \text{ lb})(a - b) = 0 \\ b = a - 7.5 \text{ ft} \quad (2) \end{aligned}$$

$$\text{EQ(1): } a = 22.5 - 2(a - 7.5)$$

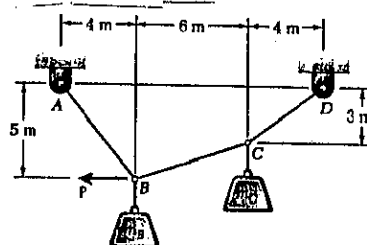
$$3a = 37.5$$

$$a = 12.5 \text{ ft}$$

$$\text{EQ(2): } b = 12.5 \text{ ft} - 7.5 \text{ ft}$$

$$b = 5 \text{ ft}$$

7.103 and 7.104



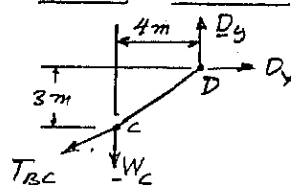
FIND: FORCE  $P$   
TO MAINTAIN  
EQUILIBRIUM

PROB. 7.103:

$$m_B = 70 \text{ kg}, m_C = 25 \text{ kg}$$

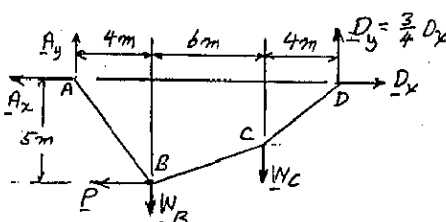
PROB. 7.104:

$$m_B = 18 \text{ kg}, m_C = 10 \text{ kg}$$



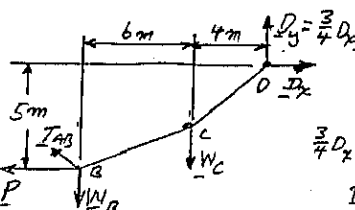
FREE BODY: PORTION CD

$$\begin{aligned} +\sum M_C = 0 \\ D_y(4 \text{ m}) - D_x(3 \text{ m}) = 0 \\ D_y = \frac{3}{4} D_x \end{aligned}$$



FREE BODY:  
ENTIRE CABLE.

$$+\sum M_A = 0: \frac{3}{4} D_x(14 \text{ m}) - W_B(4 \text{ m}) - W_C(10 \text{ m}) - P(5 \text{ m}) = 0 \quad (1)$$



FREE BODY: PORTION BCD

$$\begin{aligned} +\sum M_B = 0 \\ \frac{3}{4} D_x(10 \text{ m}) - D_x(5 \text{ m}) - W_C(6 \text{ m}) = 0 \\ D_x = 2.4 W_C \quad (2) \end{aligned}$$

$$\text{PROB. 7.103: } m_B = 70 \text{ kg} \quad m_C = 25 \text{ kg}$$

$$g = 9.81 \text{ m/s}^2: W_B = 70 \text{ g} \quad W_C = 25 \text{ g}$$

$$\text{EQ(2): } D_x = 2.4 W_C = 2.4(25 \text{ g}) = 60 \text{ g}$$

$$\text{EQ(1): } \frac{3}{4} 60 \text{ g}(14) - 70 \text{ g}(4) - 25 \text{ g}(10) - 5P = 0$$

$$100 \text{ g} - 5P = 0; P = 20 \text{ g}$$

$$P = 20(9.81) = 196.2 \text{ N}$$

$$P = 196.2 \text{ N}$$

$$\text{PROB. 7.104: } m_B = 18 \text{ kg} \quad m_C = 10 \text{ kg}$$

$$g = 9.81 \text{ m/s}^2: W_B = 18 \text{ g} \quad W_C = 10 \text{ g}$$

$$\text{EQ(2): } D_x = 2.4 W_C = 2.4(10 \text{ g}) = 24 \text{ g}$$

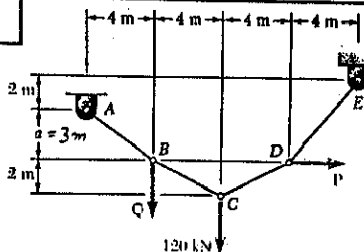
$$\text{EQ(1): } \frac{3}{4} 24 \text{ g}(14) - (18 \text{ g})(4) - (10 \text{ g})(10) - 5P = 0$$

$$80 \text{ g} = 5P: P = 16 \text{ g}$$

$$P = 16(9.81) = 156.96 \text{ N}$$

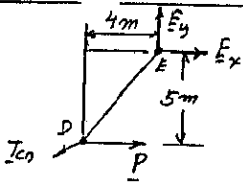
$$P = 157.0 \text{ N}$$

**7.105**



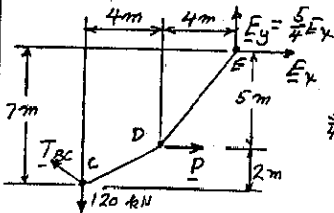
GIVEN:  
 $a = 3m$

FIND:  
MAGNITUDES  
OF P AND Q



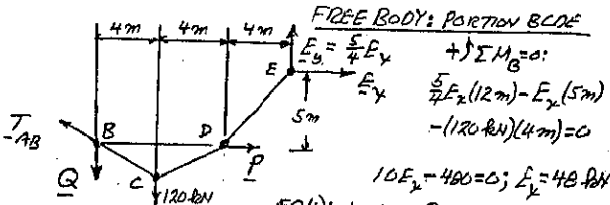
FREE BODY: PORTION DE

$$\begin{aligned} + \uparrow \sum M_D &= 0 \\ E_y(4m) - E_x(6m) &= 0 \\ E_y &= \frac{5}{4} E_x \end{aligned}$$



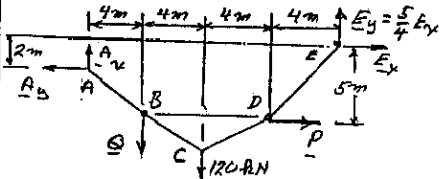
FREE BODY: PORTION CDE

$$\begin{aligned} \uparrow \Sigma M_c &= 0: \\ \frac{5}{4} F_x (8m) - F_x (7m) - P(2m) &= 0 \\ F_x &= \frac{2}{3} P \quad (1) \end{aligned}$$



FREE BODY: PORTION BCDE

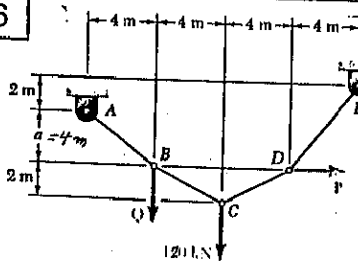
$\sum F_y = 0$   
 $\frac{5}{4} F_x (12m) - F_y (5m) - (120 \text{ kN})(4m) = 0$   
 $10 F_x - 480 = 0; F_x = 48 \text{ kN}$   
 $\sum F_x = 0$   
 $48 \text{ kN} - P = 0$   
 $P = 48 \text{ kN}$



FREE BODY:  
ENTIRE CABLE

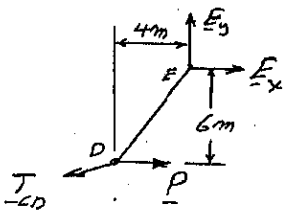
$$\begin{aligned} \uparrow \Sigma M_A = 0 &= \frac{5}{4} F_x (16\text{ m}) - F_y (2\text{ m}) + P(3\text{ m}) - Q(4\text{ m}) - (120\text{ kN})(3\text{ m}) = 0 \\ (48\text{ kN})(20\text{ m} - 2\text{ m}) + (72\text{ kN})(3\text{ m}) - Q(4\text{ m}) - 960\text{ kN}\cdot\text{m} &= 0 \\ 4Q &= 180 & Q &= 30\text{ kN} \end{aligned}$$

7.106



GIVEN:  
 $a = 4\text{ m}$

FIND:  
MAGNITUDES  
OF P AND Q



FREE BODY: PORTION DE

$$+\sum M_0 = 0$$

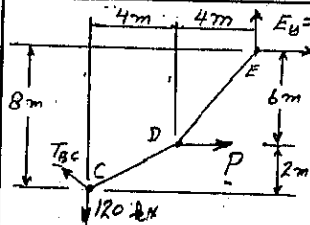
$$F_y(4m) - F_x(6m) = 0$$

$$F_y = \frac{3}{2} F_x$$

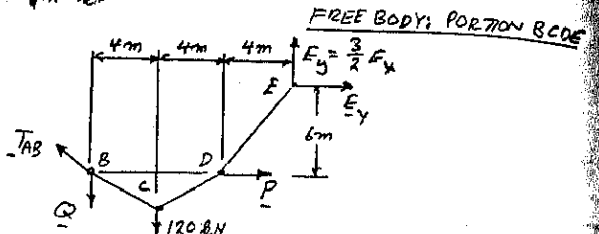
(CONTINUED)

7.106 CONTINUED

FREE BODY: PORTION C



$$F_x = \frac{1}{2} P \quad (1)$$

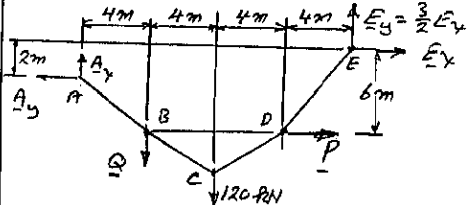


FREE BODY: PORTION BCDE

$$\uparrow \Sigma M_B = 0: \frac{3}{2} E_x (12m) - E_x (6m) + (120 \text{ kN})(4m) = 0$$

$$12E_x = 480 \quad E_x = 40 \text{ kN}$$

EQ(1):  $E_x = \frac{1}{2}P$ ;  $40 \text{ kN} = \frac{1}{2}P$   $P = 80 \text{ kN}$



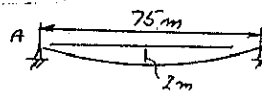
FREE BODY:  
ENTIRE CABLE

$$+\sum M_A = 0: \frac{3}{2} E_y (16m) - E_y (2m) + P(4m) - Q(4m) - (120 \text{ RN})(8m) = 0$$

$$(40 \text{ RN})(24m - 2m) + (80 \text{ RN})(4m) - Q(4m) - 960 \text{ RN} \cdot m = 0$$

$$4Q = 240 \quad Q = 60 \text{ RN}$$

7.107



GIVEN:

$$\text{MASS/METER} = 0.8 \text{ kg/m}$$

FIND: (a)  $T_{max}$

(b) LENGTH OF CABLE

$$\omega = (0.8 \text{ kg/m}) (9.8 \text{ m/s}^2) \\ = 7.848 \text{ N/m}$$

$$W = (2.848 \text{ N/m}) (37.5 \text{ m})$$

$$W = 294.3 \text{ N}$$

(a)  $\uparrow \Sigma M_B = 0$   $T_0(2m) - (W \frac{1}{2} 37.5m) = 0$   
 $T_0(2m) - (294.3N) \frac{1}{2} (37.5m) = 0$   
 $T_0 = 2759N$

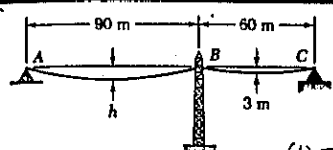
$T_m = 2770 \text{ N}$

$$(b) \quad s_B = r_B \left[ 1 + \frac{2}{3} \left( \frac{y_B}{r_B} \right)^2 + \dots \right]$$

$$= 37.5 \text{ m} \left[ 1 + \frac{2}{3} \left( \frac{2 \text{ m}}{37.5 \text{ m}} \right)^2 + \dots \right] = 37.57 \text{ m}$$

$$\text{LENGTH} = 2S_B = 2(37.57\text{m}) \quad \text{LENGTH} = 75.14\text{m}$$

108



GIVEN:  
CABLE OF 0.4 lb/ft  
FIND: (a)  $h$  FOR  
EQUAL HORIZ. COMPS.  
(b)  $T_m$  IN EACH CABLE

$W = w x_B$   
 $\sum M_B = 0$   
 $T_0 y_B - (w x_B) \frac{x_B}{2} = 0$   
 $T_0 = \frac{w x_B^2}{2 y_B}$   
 HORIZ. COMP. =  $T_0 = \frac{w x_B^2}{2 y_B}$

CABLE AB  $x_B = 45 \text{ m}$

$$T_0 = \frac{w (45 \text{ m})^2}{2 h}$$

CABLE BC  $x_B = 30 \text{ m}, y_B = 3 \text{ m}$

$$T_0 = \frac{w (30 \text{ m})^2}{2 (3 \text{ m})}$$

EQUATE  $T_0 = T_0$

$$\frac{w (45 \text{ m})^2}{2 h} = \frac{w (30 \text{ m})^2}{2 (3 \text{ m})}; \quad h = 6.75 \text{ m}$$

(b)

$$T_m^2 = T_0^2 + W^2$$

CABLE AB:  $w = (0.4 \text{ lb/ft}) (9.81 \text{ m/s}^2) = 3.924 \text{ N/m}$

$x_B = 45 \text{ m}, y_B = h = 6.75 \text{ m}$

$$T_0 = \frac{w x_B^2}{2 y_B} = \frac{(3.924 \text{ N/m}) (45 \text{ m})^2}{2 (6.75 \text{ m})} = 588.6 \text{ N}$$

$$W = w x_B = (3.924 \text{ N/m}) (45 \text{ m}) = 176.58 \text{ N}$$

$$T_m = \sqrt{(588.6 \text{ N})^2 + (176.58 \text{ N})^2}; \quad \text{FOR AB, } T_m = 615 \text{ N}$$

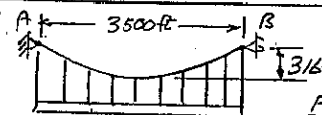
CABLE BC  $x_B = 30 \text{ m}, y_B = 3 \text{ m}$

$$T_0 = \frac{w x_B^2}{2 y_B} = \frac{(3.924 \text{ N/m}) (30 \text{ m})^2}{2 (3 \text{ m})} = 588.6 \text{ N (checks)}$$

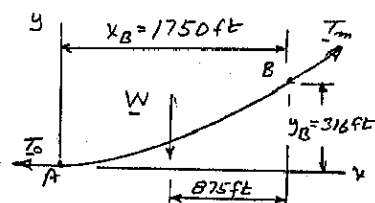
$$W = w x_B = (3.924 \text{ N/m}) (30 \text{ m}) = 117.72 \text{ N}$$

$$T_m = \sqrt{(588.6 \text{ N})^2 + (117.72 \text{ N})^2}; \quad \text{FOR BC, } T_m = 600 \text{ N}$$

7.109



GIVEN:  
 $w = 9.75 \text{ kips/ft}$   
FIND: (a)  $T_m$   
(b) LENGTH OF CABLE



$W = w x_B$   
 $= (9.75 \text{ kips/ft}) (1750 \text{ ft})$   
 $W = 17,063 \text{ kips}$

$$+\sum M_B = 0: T_0 (316 \text{ ft}) - (17,063 \text{ kips}) (875 \text{ ft}) = 0$$

(a)

$$T_0 = 47,247 \text{ kips}$$

$$T_m = \sqrt{T_0^2 + W^2} = \sqrt{(47,247 \text{ kips})^2 + (17,063 \text{ kips})^2}$$

$$T_m = 50,230 \text{ kips}$$

(b)

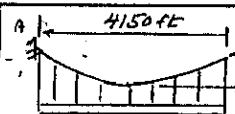
$$S_B = x_B \left[ 1 + \frac{2}{3} \left( \frac{y_B}{x_B} \right)^2 - \frac{2}{5} \left( \frac{y_B}{x_B} \right)^4 + \dots \right]$$

$$= (1750 \text{ ft}) \left[ 1 + \frac{2}{3} (0.18057)^2 - \frac{2}{5} (0.18057)^4 + \dots \right]$$

$$S_B = 1787.3 \text{ ft}; \quad \text{LENGTH} = 2S_B = 3574.6 \text{ ft}$$

$$\text{LENGTH} = 3575 \text{ ft}$$

7.110



GIVEN:  
 $w = 11.1 \text{ kips/ft}$   
FIND: (a)  $T_m$   
(b) LENGTH OF CABLE

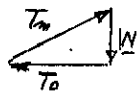
EQ. (2.8) Page 374:

$$\text{AT } B: y_B = \frac{w x_B^2}{2 T_0}$$

$$T_0 = \frac{w x_B^2}{2 y_B} = \frac{(11.1 \text{ kips/ft}) (2075 \text{ ft})^2}{2 (464 \text{ ft})}$$

$$T_0 = 57,500 \text{ kips}$$

$$W = w x_B = (11.1 \text{ kips/ft}) (2075 \text{ ft}) = 23,033 \text{ kips}$$



$$T_m = \sqrt{T_0^2 + W^2} = \sqrt{(57,500 \text{ kips})^2 + (23,033 \text{ kips})^2}$$

$$T_m = 56,420 \text{ kips}$$

(b)

$$S_B = x_B \left[ 1 + \frac{2}{3} \left( \frac{y_B}{x_B} \right)^2 - \frac{2}{5} \left( \frac{y_B}{x_B} \right)^4 + \dots \right]$$

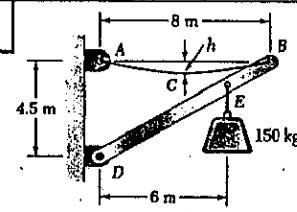
$$\frac{y_B}{x_B} = \frac{464 \text{ ft}}{2075 \text{ ft}} = 0.22361$$

$$S_B = (2075 \text{ ft}) \left[ 1 + \frac{2}{3} (0.22361)^2 - \frac{2}{5} (0.22361)^4 + \dots \right] = 2142.1 \text{ ft}$$

$$\text{LENGTH} = 2S_B = 2(2142.1 \text{ ft})$$

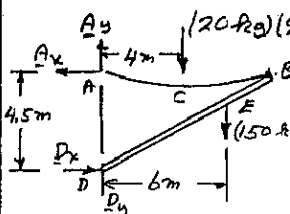
$$\text{LENGTH} = 4284 \text{ ft}$$

7.111



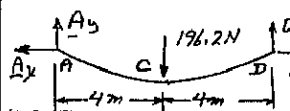
GIVEN: MASS OF  
CABLE ABC IS 20 kg

FIND: (a) SAG  $h$   
(b) SLOPE OF  
CABLE AT A



$$+\sum M_D = 0: A_x (4.5 \text{ m}) - (196.2 \text{ N}) (4 \text{ m}) - (1471.5 \text{ N}) (6 \text{ m}) = 0$$

$$A_x = 2136.4 \text{ N}$$



FREE BODY: ENTIRE FRAME

$$+\sum M_D = 0$$

$$A_y (8 \text{ m}) - (196.2 \text{ N}) (4 \text{ m}) = 0$$

$$A_y = 98.1 \text{ N}$$

(a)

FREE BODY: PORTION AC

$$\sum F_x = 0: T_0 = A_x = 2136.4 \text{ N}$$

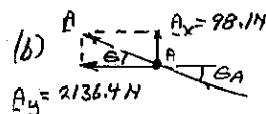
$$+\sum M_A = 0$$

$$T_0 h - (98.1 \text{ N}) (2 \text{ m}) = 0$$

$$(2136.4 \text{ N}) h - 196.2 \text{ N} \cdot \text{m} = 0$$

$$h = 0.09183 \text{ m}$$

$$h = 91.8 \text{ mm}$$



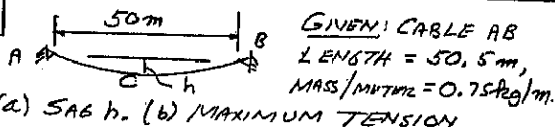
$$\tan \theta_A = \frac{A_y}{A_x} = \frac{98.1 \text{ N}}{2136.4 \text{ N}}$$

$$\tan \theta_A = 0.045918$$

$$\theta_A = 2.62^\circ$$

$$\theta_A = 2.63^\circ$$

7.112



FIRST TWO TERMS OF EQ. 7.10, page 374

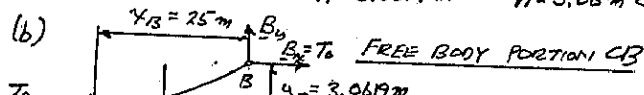
(a)  $S_B = \frac{1}{2}(50.5 \text{ m}) = 25.25 \text{ m}$ ,  $x_B = \frac{1}{2}(50 \text{ m}) = 25 \text{ m}$

$$S_B = x_B \left[ 1 + \frac{2}{3} \left( \frac{y_B}{x_B} \right)^2 \right]$$

$$25.25 \text{ m} = 25 \text{ m} \left[ 1 + \frac{2}{3} \left( \frac{y_B}{x_B} \right)^2 \right]; \left( \frac{y_B}{x_B} \right)^2 = 0.01 \left( \frac{3}{2} \right)^2 = 0.015$$

$$\frac{y_B}{x_B} = 0.12247; \frac{h}{25 \text{ m}} = 0.12247$$

$$h = 3.0619 \text{ m} \quad h = 3.06 \text{ m} \quad \blacktriangleleft$$



$$W = (0.75 \text{ kg/m})(9.81 \text{ m/s}^2) = 7.3575 \text{ N/m}$$

$$W = S_B w = (25.25 \text{ m})(7.3575 \text{ N/m})$$

$$W = 185.78 \text{ N}$$

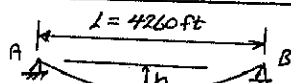
$$+\sum M_B = 0: T_0(3.0619 \text{ m}) - (185.78 \text{ N})(12.5 \text{ m}) = 0$$

$$T_0 = 758.4 \text{ N} \quad B_x = T_0 = 758.4 \text{ N}$$

$$+\uparrow \sum F_y = 0: B_y - 185.78 \text{ N} = 0 \quad B_y = 185.78 \text{ N}$$

$$T_m = \sqrt{B_x^2 + B_y^2} = \sqrt{(758.4 \text{ N})^2 + (185.78 \text{ N})^2}; \quad T_m = 781 \text{ N} \quad \blacktriangleleft$$

7.113



EQ. 7.10, page 374:  $S_B = x_B \left[ 1 + \frac{2}{3} \left( \frac{y_B}{x_B} \right)^2 - \frac{2}{5} \left( \frac{y_B}{x_B} \right)^4 + \dots \right]$

WINTER:  $y_B = h = 386 \text{ ft}$ ,  $x_B = \frac{1}{2}L = 2130 \text{ ft}$

$$S_B = (2130) \left[ 1 + \frac{2}{3} \left( \frac{386}{2130} \right)^2 - \frac{2}{5} \left( \frac{386}{2130} \right)^4 + \dots \right] = 2177.59 \text{ ft}$$

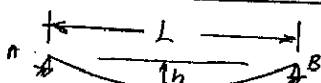
SUMMER:  $y_B = h = 394 \text{ ft}$ ,  $x_B = \frac{1}{2}L = 2130 \text{ ft}$

$$S_B = (2130) \left[ 1 + \frac{2}{3} \left( \frac{394}{2130} \right)^2 - \frac{2}{5} \left( \frac{394}{2130} \right)^4 + \dots \right] = 2175.715 \text{ ft}$$

$$\Delta = 2(\Delta S_B) = 2(2177.59 \text{ ft} - 2175.715 \text{ ft}) = 2(1.875 \text{ ft})$$

$$\text{CHANGE IN LENGTH} = 3.75 \text{ ft} \quad \blacktriangleleft$$

7.114



EQ. 7.10, page 374: (FIRST TWO TERMS)

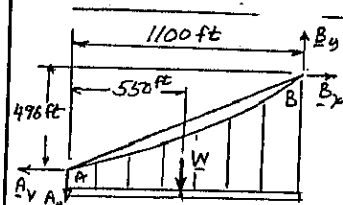
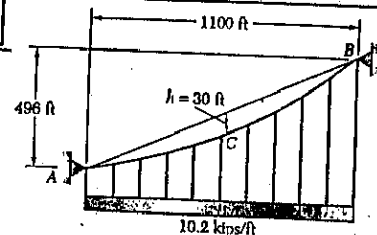
(a)  $S_B = x_B \left[ 1 + \frac{2}{3} \left( \frac{y_B}{x_B} \right)^2 \right]$   $x_B = \frac{1}{2}L$ ,  $S_B = \frac{1}{2}(L + A)$   
 $y_B = h$

$$\frac{1}{2}(L + A) = \frac{1}{2}L \left[ 1 + \frac{2}{3} \left( \frac{h}{L/2} \right)^2 \right]$$

$$\frac{A}{2} = \frac{4}{3} \frac{h^2}{L}; \quad h^2 = \frac{3}{8} LA; \quad h = \sqrt{\frac{3}{8} LA}$$

(b)  $L = 100 \text{ ft}$ ,  $h = 4 \text{ ft}$ .  $h = \sqrt{\frac{3}{8}(100)(4)}; \quad h = 12.25 \text{ ft} \quad \blacktriangleleft$

7.115

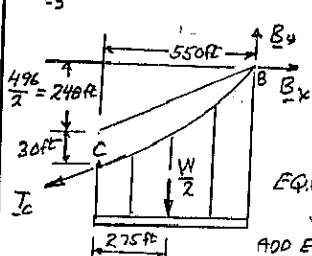


FREE BODY: ENTIRE CABLE

$$+\sum M_A = 0: B_y(1100 \text{ ft})$$

$$- B_x(496 \text{ ft}) - W(550 \text{ ft}) = 0$$

$$1100 B_y - 496 B_x - 550 W = 0 \quad (1)$$



FREE BODY: PORTION CB

$$+\sum M_C = 0: - B_y(550 \text{ ft})$$

$$+ B_x(278 \text{ ft}) + \frac{W}{2}(275 \text{ ft}) = 0$$

$$- 550 B_y + 278 B_x + 137.5 W = 0 \quad (2)$$

EQ. (1) X 0.5:

$$550 B_y - 248 B_x - 275 W = 0 \quad (3)$$

ADD EOS. (2) AND (3)

$$30 B_x - 137.5 W = 0 \quad (4)$$

$$W = WL = (10.2 \text{ kips/ft})(1100 \text{ ft}) = 11,220 \text{ kips}$$

$$\text{EQ. (4): } 30 B_x - (137.5)(11,220) = 0$$

$$B_x = 51,425 \text{ kips}$$

$$\text{EQ. (1): } 1100 B_y - 496(51,425) - 550(11,220) = 0$$

$$B_y = 28,798 \text{ kips}$$

$$B_y = 28,798 \text{ kips}$$

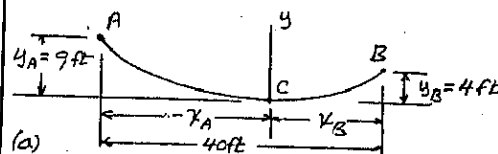
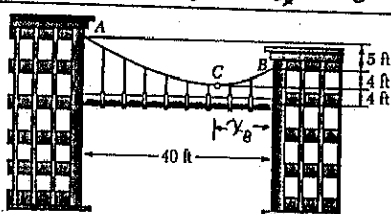
$$T_m = \sqrt{B_x^2 + B_y^2}$$

$$T_m = 58,940 \text{ kips}$$

$$\theta_B = \tan^{-1} \frac{B_y}{B_x}$$

$$\theta_B = 29.2^\circ \quad \blacktriangleleft$$

7.116



USE EQ. 7.8 page 374:

POINT A:  $y_A = \frac{w x_A^2}{2 T_0}; \quad 9 = \frac{w (x_B - 40)^2}{2 T_0} \quad (1)$

POINT B:  $y_B = \frac{w x_B^2}{2 T_0}; \quad 4 = \frac{w x_B^2}{2 T_0} \quad (2)$

DIVIDING (1) BY (2):  $\frac{9}{4} = \frac{(x_B - 40)^2}{x_B^2}; \quad x_B = 16 \text{ ft}$

POINT C IS 16 ft TO LEFT OF B

(b) MAXIMUM SLOPE AND THUS  $T_{max}$  IS AT A

$$x_A = x_B - 40 = 16 - 40 = -24 \text{ ft}$$

$$y_A = \frac{w x_A^2}{2 T_0}; \quad 9 \text{ ft} = \frac{(50 \text{ lb/ft})(-24 \text{ ft})^2}{2 T_0}; \quad T_0 = 1600 \text{ lb}$$

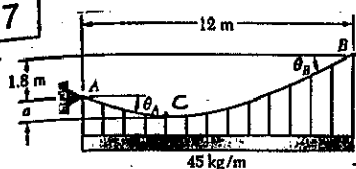
$$W_{AC} = (50 \text{ lb/ft})(24 \text{ ft}) = 1200 \text{ lb}$$

$$T_{max} = A \quad A_y = W_{AC} = 1200 \text{ lb}$$

$$A_x = T_0 = 1600 \text{ lb} \quad A$$

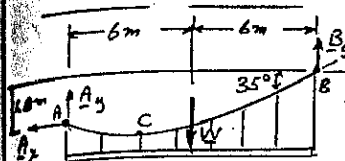
$$T_{max} = 2000 \text{ lb} \quad \blacktriangleleft$$

7.117



GIVEN:  $\theta_B = 35^\circ$

FIND: (a)  $T_m$   
(b) VERTICAL DISTANCE TO LOWEST POINT



FREE BODY: ENTIRE CABLE  
 $B_y = B_x \tan 35^\circ$

$$W = (45 \text{ kg/m})(12 \text{ m})(9.81 \text{ m/s}^2)$$

$$W = 5297.4 \text{ N}$$

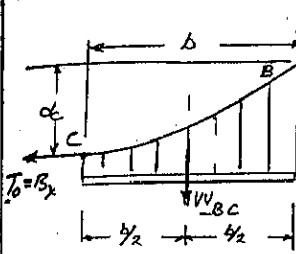
$$+\uparrow \Sigma M_A = 0: W(6 \text{ m}) + B_x(1.8 \text{ m}) - B_y(12 \text{ m}) = 0$$

$$(5297.4)(6) + 1.8 B_x - B_y \tan 35^\circ (12) = 0$$

$$B_x = 4814 \text{ N}$$

$$B_y = (4814 \text{ N}) \tan 35^\circ = 3370.8 \text{ N}$$

FREE BODY: PORTION CB



$$B_y = 3370.8 \text{ N}$$

$$B_x = 4814 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: B_y - W_{BC} = 0$$

$$W_{BC} = B_y = 3370.8 \text{ N}$$

$$W_{BC} = (45 \text{ kg/m})(9.81 \text{ m/s}^2) b$$

$$3370.8 \text{ N} = (441.45 \text{ N/m}) b$$

$$b = 7.6357 \text{ m}$$

$$+\uparrow \Sigma M_B = 0: T_0 d_c - W_{BC}(\frac{1}{2} b) = 0$$

$$(4814 \text{ N}) d_c - (3370.8 \text{ N})(\frac{1}{2})(7.6357 \text{ m}) = 0$$

$$d_c = 2.6733 \text{ m}$$

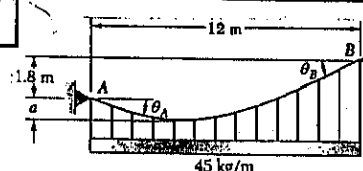
$$(a) d_c = 1.8 \text{ m} + a; 2.6733 \text{ m} = 1.8 \text{ m} + a; a = 0.873 \text{ m}$$

$$(b) T_m = B = \sqrt{B_x^2 + B_y^2} = \sqrt{(4814 \text{ N})^2 + (3370.8 \text{ N})^2}$$

$$T_m = 5877 \text{ N}$$

$$T_m = 5880 \text{ N}$$

7.118

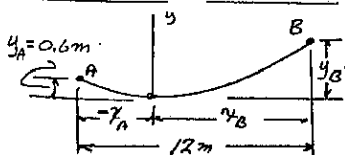


GIVEN:

$$a = 0.6 \text{ m}$$

FIND: (a)  $T_m$

(b)  $\theta_B$



NOTE:

$$x_B - x_A = 12 \text{ m}$$

OR:

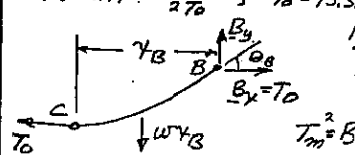
$$x_A = x_B - 12 \text{ m}$$

$$\text{POINT A: } y_A = \frac{w x_A^2}{2 T_0}; 0.6 = \frac{w (x_B - 12)^2}{2 T_0} \quad (1)$$

$$\text{POINT B: } y_B = \frac{w x_B^2}{2 T_0}; 2.4 = \frac{w x_B^2}{2 T_0} \quad (2)$$

$$\text{DIVIDING (1) BY (2): } \frac{0.6}{2.4} = \frac{(x_B - 12)^2}{x_B^2}; x_B = 8 \text{ m}$$

$$\text{EQ (2): } 2.4 = \frac{w (8)^2}{2 T_0}; T_0 = 13.333 \text{ W}$$



FREE BODY: PORTION CB

$$\Sigma F_y = 0 \quad B_y = w x_B$$

$$B_y = B W$$

$$T_m^2 = B_x^2 + B_y^2; T_m^2 = (13.333 w)^2 + (8 w)^2$$

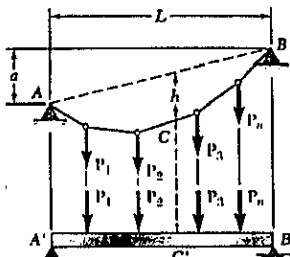
$$T_m = 15.549 \text{ W} = 15.549 (45) (9.81)$$

$$T_m = 6880 \text{ N}$$

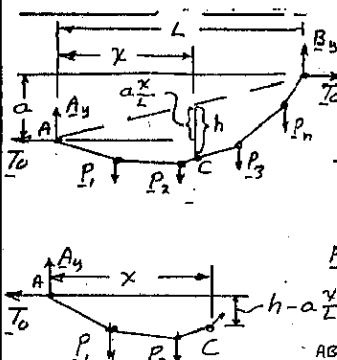
$$\theta_B = \tan^{-1} B_y / B_x = \tan^{-1} 8 w / 13.333 w$$

$$\theta_B = 31.0^\circ$$

\* 7.119



SHOW THAT  $M_C$  IN BEAM IS EQUAL TO  $T_0 h$  WHERE  $T_0$  IS HORIZ. COMPONENT OF CABLE TENSION



FREE BODY: ENTIRE CABLE

DEFINE BY  $\Sigma M_B^L$

THE SUM OF THE MOMENTS OF ALL LOADS ABOUT B.

$$+\uparrow \Sigma M_B = 0: -A_y L - T_0 a + \Sigma M_B^L = 0$$

$$A_y = \frac{1}{L} \Sigma M_B^L - T_0 \frac{a}{L} \quad (1)$$

FREE BODY: PORTION AC

DEFINE BY  $\Sigma M_C^x$  THE

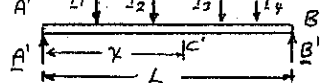
SUM OF THE MOMENTS ABOUT C OF LOADS BETWEEN A AND C.

$$+\uparrow \Sigma M_C = 0: -A_y x + T_0 (h - a \frac{x}{L}) + \Sigma M_C^x = 0 \quad (2)$$

SUBSTITUTE FOR  $A_y$  FROM (1) AND SOLVE (2) FOR  $T_0 h$ :

$$T_0 h = \frac{x}{L} \Sigma M_B^L - \Sigma M_C^x \quad (3)$$

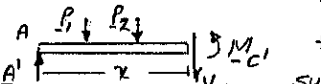
FREE BODY: ENTIRE BEAM



$$+\uparrow \Sigma M_B = 0: -A' L + \Sigma M_B^L = 0$$

$$A' = \frac{1}{L} \Sigma M_B^L \quad (4)$$

FREE BODY: PORTION AC



$$+\uparrow \Sigma M_C = 0: M_C - A' x + \Sigma M_C^x = 0$$

SUBSTITUTE FOR  $A'$  FROM (4):

$$M_C = \frac{x}{L} \Sigma M_B^L - \Sigma M_C^x \quad (5)$$

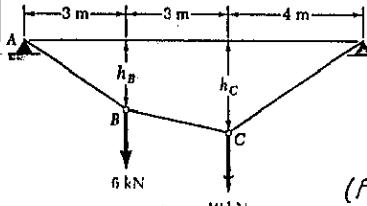
COMPARING (3) AND (5) AND NOTING THAT

$$\Sigma M_B^L = \Sigma M_B^L$$

$$\Sigma M_C^x = \Sigma M_C^x$$

$$\text{WE HAVE: } M_C = T_0 h \quad (\text{Q.E.D.})$$

7.120



GIVEN:

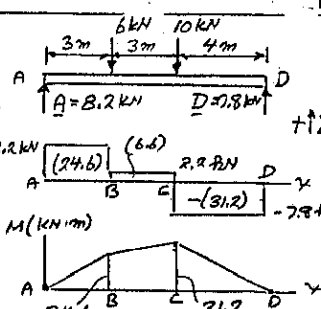
$$T_m = 15 \text{ kN}$$

FIND:

DISTANCES

$h_B$  AND  $h_C$

(PROB. 7.94 a)



$$+\uparrow \Sigma M_B = 0: A(10 \text{ m}) - (6 \text{ kN})(7 \text{ m}) - (10 \text{ kN})(4 \text{ m}) = 0$$

$$A = 8.2 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: B.2 \text{ kN} - 6 \text{ kN} - 10 \text{ kN} + B = 0$$

$$B = 7.8 \text{ kN}$$

$$\text{AT A: } T_m^2 = T_0^2 + A^2$$

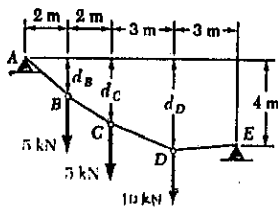
$$15^2 = T_0^2 + 8.2^2$$

$$T_0 = 12.56 \text{ kN}$$

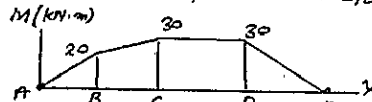
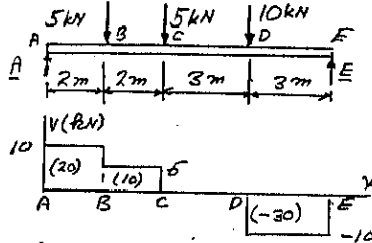
$$\text{AT B: } M_B = T_0 h_B; 24.6 \text{ kN} \cdot \text{m} = (12.56 \text{ kN}) h_B; h_B = 1.957 \text{ m}$$

$$\text{AT C: } M_C = T_0 h_C; 31.2 \text{ kN} \cdot \text{m} = (12.56 \text{ kN}) h_C; h_C = 2.48 \text{ m}$$

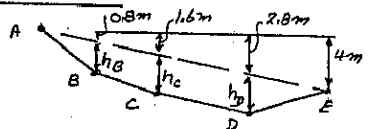
7.121

GIVEN:  $d_C = 3\text{m}$ FIND:  $d_B$  AND  $d_D$ 

(PROB. 7.97a)



GEOMETRY:



$$d_C = 1.6\text{m} + h_C$$

$$3\text{m} = 1.6\text{m} + h_C$$

$$h_C = 1.4\text{m}$$

SINCE  $M = T_0 h$ ,  $h$  IS PROPORTIONAL TO  $M$ , THUS

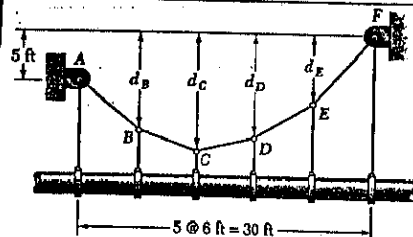
$$\frac{h_B}{M_B} = \frac{h_C}{M_C} = \frac{h_D}{M_D}; \quad \frac{h_B}{20\text{ kN}\cdot\text{m}} = \frac{h_C}{30\text{ kN}\cdot\text{m}} = \frac{h_D}{30\text{ kN}\cdot\text{m}}$$

$$h_B = 1.4 \left( \frac{20}{30} \right) = 0.9333\text{m} \quad h_D = 1.4 \left( \frac{30}{30} \right) = 1.4\text{m}$$

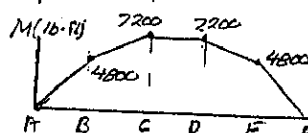
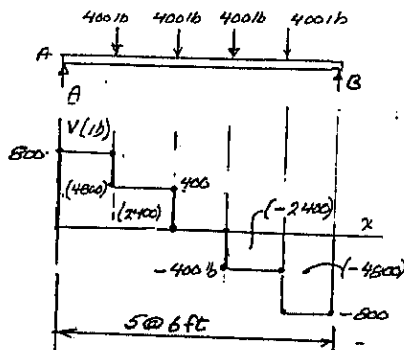
$$d_B = 0.8\text{m} + 0.9333\text{m} \quad d_D = 2.8\text{m} + 1.4\text{m}$$

$$d_B = 1.733\text{m} \quad d_D = 4.2\text{m}$$

7.122



(PROB. 7.99b)

GIVEN:  $d_C = 12\text{ft}$ , TENSION IN HANGERS = 400lbFIND:  $d_D$ 

$$A = B = \frac{1}{2}(4 \times 400) = 800\text{lb}$$

(CONTINUED)

7.122 CONTINUED

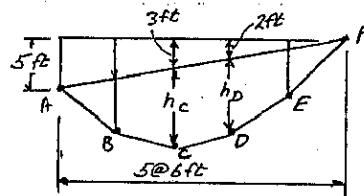
GEOMETRY

$$d_C = h_C + 3\text{ft}$$

$$12\text{ft} = h_C + 3\text{ft}$$

$$h_C = 9\text{ft}$$

$$d_D = h_D + 2\text{ft}$$

AT C:  $M_C = T_0 h_C$ 

$$7200\text{ lb}\cdot\text{ft} = T_0 (9\text{ft})$$

$$T_0 = 800\text{ lb}$$

AT D:  $M_D = T_0 h_D$ 

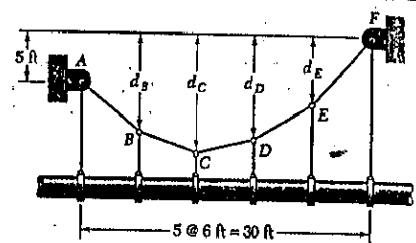
$$7200\text{ lb}\cdot\text{ft} = (800\text{ lb}) h_D$$

$$h_D = 9\text{ft}$$

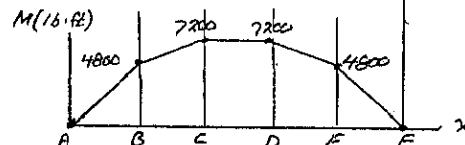
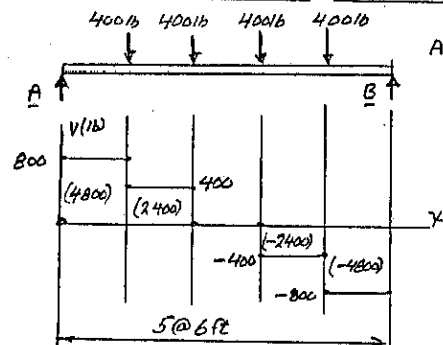
$$\text{EQ. (1): } d_D = 9\text{ft} + 2\text{ft}$$

$$d_D = 11\text{ft}$$

7.123

GIVEN:  $d_C = 9\text{ft}$ , TENSION IN HANGERS = 400lbFIND:  $d_D$ 

(PROB. 7.100b)



$$A = B = \frac{1}{2}(4 \times 400)$$

$$A = B = 800\text{lb}$$

AT ANY POINT:  $M = T_0 h$ WE NOTE THAT SINCE  $M_C = M_D$ , WE HAVE  $h_C = h_D$ 

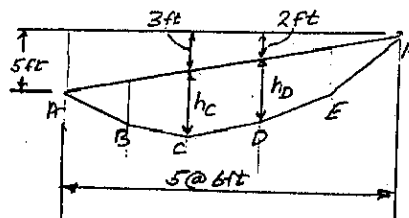
GEOMETRY

$$d_C = h_C + 3\text{ft}$$

$$9\text{ft} = h_C + 3\text{ft}$$

$$h_C = 6\text{ft}$$

$$\text{AND } h_D = 6\text{ft}$$



$$d_D = h_D + 2\text{ft} = 6\text{ft} + 2\text{ft}$$

$$d_D = 8\text{ft}$$

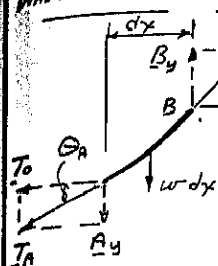


7.124

FOR A CABLE PROVE THAT

$$\frac{d^2y}{dx^2} = \frac{w(x)}{T_0}$$

WHERE  $w(x)$  IS THE DISTRIBUTED LOAD



FREE BODY: DIFFERENTIAL ELEMENT OF CABLE

$w = w(x)$  = LOAD AS FUNCTION OF  $x$

$$A_y = T_0 \tan \theta_A \quad B_y = T_0 \tan \theta_B$$

$$\tan \theta_A = \frac{dy}{dx} \Big|_A = \frac{dy}{dx}$$

$$\tan \theta_B = \frac{dy}{dx} \Big|_B = \frac{dy}{dx} + \frac{d^2y}{dx^2} dx$$

$$\text{OR} \quad \tan \theta = \frac{dy}{dx} + \frac{d^2y}{dx^2} dx$$

$$+\uparrow \Sigma F_y = 0: -A_y + B_y - w dx = 0$$

$$-T_0 \tan \theta_A + T_0 \tan \theta_B - w dx = 0$$

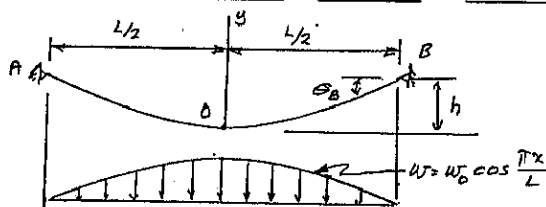
$$-T_0 \frac{dy}{dx} + T_0 \left( \frac{dy}{dx} + \frac{d^2y}{dx^2} dx \right) - w dx = 0$$

$$T_0 \frac{d^2y}{dx^2} dx - w dx = 0 \quad \frac{d^2y}{dx^2} = \frac{w}{T_0} \text{ (Q.E.D.)}$$

7.125

GIVEN:  $w = w_0 \cos(\pi x/L)$ ,  $L = \text{SPAN}$   
 $h = \text{SAG}$ , ORIGIN AT MID-SPAN.

FIND: EQUATION OF CABLE SHAPE, AND  $T_0$  AND  $T_m$



$$\frac{d^2y}{dx^2} = \frac{w}{T_0}$$

$$\frac{d^2y}{dx^2} = \frac{w_0}{T_0} \cos \frac{\pi x}{L}$$

$$\frac{dy}{dx} = \frac{w_0 L}{T_0 \pi} \sin \frac{\pi x}{L} + C_1$$

$$y = -\frac{w_0 L^2}{T_0 \pi^2} \cos \frac{\pi x}{L} + C_1 x + C_2$$

BOUNDARY CONDITIONS  $x=0, \frac{dy}{dx} = 0, \therefore C_1 = 0$

$$x=0, y=0 = -\frac{w_0 L^2}{T_0 \pi^2} \cos 0 + C_2 \quad C_2 = +\frac{w_0 L^2}{T_0 \pi^2}$$

$$y = +\frac{w_0 L^2}{T_0 \pi^2} (1 - \cos \frac{\pi x}{L})$$

(1)

$$x = \frac{L}{2}, y = h \quad h = \frac{w_0 L^2}{T_0 \pi^2} (1 - \cos \frac{\pi}{2}) \quad h = \frac{w_0 L^2}{T_0 \pi^2}$$

$$\text{EQ. (1)} \quad y = h (1 - \cos \frac{\pi x}{L})$$

$$\frac{dy}{dx} = h \frac{\pi}{L} \sin \frac{\pi x}{L} \quad \tan \theta_B = \frac{dy}{dx} \Big|_B = h \frac{\pi}{L} \sin \frac{\pi}{2} = h \frac{\pi}{L}$$

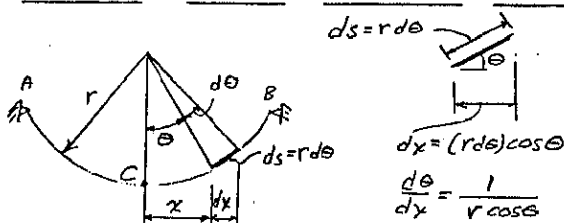
$$\frac{1}{1 + \tan^2 \theta} \quad \tan \theta_B = \frac{1}{\cos \theta_B} = \frac{1}{\sqrt{1 + \frac{h^2 \pi^2}{L^2}}}$$

$$T_m = \frac{T_0}{\cos \theta_B} = \left( \frac{w_0 L^2}{T_0 \pi^2} \right) \sqrt{1 + \frac{h^2 \pi^2}{L^2}}$$

$$\text{OR:} \quad T_{max} = \frac{w_0 L}{\pi} \sqrt{\frac{L^2}{h^2 \pi^2} + 1}$$

7.126

IF  $w = w_0 / \cos \theta$ , PROVE CURVE FORMED BY A CABLE IS A CIRCULAR ARC.



$$\frac{d\theta}{dx} = \frac{1}{r \cos \theta} \quad (1)$$

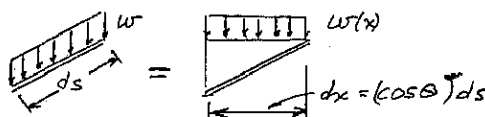
$$\frac{dy}{dx} = \tan \theta; \quad \frac{d^2y}{dx^2} = + \sec^2 \theta \frac{d\theta}{dx} \quad (2)$$

$$\text{SUBSTITUTE FROM (1):} \quad \frac{d^2y}{dx^2} = \frac{\sec^2 \theta}{r \cos \theta} = \frac{1}{r \cos^3 \theta}$$

FROM PROB 7.124:

$$\frac{d^2y}{dx^2} = \frac{w(x)}{T_0}; \quad \frac{1}{r \cos^3 \theta} = \frac{w(x)}{T_0}$$

$$w(x) = \frac{T_0}{r \cos^3 \theta} = \text{LOADING PER HORIZONTAL UNIT}$$



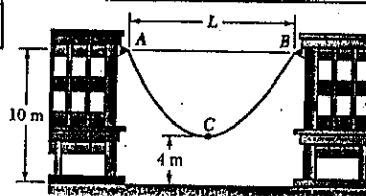
$$w ds = w(x) dx$$

$$w ds = \frac{T_0}{r \cos^3 \theta} \cos \theta ds \quad w = \frac{T_0}{r \cos^2 \theta}$$

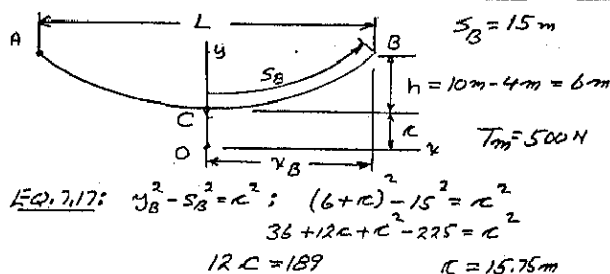
WE NOTE THAT SMALLEST VALUE OF  $w$  OCCURS AT  $\theta=0$ , DENOTING SMALLEST VALUE BY  $w_0$ , WE FIND

$$w_0 = \frac{T_0}{r} \quad w = \frac{w_0}{\cos^2 \theta} \quad (\text{Q.E.D.})$$

7.127



GIVEN:  
30-m CABLE,  
 $T_m = 500 \text{ N}$   
FIND: (a)  $L$   
(b) MASS OF CABLE



$$\text{EQ. 7.17:} \quad y_B^2 - s_B^2 = r^2; \quad (6 + r)^2 - 15^2 = r^2$$

$$36 + 12r + r^2 - 225 = r^2$$

$$12r = 189 \quad r = 15.75 \text{ m}$$

$$\text{EQ. 7.15:} \quad s_B = r \sinh \frac{y_B}{r}; \quad 15 = (15.75) \sinh \frac{y_B}{r}$$

$$\sinh \frac{y_B}{r} = 0.95238 \quad \frac{y_B}{r} = 0.8473$$

$$(a) \quad y_B = 0.8473 (15.75) = 13.345 \text{ m}; \quad L = 2y_B \quad L = 26.7 \text{ m}$$

$$(b) \text{EQ. 7.18:} \quad T_m = w y_B; \quad 500 \text{ N} = w (6 + 15.75)$$

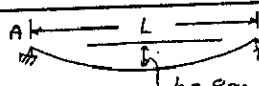
$$w = 22.99 \text{ N/m}$$

$$W = 2s_B w = (30 \text{ m})(22.99 \text{ N/m}) = 689.7 \text{ N}$$

$$m = \frac{W}{g} = \frac{689.7 \text{ N}}{9.81 \text{ m/s}^2}$$

$$\text{TOTAL MASS} = 70.3 \text{ kg}$$

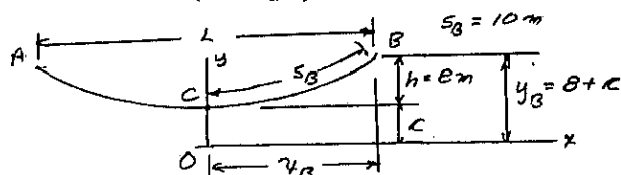
7.128



GIVEN: 20-m  
CHAIN OF MASS 12 lb  
FIND: (a) L, (b)  $T_m$

$$\text{MASS/METER} = (12 \text{ lb}) / (20 \text{ m}) = 0.6 \text{ lb/m}$$

$$W = (0.6 \text{ lb/m})(9.81 \text{ m/s}^2) = 5.886 \text{ N/m}$$



$$\text{EQ. 7.17: } y_B^2 - S_B^2 = c^2; (8+c)^2 - 10^2 = c^2$$

$$64 + 16c + c^2 - 100 = c^2$$

$$16c = 36 \quad c = 2.25 \text{ m}$$

$$\text{EQ. 7.18: } T_m = W y_B = (5.886 \text{ N/m})(8 \text{ m} + 2.25 \text{ m})$$

$$T_m = 60.33 \text{ N} \quad T_m = 60.3 \text{ N}$$

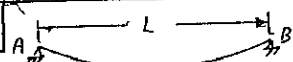
$$\text{EQ. 7.15: } S_B = c \sinh \frac{y_B}{c}; 10 \text{ m} = (2.25 \text{ m}) \sinh \frac{y_B}{c}$$

$$\sinh \frac{y_B}{c} = 4.444; \frac{y_B}{c} = 2.197$$

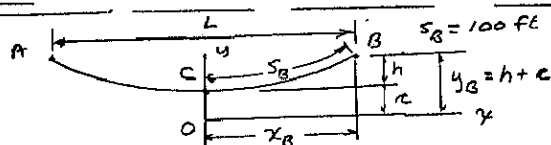
$$y_B = 2.197(2.25 \text{ m}) = 4.944 \text{ m}; L = 2y_B = 2(4.944 \text{ m}) = 9.888 \text{ m}$$

$$L = 9.89 \text{ m}$$

7.129



GIVEN: 200-ft TAPE WEIGHS 16 lb,  $T_m = 16 \text{ lb}$   
FIND: SPAN L



$$W = (16 \text{ lb}) / (200 \text{ ft}) = 0.08 \text{ lb/ft} \quad T_m = 16 \text{ m}$$

$$\text{EQ. 7.18: } T_m = W y_B; 16 \text{ lb} = (0.08 \text{ lb/ft}) y_B; y_B = 200 \text{ ft}$$

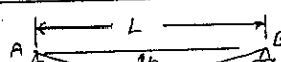
$$\text{EQ. 7.17: } y_B^2 - S_B^2 = c^2; (200)^2 - (100)^2 = c^2; c = 79.73 \text{ ft}$$

$$\text{EQ. 7.15: } S_B = c \sinh \frac{y_B}{c}; 100 = 79.73 \sinh \frac{y_B}{c}$$

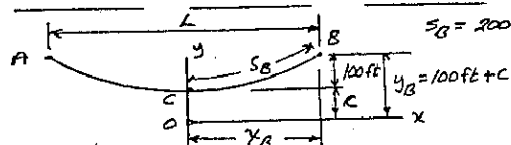
$$\frac{y_B}{c} = 0.12586; y_B = 9.9737 \text{ ft}$$

$$L = 2y_B = 2(9.9737 \text{ ft}); L = 19.947 \text{ ft}$$

7.130



GIVEN: 400-ft CABLE,  
 $h = 100 \text{ ft}$ ,  $W = 2.5 \text{ lb/ft}$ .  
FIND: SPAN L AND  $T_m$



$$\text{EQ. 7.17: } y_B^2 - S_B^2 = c^2; (100+c)^2 - 200^2 = c^2$$

$$10000 + 200c + c^2 - 40000 = c^2; c = 150 \text{ ft}$$

$$\text{EQ. 7.15: } S_B = c \sinh \frac{y_B}{c}; 200 = 150 \sinh \frac{y_B}{c}$$

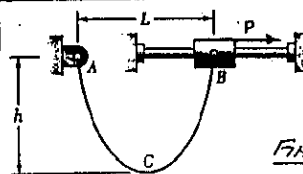
$$\sinh \frac{y_B}{c} = \frac{4}{3}; \frac{y_B}{c} = 1.0986; y_B = (150)(1.0986) = 164.79 \text{ ft}$$

$$L = 2y_B = 2(164.79 \text{ ft}) = 329.58 \text{ ft}; L = 330 \text{ ft}$$

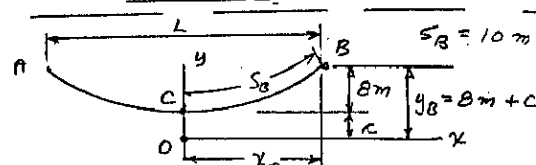
$$\text{EQ. 7.18: } T_m = W y_B = (2.5 \text{ lb/ft})(100 \text{ ft} + 150 \text{ ft})$$

$$T_m = 625 \text{ lb}$$

7.131



GIVEN: 20-m  
CABLE ACB OF  
UNIT MASS =  
0.2 lb/m,  $h = 10$   
FIND: (a) P, (b) L.



$$\text{EQ. 7.17: } y_B^2 - S_B^2 = c^2; (8+c)^2 - 10^2 = c^2$$

$$64 + 16c + c^2 - 100 = c^2$$

$$16c = 36 \quad c = 2.25 \text{ m}$$

$$\text{EQ. 7.15: } S_B = c \sinh \frac{y_B}{c}; 10 \text{ m} = (2.25 \text{ m}) \sinh \frac{y_B}{c}$$

$$\sinh \frac{y_B}{c} = 4.444; \frac{y_B}{c} = 2.197; y_B = (2.197)(2.25 \text{ m}) = 4.944 \text{ m}$$

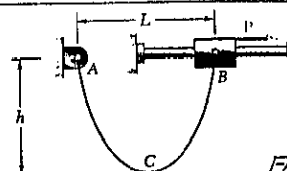
$$L = 2y_B = 2(4.944 \text{ m}) = 9.888 \text{ m}; L = 9.89 \text{ m}$$

NOTE THAT P = HORIZ. COMP. OF CABLE TENSION,  $\therefore T_0 = P$

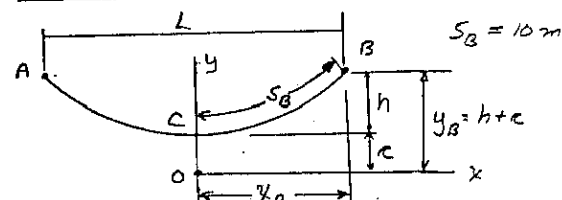
$$\text{EQ. 7.18: } T_0 = P = W c; P = (0.2 \text{ lb/m})(9.81 \text{ m/s}^2)(2.25 \text{ m})$$

$$P = 4.414 \text{ N} \quad P = 4.41 \text{ N}$$

7.132



GIVEN: 20-m  
CABLE ACB OF  
UNIT MASS =  
0.2 lb/m,  $P = 20 \text{ N}$ .  
FIND: (a) h, (b) L.



$$\text{TOTAL CABLE: } W = (0.2 \text{ lb/m})(9.81 \text{ m/s}^2)(20 \text{ m}) = 39.24 \text{ N}$$

$$\text{COLLAR AT B: } B_y = \frac{1}{2} W = 19.62 \text{ N}$$

$$T_m = \sqrt{(20 \text{ N})^2 + (19.62 \text{ N})^2}$$

$$T_m = 28.017 \text{ N}$$

$$\text{EQ. 7.18: } T_m = W y_B; 28.017 \text{ N} = (0.2 \text{ lb/m})(9.81 \text{ m/s}^2) y_B$$

$$y_B = 14.280 \text{ m}$$

$$\text{EQ. 7.17: } y_B^2 - S_B^2 = c^2; (14.280 \text{ m})^2 - (10 \text{ m})^2 = c^2$$

$$c^2 = 103.92 \quad c = 10.194 \text{ m}$$

$$y_B = h + c$$

$$14.280 \text{ m} = h + 10.194 \text{ m}$$

$$h = 4.086 \text{ m}$$

$$h = 4.09 \text{ m}$$

$$\text{EQ. 7.15: } S_B = c \sinh \frac{y_B}{c}$$

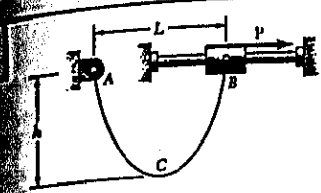
$$10 \text{ m} = (10.194 \text{ m}) \sinh \frac{y_B}{c}$$

$$\sinh \frac{y_B}{c} = 0.981 \quad \frac{y_B}{c} = 0.8678$$

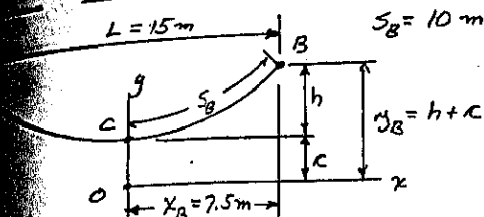
$$y_B = 0.8678(10.194 \text{ m}) = 8.847 \text{ m}$$

$$L = 2y_B = 2(8.847 \text{ m}) = 17.694 \text{ m}$$

$$L = 17.69 \text{ m}$$



GIVEN: 20-m  
CABLE ACB OF  
UNIT MASS =  
0.2 kg/m,  $L=15$  m  
FIND: (a) SAG  $h$ ,  
(b) FORCE  $P$



$$S_B = c \sinh \frac{x_B}{c}$$

$$10 = c \sinh \frac{7.5}{c}; \quad c = 5.5504 \text{ m}$$

$$y_B = c \cosh \frac{x_B}{c} = (5.5504) \cosh \frac{7.5}{5.5504}$$

$$y_B = 11.437 \text{ m}; \quad y_B = h + c$$

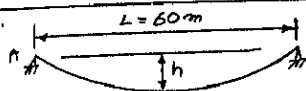
$$11.437 = h + 5.5504; \quad h = 5.89 \text{ m}$$

$$T_0 = w c = (0.2 \text{ kg/m})(9.81 \text{ m/s}^2)(5.5504 \text{ m})$$

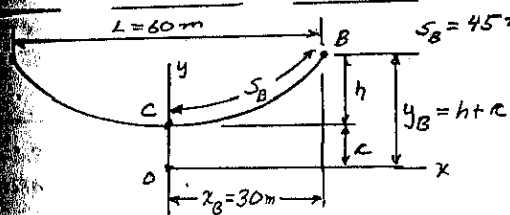
$$T_0 = 10.89 \text{ N}; \quad P = T_0 = \text{HORIZ. COMP. OF TENSION}$$

$$P = 10.89 \text{ N} \rightarrow$$

34



GIVEN:  $T_m = 300$  N,  
90-m WIRE  
FIND: (a) SAG  $h$ ,  
(b) TOTAL MASS OF CABLE



$$S_B = c \sinh \frac{x_B}{c}$$

$$45 = c \sinh \frac{30}{c}; \quad c = 18.494 \text{ m}$$

$$y_B = c \cosh \frac{x_B}{c}$$

$$y_B = (18.494) \cosh \frac{30}{18.494}; \quad y_B = 48.652 \text{ m}$$

$$y_B = h + c$$

$$48.652 = h + 18.494;$$

$$h = 30.158 \text{ m}$$

$$h = 30.2 \text{ m}$$

7.135

$$T_m = w y_B$$

$$300 \text{ N} = w (48.652 \text{ m}); \quad w = 6.166 \text{ N/m}$$

TOTAL WEIGHT OF CABLE

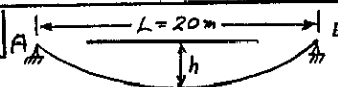
$$W = w (\text{LENGTH}) = (6.166 \text{ N/m})(90 \text{ m}) = 554.96 \text{ N}$$

TOTAL MASS OF CABLE

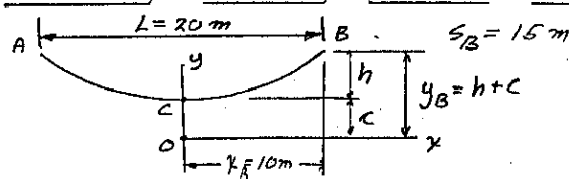
$$m = \frac{W}{g} = \frac{554.96 \text{ N}}{9.81 \text{ m/s}^2} = 56.57 \text{ kg}$$

$$m = 56.6 \text{ kg}$$

7.135



GIVEN: CABLE  
OF LENGTH 20 m.  
FIND: SAG  $h$



$$\text{EQ. 7.17: } S_B = c \sinh \frac{x_B}{c}$$

$$15 = c \sinh \frac{10}{c}; \quad c = 6.1647 \text{ m}$$

$$\text{EQ. 7.16: } y_B = c \cosh \frac{x_B}{c}$$

$$y_B = (6.1647 \text{ m}) \cosh \frac{10}{6.1647}; \quad y_B = 16.2174 \text{ m}$$

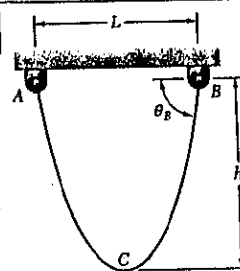
$$y_B = h + c$$

$$16.2174 \text{ m} = h + 6.1647 \text{ m}$$

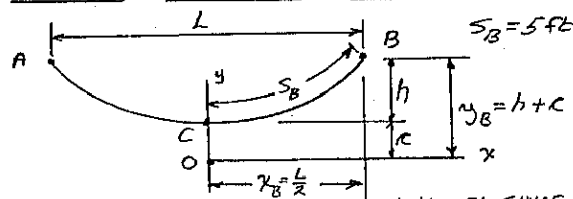
$$h = 10.0527 \text{ m}$$

$$h = 10.05 \text{ m}$$

7.136



GIVEN: ROPE OF  
LENGTH 10 ft WITH  
SPAN  $L$  EQUAL TO  
SAG  $h$   
FIND: (a) SPAN  $L$ ,  
(b) ANGLE  $\theta_B$ .



$$\text{EQ. 7.16: } y_B = c \cosh \frac{x_B}{c}$$

$$h + c = c \cosh \frac{h/2}{c}$$

$$\frac{h}{c} + 1 = \cosh \left( \frac{1}{2} \frac{h}{c} \right)$$

$$\text{SOLVE FOR } h/c: \quad \frac{h}{c} = 4.933$$

$$\text{EQ. 7.16: } y = c \cosh \frac{x}{c}$$

$$\frac{dy}{dx} = \sinh \frac{x}{c}$$

$$\text{AT } B: \quad \tan \theta_B = \frac{dy}{dx} \Big|_B = \sinh \frac{x_B}{c}$$

$$\text{SUBSTITUTE } x_B = \frac{h}{2}; \quad \tan \theta_B = \sinh \left( \frac{1}{2} \frac{h}{c} \right) = \sinh \left( \frac{1}{2} \times 4.933 \right)$$

$$\tan \theta_B = 5.848$$

$$\theta_B = 80.3^\circ$$

$$\text{EQ. 7.17: } S_B = c \sinh \frac{x_B}{c} = c \sinh \left( \frac{1}{2} \frac{h}{c} \right)$$

$$5 \text{ ft} = c \sinh \left( \frac{1}{2} \times 4.933 \right)$$

$$5 \text{ ft} = c (5.848)$$

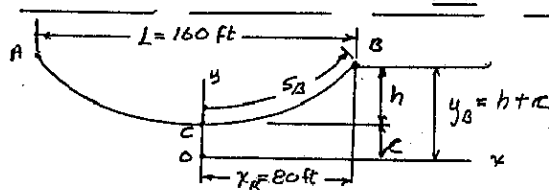
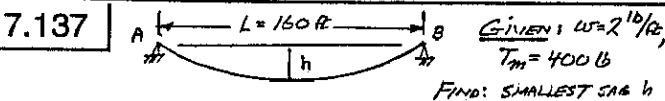
$$c = 0.855$$

$$\text{RECALL THAT } \frac{h}{c} = 4.933$$

$$h = 4.933 (0.855) = 4.218$$

$$h = 4.22 \text{ ft}$$

7.137



EQ. 7.18:  $T_m = w y_B$ ;  $400 \text{ lb} = (2 \text{ lb/ft}) y_B$ ;  $y_B = 200 \text{ ft}$

EQ. 7.16:  $y_B = c \cosh \frac{x_B}{c}$   
 $200 \text{ ft} = c \cosh \frac{80 \text{ ft}}{c}$

SOLVE FOR  $c$ :  $c = 182.148 \text{ ft}$  AND  $L = 31.592 \text{ ft}$

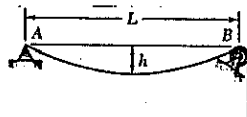
$y_B = h + c$ ;  $h = y_B - c$

FOR  $c = 182.148 \text{ ft}$ ;  $h = 200 - 182.148 = 17.852 \text{ ft}$   $\triangleleft$

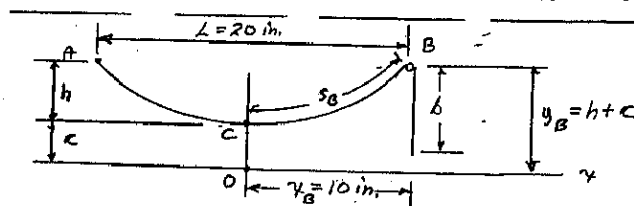
FOR  $c = 31.592 \text{ ft}$ ;  $h = 200 - 31.592 = 168.408 \text{ ft}$   $\triangleleft$

FOR  $T_m \leq 400 \text{ lb}$ : SMALLEST  $h = 17.85 \text{ ft}$   $\triangleleft$

7.138



GIVEN: 50-in. cord,  
 $L = 20 \text{ in.}$   
 FIND: TWO VALUES  
 OF  $h$  FOR EQUILIBRIUM



LENGTH OF OVER HANG:  $b = 50 \text{ in.} - 2s_B$

WEIGHT OF OVER HANG EQUALS MAX. TENSION

$T_m = T_B = w b = w(50 \text{ in.} - 2s_B)$

EQ. 7.15:  $s_B = c \sinh \frac{x_B}{c}$

EQ. 7.16:  $y_B = c \cosh \frac{x_B}{c}$

EQ. 7.18:  $T_m = w y_B$   
 $w(50 \text{ in.} - 2s_B) = w y_B$   
 $w(50 \text{ in.} - 2c \sinh \frac{x_B}{c}) = w c \cosh \frac{x_B}{c}$

$x_B = 10$ :  $50 - 2c \sinh \frac{10}{c} = c \cosh \frac{10}{c}$

SOLVE BY TRIAL + ERROR:

$c = 5.549 \text{ in.}$  AND  $c = 27.742 \text{ in.}$

FOR  $c = 5.549 \text{ in.}$

$y_B = (5.549 \text{ in.}) \cosh \frac{10 \text{ in.}}{5.549 \text{ in.}} = 17.277 \text{ in.}$

$y_B = h + c$ ;  $17.277 \text{ in.} = h + 5.549 \text{ in.}$

$h = 11.728 \text{ in.}$   $h = 11.73 \text{ in.}$   $\triangleleft$

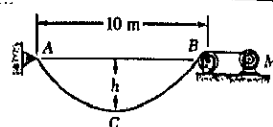
FOR  $c = 27.742 \text{ in.}$

$y_B = (27.742 \text{ in.}) \cosh \frac{10 \text{ in.}}{27.742 \text{ in.}} = 29.564 \text{ in.}$

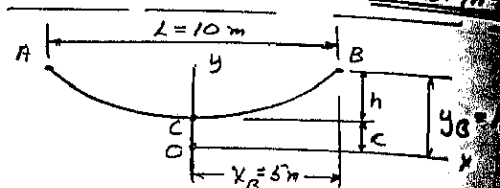
$y_B = h + c$ ;  $29.564 \text{ in.} = h + 27.742 \text{ in.}$

$h = 1.8219 \text{ in.}$   $h = 1.822 \text{ in.}$   $\triangleleft$

7.139 and 7.140



GIVEN: UNIT  
 CABLE = 0.4  
 FIND: MAX  
 IN-CABLE  
 PROB. 7.139  
 PROB. 7.140



PROB. 7.139  $h = 5 \text{ m}$   $y_B = 5 \text{ m} + c$

EQ. 7.16:  $y_B = c \cosh \frac{x_B}{c}$   
 $5 \text{ m} + c = c \cosh \frac{5 \text{ m}}{c}$

SOLVE BY TRIAL:  $c = 3.0938 \text{ m}$

EQ. 7.18:  $T_m = w y_B = w(h + c)$   
 $= (0.4 \text{ kg/m})(9.81 \text{ m/s}^2)(5 \text{ m} + 3.0938 \text{ m})$   
 $T_m = 31.76 \text{ N}$   $T_m = 31.76 \text{ N}$

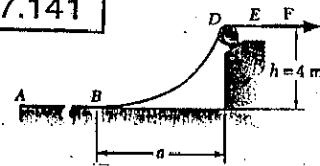
PROB. 7.140  $h = 3 \text{ m}$   $y_B = 3 \text{ m} + c$

EQ. 7.16:  $y_B = c \cosh \frac{x_B}{c}$   
 $3 \text{ m} + c = c \cosh \frac{10 \text{ m}}{c}$

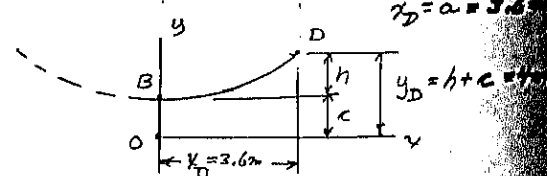
SOLVE BY TRIAL:  $c = 4.5945 \text{ m}$

EQ. 7.18:  $T_m = w y_B = w(h + c)$   
 $= (0.4 \text{ kg/m})(9.81 \text{ m/s}^2)(3 \text{ m} + 4.5945 \text{ m})$   
 $T_m = 29.80 \text{ N}$   $T_m = 29.80 \text{ N}$

7.141



GIVEN:  $w = 2 \text{ kg/m}$   
 UNIT MASS OF  
 $= 2 \text{ kg/m}$   
 FIND: FORCE



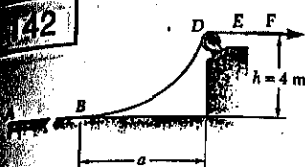
EQ. 7.16:  $y_D = c \cosh \frac{x_D}{c}$   
 $4 \text{ m} + c = c \cosh \frac{3.6 \text{ m}}{c}$

SOLVE BY TRIAL:  $c = 2.0712 \text{ m}$

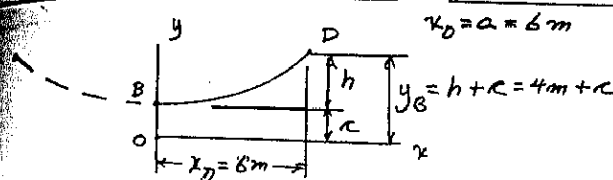
NOTE:  $F = T_m$

EQ. 7.18:  $F = T_m = w y_D = w(4 \text{ m} + c)$   
 $F = (2 \text{ kg/m})(9.81 \text{ m/s}^2)(4 \text{ m} + 2.0712 \text{ m})$   
 $F = 119.12 \text{ N}$   $F = 119.1 \text{ N}$   $\rightarrow$

142



GIVEN:  $a = 6m$ ,  
UNIT MASS OF CABLE  
 $= 2 \text{ lb/ft}$ .  
FIND: FORCE  $P$



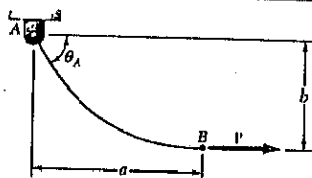
EQ. 7.116:  $y_D = c \cosh \frac{x_D}{c}$   
 $4m + c = c \cosh \frac{6m}{c}$

SOLVE BY TRIAL:  $c = 5.054m$

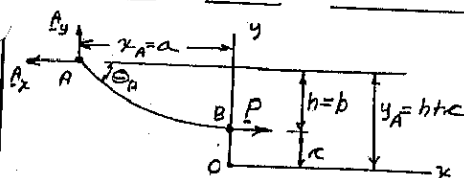
NOTE:  $F = T_m$

EQ. 7.118:  $F = T_m = w y_D = w(4m + c)$   
 $F = (2 \text{ lb/ft})(9.81 \text{ m/s}^2)(4m + 5.054m)$   
 $F = 177.64N$

7.143



GIVEN:  $w = 3 \text{ lb/ft}$ ,  
 $\theta_A = 60^\circ$ ,  $P = 180 \text{ lb}$ .  
FIND: (a) DISTANCES  
a AND b. (b)  
LENGTH OF CABLE



EQ. 7.118:  $T_0 = P = cw$   
 $c = \frac{P}{w} = \frac{180 \text{ lb}}{3 \text{ lb/ft}}; c = 60 \text{ ft}$

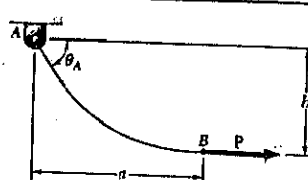
AT A:  $T_m = \frac{P}{\cos 60^\circ} = \frac{cw}{0.5} = 2cw$   
 $T_m = w(h + c)$   
 $2c = h + c; h = b = c; b = 60 \text{ ft}$

EQ. 7.118:  $T_m = w(h + c)$   
 $2cw = w(h + c)$   
 $2c = h + c; h = b = c; b = 60 \text{ ft}$

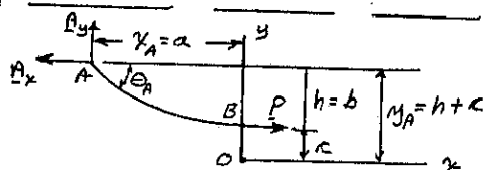
EQ. 7.16:  $y_A = c \cosh \frac{x_A}{c}$   
 $h + c = c \cosh \frac{x_A}{c}$   
 $(60 \text{ ft} + 60 \text{ ft}) = (60 \text{ ft}) \cosh \frac{x_A}{60}$   
 $\cosh \frac{x_A}{60 \text{ m}} = 2; \frac{x_A}{60 \text{ m}} = 1.3170$   
 $x_A = 79.02 \text{ ft} \quad a = 79.0 \text{ ft}$

EQ. 7.15:  $S_A = c \sinh \frac{x_B}{c} = (60 \text{ ft}) \sinh \frac{79.02 \text{ ft}}{60 \text{ ft}}$   
 $S_A = 103.92 \text{ ft}$   
LENGTH =  $S_A$   $S_A = 103.9 \text{ ft}$

7.144



GIVEN:  $w = 3 \text{ lb/ft}$ ,  
 $\theta_A = 60^\circ$ ,  $P = 150 \text{ lb}$ .  
FIND: (a) DISTANCES  
a AND b. (b)  
LENGTH OF CABLE.



EQ. 7.118:  $T_0 = P = cw$   $c = \frac{P}{w} = \frac{150 \text{ lb}}{3 \text{ lb/ft}} = 50 \text{ ft}$

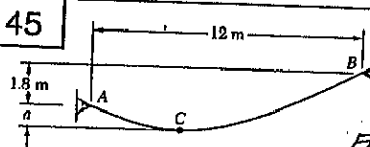
AT A:  $T_m = \frac{P}{\cos 60^\circ} = \frac{cw}{0.5} = 2cw$   
 $T_m = w(h + c)$   
 $2c = h + c; h = b = c; b = 50 \text{ ft}$

EQ. 7.118:  $T_m = w(h + c)$   
 $2cw = w(h + c)$   
 $2c = h + c; h = b = c; b = 50 \text{ ft}$

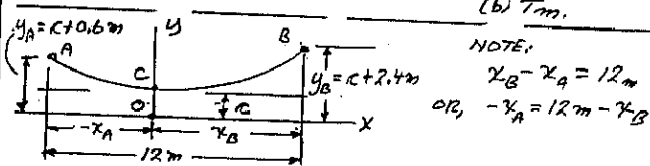
EQ. 7.16:  $y_A = c \cosh \frac{x_A}{c}$   
 $h + c = c \cosh \frac{x_A}{c}$   
 $(50 \text{ ft} + 50 \text{ ft}) = (50 \text{ ft}) \cosh \frac{x_A}{c}$   
 $\cosh \frac{x_A}{c} = 2; \frac{x_A}{c} = 1.3170$   
 $x_A = 1.3170(50 \text{ ft}) = 65.85 \text{ ft}; a = 65.8 \text{ ft}$

EQ. 7.15:  $S_A = c \sinh \frac{x_B}{c} = (50 \text{ ft}) \sinh \frac{65.85 \text{ ft}}{50 \text{ ft}}$   
 $S_A = 86.6 \text{ ft}; \text{LENGTH} = S_A = 86.6 \text{ ft}$

7.145



GIVEN:  $a = 0.6m$   
UNIT MASS OF  
CABLE =  $0.45 \text{ lb/ft}$ .  
FIND: (a) LOCATION OF C.  
(b)  $T_m$ .



POINT A:  $y_A = c \cosh \frac{x_A}{c}; c + 0.6 = c \cosh \frac{12 - x_B}{c} \quad (1)$

POINT B:  $y_B = c \cosh \frac{x_B}{c}; c + 2.4 = c \cosh \frac{x_B}{c} \quad (2)$

FROM (1):  $\frac{12 - x_B}{c} = \cosh^{-1} \left( \frac{c + 0.6}{c} \right) \quad (3)$

FROM (2):  $\frac{x_B}{c} = \cosh^{-1} \left( \frac{c + 2.4}{c} \right) \quad (4)$

ADD (3) + (4):  $\frac{12}{c} = \cosh^{-1} \left( \frac{c + 0.6}{c} \right) + \cosh^{-1} \left( \frac{c + 2.4}{c} \right)$

SOLVE BY TRIAL + ERROR:  $c = 13.6214m$

EQ. (2)  $13.6214 + 2.4 = 13.6214 \cosh \frac{x_B}{c}$   
 $\cosh \frac{x_B}{c} = 1.1762; \frac{x_B}{c} = 0.58523$

$x_B = 0.58523(13.6214m) = 7.9717m$

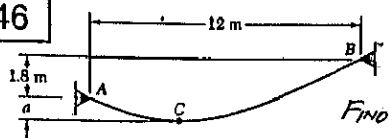
POINT C IS 7.97m TO LEFT OF B

$y_B = c + 2.4 = 13.6214 + 2.4 = 16.0214m$

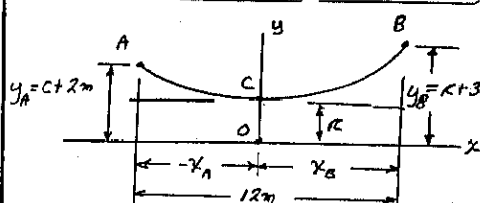
EQ. 7.18:  $T_m = w y_B = (0.45 \text{ lb/ft})(9.81 \text{ m/s}^2)(16.0214m)$   
 $T_m = 70.726N$

$T_m = 70.7N$

7.146



GIVEN:  $\alpha = 2\text{ m}$ ,  
UNIT MASS OF  
CABLE =  $0.45\text{ kg/m}$ .  
FIND: (a) LOCATION OF C,  
(b)  $T_m$



NOTE:  
 $x_B - x_A = 12\text{ m}$   
OR  
 $-x_A = 12\text{ m} - x_B$

POINT A:  $y_A = c \cosh \frac{-x_A}{c}$ ;  $c + 2 = c \cosh \frac{12 - x_B}{c}$  (1)

POINT B:  $y_B = c \cosh \frac{x_B}{c}$ ;  $c + 3.8 = c \cosh \frac{x_B}{c}$  (2)

FROM (1):  $\frac{12}{c} - \frac{x_B}{c} = \cosh^{-1} \left( \frac{c + 2}{c} \right)$  (3)

FROM (2):  $\frac{x_B}{c} = \cosh^{-1} \left( \frac{c + 3.8}{c} \right)$  (4)

ADD (3) + (4):  $\frac{12}{c} = \cosh^{-1} \left( \frac{c + 2}{c} \right) + \cosh^{-1} \left( \frac{c + 3.8}{c} \right)$

SOLVE BY TRIAL AND ERROR:  $c = 6.8154\text{ m}$

EQ. (2):  $6.8154\text{ m} + 3.8\text{ m} = (6.8154\text{ m}) \cosh \frac{x_B}{c}$

$\cosh \frac{x_B}{c} = 1.5576$   $\frac{x_B}{c} = 1.0122$

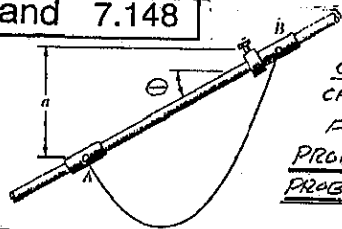
$x_B = 1.0122(6.8154\text{ m}) = 6.899\text{ m}$

POINT C IS 6.90 m TO LEFT OF B

$y_B = c + 3.8 = 6.8154 + 3.8 = 10.6154\text{ m}$

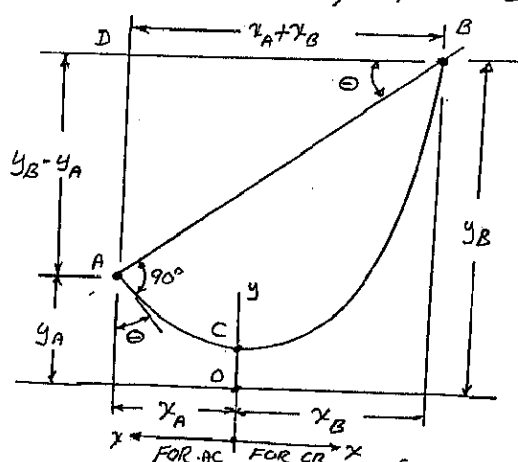
EQ. (7.18):  $T_m = w y_B = (0.45\text{ kg/m})(9.81\text{ m/s}^2)(10.6154\text{ m})$   
 $T_m = 46.86\text{ N}$   $T_m = 46.9\text{ N}$

\*7.147 and 7.148



GIVEN: LENGTH OF  
CABLE AB IS 10 ft.  
FIND: DISTANCE  $a$ ;  
PROB. 7.147: FOR  $\theta = 30^\circ$ .  
PROB. 7.148: FOR  $\theta = 45^\circ$ .

COLLAR AT A: SINCE  $\psi = 0$ , CABLE  $\perp$  ROD



(CONTINUED)

\*7.147 and 7.148 CONTINUED

POINT A:  $y = c \cosh \frac{x}{c}$ ;  $\frac{dy}{dx} = \sinh \frac{x}{c}$

$\tan \theta = \frac{dy}{dx} \bigg|_B = \sinh \frac{x_B}{c}$

$\therefore \frac{x_B}{c} = \sinh^{-1}(\tan \theta)$ ;  $x_B = c \sinh^{-1}(\tan \theta)$  (1)

LENGTH OF CABLE = 10 ft

10 ft = AC + CB

$10 = c \sinh \frac{x_B}{c} + c \sinh \frac{x_A}{c}$

$\sinh \frac{x_B}{c} = \frac{10}{c} - \sinh \frac{x_A}{c}$

$x_B = c \sinh^{-1} \left[ \frac{10}{c} - \sinh \frac{x_A}{c} \right]$  (2)

$y_A = c \cosh \frac{x_A}{c}$   $y_B = c \cosh \frac{x_B}{c}$  (3)

IN  $\triangle ABD$ :  $\tan \theta = \frac{y_B - y_A}{x_B + x_A}$  (4)

METHOD OF SOLUTION:

FOR GIVEN VALUE OF  $\theta$ , CHOOSE TRIAL  
VALUE OF  $c$  AND CALCULATE:

FROM EQ. (1):  $x_A$

USING VALUE OF  $x_A$  AND  $c$ , CALCULATE:

FROM EQ. (2):  $x_B$

FROM EQ. (3):  $y_A$  AND  $y_B$

SUBSTITUTE VALUES OBTAINED FOR  $x_A$ ,  $x_B$ ,  $y_A$ ,  $y_B$   
INTO EQ. (4) AND CALCULATE  $\theta$

CHOOSE NEW TRIAL VALUE OF  $c$  AND  
REPEAT ABOVE PROCEDURE UNTIL CALCULATED  
VALUE OF  $\theta$  IS EQUAL TO GIVEN VALUE OF  $\theta$ .

PROB. 7.147: GIVEN VALUE:  $\theta = 30^\circ$

RESULT OF TRIAL AND ERROR PROCEDURE

$c = 1.803\text{ m}$

$x_A = 2.3745\text{ m}$

$x_B = 3.6937\text{ m}$

$y_A = 3.606\text{ m}$

$y_B = 7.109\text{ m}$

$a = y_B - y_A = 7.109\text{ m} - 3.606\text{ m} = 3.503\text{ m}$

$a = 3.50\text{ m}$

PROB. 7.148: GIVEN VALUE:  $\theta = 45^\circ$

RESULT OF TRIAL AND ERROR PROCEDURE

$c = 1.8652\text{ m}$

$x_A = 1.644\text{ m}$

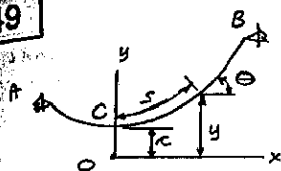
$x_B = 4.064\text{ m}$

$y_A = 2.638\text{ m}$

$y_B = 8.346\text{ m}$

$a = y_B - y_A = 8.346\text{ m} - 2.638\text{ m} = 5.708\text{ m}$

$a = 5.71\text{ m}$



GIVEN: UNIFORM CABLE

PROVE: (a)  $s = c \tan \theta$   
(b)  $y = c \sec \theta$

(a) EQ. 7.16:  $y = c \cosh \frac{x}{c}$

$$\tan \theta = \frac{dy}{dx} = \sinh \frac{x}{c}$$

EQ. 7.15:  $s = c \sinh \frac{x}{c}$  ;  $s = c \tan \theta$  ◀

(b) EQ. 7.14:  $\cosh^2 \frac{x}{c} - \sinh^2 \frac{x}{c} = 1$

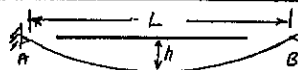
$$\cosh \frac{x}{c} = \sqrt{1 + \sinh^2 \frac{x}{c}} = \sqrt{1 + \tan^2 \theta} \quad (1)$$

$$\cos \theta = \frac{1}{\sqrt{1 + \tan^2 \theta}} \quad (2)$$

SUBSTITUTE (2) INTO (1):  $\cosh \frac{x}{c} = \frac{1}{\cos \theta} \quad (3)$

EQ. 7.16:  $y = c \cosh \frac{x}{c} = c \frac{1}{\cos \theta}$  ;  $y = c \sec \theta$  ◀

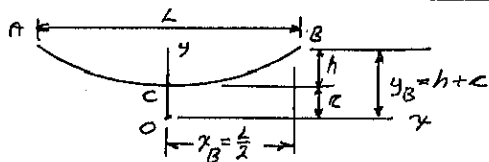
\* 7.150



GIVEN: UNIFORM CABLE,  $w = 16/\text{ft}$

FIND: (a) MAXIMUM SPAN FOR GIVEN VALUE  $T_m$

(b) MAXIMUM SPAN FOR  $w = 0.2516/\text{ft}$  AND  $T_m = 8000\text{ lb}$



(a)  $T_m = w y_B = w c \cosh \frac{x_B}{c} = w x_B \left( \frac{1}{x_B/c} \right) \cosh \frac{x_B}{c}$

WE SHALL FIND RATIO  $(x_B/c)$  FOR WHICH  $T_m$  IS MINIMUM

$$\frac{dT_m}{d(x_B/c)} = w x_B \left[ \frac{1}{x_B/c} \sinh \frac{x_B}{c} - \left( \frac{1}{x_B/c} \right)^2 \cosh \frac{x_B}{c} \right] = 0$$

$$\frac{\sinh \frac{x_B}{c}}{\cosh \frac{x_B}{c}} = \frac{1}{x_B/c} ; \quad \tanh \frac{x_B}{c} = \frac{c}{x_B}$$

SOLVE BY TRIAL AND ERROR FOR:  $\frac{x_B}{c} = 1.200 \quad (1)$

$$s_B = c \sinh \frac{x_B}{c} = c \sinh(1.200) ; \quad \frac{s_B}{c} = 1.509$$

EQ. 7.17:  $y_B^2 - s_B^2 = c^2$  ;  $y_B^2 = c^2 \left[ 1 + \left( \frac{s_B}{c} \right)^2 \right] = c^2 (1 + 1.509^2)$   
 $y_B = 1.810 c$

EQ. 7.18:  $T_m = w y_B = 1.810 w c$   
 $c = \frac{T_m}{1.810 w}$

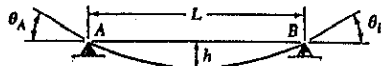
EQ. (1):  $x_B = 1.509 c = 1.509 T_m / 1.810 w = 0.6630 \frac{T_m}{w}$

SPAN:  $L = 2 x_B = 2(0.6630) \frac{T_m}{w} \quad L = 1.326 \frac{T_m}{w}$  ◀

(b) FOR  $w = 0.2516/\text{ft}$  AND  $T_m = 8000\text{ lb}$

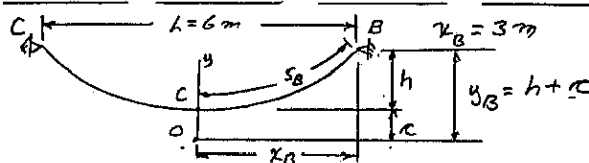
$$L = 1.326 \frac{8000\text{ lb}}{0.2516/\text{ft}} = 42,432\text{ ft} ; L = 8.04\text{ miles}$$
 ◀

\* 7.151



GIVEN: UNIT MASS =  $3\text{ lb}_m/\text{m}$ ,  $L = 6\text{ m}$

FIND: TWO VALUES OF  $h$  FOR WHICH  $T_m = 350\text{ N}$



$$w = (3\text{ lb}_m/\text{m})(9.81\text{ m/s}^2) = 29.43\text{ N/m}$$

EQ. 7.18:  $T_m = w y_B$  ;  $350\text{ N/m} = (29.43\text{ N/m}) y_B$  ;  $y_B = 11.893\text{ m}$

EQ. 7.16:  $y_B = c \cosh \frac{x_B}{c}$   
 $9.81\text{ m/s}^2 = c \cosh \frac{3\text{ m}}{c}$

SOLVE BY TRIAL AND ERROR FOR TWO VALUES OF  $c$

$$c = 0.924\text{ m}$$

$$h = y_B - c$$

$$h = 11.893\text{ m} - 0.924\text{ m}$$

$$h = 10.97\text{ m}$$
 ◀

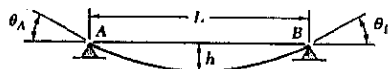
$$c = 11.499\text{ m}$$

$$h = y_B - c$$

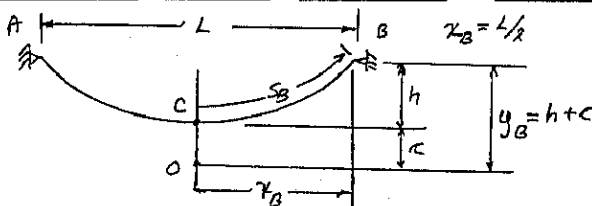
$$h = 11.893\text{ m} - 11.499\text{ m}$$

$$h = 0.394\text{ m}$$
 ◀

\* 7.152



FIND THE  $h/L$  RATIO FOR TOTAL WEIGHT EQUAL TO  $T_m$ .



TOTAL WEIGHT:  $W = (2 s_B) w$  ;  $\therefore T_m = 2 s_B w$

EQ. 7.15:  $s_B = c \sinh \frac{x_B}{c}$

EQ. 7.16:  $y_B = c \cosh \frac{x_B}{c}$

EQ. 7.18:  $T_m = w y_B$   
 $2 s_B w = w y_B$   
 $2 c \sinh \frac{x_B}{c} = c \cosh \frac{x_B}{c}$

$$\tanh \frac{x_B}{c} = \frac{1}{2} ; \quad \frac{x_B}{c} = 0.5493 \quad (1)$$

$$h = y_B - c = c \cosh \frac{x_B}{c} - c = c [\cosh(0.5493) - 1]$$

$$h = c(1.1547 - 1) = 0.1547 c$$

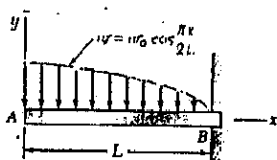
FROM (1):  $c = \frac{x_B}{0.5493}$

THUS:  $h = (0.1547) \frac{x_B}{0.5493} = 0.2816 x_B$

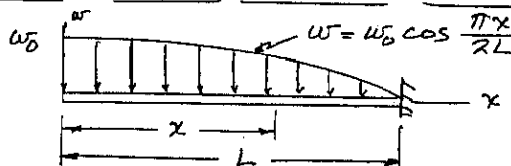
RECALL:  $L = 2 x_B$  ;  $h = (0.2816) \frac{L}{2}$

$$\frac{h}{L} = 0.1408$$
 ◀

7.159



WRITE EQUATIONS  
OF V AND M CURVES.  
FIND:  $M_{max}$



$$\frac{dV}{dx} = -w = -w_0 \cos \frac{\pi x}{2L}$$

$$V = -\int w dx = -w_0 \left( \frac{2L}{\pi} \right) \sin \frac{\pi x}{2L} + C_1 \quad (1)$$

$$\frac{dM}{dx} = V = -w_0 \left( \frac{2L}{\pi} \right) \sin \frac{\pi x}{2L} + C_1$$

$$M = \int V dx = +w_0 \left( \frac{2L}{\pi} \right)^2 \cos \frac{\pi x}{2L} + C_1 x + C_2 \quad (2)$$

BOUNDARY CONDITIONS

AT  $x=0$ :  $V = C_1 = 0$   $C_1 = 0$

AT  $x=0$ :  $M = +w_0 \left( \frac{2L}{\pi} \right)^2 \cos(0) + C_2 = 0$

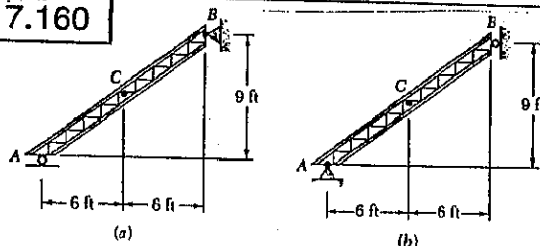
$$C_2 = -w_0 \left( \frac{2L}{\pi} \right)^2$$

Eq (1)  $V = -w_0 \left( \frac{2L}{\pi} \right) \sin \frac{\pi x}{2L}$

$$M = w_0 \left( \frac{2L}{\pi} \right)^2 (-1 + \cos \frac{\pi x}{2L})$$

$M_{max}$  at  $x=L$ :  $|M_{max}| = w_0 \left( \frac{2L}{\pi} \right)^2 |-1+0| = \frac{4}{\pi^2} w_0 L^2$

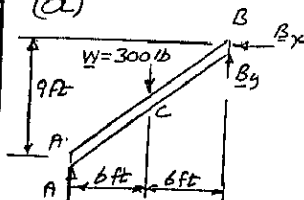
7.160



GIVEN: CHANNEL WEIGHS 20 lb/ft

FIND: INTERNAL FORCES AT C FOR EACH SUPPORT

(a)



FREE BODY: AB

$$AB = \sqrt{9^2 + 12^2} = 15 \text{ ft}$$

$$W = (20 \text{ lb/ft})(15 \text{ ft}) = 300 \text{ lb}$$

$$+\sum M_B = 0:$$

$$A(12 \text{ ft}) - (300 \text{ lb})(6 \text{ ft}) = 0$$

$$A = +150 \text{ lb} \quad A = 150 \text{ lb} \uparrow$$

FREE BODY: AC

(150-lb FORCES FORM A COUPLE)

$$+\sum F_x = 0 \quad F = 0$$

$$+\sum F_y = 0 \quad V = 0$$

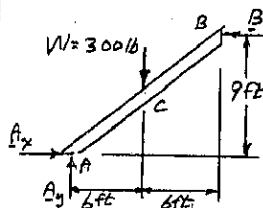
$$+\sum M_C = 0: M - (150 \text{ lb})(3 \text{ ft}) = 0$$

$$M = +450 \text{ lb} \cdot \text{ft}$$

$$M = 450 \text{ lb} \cdot \text{ft} \uparrow$$

(CONTINUED)

7.160 CONTINUED



(b) FREE BODY: AB

$$+\sum M_A = 0:$$

$$B(9 \text{ ft}) - (300 \text{ lb})(6 \text{ ft}) = 0$$

$$B = +200 \text{ lb}$$

$$B = 200 \text{ lb} \leftarrow$$

FREE BODY: CB

$$+\sum M_C = 0: (200 \text{ lb})(4.5 \text{ ft})$$

$$- (150 \text{ lb})(3 \text{ ft}) - M = 0$$

$$M = +450 \text{ lb} \cdot \text{ft}$$

$$M = 450 \text{ lb} \cdot \text{ft} \uparrow$$

$$\tan \theta = \frac{9}{12} = \frac{3}{4}; \sin \theta = \frac{3}{5}; \cos \theta = \frac{4}{5}$$

$$+\sum F_x = 0: F - \frac{3}{5}(150 \text{ lb}) - \frac{4}{5}(200 \text{ lb}) = 0$$

$$F = +250 \text{ lb} \quad F = 250 \text{ lb} \nearrow$$

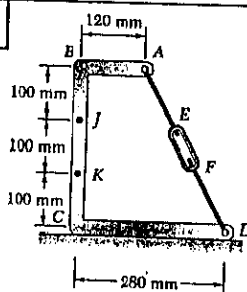
$$+\sum F_y = 0: V - \frac{4}{5}(150 \text{ lb}) + \frac{3}{5}(200 \text{ lb}) = 0$$

$$V = 0 \quad V = 0$$

ON PORTION AC INTERNAL FORCES ARE

$$M = 450 \text{ lb} \cdot \text{ft} \uparrow, F = 250 \text{ lb} \nearrow, V = 0$$

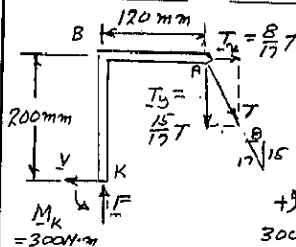
7.161



GIVEN:  $M_K = 300 \text{ N} \cdot \text{m}$

FIND: (a) TENSION IN RODS

(b) INTERNAL FORCES AT J.



FREE BODY: ABK

$$300 \text{ N} \cdot \text{m} = 340 \Rightarrow 17/15$$

$$+\sum M_K = 0$$

$$300 \text{ N} \cdot \text{m} - \frac{8}{17} T (0.2 \text{ m}) - \frac{15}{17} T (0.12 \text{ m}) = 0$$

$$T = 1500 \text{ N}$$

FREE BODY: AJ

$$T_x = \frac{8}{17} T = \frac{8}{17} (1500 \text{ N}) = 705.88 \text{ N}$$

$$T_y = \frac{15}{17} T = \frac{15}{17} (1500 \text{ N}) = 1323.53 \text{ N}$$

INTERNAL FORCES ON ABJ

$$+\sum F_x = 0: 705.88 \text{ N} - V = 0$$

$$V = +705.88 \text{ N} \quad V = 706 \text{ N} \leftarrow$$

$$+\sum F_y = 0: F - 1323.53 \text{ N} = 0$$

$$F = +1323.53 \text{ N} \quad F = 1324 \text{ N} \uparrow$$

$$+\sum M_J = 0:$$

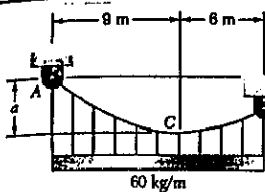
$$M - (705.88 \text{ N})(0.1 \text{ m}) - (1323.53 \text{ N})(0.12 \text{ m}) = 0$$

$$M = +229.4 \text{ N} \cdot \text{m}$$

$$M = 229 \text{ N} \cdot \text{m} \uparrow$$

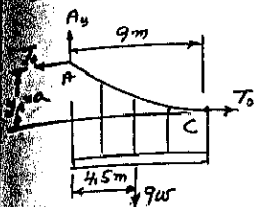


7.162



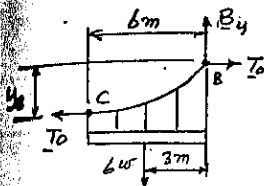
**FIND:**  
(a) DISTANCE  $a$   
(b) LENGTH ACB  
(c) COMPONENTS OF REACTION AT A.

**FREE BODY: PORTION AC**



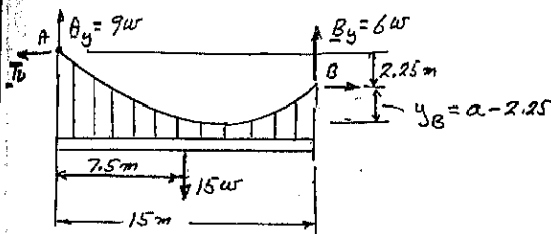
$$\begin{aligned} \uparrow \sum F_y = 0: & A_y - 9w = 0 \\ & A_y = 9w \uparrow \\ \rightarrow \sum M_A = 0: & T_B a - (9w)(4.5m) = 0 \quad (1) \end{aligned}$$

**FREE BODY: PORTION CB**



$$\begin{aligned} \uparrow \sum F_y = 0: & B_y - 6w = 0 \\ & B_y = 6w \uparrow \\ \rightarrow \sum M_B = 0: & T_C a - 6w(3m) = 0 \quad (2) \end{aligned}$$

**FREE BODY: ENTIRE CABLE**



$$\begin{aligned} \rightarrow \sum M_A = 0: & 15w(7.5m) - 6w(15m) - T_B(2.25m) = 0 \\ & T_B = 10w \end{aligned}$$

(a)

$$\begin{aligned} \text{Eq (1): } T_B a - (9w)(4.5m) &= 0 \\ 10w a &= (9w)(4.5) = 0 \\ & a = 4.05m \end{aligned}$$

(b) LENGTH = AC + CB

PORTION AC:  $x_A = 9m, y_A = a = 4.05m; \frac{y_A}{x_A} = \frac{4.05}{9} = 0.45$

$$S_{AC} = x_B \left[ 1 + \frac{2}{3} \left( \frac{y_A}{x_A} \right)^2 - \frac{2}{5} \left( \frac{y_A}{x_A} \right)^4 + \dots \right]$$

$$S_{AC} = 9m \left( 1 + \frac{2}{3} (0.45)^2 - \frac{2}{5} (0.45)^4 + \dots \right) = 10.067m$$

PORTION CB:  $x_B = 6m, y_B = 4.05 - 2.25 = 1.8m; \frac{y_B}{x_B} = 0.3$

$$S_{CB} = 6m \left( 1 + \frac{2}{3} (0.3)^2 - \frac{2}{5} (0.3)^4 + \dots \right) = 6.341m$$

$$\text{TOTAL LENGTH} = 10.067m + 6.341m$$

$$\text{TOTAL LENGTH} = 16.45m$$

(c) COMPONENTS OF REACTION AT A.

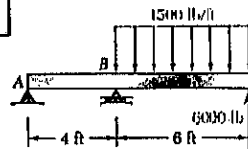
$$T_m \quad A_y = 9w = 9(60 \text{ kg/m})(9.81 \text{ m/s}^2) = 5297.4 \text{ N}$$

$$A_x = T_B = 10w = 10(60 \text{ kg/m})(9.81 \text{ m/s}^2) = 5886 \text{ N}$$

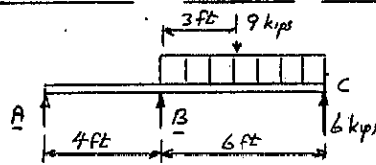
$$A_x = 5890 \text{ N} \leftarrow$$

$$A_y = 5300 \text{ N} \uparrow$$

7.163



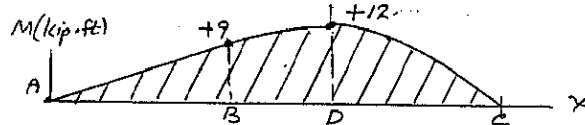
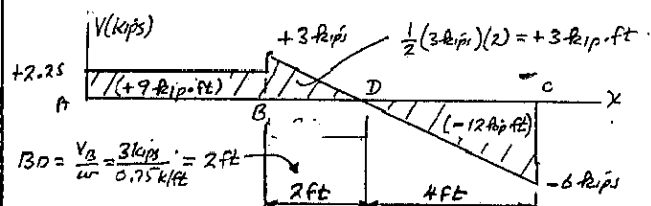
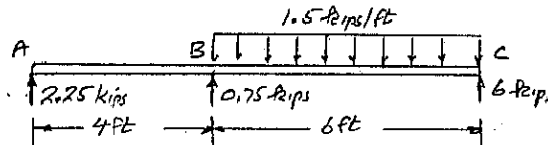
(a) DRAW  $V$  &  $M$  DIAGRAMS  
(b) FIND  $M_{max}$



**FREE BODY: ENTIRE BEAM**

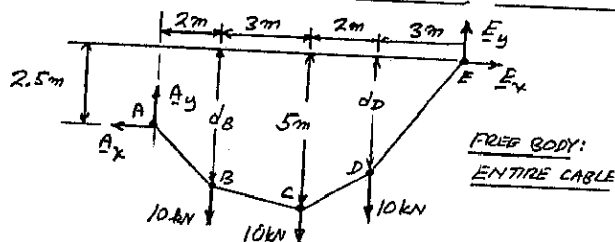
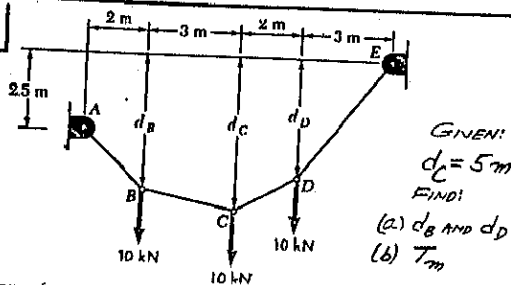
$$\begin{aligned} \rightarrow \sum M_A = 0: & (6 \text{ kips})(10 \text{ ft}) - (9 \text{ kips})(7 \text{ ft}) + B(4 \text{ ft}) = 0 \\ & B = +0.75 \text{ kips} \\ & B = 0.75 \text{ kips} \uparrow \end{aligned}$$

$$\begin{aligned} \uparrow \sum F = 0: & A + 0.75 \text{ kips} - 9 \text{ kips} + 6 \text{ kips} = 0 \\ & A = +2.25 \text{ kips} \quad A = 2.25 \text{ kips} \uparrow \end{aligned}$$



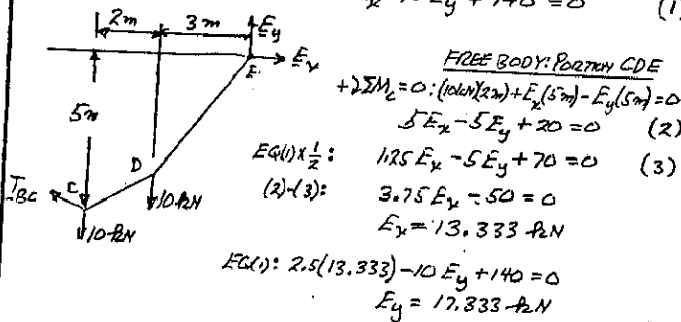
$$M_{max} = 12 \text{ kip-ft}$$

7.164

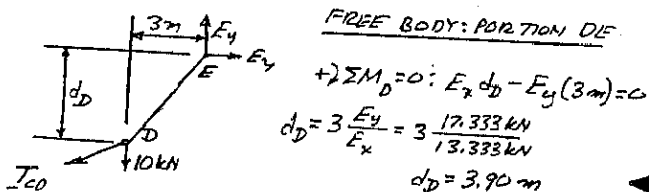


$$+\uparrow \Sigma M_A = 0: (10\text{kN})(2\text{m}) + (10\text{kN})(5\text{m}) + (10\text{kN})(7\text{m}) + E_x(2.5\text{m}) - E_y(10\text{m}) = 0$$

$$2.5E_x - 10E_y + 140 = 0 \quad (1)$$



$$T_m = \sqrt{E_x^2 + E_y^2} = \sqrt{(13.333)^2 + (17.333)^2} \quad T_m = 21.9\text{ kN}$$



RETURN TO FREE BODY OF ENTIRE CABLE AND WRITE

$$+\uparrow \Sigma F_y = 0: A_y - 3(10\text{kN}) + E_y = 0$$

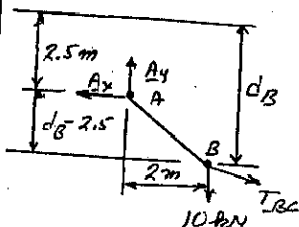
$$A_y - 30\text{ kN} + 17.333\text{ kN} = 0$$

$$A_y = 12.667\text{ kN}$$

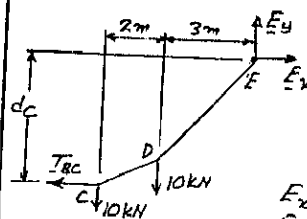
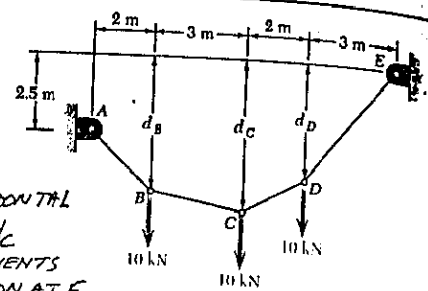
$$\pm \Sigma F_x = 0: -A_x + E_x = 0$$

$$A_x = 13.333\text{ kN}$$

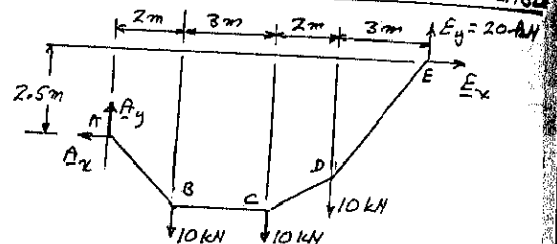
FREE BODY: PORTION AB



7.165



FREE BODY: ENTIRE CABLE

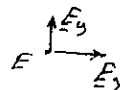


$$\text{EQ (1)}: E_x d_C = 80\text{ kN} \cdot \text{m}$$

$$(24\text{ kN} \cdot \text{m}) d_C = 80\text{ kN} \cdot \text{m}$$

$$d_C = 3.333\text{ m} \quad d_C = 3.33\text{ m}$$

AT E:

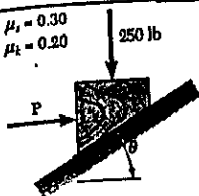


$$E_x = 24\text{ kN} \rightarrow$$

$$E_y = 20\text{ kN} \uparrow$$

$$\mu_s = 0.30$$

$$\mu_k = 0.20$$



**GIVEN:**  $\theta = 30^\circ$   
 $P = 50 \text{ lb}$

**FIND:** FRICTION  
 FORCE ACTING ON  
 BLOCK.

ASSUME EQUILIBRIUM

$$+\uparrow \Sigma F_y = 0: N - (250 \text{ lb}) \cos 30^\circ - (50 \text{ lb}) \sin 30^\circ = 0$$

$$N = 241.5 \text{ lb} \quad N = 241.5 \text{ lb} \uparrow$$

$$+\rightarrow \Sigma F_x = 0: F - (250 \text{ lb}) \sin 30^\circ + (50 \text{ lb}) \cos 30^\circ = 0$$

$$F = 81.7 \text{ lb} \quad F = 81.7 \text{ lb} \rightarrow$$

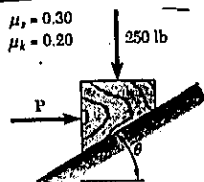
MAXIMUM FRICTION FORCE:  $F_m = \mu_s N = 0.3(241.5 \text{ lb}) = 72.5 \text{ lb}$   
 SINCE  $F > F_m$ , BLOCK MOVES DOWN

FRICTION FORCE:  $F = \mu_k N = (0.20)(241.5 \text{ lb}) = 48.3 \text{ lb}$   
 $F = 48.3 \text{ lb} \leftarrow$

**8.2**

$$\mu_s = 0.30$$

$$\mu_k = 0.20$$



**GIVEN:**  $\theta = 35^\circ$   
 $P = 100 \text{ lb}$

**FIND:** FRICTION  
 FORCE ACTING ON  
 BLOCK.

ASSUME EQUILIBRIUM

$$+\uparrow \Sigma F_y = 0: N - (250 \text{ lb}) \cos 35^\circ - (100 \text{ lb}) \sin 35^\circ = 0$$

$$N = 262.15 \text{ lb} \quad N = 262.15 \text{ lb} \uparrow$$

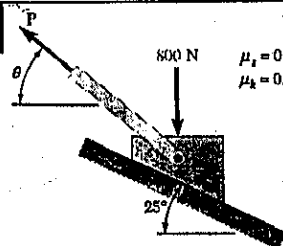
$$+\rightarrow \Sigma F_x = 0: F - (250 \text{ lb}) \sin 35^\circ + (100 \text{ lb}) \cos 35^\circ = 0$$

$$F = 61.48 \text{ lb} \quad F = 61.48 \text{ lb} \rightarrow$$

MAXIMUM FRICTION FORCE:  $F_m = \mu_s N = 0.3(262.15 \text{ lb}) = 78.64 \text{ lb}$   
 SINCE  $F < F_m$ , BLOCK IS IN EQUILIBRIUM

FRICTION FORCE:  $F = 61.5 \text{ lb} \rightarrow$

**8.3**



$$\mu_s = 0.20$$

$$\mu_k = 0.15$$

**GIVEN:**  $\theta = 40^\circ$   
 $P = 400 \text{ N}$

**FIND:** FRICTION  
 FORCE ACTING ON  
 BLOCK.

ASSUME EQUILIBRIUM

$$+\uparrow \Sigma F_y = 0: N - (800 \text{ N}) \cos 25^\circ - (400 \text{ N}) \sin 15^\circ = 0$$

$$N = 621.5 \text{ N} \quad N = 621.5 \text{ N} \uparrow$$

$$+\rightarrow \Sigma F_x = 0: F - (800 \text{ N}) \sin 25^\circ - (400 \text{ N}) \cos 15^\circ = 0$$

$$F = 48.3 \text{ N} \quad F = 48.3 \text{ N} \leftarrow$$

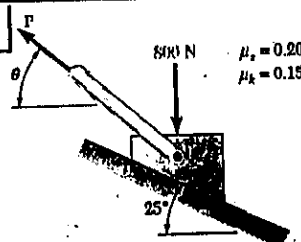
MAXIMUM FRICTION FORCE:

$$F_m = \mu_s N = 0.20(621.5 \text{ N}) = 124.3 \text{ N}$$

SINCE  $F < F_m$ , BLOCK IS IN EQUILIBRIUM

$$F = 48.3 \text{ N} \leftarrow$$

**8.4**



$$\mu_s = 0.20$$

$$\mu_k = 0.15$$

**GIVEN:**  $\theta = 35^\circ$   
 $P = 200 \text{ N}$

**FIND:** FRICTION  
 FORCE ACTING ON  
 BLOCK.

ASSUME EQUILIBRIUM

$$+\uparrow \Sigma F_y = 0: N - (800 \text{ N}) \cos 25^\circ + (200 \text{ N}) \sin 10^\circ = 0$$

$$N = 690.3 \text{ N} \quad N = 690.3 \text{ N} \uparrow$$

$$+\rightarrow \Sigma F_x = 0: -F + (800 \text{ N}) \sin 25^\circ - (200 \text{ N}) \cos 10^\circ = 0$$

$$F = 141.13 \text{ N} \quad F = 141.13 \text{ N} \leftarrow$$

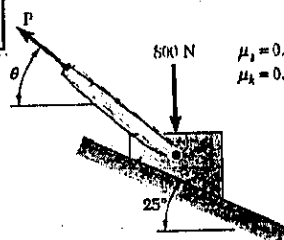
MAXIMUM FRICTION FORCE:

$$F_m = \mu_s N = (0.20)(690.3 \text{ N}) = 138.06 \text{ N}$$

SINCE  $F > F_m$ , BLOCK MOVES DOWN

FRICTION FORCE:  $F = \mu_k N = (0.15)(690.3 \text{ N}) = 103.54 \text{ N}$   
 $F = 103.5 \text{ N} \leftarrow$

**8.5**



$$\mu_s = 0.20$$

$$\mu_k = 0.15$$

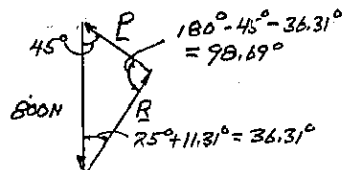
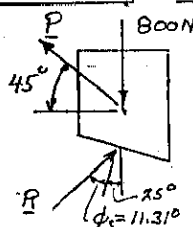
**GIVEN:**  $\theta = 45^\circ$

**FIND:** RANGE OF VALUES  
 OF P FOR EQUILIBRIUM

TO START BLOCK UP THE INCLINE

$$\mu_s = 0.20$$

$$\phi_s = \tan^{-1} 0.20 = 11.31^\circ$$

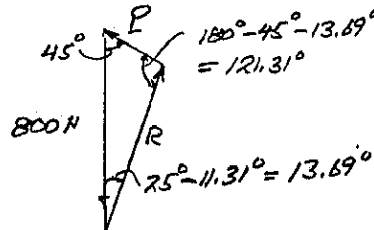
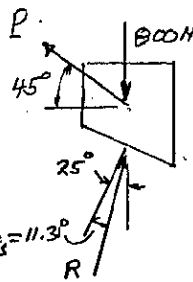


FROM FORCE TRIANGLE

$$\frac{P}{\sin 36.31^\circ} = \frac{800 \text{ N}}{\sin 98.69^\circ}$$

$$P = 479.2 \text{ N}$$

TO PREVENT BLOCK FROM MOVING DOWN



FROM FORCE TRIANGLE

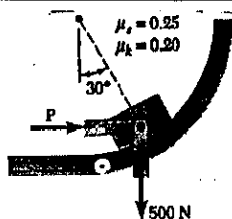
$$\frac{P}{\sin 13.69^\circ} = \frac{800 \text{ N}}{\sin 121.31^\circ}$$

$$P = 221.6 \text{ N}$$

EQUILIBRIUM IS MAINTAINED FOR

$$222 \text{ N} \leq P \leq 479 \text{ N}$$

8.6

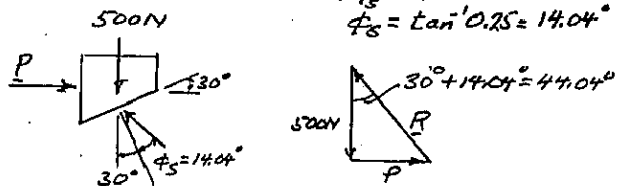


FIND: RANGE OF  
VALUES OF P FOR  
WHICH EQUILIBRIUM  
IS MAINTAINED.

TO START BLOCK UP THE SLOPE

$$\mu_s = 0.25$$

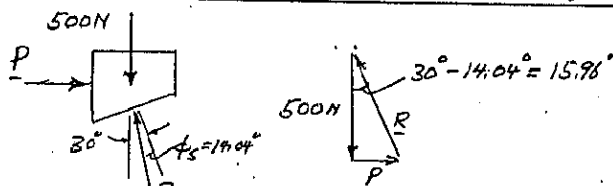
$$\phi_s = \tan^{-1} 0.25 = 14.04^\circ$$



FROM FORCE TRIANGLE:

$$P = (500 \text{ N}) \tan 44.04^\circ; P = 493 \text{ lb}$$

TO PREVENT BLOCK FROM MOVING DOWN

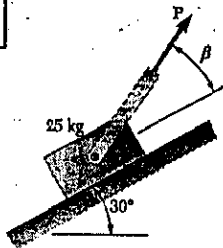


FROM FORCE TRIANGLE:

$$P = (500 \text{ N}) \tan 15.96^\circ; P = 143.0 \text{ lb}$$

EQUILIBRIUM MAINTAINED FOR:  $143.0 \text{ lb} \leq P \leq 493 \text{ lb}$

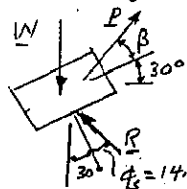
8.7



GIVEN:  $\mu_s = 0.25$

FIND: (a) SMALLEST  
VALUE OF P TO START  
BLOCK UP THE INCLINE.  
(b) CORRESPONDING  
VALUE OF  $\beta$

TO START BLOCK MOVING UP THE INCLINE



$$\mu_s = 0.25$$

$$\phi_s = \tan^{-1} 0.25 = 14.04^\circ$$

FORCE TRIANGLE  
FOR SMALLEST P  
WE CHOOSE  $P \perp R$ .

$$30^\circ + \phi_s = 30^\circ + \beta$$

$$\therefore \beta = \phi_s = 14.04^\circ$$

$$\beta = 14.0^\circ$$

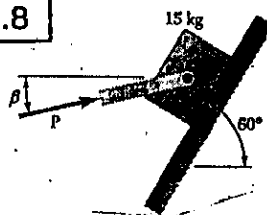
$$P = W \sin(30^\circ + \phi_s)$$

$$= (245.25 \text{ N}) \sin 44.04^\circ$$

$$= 170.49 \text{ N}$$

$$P = 170.5 \text{ N}$$

8.8



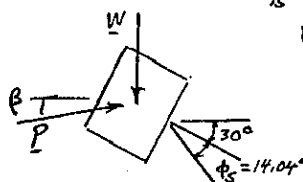
GIVEN:  $\mu_s = 0.25$

FIND: (a) SMALLEST  
VALUE OF P FOR  
EQUILIBRIUM.  
(b) CORRESPONDING  
VALUE OF  $\beta$ .

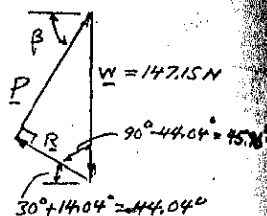
TO PREVENT BLOCK FROM MOVING DOWN THE INCLINE

$$\mu_s = 0.25; \phi_s = \tan^{-1} 0.25 = 14.04^\circ$$

$$W = (15 \text{ kg})(9.81 \text{ m/s}^2) = 147.15 \text{ N}$$



FORCE TRIANGLE  
FOR SMALLEST P WE  
CHOOSE  $P \perp R$ .

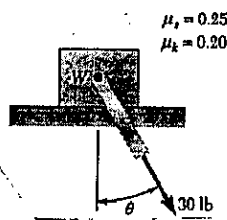


$$\beta = 45.96^\circ, \beta = 46.0^\circ$$

$$P = (147.15 \text{ N}) \sin 45.96^\circ = 105.78 \text{ N}$$

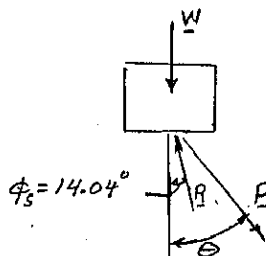
$$P = 105.8 \text{ N}$$

8.9



$\mu_s = 0.25$   
 $\mu_k = 0.20$

FIND: For  $\theta < 90^\circ$ ,  
THE VALUE OF  $\theta$   
REQUIRED TO START  
BLOCK MOVING TO RIGHT  
WHEN (a)  $W = 75 \text{ lb}$   
(b)  $W = 100 \text{ lb}$



$$\mu_s = 0.25$$

$$\phi_s = \tan^{-1} 0.25$$

$$\tan \phi_s = 0.25$$

FORCE TRIANGLE

$$\tan \phi_s = \frac{P \sin \theta}{W + P \cos \theta}$$

$$0.25(W + P \cos \theta) = P \sin \theta$$

$$\frac{W}{P} + \cos \theta = 4 \sin \theta$$

$$(a) \text{ For } W = 75 \text{ lb}, P = 30 \text{ lb}; \frac{W}{P} = 2.5$$

$$2.5 + \cos \theta = 4 \sin \theta$$

SOLVE BY TRIAL & ERROR

$$\theta = 57.36^\circ$$

$$\theta = 57.4^\circ$$

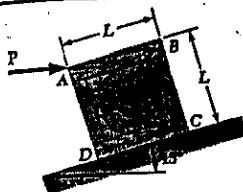
$$(b) \text{ For } W = 100 \text{ lb}, P = 30 \text{ lb}; \frac{W}{P} = 3.33$$

$$3.33 + \cos \theta = 4 \sin \theta$$

$$\theta = 67.28^\circ$$

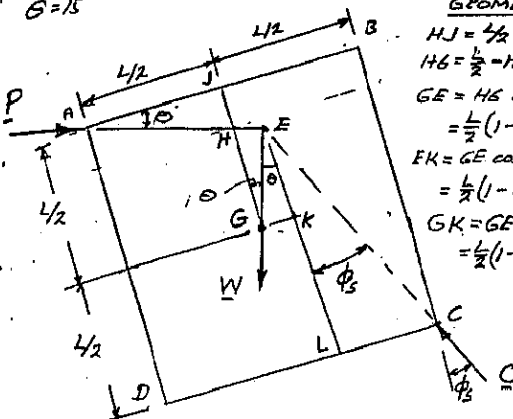
$$\theta = 68.0^\circ$$

8.17



GIVEN: MASS OF CRATE = 30 kg.  
FIND: (a) LARGEST  $\mu_s$  FOR WHICH CRATE CAN BE STARTED UP WITH NO TIPPING. (b) CORRESPONDING MAGNITUDE OF HORIZONTAL FORCE  $P$ .

FOR TIPPING TO BE IMPENDING REACTION IS AT C.  
FREE BODY: CRATE THREE-FORCE BODY. REACTION  $S$  MUST PASS THROUGH E WHERE  $P+W$  INTERSECT.

(a)  $\theta = 15^\circ$ 

GEOMETRY  
 $HJ = \frac{1}{2} \tan \theta$   
 $HG = \frac{1}{2} - HJ = \frac{1}{2}(1 - \tan \theta)$   
 $GE = HG \cos \theta = \frac{1}{2}(1 - \tan \theta) \cos \theta$   
 $EK = GE \cos \theta = \frac{1}{2}(1 - \tan \theta) \cos^2 \theta$   
 $GK = GE \sin \theta = \frac{1}{2}(1 - \tan \theta) \cos \theta \sin \theta$

$$EL = \frac{L}{2} + EK = \frac{L}{2} + \frac{1}{2}(1 - \tan \theta) \cos^2 \theta = \frac{L}{2} + \frac{1}{2}(\cos^2 \theta - \sin \theta \cos \theta)$$

$$EL = \frac{L}{2}(1 + \cos^2 15^\circ - \sin 15^\circ \cos 15^\circ) = 0.84151 L$$

$$LC = \frac{L}{2} - GK = \frac{L}{2} - \frac{1}{2}(1 - \tan \theta) \cos \theta \sin \theta$$

$$= \frac{1}{2} - \frac{1}{2}(\cos \theta \sin \theta - \frac{\sin \theta}{\cos \theta} \cos \theta \sin \theta)$$

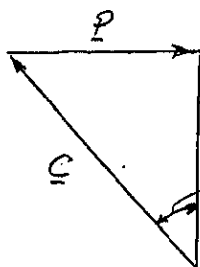
$$= \frac{1}{2}(1 - \cos \theta \sin \theta + \sin^2 \theta)$$

$$LC = \frac{L}{2}(1 - \cos 15^\circ \sin 15^\circ + \sin^2 15^\circ) = 0.40849 L$$

$$\tan \phi_s = \frac{LC}{EL} = \frac{0.40849 L}{0.84151 L} = 0.48543; \phi_s = 25.89^\circ$$

$$\mu_s = \tan \phi_s = 0.485$$

(b) FORCE TRIANGLE



$$W = 294.3 \text{ N}$$

$$\theta + \phi_s = 15^\circ + 26.89^\circ = 41.89^\circ$$

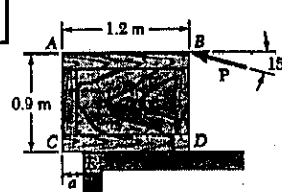
$$P = W \tan(\theta + \phi_s)$$

$$= (294.3 \text{ N}) \tan 41.89^\circ$$

$$= 254.8 \text{ N}$$

$$P = 255 \text{ N} \rightarrow$$

8.18

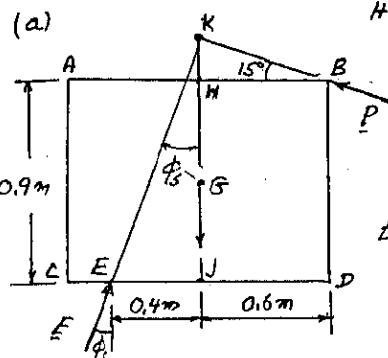


GIVEN: MASS OF CRATE = 50 kg  
FOR  $\alpha = 0.2 \text{ m}$  TIPPING IMPENDING  
FIND (a)  $\mu_s$   
(b) MAGNITUDE OF  $P$

FREE BODY: CRATE THREE-FORCE BODY.  
REACTION  $E$  MUST PASS THROUGH K WHERE  $P$  AND  $W$  INTERSECT

GEOMETRY

$$HK = (0.6 \text{ m}) \tan 15^\circ = 0.16077 \text{ m}$$



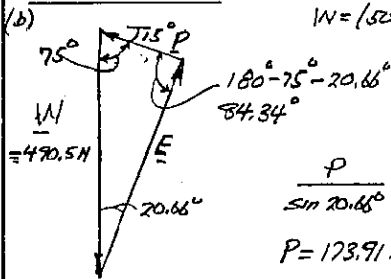
$$JK = 0.9 \text{ m} + HK = 1.06077 \text{ m}$$

$$\tan \phi_s = \frac{0.4 \text{ m}}{1.06077 \text{ m}} = 0.37705$$

$$\mu_s = \tan \phi_s = 0.377$$

$$\phi_s = 20.66^\circ$$

FORCE TRIANGLE



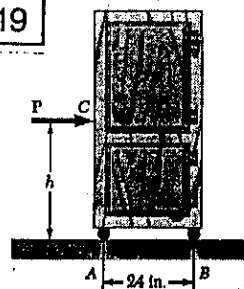
$$W = (50 \text{ kg})(9.8 \text{ m/s}^2) = 490.5 \text{ N}$$

LAW OF SINES

$$\frac{P}{\sin 20.66^\circ} = \frac{490.5 \text{ N}}{\sin 84.34^\circ}$$

$$P = 173.91 \text{ N} \quad P = 173.9 \text{ N}$$

8.19



GIVEN: 120-lb CABINET  
 $h = 32 \text{ in}$ ,  $\mu_s = 0.30$

FIND: FORCE  $P$  REQUIRED TO MOVE CABINET, WHEN  
(a) ALL CASTERS ARE LOCKED  
(b) CASTERS B ARE LOCKED AND CASTERS A ARE FREE  
(c) CASTERS A ARE LOCKED AND CASTER B ARE FREE

(a) ALL CASTERS LOCKED

$$+\uparrow \Sigma F_y = 0: N_A + N_B - W = 0$$

$$N_A + N_B = W = 120 \text{ lb}$$

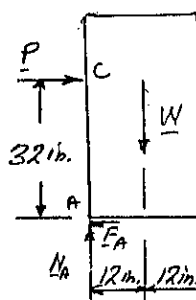
$$F_A + F_B = \mu_s N_A + \mu_s N_B = \mu_s (N_A + N_B)$$

$$= 0.30(120 \text{ lb}) = 36 \text{ lb}$$

$$+\rightarrow \Sigma F_x = 0: P - F_A - F_B = 0$$

$$P = F_A + F_B$$

$$P = 36 \text{ lb} \rightarrow$$



CHECK FOR TIPPING

$$+\rightarrow \Sigma M_B = 0$$

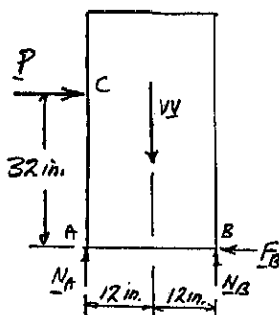
$$(120 \text{ lb})(12 \text{ in}) - (36 \text{ lb})(32 \text{ in}) - N_A(24 \text{ in}) = 0$$

$$N_A = 112 \text{ lb} > 0 \quad \text{OK}$$

(CONTINUED)

# 8.19 CONTINUED

$$W = 120 \text{ lb}$$



(b) CASTERS LOCKED AT B AND FREE AT A

$$F_B = \mu_s N_B = 0.3 N_B$$

$$\sum F_x = 0: P = F_B = 0.3 N_B \quad (1)$$

$$+\uparrow \sum M_A = 0:$$

$$-P(32 \text{ in}) - (120 \text{ lb})(12 \text{ in}) + N_B(24 \text{ in}) = 0$$

$$-0.3 N_B(32 \text{ in}) + N_B(24 \text{ in}) = (120 \text{ lb})(12 \text{ in}) = 0$$

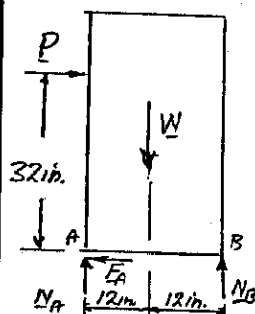
$$14.4 N_B = (120 \text{ lb})(12 \text{ in})$$

$$N_B = 100 \text{ lb}$$

$$\text{Eq. (1)} \quad P = 0.3(100 \text{ lb}) = 30 \text{ lb}$$

$$P = 30 \text{ lb} \rightarrow$$

(c) CASTERS LOCKED AT A AND FREE AT B



$$F_A = \mu_s N_A = 0.3 N_A$$

$$\sum F_x = 0: P = F_A = 0.3 N_A \quad (2)$$

$$+\uparrow \sum M_B = 0:$$

$$-P(32 \text{ in}) + (120 \text{ lb})(12 \text{ in}) - N_A(24 \text{ in}) = 0$$

$$-0.3 N_A(32 \text{ in}) - N_A(24 \text{ in}) + (120 \text{ lb})(12 \text{ in}) = 0$$

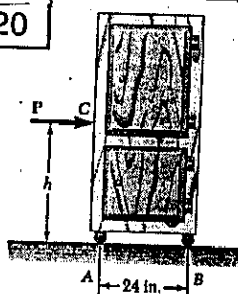
$$33.6 N_A = (120 \text{ lb})(12 \text{ in})$$

$$N_A = 42.857 \text{ lb}$$

$$\text{Eq. (2)}: P = 0.3(42.857) = 12.86 \text{ lb}$$

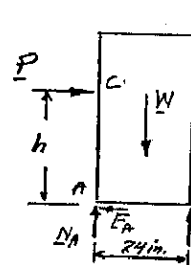
$$P = 12.86 \text{ lb} \rightarrow$$

# 8.20



GIVEN: 120-lb CABINET  
 $\mu_s = 0.30$   
 ALL CASTERS ARE LOCKED.

FIND: (a) FORCE  $P$  TO MOVE CABINET  
 (b) MAXIMUM  $h$  IF CABINET IS NOT TO TIP



(a)  $W = 120 \text{ lb}$

$$+\uparrow \sum F_y = 0: N_A + N_B - W = 0$$

$$N_A + N_B = 120 \text{ lb}$$

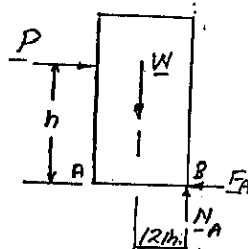
$$F_A + F_B = \mu_s N_A + \mu_s N_B = \mu_s (N_A + N_B)$$

$$F_A + F_B = 0.3(120 \text{ lb}) = 36 \text{ lb}$$

$$+\uparrow \sum F_x = 0: P - F_A - F_B = 0$$

$$P = F_A + F_B \quad P = 36 \text{ lb} \rightarrow$$

(b) LARGEST ALLOWABLE VALUE OF  $h$ .



WHEN TIPPING IMPENDS THERE IS NO REACTION AT A.  $N_A = 0$

$$+\uparrow \sum M_B = 0:$$

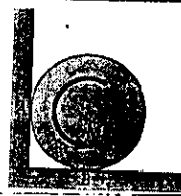
$$W(12 \text{ in}) - Ph = 0$$

$$h = \frac{W}{P}(12 \text{ in})$$

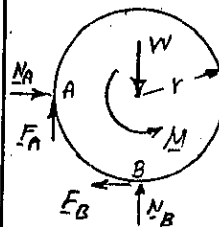
$$= \frac{120 \text{ lb}}{36 \text{ lb}}(12 \text{ in}) = 40 \text{ in}$$

$$h = 40 \text{ in}$$

# 8.21



GIVEN:  $r$  = RADIUS,  
 $W$  = WEIGHT,  
 $\mu_s$  IS SAME AT A AND B.  
 FIND: LARGEST  $M$  IF CYLINDER IS NOT TO ROTATE



SINCE MOTION WILL IMPEND,  
 $F_A = \mu_s N_A$        $F_B = \mu_s N_B$

$$+\uparrow \sum M_B = 0: M - rF_A - rN_A = 0$$

$$M = rN_A + rF_A = rN_A + r\mu_s N_A$$

$$M = rN_A(1 + \mu_s) \quad (1)$$

$$+\uparrow \sum F_x = 0: N_A - F_B = 0; \quad N_A = \mu_s N_B \quad (2)$$

$$+\uparrow \sum F_y = 0: N_B + F_A - W = 0; \quad N_B = W - \mu_s N_A \quad (3)$$

SUBSTITUTE FOR  $N_B$  FROM (3) INTO (2):

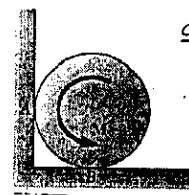
$$N_A = \mu_s (W - \mu_s N_A)$$

$$N_A(1 + \mu_s^2) = \mu_s W \quad N_A = \frac{\mu_s W}{1 + \mu_s^2}$$

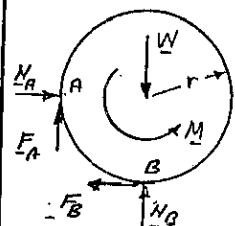
SUBSTITUTE FOR  $N_A$  INTO (1):

$$M = r \frac{\mu_s W}{(1 + \mu_s^2)} (1 + \mu_s) \quad M = W r \mu_s \frac{(1 + \mu_s)}{(1 + \mu_s^2)}$$

# 8.22



GIVEN:  $r$  = RADIUS  
 $W$  = WEIGHT  
 FIND: LARGEST  $M$  IF CYLINDER IS NOT TO ROTATE  
 (a) FOR  $\mu_A = 0, \mu_B = 0.30$ ,  
 (b) FOR  $\mu_A = 0.25, \mu_B = 0.30$ .



SINCE MOTION WILL IMPEND  
 $F_A = \mu_A N_A$        $F_B = \mu_B N_B$

$$+\uparrow \sum M_B = 0: M - rF_A - rN_A = 0$$

$$M = rN_A + rF_A = rN_A + r\mu_A N_A$$

$$M = rN_A(1 + \mu_A) \quad (1)$$

$$+\uparrow \sum F_x = 0: N_A - F_B = 0 \quad N_A = \mu_B N_B \quad (2)$$

$$+\uparrow \sum F_y = 0: N_B + F_A - W = 0 \quad N_B = W - \mu_A N_A \quad (3)$$

SUBSTITUTE FOR  $N_B$  FROM (3) INTO (2)

$$N_A = \mu_B (W - \mu_A N_A)$$

$$N_A(1 + \mu_A \mu_B) = \mu_B W \quad N_A = \frac{\mu_B W}{1 + \mu_A \mu_B}$$

Eq. (1):  $M = r \frac{\mu_B W}{1 + \mu_A \mu_B} (1 + \mu_A) \quad M = W r \frac{\mu_B (1 + \mu_A)}{1 + \mu_A \mu_B}$

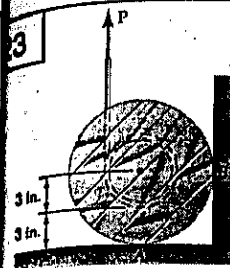
(a) FOR  $\mu_A = 0$  AND  $\mu_B = 0.30$ :

$$M = W r \frac{0.30}{1} \quad M = 0.300 W r$$

(b) FOR  $\mu_A = 0.25$  AND  $\mu_B = 0.30$ :

$$M = W r \frac{(0.30)(1 + 0.25)}{1 + (0.25)(0.30)} = W r \frac{(0.30)(1.25)}{1.075}$$

$$M = 0.3488 W r \quad M = 0.349 W r$$



**GIVEN:**  $W = 20 \text{ lb}$   
 AT A AND B,  
 $\mu_s = 0.40$ ,  $\mu_k = 0.30$

**FIND:** MAGNITUDE OF  $P$   
 TO DRAW WIRE AT A  
 CONSTANT RATE.

SINCE SPOOL IS ROTATING  
 $F_A = \mu_k N_A$   $F_B = \mu_k N_B$

$$+2 \Sigma M_G = 0:$$

$$P(3 \text{ in.}) - F_A(6 \text{ in.}) - F_B(6 \text{ in.}) = 0$$

$$3P - 6\mu_k(N_A + N_B) = 0 \quad (1)$$

$$+\uparrow \Sigma F_x = 0: F_A - N_B = 0$$

$$N_B = \mu_k N_A \quad (2)$$

$$+\uparrow \Sigma F_y = 0: P + N_A + F_B - 20 \text{ lb} = 0$$

$$P + N_A + \mu_k N_B - 20 = 0$$

$$P + N_A + \mu_k^2 N_A - 20 = 0 \quad (3)$$

SUBSTITUTE FOR  $N_B$  FROM (2):

$$N_A = \frac{20 - P}{1 + \mu_k^2} \quad (3)$$

SUBSTITUTE FROM (2) INTO (1):  $3P - 6\mu_k(N_A + \mu_k N_A) = 0$

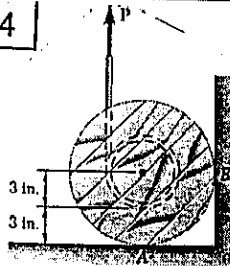
$$N_A = \frac{1}{2} \frac{P}{\mu_k(1 + \mu_k)} \quad (4)$$

$$(4): \frac{20 - P}{1 + \mu_k^2} = \frac{P}{2(\mu_k + \mu_k^2)}$$

$$\text{SUBSTITUTE } \mu_k = 0.30: \frac{20 - P}{1 + (0.3)^2} = \frac{P}{2(0.3)(1.03)}$$

$$20 - P = 1.3974P; 2.3974P = 20; P = 8.34 \text{ lb}$$

8.24



**GIVEN:**  $W = 20 \text{ lb}$   
 AT A:  $\mu_s = 0.40$ ,  $\mu_k = 0.30$   
 AT B:  $\mu_s = \mu_k = 0$

**FIND:** MAGNITUDE OF  $P$   
 TO DRAW WIRE AT A  
 CONSTANT RATE

SINCE SPOOL IS ROTATING

$$F_A = \mu_k N_A$$

$$+2 \Sigma M_G = 0$$

$$P(3 \text{ in.}) - F_A(6 \text{ in.}) = 0$$

$$P = 2F_A = 2\mu_k N_A \quad (1)$$

$$+\uparrow \Sigma F_y = 0: P - 20 \text{ lb} + N_A = 0$$

$$N_A = 20 - P \quad (2)$$

SUBSTITUTE FOR  $N_A$  FROM (2) INTO (1):

$$P = 2\mu_k(20 - P)$$

$$\text{SUBSTITUTE } \mu_k = 0.30: P = 2(0.3)(20 - P)$$

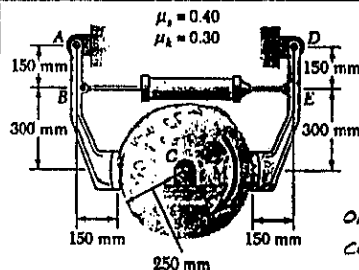
$$6.667P = 20 - P$$

$$7.667P = 20$$

$$P = 2.609 \text{ lb}$$

$$P = 2.61 \text{ lb}$$

8.25



**GIVEN:**

CYLINDER  
 EXERTS 3-RN  
 ON B AND ON E.

**FIND:** MAGNITUDE  
 OF  $M$  REQUIRED FOR  
 CONSTANT ROTATION

**FREE BODY: DRUM**

$$+2 \Sigma M_C = 0: M - (0.25 \text{ m})(F_L + F_R) = 0$$

$$M = (0.25 \text{ m})(F_L + F_R) \quad (1)$$

SINCE DRUM IS ROTATING,

$$F_L = \mu_k N_L = 0.3N_L \quad F_R = \mu_k N_R = 0.3N_R$$

**FREE BODY: LEFT ARM ABL**

$$+\uparrow \Sigma M_A = 0$$

$$(3 \text{ kN})(0.15 \text{ m}) + F_L(0.15 \text{ m}) - N_L(0.45 \text{ m}) = 0$$

$$0.45 \text{ kN} \cdot \text{m} + (0.3N_L)(0.15 \text{ m}) - N_L(0.45 \text{ m}) = 0$$

$$0.405 N_L = 0.45$$

$$N_L = 1.111 \text{ kN}$$

$$F_L = 0.3N_L = 0.3(1.111 \text{ kN}) = 0.3333 \text{ kN} \quad (2)$$

**FREE BODY: RIGHT ARM DER**

$$+2 \Sigma M_D = 0: (3 \text{ kN})(0.15 \text{ m}) - F_R(0.15 \text{ m}) - N_R(0.45 \text{ m}) = 0$$

$$0.45 \text{ kN} \cdot \text{m} - (0.3N_R)(0.15 \text{ m}) - N_R(0.45 \text{ m}) = 0$$

$$0.495 N_R = 0.45$$

$$N_R = 0.9091 \text{ kN}$$

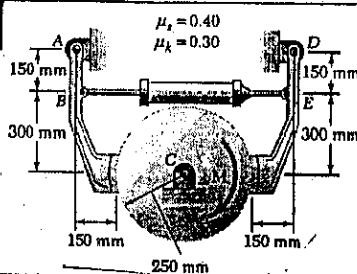
$$F_R = \mu_k N_R = 0.3(0.9091 \text{ kN}) = 0.2727 \text{ kN} \quad (3)$$

SUBSTITUTE FOR  $F_L$  AND  $F_R$  INTO (1):  $M = (0.25 \text{ m})(0.3333 \text{ kN} + 0.2727 \text{ kN})$

$$M = 0.1515 \text{ kN} \cdot \text{m}$$

$$M = 151.5 \text{ N} \cdot \text{m}$$

8.26



**GIVEN:**

$$M = 100 \text{ N} \cdot \text{m}$$

**FIND:** SMALLEST  
 FORCE EXERTED  
 BY CYLINDER FOR  
 NO ROTATION OF  
 DRUM.

**FREE BODY: DRUM**

$$+2 \Sigma M_C = 0: 100 \text{ N} \cdot \text{m} - (0.25 \text{ m})(F_L + F_R) = 0$$

$$F_L + F_R = 400 \text{ N} \quad (1)$$

SINCE MOTION IMPENDS

$$F_L = \mu_s N_L = 0.4N_L \quad F_R = \mu_s N_R = 0.4N_R$$

**FREE BODY: LEFT ARM ABL**

$$+\uparrow \Sigma M_A = 0: T(0.15 \text{ m}) + F_L(0.15 \text{ m}) - N_L(0.45 \text{ m}) = 0$$

$$0.15 T + (0.4N_L)(0.15 \text{ m}) - N_L(0.45 \text{ m}) = 0$$

$$0.39 N_L = 0.15 T; N_L = 0.38462 T$$

$$F_L = 0.4N_L = 0.4(0.38462 T); F_L = 0.15385 T \quad (2)$$

**FREE BODY: RIGHT ARM DER**

$$+\uparrow \Sigma M_D = 0: T(0.15 \text{ m}) - F_R(0.15 \text{ m}) - N_R(0.45 \text{ m}) = 0$$

$$0.15 T - (0.4N_R)(0.15 \text{ m}) - N_R(0.45 \text{ m}) = 0$$

$$0.51 N_R = 0.15 T; N_R = 0.29412 T$$

$$F_R = 0.4N_R = 0.4(0.29412 T); F_R = 0.11765 T \quad (3)$$

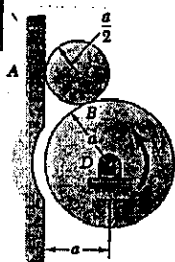
SUBSTITUTE FOR  $F_L$  AND  $F_R$  INTO (1):

$$0.15385 T + 0.11765 T = 400$$

$$T = 1473.3 \text{ N}$$

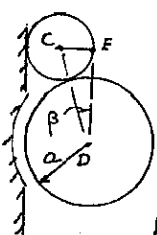
$$T = 1473 \text{ N}$$

\*8.27



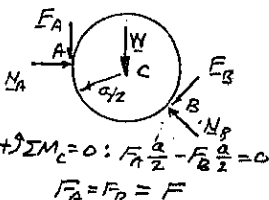
GIVEN:  $\mu_s = 0.25$  AT A  
AND AT B,  
CYLINDER C WEIGHS  $W$ .

FIND: LARGEST  
COUNTERCLOCKWISE  $M$  IF  
CYLINDER D IS NOT  
TO ROTATE.



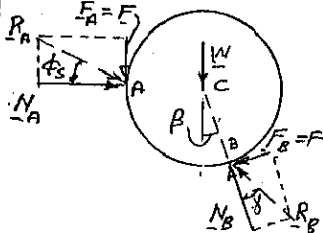
GEOMETRY  
 $CE = a/2$   
 $CD = 3a/2$   
 $\sin \beta = \frac{CE}{CD}$   
 $= \frac{a/2}{3a/2} = \frac{1}{3}$   
 $\beta = 19.47^\circ$

FREE BODY: CYLINDER C

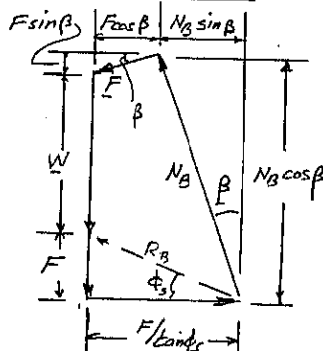


$\sum M_C = 0: F \frac{a}{2} - N_B \frac{a}{2} = 0$   
 $F_A = F_B = F$

ASSUME MOTION IMPENDS AT A:  $\tan \phi_s = 0.25$ ;  $\phi_s = 14.04^\circ$



FORCE POLYGON



ASSUME NO SLIPPING  
AT B, THAT IS  $\delta < \phi_s$ .  
SEE BELOW FOR  
VALUE OF  $\gamma$

VERTICAL COMPONENTS:  $N_B \cos \beta = W + F \sin \beta + F$

$N_B = \frac{W + F(1 + \sin \beta)}{\cos \beta}$  (1)

HORIZONTAL COMPONENTS:  $\frac{F}{\tan \phi_s} = F \cos \beta + N_B \sin \beta$  (2)

(1)  $\rightarrow$  (2):  $\frac{F}{\tan \phi_s} = F \cos \beta + [W + F(1 + \sin \beta)] \frac{\sin \beta}{\cos \beta}$   
 $\frac{F}{\tan \phi_s} = F \cos \beta + W \tan \beta + F(1 + \sin \beta) \tan \beta$

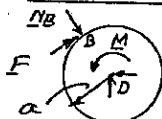
$F \left[ \frac{1}{\tan \phi_s} - \cos \beta - \tan \beta (1 + \sin \beta) \right] = W \tan \beta$

RECALL:  $\beta = 19.47^\circ$ ,  $\tan \phi_s = 0.25$

$F \left[ \frac{1}{0.25} - \cos 19.47^\circ - \tan 19.47^\circ (1 + \sin 19.47^\circ) \right] = W \tan 19.47^\circ$

$F(2.5857) = 0.35355W$   $F = 0.13673W$

FREE BODY: CYLINDER D



$\sum M_D = 0: M - Fa = 0$

$M = Fa = 0.13673Wa$

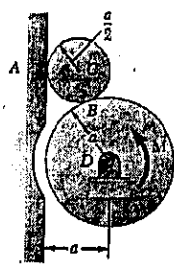
$M = 0.1367Wa$

VALUE OF  $\gamma$ : EQ. (1)  $N_B = \frac{W + 0.13673W(1 + \sin 19.47^\circ)}{\cos 19.47^\circ} = 1.254W$

$\tan \gamma = \frac{F}{N_B} = \frac{0.1367W}{1.254W} = 0.10902$   $\gamma = 6.22^\circ < \phi_s$

WE FIND NO SLIPPING AT B. OK

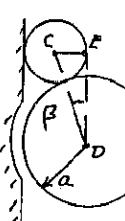
\*8.28



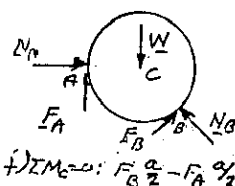
GIVEN:  $\mu_s = 0.25$   
AND AT B,  
CYLINDER C WEIGHS  $W$ .

FIND: LARGEST  
CLOCKWISE  $M$  IF  
CYLINDER D IS NOT  
TO ROTATE

FREE BODY: CYLINDER C

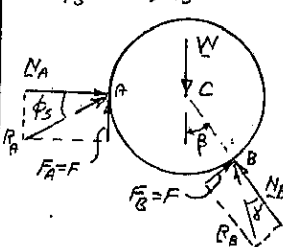


GEOMETRY  
 $CE = a/2$   
 $CD = 3a/2$   
 $\sin \beta = \frac{CE}{CD}$   
 $= \frac{a/2}{3a/2} = \frac{1}{3}$   
 $\beta = 19.47^\circ$

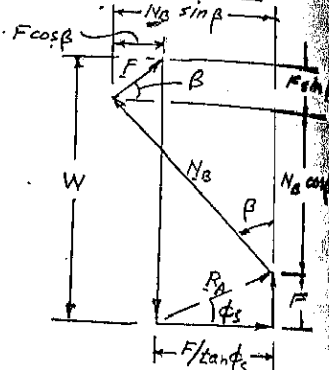


$\sum M_C = 0: F \frac{a}{2} - N_B \frac{a}{2} = 0$   
 $F_A = F_B = F$

ASSUME MOTION IMPENDS AT A:  
 $\tan \phi_s = 0.25$ ;  $\phi_s = 14.04^\circ$



FORCE POLYGON



ASSUME NO SLIPPING  
AT B, THAT IS,  $\delta < \phi_s$ .  
SEE BELOW FOR  
VALUE OF  $\gamma$ .

VERTICAL COMPONENTS:  $N_B \cos \beta + F + F \sin \beta = W$

$N_B = \frac{W - F(1 + \sin \beta)}{\cos \beta}$  (1)

HORIZONTAL COMPONENTS:  $\frac{F}{\tan \phi_s} = N_B \sin \beta - F \cos \beta$  (2)

(1)  $\rightarrow$  (2):  $\frac{F}{\tan \phi_s} = [W - F(1 + \sin \beta)] \frac{\sin \beta}{\cos \beta} - F \cos \beta$

$\frac{F}{\tan \phi_s} = W \tan \beta - F(1 + \sin \beta) \tan \beta - F \cos \beta$

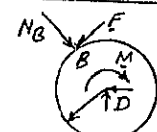
$F \left[ \frac{1}{\tan \phi_s} + \cos \beta + \tan \beta (1 + \sin \beta) \right] = W \tan \beta$

RECALL:  $\beta = 19.47^\circ$ ,  $\tan \phi_s = 0.25$

$F \left[ \frac{1}{0.25} + \cos 19.47^\circ + \tan 19.47^\circ (1 + \sin 19.47^\circ) \right] = W \tan 19.47^\circ$

$F(5.4121) = 0.35355W$   $F = 0.0653W$

FREE BODY: CYLINDER D



$\sum M_D = 0: M - Fa = 0$

$M = Fa = 0.0653Wa$

$M = 0.0653Wa$

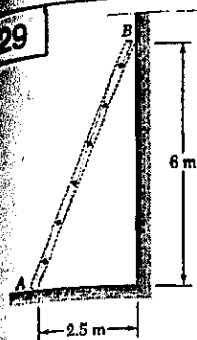
VALUE OF  $\gamma$ : EQ. (1):  $N_B = \frac{W - 0.0653W(1 + \sin 19.47^\circ)}{\cos 19.47^\circ} = 0.9683W$

$\tan \gamma = \frac{F}{N_B} = \frac{0.0653W}{0.9683W} = 0.0674$   $\gamma = 3.86^\circ < \phi_s$

WE FIND NO SLIPPING AT B. OK



8.29



GIVEN:  $\mu_s = 0$  AT B

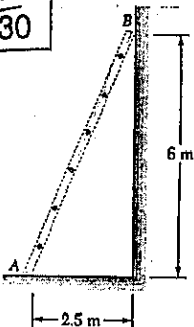
FIND: SMALLEST  $\mu_s$  AT A  
FOR WHICH EQUILIBRIUM  
IS MAINTAINED

FREE BODY: LADDER  
THREE-FORCE BODY. LINE  
OF ACTION OF  $A$  MUST PASS  
THROUGH D, WHERE  $W$  AND  
 $B$  INTERSECT

$$\text{AT A: } \mu_s = \tan \phi_s = \frac{1.25 \text{ m}}{6 \text{ m}} = 0.2083$$

$$\mu_s = 0.208$$

8.30



GIVEN: SAME VALUE  
OF  $\mu_s$  AT A AND AT B.

FIND: SMALLEST VALUE  
OF  $\mu_s$  FOR WHICH  
EQUILIBRIUM IS  
MAINTAINED

FREE BODY: LADDER  
MOTION IMPENDING  
 $F_A = \mu_s N_A$   $F_B = \mu_s N_B$

$$+\sum M_A = 0: W(1.25 \text{ m}) - N_B(6 \text{ m}) - \mu_s N_B(2.5 \text{ m}) = 0$$

$$N_B = \frac{1.25 W}{6 + 2.5 \mu_s} \quad (1)$$

$$+\uparrow \sum F_y = 0: N_A + \mu_s N_B - W = 0$$

$$N_A = W - \mu_s N_B$$

$$N_A = W - \frac{1.25 \mu_s W}{6 + 2.5 \mu_s} \quad (2)$$

$$+\sum F_x = 0: \mu_s N_A - N_B = 0$$

SUBSTITUTE FOR  $N_A$  AND  $N_B$  FROM EGS. (1) AND (2).

$$\mu_s W - \frac{1.25 \mu_s^2 W}{6 + 2.5 \mu_s} = \frac{1.25 W}{6 + 2.5 \mu_s}$$

$$6 \mu_s + 2.5 \mu_s^2 - 1.25 \mu_s^2 = 1.25$$

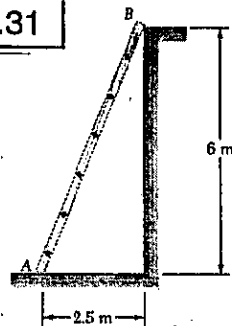
$$1.25 \mu_s^2 + 6 \mu_s - 1.25 = 0$$

$$\mu_s = 0.2$$

$$\text{AND } \mu_s = -5 \text{ (DISCARD)}$$

$$\mu_s = 0.200$$

8.31



GIVEN: SAME VALUE  
OF  $\mu_s$  AT A AND AT B.

FIND: SMALLEST VALUE  
OF  $\mu_s$  FOR WHICH  
EQUILIBRIUM IS  
MAINTAINED

$$AB = 6.5 \text{ m}$$

$$6.5 \triangle 6 \Rightarrow \frac{13}{5} \triangle 12$$

FREE BODY: LADDER  
MOTION IMPENDING  
 $F_A = \mu_s N_A$   $F_B = \mu_s N_B$

$$+\sum M_A = 0: W(1.25 \text{ m}) - N_B(6.5 \text{ m}) = 0$$

$$N_B = \frac{1.25 W}{6.5} \quad (1)$$

$$+\uparrow \sum F_y = 0: N_A + \frac{5}{13} N_B + \frac{12}{13} \mu_s N_B - W = 0$$

$$N_A + \frac{5}{13} \left( \frac{1.25 W}{6.5} \right) + \frac{12}{13} \mu_s \left( \frac{1.25 W}{6.5} \right) - W = 0$$

$$N_A = W \left( 1 - \frac{6.25}{84.5} - \frac{15}{84.5} \mu_s \right) \quad (2)$$

$$+\sum F_x = 0: \mu_s N_A - \frac{12}{13} N_B + \frac{5}{13} \mu_s N_B = 0$$

SUBSTITUTE FOR  $N_A$  AND  $N_B$  FROM EGS. (1) AND (2):

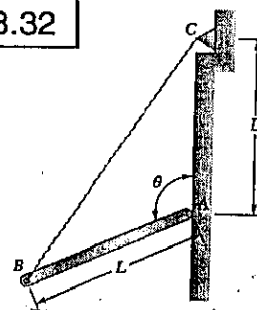
$$\mu_s \left( \frac{78.25 - 15 \mu_s}{84.5} \right) = \left( \frac{12 - 5 \mu_s}{13} \right) \left( \frac{1.25 W}{6.5} \right)$$

$$84.5 \mu_s - 15 \mu_s^2 - 15 = 0$$

$$\mu_s = 0.18349 \quad \text{AND } \mu_s = -5.45 \text{ (DISCARD)}$$

$$\mu_s = 0.1835$$

8.32



GIVEN:  $\mu_s = 0.40$   
 $\mu_k = 0.30$

FIND: (a) VALUE OF  $\theta$   
FOR IMPENDING MOTION.  
(b) CORRESPONDING  
TENSION IN CORD BC

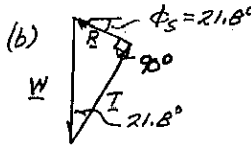
FREE-BODY DIAGRAM

THREE-FORCE BODY. LINE OF  
ACTION OF  $R$  MUST PASS  
THROUGH D, WHERE T AND  
R INTERSECT

MOTION IMPENDING

$$\tan \phi_s = 0.4 \quad \phi_s = 21.80^\circ$$

(a) SINCE  $BG = GA$ , IT FOLLOWS  
THAT  $BD = DC$  AND  $AD$  BISECTS  $\angle BAC$   
 $\therefore \theta/2 + \phi_s = 90^\circ$   $\theta/2 + 21.8^\circ = 90^\circ$   $\theta = 136.4^\circ$

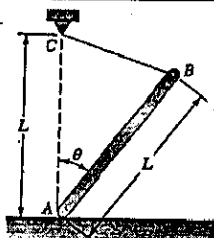


FORCE TRIANGLE (RIGHT TRIANGLE)

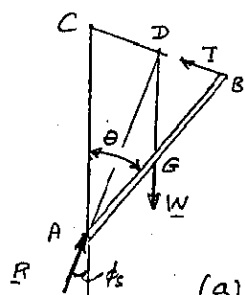
$$T = W \cos 21.8^\circ$$

$$T = 0.928 W$$

8.33



GIVEN:  $\mu_s = 0.40$   
 $\mu_k = 0.30$   
 FIND: (a) VALUE OF  $\theta$   
 FOR IMPENDING MOTION  
 (b) CORRESPONDING  
 TENSION IN CORD BC

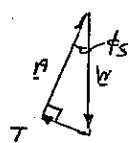


FREE-BODY DIAGRAM  
 ROD AB IS A THREE-FORCE  
 BODY. THUS, LINE OF ACTION  
 OF R MUST PASS THROUGH D,  
 WHERE W AND T INTERSECT.

SINCE  $AG = GB$ ,  $CD = DB$   
 AND THE MEDIAN AD OF THE  
 ISOSCELES TRIANGLE ABC  
 BISECTS THE ANGLE  $\theta$ ,  
 THUS  $\phi_s = \frac{1}{2}\theta$

SINCE MOTION IMPENDS  $\phi_s = \tan^{-1} 0.40 = 21.80^\circ$   
 $\theta = 2\phi_s = 2(21.8^\circ) \quad \theta = 43.6^\circ$

(b)



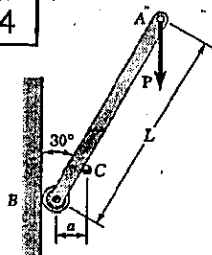
FORCE TRIANGLE

THIS IS A RIGHT TRIANGLE

$T = W \sin \phi_s = W \sin 21.8^\circ$

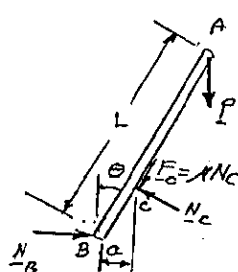
$T = 0.371 W$

8.34



GIVEN: BETWEEN  
 PIN C AND ROD;  
 $\mu_s = 0.15$

FIND: RANGE OF VALUES  
 OF  $L/a$  FOR WHICH  
 EQUILIBRIUM IS MAINTAINED.



FREE-BODY DIAGRAM: FOR  
 MOTION OF B IMPENDING UPWARD.

$$+\sum M_B = 0$$

$$PL \sin \theta - N_C \left( \frac{a}{\sin \theta} \right) = 0$$

$$N_C = \frac{PL}{a} \sin^2 \theta \quad (1)$$

$$+\uparrow \sum F_y = 0: N_C \sin \theta - \mu_s N_C \cos \theta - P = 0$$

$$N_C (\sin \theta - \mu_s \cos \theta) = P$$

SUBSTITUTE FOR  $N_C$  FROM (1), AND SOLVE FOR  $a/L$

$$a/L = \sin^2 \theta (\sin \theta - \mu_s \cos \theta) \quad (2)$$

FOR  $\theta = 30^\circ$  AND  $\mu_s = 0.15$ :

$$a/L = \sin^2 30^\circ (\sin 30^\circ - 0.15 \cos 30^\circ)$$

$$a/L = 0.092524$$

$$L/a = 10.808$$

FOR MOTION OF B IMPENDING DOWNWARD, REVERSE  
 SENSE OF FRICTION FORCE  $F_f$ . TO DO THIS  
 WE MAKE  $\mu_s = -0.15$  IN EQ.(2).

$$\text{EQ.(2): } a/L = \sin^2 30^\circ (\sin 30^\circ - (-0.15) \cos 30^\circ)$$

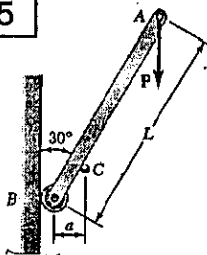
$$a/L = 0.15748$$

$$L/a = 6.350$$

RANGE OF VALUES OF  $L/a$  FOR EQUILIBRIUM:

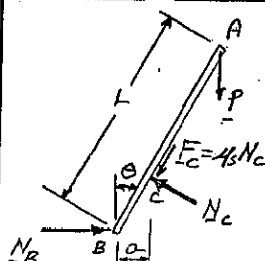
$$6.35 \leq \frac{L}{a} \leq 10.81$$

8.35



GIVEN: BETWEEN PIN C  
 AND ROD:  $\mu_s = 0.60$

FIND: RANGE OF VALUES  
 OF  $L/a$  FOR WHICH  
 EQUILIBRIUM IS  
 MAINTAINED.



FREE-BODY DIAGRAM: FOR  
 MOTION OF B IMPENDING UPWARD

$$+\sum M_B = 0$$

$$PL \sin \theta - N_C \left( \frac{a}{\sin \theta} \right) = 0$$

$$N_C = \frac{PL}{a} \sin^2 \theta \quad (1)$$

$$+\uparrow \sum F_y = 0: N_C \sin \theta - \mu_s N_C \cos \theta - P = 0$$

$$N_C (\sin \theta - \mu_s \cos \theta) = P$$

SUBSTITUTE FOR  $N_C$  FROM (1), AND SOLVE FOR  $a/L$

$$a/L = \sin^2 \theta (\sin \theta - \mu_s \cos \theta)$$

FOR  $\theta = 30^\circ$  AND  $\mu_s = 0.60$ :

$$a/L = \sin^2 30^\circ (\sin 30^\circ - 0.60 \cos 30^\circ)$$

$$a/L = -0.0049 < 0$$

THUS, SLIPPING OF B UPWARD DOES NOT OCCUR  
 FOR MOTION OF B IMPENDING DOWNWARD,  
 REVERSE SENSE OF FRICTION FORCE  $F_f$ . TO DO  
 THIS WE MAKE  $\mu_s = -0.60$  IN EQ.(2).

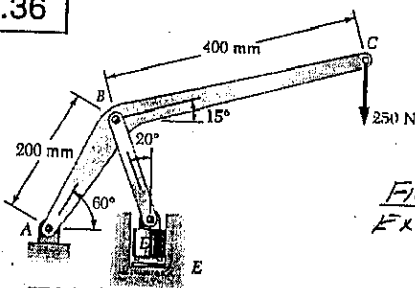
$$a/L = \sin^2 30^\circ (\sin 30^\circ - (-0.60) \cos 30^\circ)$$

$$a/L = 0.2459$$

$$L/a = 3.923$$

RANGE OF  $L/a$  FOR EQUILIBRIUM:  $L/a \geq 3.92$

8.36



GIVEN: BETWEEN  
 DIE D AND  
 GUIDE E:  
 $\mu_s = 0.30$

FIND: FORCE  
 EXERTED ON SEAL

DIMENSIONS IN mm

FREE BODY: MEMBER AC

$$+\sum M_B = 0$$

$$F_{BD} \cos 20^\circ (100) + F_{BD} \sin 20^\circ (173.21) - (250)(100 + 386.37) = 0$$

$$F_{BD} = 793.6 \text{ N}$$

FREE BODY: DIE D

$$\phi_s = \tan^{-1} 0.30 = 16.70^\circ$$

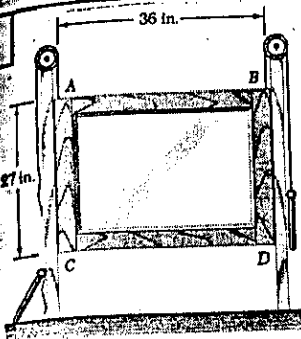
FORCE TRIANGLE

$$\frac{D}{\sin 53.3^\circ} = \frac{793.6 \text{ N}}{\sin 106.7^\circ}$$

$$D = 664.3 \text{ N}$$

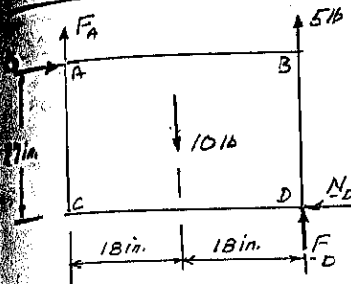
ON SEAL:

$$D = 664 \text{ N} \downarrow$$



**GIVEN:**  
10-lb WINDOW SASH  
5-lb SASH WEIGHT

**FIND:** SMALLEST  
VALUE OF  $\mu_s$  FOR  
WHICH WINDOW  
WILL STAY OPEN.



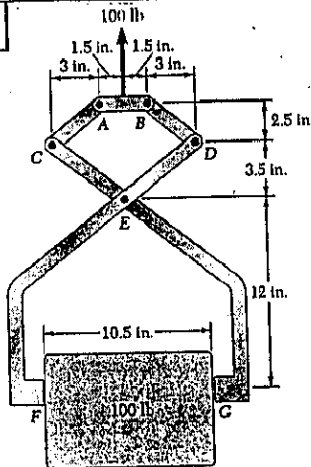
**FREE BODY: SASH**  
MOTION IMPENDS  
 $F_A = \mu_s N_A$   $F_D = \mu_s N_D$

$$\begin{aligned}\sum F_x = 0: & N_A = N_D \\ \uparrow \sum F_y = 0: & F_A + F_D - 10\text{ lb} + 5\text{ lb} = 0 \\ & F_A + F_D = 5\text{ lb} \\ & \mu_s N_A + \mu_s N_D = 5\text{ lb} \\ & 2\mu_s N_A = 5\text{ lb} \\ & F_A = \mu_s N_A = 2.5\text{ lb}\end{aligned}$$

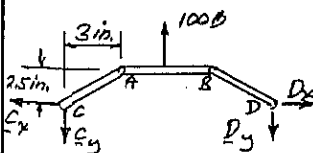
$$\begin{aligned}\uparrow \sum M_D = 0: & -N_A(27\text{ in}) - F_A(36\text{ in}) + (10\text{ lb})(18\text{ in}) = 0 \\ & -N_A(27\text{ in}) - (2.5\text{ lb})(36\text{ in}) + (10\text{ lb})(18\text{ in}) = 0 \\ & 27 N_A = 90 \quad N_A = 3.333\text{ lb} \\ & \mu_s = \frac{F_A}{N_A} = \frac{2.5\text{ lb}}{3.333\text{ lb}} = 0.75\end{aligned}$$

$$\mu_s = 0.75$$

8.38



**FIND:** SMALLEST  
 $\mu_s$  FOR BLOCK  
TO BE SUPPORTED.



**FREE BODY:**  
MEMBERS CA, AB, BD

BY SYMMETRY:  $C_y = D_y = \frac{1}{2}(100\text{ lb}) = 50\text{ lb}$

SINCE CA IS A TWO-FORCE MEMBER

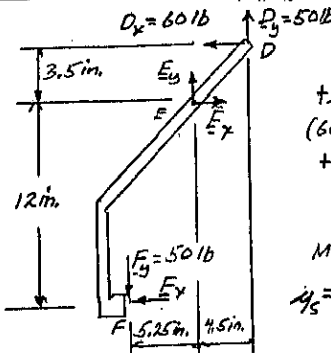
$$\frac{C_x}{3\text{ in}} = \frac{C_y}{2.5\text{ in}}; \quad \frac{C_x}{3\text{ in}} = \frac{50\text{ lb}}{2.5\text{ in}}; \quad C_x = 60\text{ lb}$$

$$\sum F_x = 0: D_x = C_x \quad D_x = 60\text{ lb}$$

(CONTINUED)

8.38 CONTINUED

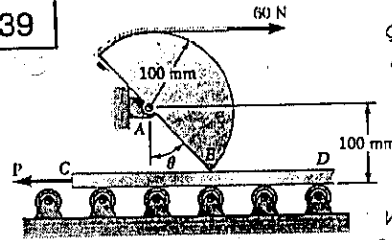
**FREE BODY: TANG DEF**



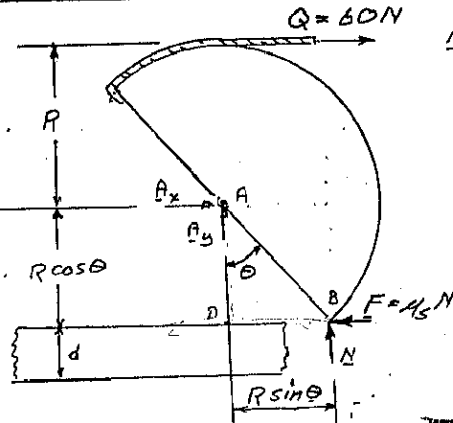
$$\begin{aligned}\uparrow \sum M_E = 0: & (60\text{ lb})(3.5\text{ in}) + (50\text{ lb})(4.5\text{ in}) \\ & + (50\text{ lb})(5.25\text{ in}) - F_x(12\text{ in}) = 0\end{aligned}$$

$$\begin{aligned}F_x &= +58.125\text{ lb} \\ \text{MINIMUM VALUE OF } \mu_s: & \\ \mu_s = \frac{F_y}{F_x} &= \frac{50\text{ lb}}{58.125\text{ lb}}; \mu_s = 0.8602 \\ \mu_s &= 0.86\end{aligned}$$

8.39



**GIVEN:** AT B,  $\mu_s = 0.45$   
d = PLATE THICKNESS  
**FIND:** (a) FORCE  
P TO MOVE PLATE  
IF d = 20 mm  
(b) LARGEST d FOR  
WHICH PLATE CANNOT  
BE MOVED IF P → ∞.

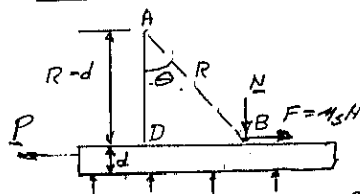


**FREE BODY: CAM**

FOR IMPENDING  
MOTION  
 $F = \mu_s N$

$$\begin{aligned}\uparrow \sum M_A = 0: & QR - NR \sin \theta + (\mu_s N) R \cos \theta = 0 \\ N &= \frac{Q}{\sin \theta - \mu_s \cos \theta}\end{aligned} \quad (1)$$

**FREE BODY: PLATE**  $\sum F_x = 0$   $P = \mu_s N$  (2)



**GEOMETRY**

IN  $\triangle ABD$   
WITH  $R = 100\text{ mm}$   
AND  $d = 20\text{ mm}$

$$\cos \theta = \frac{R-d}{R} = \frac{80\text{ mm}}{100\text{ mm}} = 0.8$$

$$\sin \theta = \sqrt{1 - \cos^2 \theta} = 0.6$$

EQ.(1) USING  $Q = 60\text{ N}$  AND  $\mu_s = 0.45$

$$N = \frac{60\text{ N}}{0.6 - (0.45)(0.8)} = \frac{60}{0.24} = 250\text{ N}$$

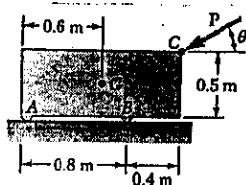
$$\text{EQ.(2)} \quad P = \mu_s N = (0.45)(250\text{ N}); \quad P = 112.5\text{ N}$$

(b) FOR  $P = \infty$ ,  $N = \infty$ . DENOMINATOR IS ZERO IN EQ.(1)

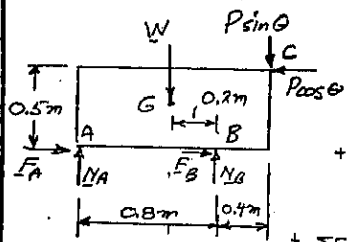
$$\sin \theta - \mu_s \cos \theta = 0; \quad \tan \theta = \mu_s = 0.45; \quad \theta = 24.23^\circ$$

$$\cos \theta = \frac{R-d}{R}; \quad \cos 24.23^\circ = \frac{100-d}{100}; \quad d = 2.81\text{ mm}$$

8.40



GIVEN: MASS = 75 kg,  
 $P = 500 \text{ N}$ ,  $\mu_s = 0.30$ .  
 FIND: RANGE OF  
 VALUES OF  $\theta$  FOR  
 BASE TO MOVE



FREE BODY: MACHINE BASE

$$W = (75 \text{ kg})(9.81 \text{ m/s}^2) = 735.75 \text{ N}$$

ASSUME SLIDING IMPENDS

$$F_A = \mu_s N_A \quad F_B = \mu_s N_B$$

$$+\uparrow \Sigma F_y = 0$$

$$N_A + N_B - W - P \sin \theta = 0$$

$$(N_A + N_B) = W + P \sin \theta \quad (1)$$

$$+\rightarrow \Sigma F_x = 0: F_A + F_B - P \cos \theta = 0$$

$$\mu_s (N_A + N_B) = P \cos \theta = 0 \quad (2)$$

$$\frac{\text{Eq. (2)}}{\text{Eq. (1)}}: \mu_s = \frac{P \cos \theta}{W + P \sin \theta}$$

$$\mu_s W + \mu_s P \sin \theta = P \cos \theta$$

$$0.30(735.75 \text{ N}) + 0.30(500 \text{ N}) \sin \theta = 500 \cos \theta$$

$$500 \cos \theta - 150 \sin \theta = 220.73$$

$$\text{SOLVE FOR } \theta: \quad \theta = 48.28^\circ$$

ASSUME TIPPING ABOUT B IMPENDS:  $\therefore N_A = 0$ 

$$+\rightarrow \Sigma M_B = 0: P \sin \theta (0.4 \text{ m}) - P \cos \theta (0.5 \text{ m}) - W(0.2 \text{ m}) = 0$$

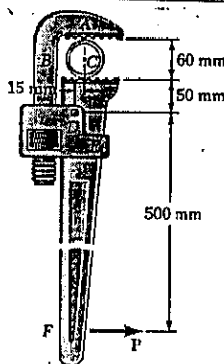
$$500 \sin \theta (0.4) - 500 \cos \theta (0.5) - 735.75 (0.2 \text{ m}) = 0$$

$$200 \sin \theta - 250 \cos \theta = 147.15$$

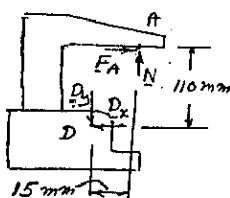
$$\text{SOLVE FOR } \theta: \quad \theta = 78.03^\circ$$

$$\text{RANGE FOR NO MOTION: } 48.3^\circ \leq \theta \leq 78.0^\circ$$

8.41



FIND: SMALLEST  
 VALUE OF  $\mu_s$   
 AT A AND C FOR  
 WRENCH TO SELF  
 LOCK ON THE PIPE



FREE BODY: PORTION ABDE

$$\Sigma F_x = 0: D_x = F_A \quad (1)$$

$$+\rightarrow \Sigma M_D = 0: N(15 \text{ mm}) - F_A(110 \text{ mm}) = 0$$

$$F_A = 0.1363 \text{ N}$$

$$\mu_s = F_A / N = 0.1363$$

$$\text{AT A: } \mu_s = 0.136$$

FREE BODY: PORTION CF

$$+\rightarrow \Sigma M_D = 0$$

$$P(50 \text{ mm}) - N(15 \text{ mm}) + F_D(50 \text{ mm}) = 0$$

$$500P - 15N + 50\mu_s N = 0$$

$$P = 0.03N - 0.1\mu_s N \quad (2)$$

$$+\rightarrow \Sigma F_x = 0: P - \mu_s N + D_x = 0$$

$$\text{USE EQ. (1)} \quad P = \mu_s N - F_A \quad (3)$$

$$\text{EQUATE (2) + (3):}$$

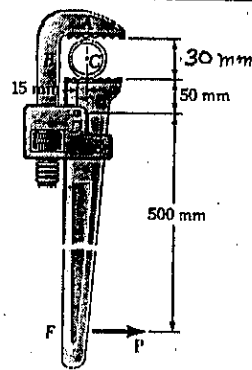
$$0.03N - 0.1\mu_s N = \mu_s N - F_A$$

$$\text{FROM EQ. (1) SUBSTITUTE } F_A = 0.1363 \text{ N:}$$

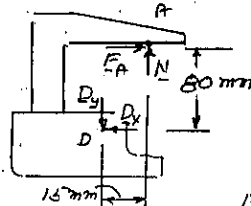
$$0.03N - 0.1\mu_s N = \mu_s N - 0.1363 \text{ N}$$

$$\text{SOLVE FOR } \mu_s: \quad \mu_s = 0.1512; \quad \text{AT C: } \mu_s = 0.151$$

8.42



FIND: SMALLEST  
 VALUES OF  $\mu_s$   
 AT A AND C FOR  
 WRENCH TO SELF  
 LOCK ON THE PIPE



FREE BODY: PORTION ABDE

$$\Sigma F_x = 0: D_x = F_A \quad (1)$$

$$+\rightarrow \Sigma M_D = 0: N(15 \text{ mm}) - F_A(80 \text{ mm}) = 0$$

$$F_A = 0.1875 \text{ N}$$

$$\mu_s = F_A / N = 0.1875$$

$$\text{AT A: } \mu_s = 0.188$$

FREE BODY PORTION CF

$$+\rightarrow \Sigma M_D = 0$$

$$P(50 \text{ mm}) - N(15 \text{ mm}) + F_D(50 \text{ mm}) = 0$$

$$500P - 15N + 50\mu_s N = 0$$

$$P = 0.03N - 0.1\mu_s N \quad (2)$$

$$+\rightarrow \Sigma F_x = 0: P - \mu_s N + D_x = 0$$

$$\text{USE EQ. (1)} \quad P = \mu_s N - F_A \quad (3)$$

$$\text{EQUATE (2) + (3):}$$

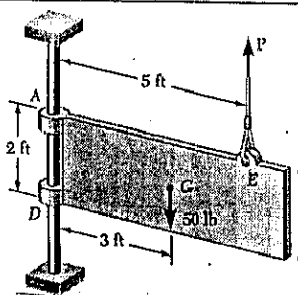
$$0.03N - 0.1\mu_s N = \mu_s N - F_A$$

$$\text{FROM EQ. (1) SUBSTITUTE } F_A = 0.1875 \text{ N:}$$

$$0.03N - 0.1\mu_s N = \mu_s N - 0.1875 \text{ N}$$

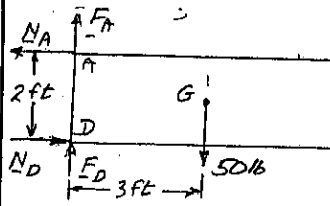
$$\text{SOLVE FOR } \mu_s: \quad \mu_s = 0.1977; \quad \text{AT B: } \mu_s = 0.198$$

8.43



GIVEN: AT A AND  
 AT B  $\mu_s = 0.40$

FIND: WHETHER  
 PLATE IS IN  
 EQUILIBRIUM IF  
 (a)  $P = 0$ ,  
 (b)  $P = 20 \text{ lb}$ .

(a)  $P = 0$ 

$$+\rightarrow \Sigma M_D = 0$$

$$N_A(2 \text{ ft}) - (50 \text{ lb})(3 \text{ ft}) = 0$$

$$N_A = 75 \text{ lb}$$

$$\Sigma F_x = 0: N_D = N_A = 75 \text{ lb}$$

$$+\rightarrow \Sigma F_y = 0: F_A + F_D - 50 \text{ lb} = 0$$

$$F_A + F_D = 50 \text{ lb}$$

$$\text{BUT: } (F_A)_m = \mu_s N_A = 0.40(75 \text{ lb}) = 30 \text{ lb}$$

$$(F_D)_m = \mu_s N_D = 0.40(75 \text{ lb}) = 30 \text{ lb}$$

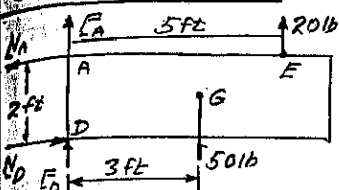
$$\text{THUS: } (F_A)_m + (F_D)_m = 60 \text{ lb}$$

$$\text{AND } (F_A)_m + (F_D)_m > F_A + F_D$$

$$\text{PLATE IS IN EQUILIBRIUM}$$

(CONTINUED)

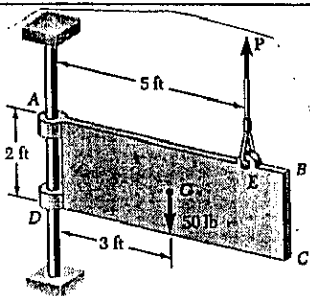
# 8.43 CONTINUED



(b)  $P = 20 \text{ lb}$   
 $\sum M_D = 0$   
 $N_A(2\text{ft}) - (50\text{lb})(3\text{ft}) + (20\text{lb})(5\text{ft}) = 0$   
 $N_A = 25 \text{ lb}$   
 $\sum F_x = 0: N_D = N_A = 25 \text{ lb}$   
 $\sum F_y = 0: F_A + F_D - 50\text{lb} + 20\text{lb} = 0$   
 $F_A + F_D = 30 \text{ lb}$

BUT:  $(F_A)_m = \mu_s N_A = 0.4(25\text{lb}) = 10 \text{ lb}$   
 $(F_D)_m = \mu_s N_D = 0.4(25\text{lb}) = 10 \text{ lb}$   
 THUS:  $(F_A)_m + (F_D)_m = 20 \text{ lb}$ , AND  $F_A + F_D > (F_A)_m + (F_D)_m$   
 PLATE MOVES DOWNWARD

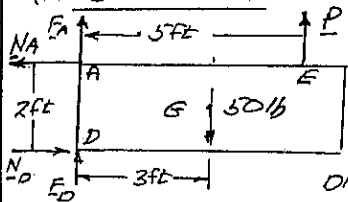
# 8.44



GIVEN: AT A  
 AND AT B:  $\mu_s = 0.40$

FIND: RANGE  
 OF VALUES OF P  
 FOR WHICH  
 PLATE WILL  
 MOVE DOWNWARD.

WE SHALL CONSIDER THE FOLLOWING TWO CASES  
 (1)  $0 < P < 30 \text{ lb}$



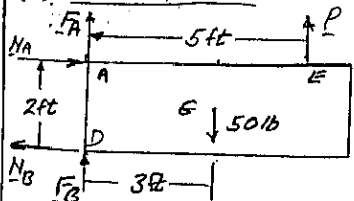
$\sum M_D = 0:$   
 $N_A(2\text{ft}) - (50\text{lb})(3\text{ft}) + P(5\text{ft}) = 0$   
 $N_A = 75\text{lb} - 2.5P$

(NOTE:  $N_A \geq 0$  AND  
 DIRECTED  $\leftarrow$  FOR  $P \leq 30 \text{ lb}$   
 AS ASSUMED HERE)

$\sum F_x = 0: N_A = N_D$   
 $\sum F_y = 0: F_A + F_D + P - 50 = 0; F_A + F_D = 50 - P$   
 BUT:  $(F_A)_m = (F_D)_m = \mu_s N_A = 0.40(75 - 2.5P) = 30 - P$

PLATE MOVES  $\downarrow$  IF  $F_A + F_D > (F_A)_m + (F_D)_m$   
 OR  $50 - P > (30 - P) + (30 - P)$   
 $P > 10 \text{ lb}$

(2)  $30 \text{ lb} < P < 50 \text{ lb}$



$\sum M_D = 0$   
 $-N_A(2\text{ft}) - (50\text{lb})(3\text{ft}) + P(5\text{ft}) = 0$   
 $N_A = 2.5P - 75$

(NOTE:  $N_A > 0$  AND DIRECTED  $\rightarrow$   
 FOR  $P > 30 \text{ lb}$  AS ASSUMED)

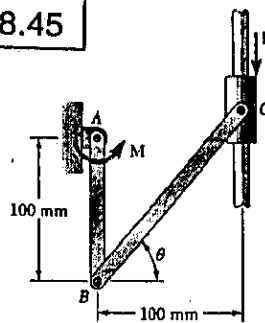
$\sum F_x = 0: N_A = N_D$   
 $\sum F_y = 0: F_A + F_D + P - 50 = 0; F_A + F_D = 50 - P$   
 BUT:  $(F_A)_m = (F_D)_m = \mu_s N_A = 0.40(2.5P - 75) = P - 30 \text{ lb}$

PLATE MOVES  $\downarrow$  IF  $F_A + F_D > (F_A)_m + (F_D)_m$   
 $50 - P > (P - 30) + (P - 30)$   
 $P < \frac{110}{3} = 36.7 \text{ lb}$

THUS: PLATE MOVE DOWNWARD FOR:  
 $10 \text{ lb} < P < 36.7 \text{ lb}$

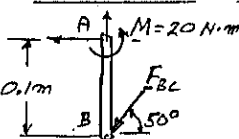
NOTE: FOR  $P > 50 \text{ lb}$ , PLATE IS IN EQUILIBRIUM

# 8.45



GIVEN:  $\mu_s = 0.35$ ,  
 $\theta = 50^\circ$ ,  $M = 20 \text{ N}\cdot\text{m}$

FIND: RANGE OF  
 VALUES OF P FOR  
 EQUILIBRIUM



FREE BODY: MEMBER AB

BC IS A TWO-FORCE MEMBER

$\sum M_A = 0: 20 \text{ N}\cdot\text{m} - F_{BC} \cos 50^\circ (0.1 \text{ m}) = 0$   
 $F_{BC} = 311.145 \text{ N}$

MOTION OF C IMPENDING UPWARD

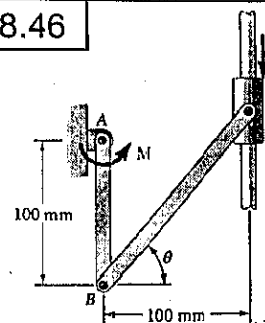
$\sum F_x = 0: (311.145 \text{ N}) \cos 50^\circ - N = 0$   
 $N = 200 \text{ N}$   
 $\sum F_y = 0: (311.145 \text{ N}) \sin 50^\circ - P - (0.35)(200 \text{ N}) = 0$   
 $P = 168.351 \text{ N}$

MOTION OF C IMPENDING DOWNWARD

$\sum F_x = 0: (311.145 \text{ N}) \cos 50^\circ - N = 0$   
 $N = 200 \text{ N}$   
 $\sum F_y = 0: (311.145 \text{ N}) \sin 50^\circ - P + (0.35)(200 \text{ N}) = 0$   
 $P = 308.35 \text{ N}$

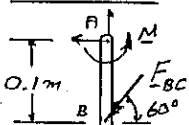
RANGE OF P:  $168.4 \text{ N} \leq P \leq 308 \text{ N}$

# 8.46



GIVEN:  $\mu_s = 0.40$ ,  
 $\theta = 60^\circ$ ,  $P = 200 \text{ N}$

FIND: RANGE OF  
 VALUES OF M FOR  
 EQUILIBRIUM



FREE BODY: MEMBER AB

BC IS A TWO-FORCE MEMBER

$\sum M_A = 0: M - F_{BC} \cos 60^\circ (0.1 \text{ m}) = 0$   
 $M = 0.05 F_{BC}$  (1)

MOTION OF C IMPENDING UPWARD

$\sum F_x = 0: F_{BC} \cos 60^\circ - N = 0$   
 $N = 0.5 F_{BC}$   
 $\sum F_y = 0: F_{BC} \sin 60^\circ - 200 \text{ N} - (0.40)(0.5 F_{BC}) = 0$   
 $F_{BC} = 300.29 \text{ N}$

EQ.(1):  $M = 0.05(300.29)$   
 $M = 15.014 \text{ N}\cdot\text{m}$

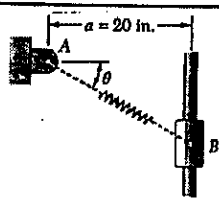
MOTION OF C IMPENDING DOWNWARD

$\sum F_x = 0: F_{BC} \cos 60^\circ - N = 0$   
 $N = 0.5 F_{BC}$   
 $\sum F_y = 0: F_{BC} \sin 60^\circ - 200 \text{ N} + (0.40)(0.5 F_{BC}) = 0$   
 $F_{BC} = 187.613 \text{ N}$

EQ.(1):  $M = 0.05(187.613)$   
 $M = 9.381 \text{ N}\cdot\text{m}$

RANGE OF M:  $9.38 \text{ N}\cdot\text{m} \leq M \leq 15.01 \text{ N}\cdot\text{m}$

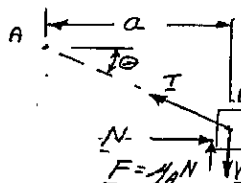
8.47



GIVEN:  $k = 15 \text{ lb/in.}$   
 $T = 0 \text{ WHEN } \theta = 0.$   
 $\mu_s = 0.40.$

FIND: RANGE OF  $W$  FOR  
 EQUILIBRIUM WHEN

(a)  $\theta = 20^\circ$ , (b)  $\theta = 30^\circ$ .



TENSION IN SPRING:  $AB = \frac{a}{\cos \theta}$

$T = k\Delta = k(AB - a) = k\left(\frac{a}{\cos \theta} - a\right)$

FOR MOTION IMPENDING DOWNWARD

$\sum F_x = 0: N - T \cos \theta = 0$

$N = T \cos \theta$

$\uparrow \sum F_y = 0: T \sin \theta - W + \mu_s N = 0$

$T \sin \theta - W + \mu_s T \cos \theta = 0$

$W = T(\sin \theta + \mu_s \cos \theta)$

$W = ka\left(\frac{1}{\cos \theta} - 1\right)(\sin \theta + \mu_s \cos \theta) \quad (1)$

FOR MOTION IMPENDING UPWARD,  $F = \mu_s N$  ACTS DOWNWARD

$\therefore \text{IN EQ. (1): } \mu_s \rightarrow -\mu_s$

$W = ka\left(\frac{1}{\cos \theta} - 1\right)(\sin \theta - \mu_s \cos \theta) \quad (2)$

(a)  $\theta = 20^\circ$ ,  $k = 15 \text{ lb/in.}$ ,  $\mu_s = 0.40$

MOTION  $\downarrow$ , WE USE EQ. (1):

$W = (15 \text{ lb/in.})(20 \text{ in.})\left(\frac{1}{\cos 20^\circ} - 1\right)(\sin 20^\circ + 0.40 \cos 20^\circ)$

$W = (300 \text{ lb})(0.06478)(0.34202 + 0.40 \times 0.93969)$

$W = 13.82 \text{ lb}$

MOTION  $\uparrow$ , WE USE EQ. (2):

$W = (300 \text{ lb})(0.06478)(0.34202 - 0.40 \times 0.93969)$

$W = -0.652 \text{ lb}$ : NEGATIVE WEIGHT, IMPOSSIBLE

RANGE WHEN  $\theta = 20^\circ$ :  $W \leq 13.82 \text{ lb}$

(b)  $\theta = 30^\circ$ ,  $k = 15 \text{ lb/in.}$ ,  $\mu_s = 0.40$

MOTION  $\downarrow$ , WE USE EQ. (1)

$W = (15 \text{ lb/in.})(20 \text{ in.})\left(\frac{1}{\cos 30^\circ} - 1\right)(\sin 30^\circ + 0.40 \cos 30^\circ)$

$W = (300 \text{ lb})(0.15470)(0.5 + 0.40 \times 0.86603)$

$W = 39.28 \text{ lb}$

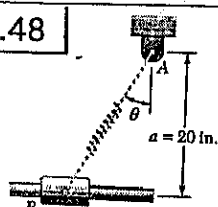
MOTION  $\uparrow$ , WE USE EQ. (2):

$W = (300 \text{ lb})(0.15470)(0.5 - 0.40 \times 0.86603)$

$W = 7.12 \text{ lb}$

RANGE WHEN  $\theta = 30^\circ$ :  $7.13 \text{ lb} \leq W \leq 39.3 \text{ lb}$

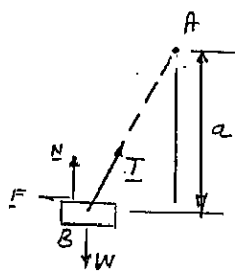
8.48



GIVEN:  $k = 15 \text{ lb/in.}$ ,  $\mu_s = 0.40$   
 $T = 0 \text{ WHEN } \theta = 0.$

FIND: RANGE OF  $W$  FOR  
 EQUILIBRIUM WHEN

(a)  $\theta = 20^\circ$ , (b)  $\theta = 30^\circ$



TENSION IN SPRING

$AB = \frac{a}{\cos \theta}$

ELONGATION OF SPRING

$\Delta = \frac{a}{\cos \theta} - a$

$T = k\Delta = ka\left(\frac{1}{\cos \theta} - 1\right) \quad (1)$

(CONTINUED)

8.48 CONTINUED

NOTE: ONLY POSSIBLE MOTION  
 IS  $\rightarrow$ ; BUT  $N$  CAN BE  $\uparrow$  OR  $\downarrow$ .

$\pm \sum F_x = 0: T \sin \theta - \mu_s N = 0$

$N = (T \sin \theta) / \mu_s$

$\uparrow \sum F_y = 0: N + T \cos \theta - W = 0$

$W = T \cos \theta + \mu_s T \sin \theta$

(a)  $\theta = 20^\circ$ ,  $k = 15 \text{ lb/in.}$ ,  $\mu_s = 0.40$

EQ. (1):  $T = (15 \text{ lb/in.})(20 \text{ in.})\left(\frac{1}{\cos 20^\circ} - 1\right) = 19.2533 \text{ lb}$

EQ. (2):  $N = (19.2533 \text{ lb})(\sin 20^\circ) / 0.40 = 16.4626 \text{ lb}$

IF  $N$  ACTS  $\uparrow$ : THAT IS,  $N = +16.4626 \text{ lb}$

EQ. (3):  $W = (19.2533 \text{ lb}) \cos 20^\circ + 16.4626 \text{ lb} = 34.555 \text{ lb}$

COLLAR IN EQUILIBRIUM WHEN:  $W \geq 35.6 \text{ lb}$

IF  $N$  ACTS  $\downarrow$ : THAT IS,  $N = -16.4626 \text{ lb}$

$W = (19.2533 \text{ lb}) \cos 20^\circ - 16.4626 \text{ lb} = 1.6296 \text{ lb}$

COLLAR IN EQUILIBRIUM WHEN:  $W \geq 1.630 \text{ lb}$

(b)  $\theta = 30^\circ$ ,  $k = 15 \text{ lb/in.}$ ,  $\mu_s = 0.40$

EQ. (1):  $T = (15 \text{ lb/in.})(20 \text{ in.})\left(\frac{1}{\cos 30^\circ} - 1\right) = 46.41 \text{ lb}$

EQ. (2):  $N = (46.41 \text{ lb}) \sin 30^\circ / 0.40 = 58.01 \text{ lb}$

IF  $N$  ACTS  $\uparrow$ : THAT IS,  $N = 58.01 \text{ lb}$

EQ. (3):  $W = (46.41 \text{ lb}) \cos 30^\circ + 58.01 \text{ lb} = 98.21 \text{ lb}$

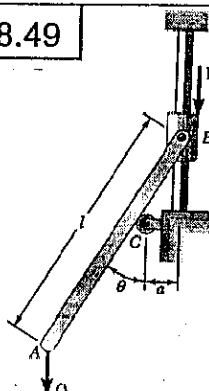
COLLAR IN EQUILIBRIUM WHEN:  $W \geq 98.2 \text{ lb}$

IF  $N$  ACTS  $\downarrow$ : THAT IS,  $N = -58.01 \text{ lb}$

$W = (46.41 \text{ lb}) \cos 30^\circ - 58.01 \text{ lb} = -17.81 \text{ lb}$

NEGATIVE WEIGHT, IMPOSSIBLE

8.49



GIVEN:  $l = 600 \text{ mm}$

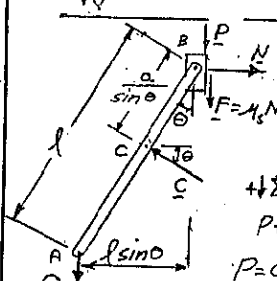
$a = 80 \text{ mm}$

$\mu_s = 0.25$

$Q = 100 \text{ N}$

$\theta = 30^\circ$

FIND: RANGE OF VALUES  
 OF  $P$  FOR EQUILIBRIUM



FOR MOTION OF COLLAR AT B  
 IMPENDING UPWARD,  $F = \mu_s N \uparrow$

$\uparrow \sum M_B = 0: Ql \sin \theta - Ca / \sin \theta = 0$

$C = Q(l/a) \sin^2 \theta$

$\sum F_x = 0: N = C \cos \theta = Q(l/a) \sin^2 \theta \cos \theta$

$\uparrow \sum F_y = 0: P + Q - C \sin \theta - \mu_s N = 0$

$P + Q - Q(l/a) \sin^2 \theta - \mu_s Q(l/a) \sin^2 \theta \cos \theta = 0$

$P = Q\left[\frac{l}{a} \sin^2 \theta (\sin \theta - \mu_s \cos \theta) - 1\right] \quad (1)$

SUBSTITUTE DATA:

$P = (100 \text{ N})\left[\frac{600 \text{ mm}}{80 \text{ mm}} \sin^2 30^\circ (\sin 30^\circ - 0.25 \cos 30^\circ) - 1\right]$

$P = -46.84 \text{ N}$  ( $P$  IS DIRECTED  $\uparrow$ )

$P = -46.8 \text{ N}$

FOR MOTION OF COLLAR IMPENDING DOWNWARD  $F = \mu_s N \uparrow$

IN EQ. (1) WE SUBSTITUTE  $-\mu_s$  FOR  $\mu_s$ .

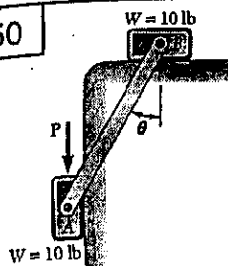
$P = Q\left[\frac{l}{a} \sin^2 \theta (\sin \theta + \mu_s \cos \theta) - 1\right]$

$P = (100 \text{ N})\left[\frac{600 \text{ mm}}{80 \text{ mm}} \sin^2 30^\circ (\sin 30^\circ + 0.25 \cos 30^\circ) - 1\right]$

$P = +34.34 \text{ N}$

FOR EQUILIBRIUM:  $-46.8 \text{ N} \leq P \leq 34.3 \text{ N}$

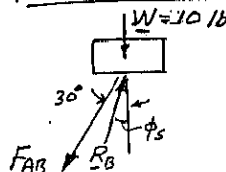
8.50



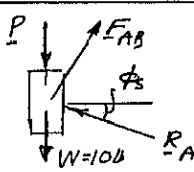
GIVEN: AT ALL SURFACES  $\mu_s = 0.30$   
 $\theta = 30^\circ$

- (a) CONFIRM EQUILIBRIUM WHEN  $P = 0$   
 (b) FIND LARGEST  $P$  FOR EQUILIBRIUM.

FOR MOTION: BLOCK A MOVES  $\downarrow$  AND BLOCK B MOVES  $\leftarrow$ .  
 ASSUME MOTION IMPENDS:  $\phi_s = \tan^{-1} 0.30 = 16.7^\circ$   
 FREE BODY: BLOCK B FREE BODY: BLOCK A



NOTE: AB IS A TWO-FORCE MEMBER



FORCE TRIANGLES

BLOCK B:  $\Delta 1, 2, 3$ 

$$\frac{F_{AB}}{\sin 16.7^\circ} = \frac{10 \text{ lb}}{\sin 13.3^\circ}$$

$$F_{AB} = 12.491 \text{ lb}$$

BLOCK A:  $\Delta 2, 4, 3$ 

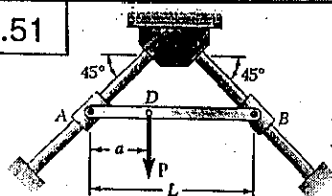
$$\frac{W + P}{\sin 76.7^\circ} = \frac{F_{AB}}{\sin 73.3^\circ}$$

$$\frac{10 \text{ lb} + P}{\sin 76.7^\circ} = \frac{12.491 \text{ lb}}{\sin 73.3^\circ}$$

$$10 \text{ lb} + P = 12.69 \text{ lb}$$

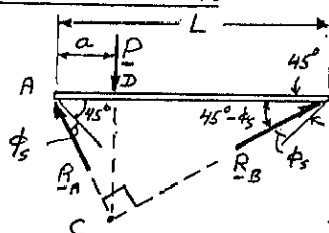
- (b)  $P = 2.69 \text{ lb}$   
 (a) EQUILIBRIUM FOR  $P < 2.69 \text{ lb}$   
 (b) EQUILIBRIUM FOR  $P = 0$

8.51

GIVEN:  $\mu_s = 0.30$ 

FIND: SMALLEST VALUE OF  $a/L$  FOR EQUILIBRIUM

FREE BODY: ROD AB



MOTION IMPENDS

$$\phi_s = \tan^{-1} 0.30$$

$$\phi_s = 16.7^\circ$$

THREE-FORCE BODY

P MUST PASS THROUGH E WHERE  $R_A$  AND  $R_B$  INTERSECT

IN RIGHT TRIANGLE ABC:

$$AC = L \sin(45 - \phi_s)$$

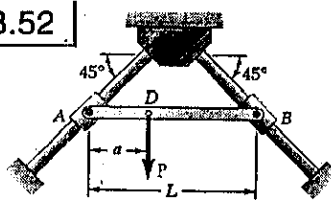
IN RIGHT TRIANGLE ADC:

$$a = AC \cos(45 + \phi_s) = L \sin(45 - \phi_s) \cos(45 + \phi_s)$$

$$\frac{a}{L} = \sin(45 - 16.7^\circ) \cos(45 + 16.7^\circ) = 0.2248$$

$$\frac{a}{L} = 0.225$$

8.52



DERIVE: EXPRESSION IN  $a/L$  FOR SMALLEST VALUE OF  $a/L$  FOR EQUILIBRIUM

FREE BODY: ROD AB

$$+\sum M_A = 0 \quad (N_B - \frac{1}{2} N_B) \frac{L}{\sqrt{2}} - Wa = 0$$

$$N_B = \frac{\sqrt{2} W a}{(1 - \frac{1}{2}) L} \quad (1)$$

$$+\sum F_x = 0: (N_B + \frac{1}{2} N_B) \frac{1}{\sqrt{2}} - (N_A - \frac{1}{2} N_A) \frac{1}{\sqrt{2}} = 0$$

$$N_A = \frac{1 + \frac{1}{2}}{1 - \frac{1}{2}} N_B \quad (2)$$

$$+\sum F_y = 0: (N_A + \frac{1}{2} N_A) \frac{1}{\sqrt{2}} + (N_B - \frac{1}{2} N_B) \frac{1}{\sqrt{2}} - W = 0$$

$$N_A = \frac{\sqrt{2} W - N_B (1 - \frac{1}{2})}{1 + \frac{1}{2}} \quad (3)$$

EQUATE (2) AND (3):  $\frac{1 + \frac{1}{2}}{1 - \frac{1}{2}} N_B = \frac{\sqrt{2} W - N_B (1 - \frac{1}{2})}{1 + \frac{1}{2}}$

$$N_B \left[ \frac{1 + \frac{1}{2}}{1 - \frac{1}{2}} + \frac{1 - \frac{1}{2}}{1 + \frac{1}{2}} \right] = \frac{\sqrt{2} W}{1 + \frac{1}{2}} \quad (4)$$

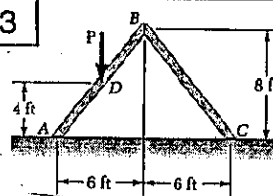
SUBSTITUTE FROM (1):

$$\frac{\sqrt{2} W a}{(1 - \frac{1}{2}) L} \left[ \frac{(1 + \frac{1}{2})^2 + (1 - \frac{1}{2})^2}{(1 - \frac{1}{2})(1 + \frac{1}{2})} \right] = \frac{\sqrt{2} W}{1 + \frac{1}{2}}$$

$$\frac{a}{L} \left[ \frac{1 + 2\frac{1}{2} + \frac{1}{4} + 1 - 2\frac{1}{2} + \frac{1}{4}}{1 - \frac{1}{4}} \right] = \frac{1 - \frac{1}{2}}{1 + \frac{1}{2}}$$

$$\frac{a}{L} \left[ \frac{2(1 + \frac{1}{2})^2}{1 - \frac{1}{4}} \right] = \frac{1 - \frac{1}{2}}{1 + \frac{1}{2}}; \quad \frac{a}{L} = \frac{1(1 - \frac{1}{2})(1 - \frac{1}{2})^2}{2(1 + \frac{1}{2})(1 + \frac{1}{2})^2}$$

8.53



GIVEN:  $\mu_s = 0.40$ , EACH BOARD WEIGHS 40 lb.  
 FIND: (a) LARGEST  $P$  FOR EQUILIBRIUM,  
 (b) WHERE MOTION IMPENDS

FREE BODY: ENTIRE FRAME

$$+\sum M_C = 0 \text{ YIELDS: } N_A = 40 + \frac{3}{4} P$$

$$+\sum M_A = 0 \text{ YIELDS: } N_C = 40 + \frac{1}{4} P$$

$$+\sum F_x = 0 \text{ YIELDS: } F_C = F_A$$

FREE BODY: BOARD AB

$$+\sum F_y = 0: 40 + \frac{3}{4} P - P - 40 + F_B = 0 \quad F_B = \frac{P}{4}$$

$$+\sum M_B = 0: (P + 40)(3 \text{ ft}) - (40 + \frac{3}{4} P)(6 \text{ ft}) + F_A(8 \text{ ft}) = 0$$

$$+\sum F_x = 0: N_B = F_A \quad F_A = 15 + \frac{3}{8} P$$

AT POINTS A, B, AND C WE EXPRESS THAT FOR IMPEND MOTION  $\mu_s = F/N$ .

AT POINT A

$$\mu_s = \frac{F_A}{N_A}$$

$$0.40 = \frac{15 + \frac{3}{8} P}{40 + \frac{3}{4} P}$$

$$P = -8.889 \text{ lb}$$

AT POINT B

$$\mu_s = \frac{F_B}{N_B} = \frac{F_B}{F_A}$$

$$0.40 = \frac{P/4}{15 + \frac{3}{8} P}$$

$$P = 34.79 \text{ lb}$$

AT POINT C

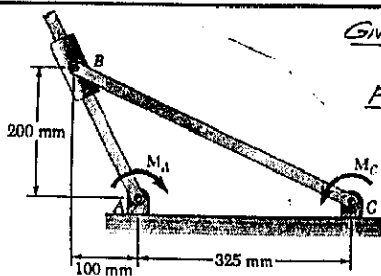
$$\mu_s = \frac{F_C}{N_C} = \frac{F_A}{N_C}$$

$$0.40 = \frac{15 + \frac{3}{8} P}{40 + \frac{1}{4} P}$$

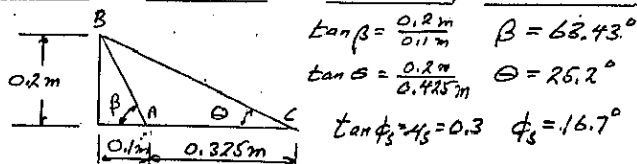
$$P = 11.429 \text{ lb}$$

- (a)  $P_{\max} = 11.43 \text{ lb}$  (b) MOTION IMPENDS AT C.

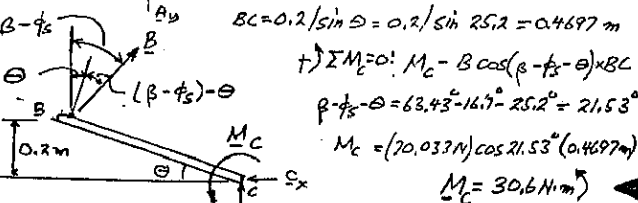
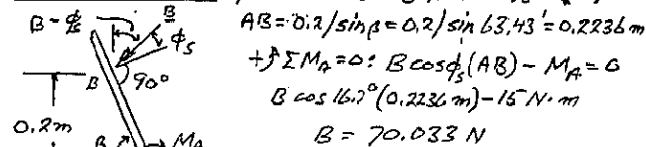
8.54



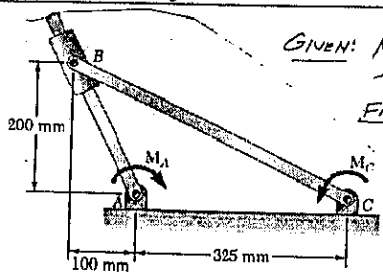
GIVEN:  $M_A = 15 \text{ N}\cdot\text{m}$   
 $\mu_s = 0.30$   
 FIND: LARGEST  
 $M_C$  FOR WHICH  
 EQUILIBRIUM  
 IS MAINTAINED



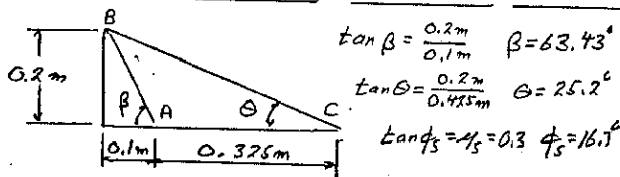
FOR LARGEST  $M_C$ , MOTION OF B IMPENDS



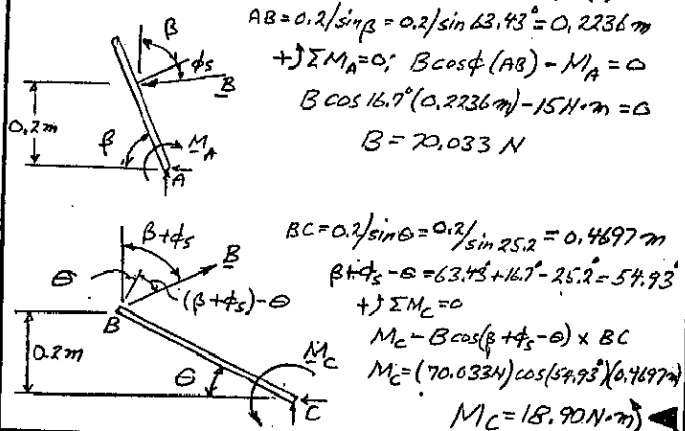
8.55



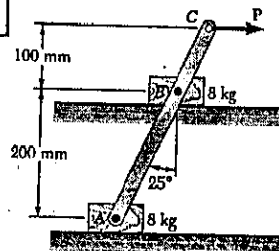
GIVEN:  $M_A = 15 \text{ N}\cdot\text{m}$   
 $\mu_s = 0.30$   
 FIND: SMALLEST  
 $M_C$  FOR WHICH  
 EQUILIBRIUM  
 IS MAINTAINED



FOR SMALLEST  $M_C$ , MOTION OF B IMPENDS

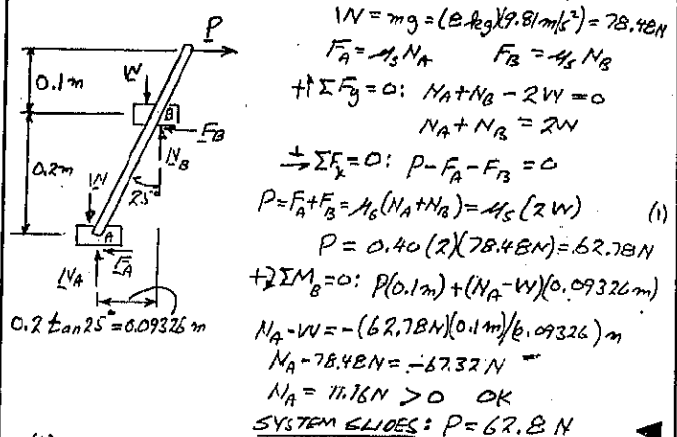


8.56



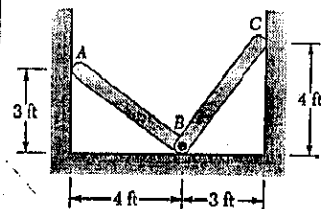
FIND: VALUE OF  $P$   
 FOR WHICH  
 MOTION OCCURS  
 AND WHAT MOTION IS  
 FOR (a)  $\mu_s = 0.40$   
 (b)  $\mu_s = 0.50$

(a)  $\mu_s = 0.40$ : ASSUME BLOCKS SLIDE TO RIGHT

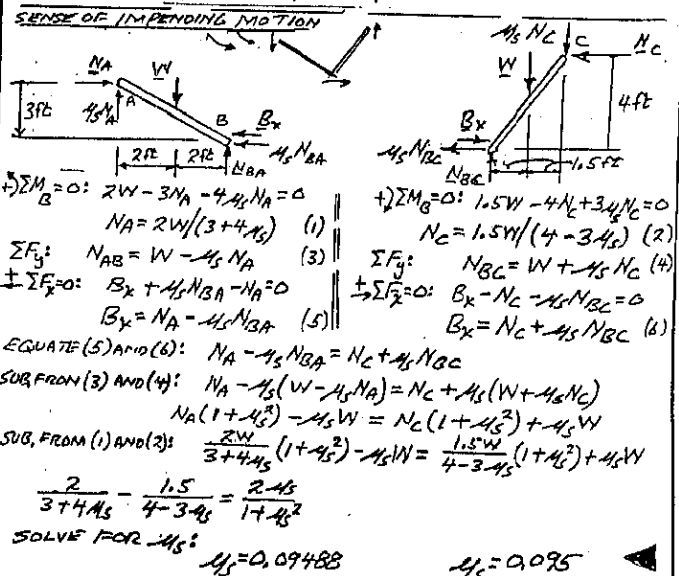


(b)  $\mu_s = 0.50$ : SEE PART a.  
 For (1)  $P = 0.5(2)(78.48 \text{ N}) = 78.48 \text{ N}$   
 $\sum M_B = 0: P(0.1 \text{ m}) + (N_A - W)(0.09326 \text{ m}) = 0$   
 $N_A - W = -(78.48 \text{ N})(0.1 \text{ m}) / (0.09326 \text{ m})$   
 $N_A - 78.48 \text{ N} = -84.15 \text{ N}$   
 $N_A = -5.67 \text{ N} < 0$  UPLIFT, ROTATION ABOUT B  
 FOR  $N_A = 0: \sum M_B = 0: P(0.1 \text{ m}) - W(0.09326 \text{ m}) = 0$   
 $P = (78.48 \text{ N})(0.09326 \text{ m}) / (0.1) = 73.19$   
 SYSTEM ROTATES ABOUT B:  $P = 73.2 \text{ N}$

8.57

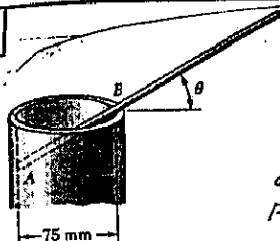


GIVEN:  $\mu_s$  AT  
 A, B, AND C  
 FIND: SMALLEST  
 $\mu_s$  FOR  
 EQUILIBRIUM



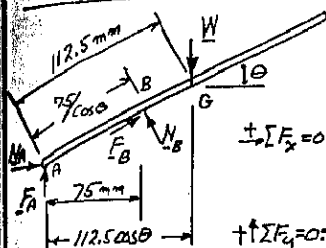


8.58



GIVEN: LENGTH OF ROD = 225 mm,  
 $\mu_s = 0.20$ ,  
 FIND: LARGEST VALUE OF  $\theta$  FOR ROD TO NOT FALL INTO THE PIPE.

MOTION OF ROD IMPENDS DOWN AT A AND TO LEFT AT B.  
 $F_A = \mu_s N_A$   $F_B = \mu_s N_B$



$$\begin{aligned} \sum F_x = 0: N_A - N_B \sin \theta + F_B \cos \theta &= 0 \\ N_A - N_B \sin \theta + \mu_s N_B \cos \theta &= 0 \\ N_A &= N_B (\sin \theta - \mu_s \cos \theta) \quad (1) \\ \sum F_y = 0: F_A + N_B \cos \theta + F_B \sin \theta - W &= 0 \\ \mu_s N_A + N_B \cos \theta + \mu_s N_B \sin \theta - W &= 0 \quad (2) \end{aligned}$$

SUBSTITUTE FOR  $N_A$  FROM (1) INTO (2):

$$\mu_s N_B (\sin \theta - \mu_s \cos \theta) + N_B \cos \theta + \mu_s N_B \sin \theta - W = 0$$

$$N_B = \frac{W}{(1 - \mu_s^2) \cos \theta + 2 \mu_s \sin \theta} = \frac{W}{(1 - 0.2^2) \cos \theta + 2(0.2) \sin \theta} \quad (3)$$

$$\sum M_A = 0: N_B (75 \cos \theta) - W (112.5 \cos \theta) = 0$$

SUBSTITUTE FOR  $N_B$  FROM (3), CANCEL  $W$ , AND SIMPLIFY TO FIND

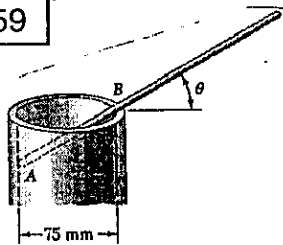
$$9.6 \cos^3 \theta + 4 \sin \theta \cos^2 \theta - 6.667 = 0$$

$$\cos^3 \theta (2.4 + \tan \theta) = 1.667$$

SOLVE BY TRIAL + ERROR:

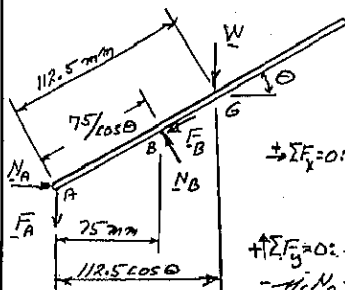
$$\theta = 35.8^\circ$$

8.59



GIVEN: LENGTH OF ROD = 225 mm,  
 $\mu_s = 0.20$ ,  
 FIND: SMALLEST VALUE OF  $\theta$  FOR ROD TO NOT FALL OUT OF THE PIPE.

MOTION OF ROD IMPENDS UP AT A AND RIGHT AT B  
 $F_A = \mu_s N_A$   $F_B = \mu_s N_B$



$$\begin{aligned} \sum F_x = 0: N_A - N_B \sin \theta - F_B \cos \theta &= 0 \\ N_A - N_B \sin \theta - \mu_s N_B \cos \theta &= 0 \\ N_A &= N_B (\sin \theta + \mu_s \cos \theta) \quad (1) \\ \sum F_y = 0: -F_A + N_B \cos \theta - F_B \sin \theta - W &= 0 \\ -\mu_s N_A + N_B \cos \theta - \mu_s N_B \sin \theta - W &= 0 \quad (2) \end{aligned}$$

SUBSTITUTE FOR  $N_A$  FROM (1) INTO (2):

$$-\mu_s N_B (\sin \theta + \mu_s \cos \theta) + N_B \cos \theta - \mu_s N_B \sin \theta - W = 0$$

$$N_B = \frac{W}{(1 - \mu_s^2) \cos \theta - 2 \mu_s \sin \theta} = \frac{W}{(1 - 0.2^2) \cos \theta - 2(0.2) \sin \theta} \quad (3)$$

$$\sum M_A = 0: N_B (75 \cos \theta) - W (112.5 \cos \theta) = 0$$

SUBSTITUTE FOR  $N_B$  FROM (3), CANCEL  $W$ , AND SIMPLIFY

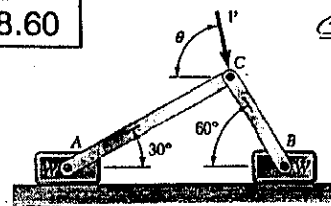
$$9.6 \cos^3 \theta - 4 \sin \theta \cos^2 \theta - 6.667 = 0$$

$$\cos^3 \theta (2.4 - \tan \theta) = 1.667$$

SOLVE BY TRIAL + ERROR:

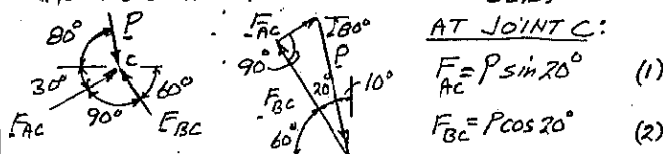
$$\theta = 20.5^\circ$$

8.60



GIVEN:  $\theta = 80^\circ$ ,  
 $\mu_s = 0.30$ ,  
 FIND: LARGEST  $P$  FOR EQUILIBRIUM

AC AND BC ARE TWO-FORCE MEMBERS

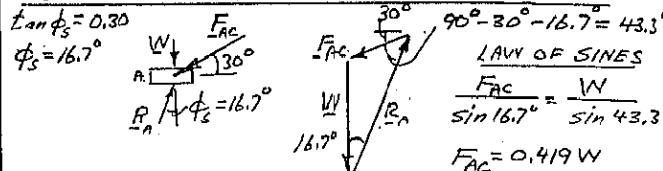


AT JOINT C:

$$F_A = P \sin 20^\circ \quad (1)$$

$$F_B = P \cos 20^\circ \quad (2)$$

ASSUME MOTION OF BLOCK A IMPENDS TO LEFT.



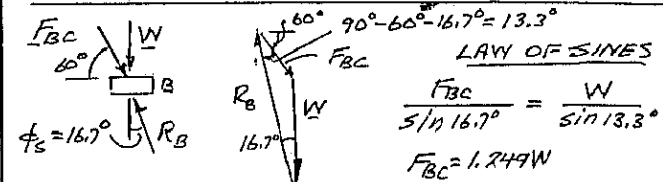
LAW OF SINES

$$\frac{F_A}{\sin 16.7^\circ} = \frac{W}{\sin 43.3^\circ}$$

$$F_A = 0.419 W$$

SUBSTITUTE INTO EQ.(1):  $F_A = 0.419 W = P \sin 20^\circ$ ;  $P = 1.225 W$

ASSUME MOTION OF BLOCK B IMPENDS TO RIGHT



LAW OF SINES

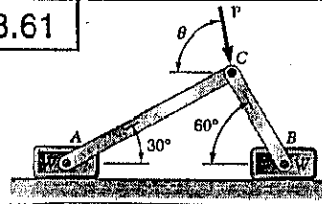
$$\frac{F_B}{\sin 16.7^\circ} = \frac{W}{\sin 13.3^\circ}$$

$$F_B = 1.249 W$$

SUBSTITUTE INTO EQ.(2):  $F_B = 1.249 W = P \cos 20^\circ$ ;  $P = 1.329 W$

LARGEST  $P$  FOR EQUILIBRIUM:  $P = 1.225 W$

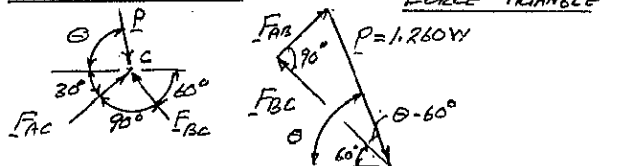
8.61



GIVEN:  $P = 1.260 W$ ,  
 $\mu_s = 0.30$ ,  
 FIND: RANGE OF  $\theta$ , BETWEEN  $0$  AND  $180^\circ$ , FOR EQUILIBRIUM

AC AND BC ARE TWO-FORCE MEMBERS  
 FREE BODY, JOINT C

FORCE TRIANGLE



FROM FORCE TRIANGLE:

$$F_A = P \sin (\theta - 60^\circ) = 1.26 W \sin (\theta - 60^\circ) \quad (1)$$

$$F_B = P \cos (\theta - 60^\circ) = 1.26 W \cos (\theta - 60^\circ) \quad (2)$$

WE SHALL, IN TURN, SEEK  $\theta$  CORRESPONDING TO IMPENDING MOTION OF EACH BLOCK

FOR MOTION OF A IMPENDING TO LEFT

FROM SOLUTION OF PROB 8.60;  $F_A = 0.419 W$

$$EQ.(1): F_A = 0.419 W = 1.26 W \sin (\theta - 60^\circ)$$

$$\sin (\theta - 60^\circ) = 0.33254$$

$$\theta - 60^\circ = 19.422^\circ$$

$$\theta = 79.42^\circ$$

(CONTINUED)

# 8.61 CONTINUED

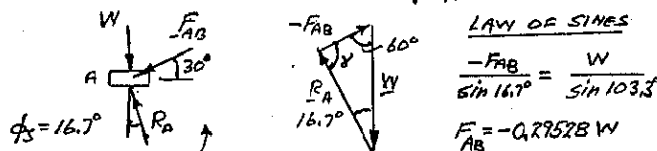
FOR MOTION OF B  
IMPENDING TO RIGHT.

FROM SOLUTION OF PROB. 8.60;  $F_{BC} = 1.249W$   
EQ.(2):  $F_{BC} = 1.249W = 1.26W \cos(\theta - 60^\circ)$   
 $\cos(\theta - 60^\circ) = 0.99127$

$\theta - 60^\circ = \pm 7.58^\circ$   
 $\theta - 60^\circ = +7.58^\circ$   $\theta = 67.6^\circ$   
 $\theta - 60^\circ = -7.58^\circ$   $\theta = 52.4^\circ$

FOR MOTION OF A IMPENDING TO RIGHT

$\gamma = 180^\circ - 60^\circ - 16.7^\circ = 103.3^\circ$



LAW OF SINES

$\frac{-F_{AB}}{\sin 16.7^\circ} = \frac{W}{\sin 103.3^\circ}$   
 $F_{AB} = -0.29528W$

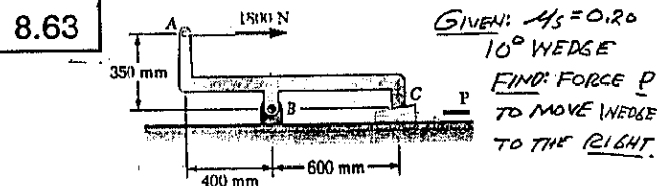
NOTE: DIRECTION OF  $+F_{AB}$  IS KEPT SAME AS IN FREE BODY OF JOINT C.

EQ.(1):  $F_{AB} = -0.29528W = 1.26W \sin(\theta - 60^\circ)$   
 $\sin(\theta - 60^\circ) = -0.23435$   
 $(\theta - 60^\circ) = -13.553^\circ$   $\theta = 46.4^\circ$

SUMMARY:	A MOVES TO RIGHT	NO MOTION	B MOVES TO RIGHT	NO MOTION	A MOVES TO LEFT
	46.4°	52.4°	67.6°	79.4°	

NO MOTION FOR:  $46.4^\circ \leq \theta \leq 52.4^\circ$  AND  $67.6^\circ \leq \theta \leq 79.4^\circ$

# 8.63



GIVEN:  $\mu_s = 0.20$   
10° WEDGE

FIND: FORCE P TO MOVE WEDGE TO THE RIGHT.

$\phi_s = \tan^{-1} 0.20 = 11.31^\circ$

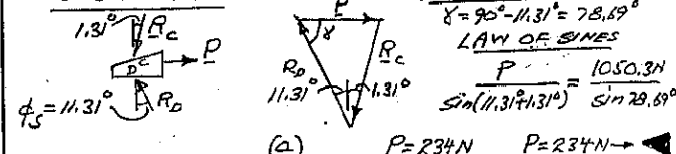
FREE BODY: PART ABC

$+\Sigma M_B = 0$   
 $(1800N)(0.35m) - R_C \cos 1.31^\circ (0.6m) = 0$   
 $R_C = 1050.3N$

FORCE TRIANGLE

$\gamma = 90^\circ - 11.31^\circ = 78.69^\circ$   
LAW OF SINES  
 $\frac{P}{\sin(11.31^\circ + 11.31^\circ)} = \frac{1050.3N}{\sin 78.69^\circ}$

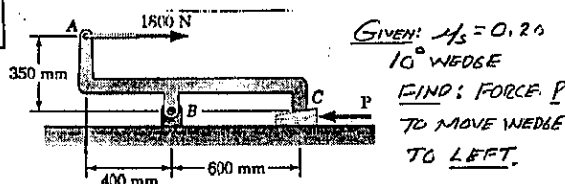
FREE BODY: WEDGE



(b) RETURN TO PART ABC:

$+\Sigma F_x = 0$ :  $B_x + 1800N + R_C \sin 1.31^\circ = 0$   
 $B_x + 1800N + (1050.3N) \sin 1.31^\circ = 0$   
 $B_x = -1824N$   
 $+\Sigma F_y = 0$ :  $B_y + R_C \cos 1.31^\circ = 0$   
 $B_y + (1050.3N) \cos 1.31^\circ = 0$   
 $B_y = -1050N$

# 8.62



GIVEN:  $\mu_s = 0.20$   
10° WEDGE

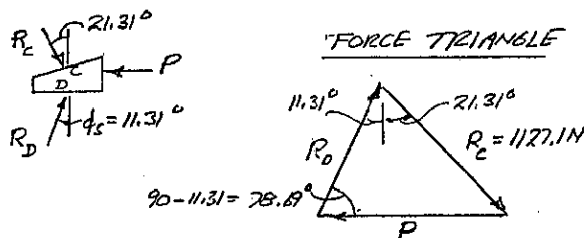
FIND: FORCE P TO MOVE WEDGE TO LEFT.

$\phi_s = \tan^{-1} 0.20 = 11.31^\circ$

FREE BODY: PART ABC

$+\Sigma M_B = 0$   
 $(1800N)(0.35m) - R_C \cos 21.31^\circ (0.6m) = 0$   
 $R_C = 1127.1N$

FREE BODY: WEDGE



(a) LAW OF SINES

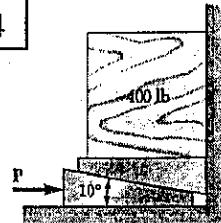
$\frac{P}{\sin(11.31^\circ + 21.31^\circ)} = \frac{1127.1N}{\sin 78.69^\circ}$

$P = 619.6N$ ;  $P = 620N \leftarrow$

(b) RETURN TO PART ABC:

$+\Sigma F_x = 0$ :  $B_x + 1800N - R_C \sin 21.31^\circ = 0$   
 $B_x + 1800N - (1127.1N) \sin 21.31^\circ = 0$   
 $B_x = -1390.4N$   $B_x = 1390N \leftarrow$   
 $+\Sigma F_y = 0$ :  $B_y + R_C \cos 21.31^\circ = 0$   
 $B_y + (1127.1N) \cos 21.31^\circ = 0$   
 $B_y = -1050N$   $B_y = 1050N \downarrow$

# 8.64

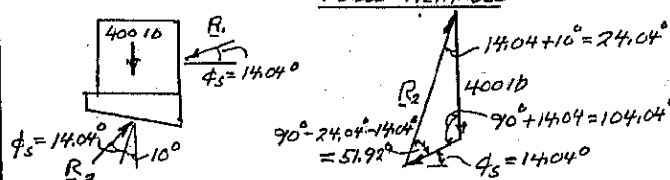


GIVEN: Two 10° WEDGES  
 $\mu_s = 0.25$

FIND: SMALLEST P TO MOVE WEDGE

FREE BODY: BLOCK AND TOP WEDGE

FORCE TRIANGLE

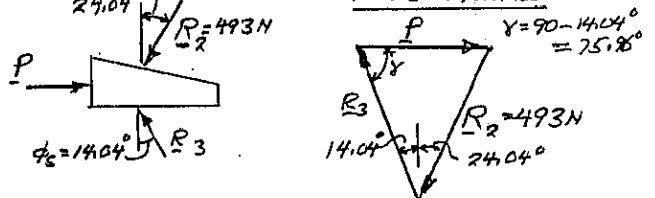


LAW OF SINES

$\frac{R_2}{\sin 104.04^\circ} = \frac{400lb}{\sin 51.92^\circ}$   
 $R_2 = 493N$

FREE BODY: LOWER WEDGE

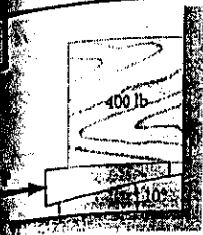
FORCE TRIANGLE



LAW OF SINES

$\frac{P}{\sin(14.04^\circ + 24.04^\circ)} = \frac{493N}{\sin 75.92^\circ}$

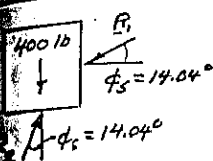
$P = 313.4N$   $P = 313N \leftarrow$



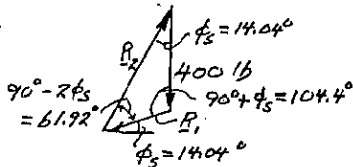
GIVEN: TWO 10° WEDGES  
 $\mu_s = 0.25$

FIND: SMALLEST  $P$   
 TO MOVE WEDGE

BODY: BLOCK



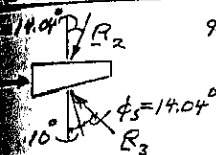
$\phi_s = \tan^{-1} 0.25 = 14.04^\circ$   
 FORCE TRIANGLE



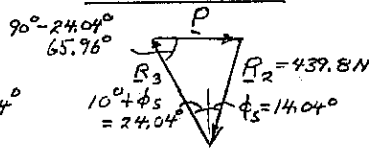
LAW OF SINES

$$\frac{R_2}{\sin 104.4^\circ} = \frac{400 \text{ lb}}{\sin 61.72^\circ} \quad R_2 = 439.8 \text{ N}$$

BODY: WEDGE

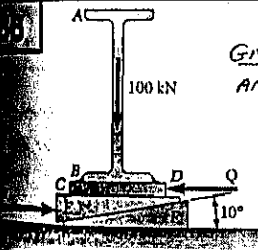


FORCE TRIANGLE



LAW OF SINES

$$\frac{P}{\sin(24.04^\circ + 14.04^\circ)} = \frac{439.8 \text{ N}}{\sin 61.72^\circ} \quad P = 297.0 \text{ N} \quad P = 297 \rightarrow$$

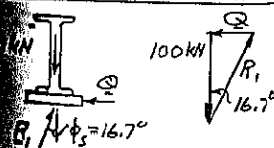


GIVEN:  $\mu_s = 0.60$  BETWEEN STEEL  
 AND CONCRETE

$\mu_s = 0.30$  BETWEEN TWO  
 STEEL SURFACES

FIND: (a)  $P$  TO RAISE BEAM  
 (b) CORRESPONDING  $Q$

BODY: BEAM AND PLATE CD



$\phi_s = \tan^{-1} 0.3 = 16.7^\circ$

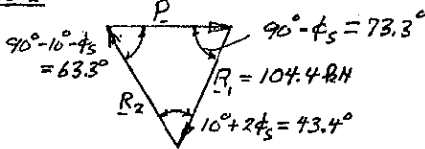
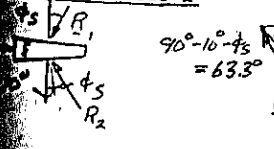
$$Q = (100 \text{ kN}) \tan 16.7^\circ$$

$$Q = 30.8 \text{ kN} \leftarrow$$

$$R_1 = (100 \text{ kN}) / \cos 16.7^\circ$$

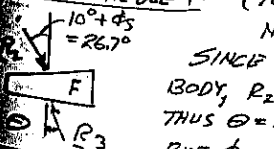
$$R_1 = 104.4 \text{ kN}$$

BODY: WEDGE E



$$\frac{P}{\sin 43.4^\circ} = \frac{104.4 \text{ kN}}{\sin 63.3^\circ}; \quad P = 80.3 \text{ kN} \rightarrow$$

BODY: WEDGE F

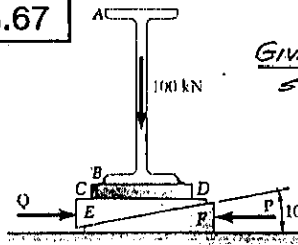


(TO CHECK THAT IT DOES  
 NOT MOVE.)

SINCE WEDGE F IS A TWO-FORCE  
 BODY,  $R_2$  AND  $R_3$  ARE COLINEAR  
 THUS  $\theta = 26.7^\circ$

BUT  $\phi_{\text{CONCRETE}} = \tan^{-1} 0.6 = 31.0^\circ > \theta$  OK

8.67

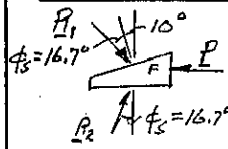


GIVEN:  $\mu_s = 0.60$  BETWEEN  
 STEEL AND CONCRETE

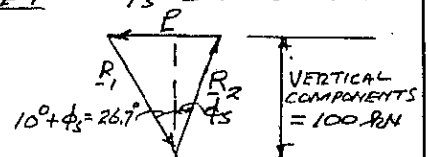
$\mu_s = 0.30$  BETWEEN TWO  
 STEEL SURFACES

FIND: (a)  $P$  TO RAISE BEAM  
 (b) CORRESPONDING  $Q$

FREE BODY: WEDGE F



$\phi_s = \tan^{-1} 0.30 = 16.7^\circ$



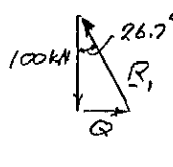
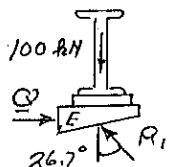
$$P = (100 \text{ kN}) \tan 26.7^\circ + (100 \text{ kN}) \tan \phi_s$$

$$P = 50.29 \text{ kN} + 30.8 \text{ kN}$$

$$P = 80.29 \text{ kN} \quad P = 80.3 \text{ kN} \leftarrow$$

$$R_1 = (100 \text{ kN}) / \cos 26.7^\circ = 111.94 \text{ kN}$$

FREE BODY: BEAM, PLATE, AND WEDGE E



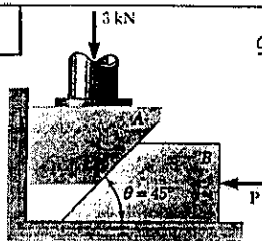
$$Q = R_1 \sin 26.7^\circ$$

$$Q = (111.94 \text{ kN}) \sin 26.7^\circ$$

$$Q = 50.29$$

$$Q = 50.3 \text{ kN} \rightarrow$$

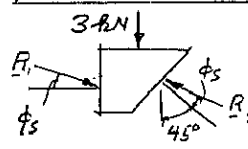
8.68



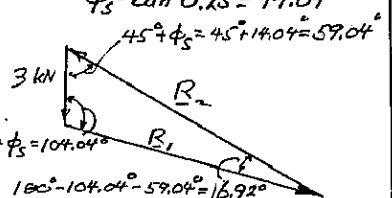
GIVEN:  $\mu_s = 0.25$  AT ALL  
 SURFACES OF CONTACT.

FIND: SMALLEST  $P$  TO  
 RAISE BLOCK A.

FREE BODY: BLOCK A



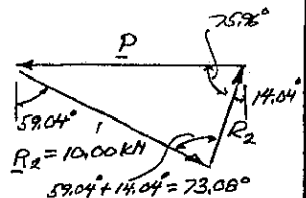
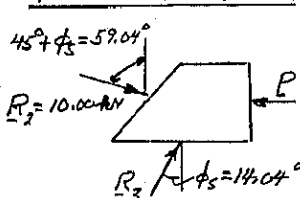
$\phi_s = \tan^{-1} 0.25 = 14.04^\circ$



LAW OF SINES

$$\frac{R_2}{\sin 104.04^\circ} = \frac{3 \text{ kN}}{\sin 16.92^\circ} \quad R_2 = 10.00 \text{ kN}$$

FREE BODY: WEDGE B



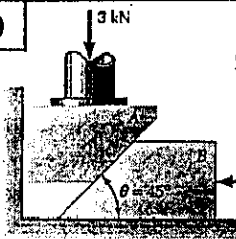
LAW OF SINES

$$\frac{P}{\sin 73.08^\circ} = \frac{10.00 \text{ kN}}{\sin 75.96^\circ}$$

$$P = 9.86 \text{ kN}$$

$$P = 9.86 \text{ kN} \leftarrow$$

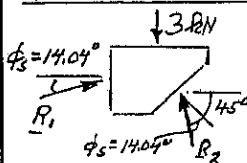
8.69



GIVEN:  $\mu_s = 0.25$  BETWEEN ALL SURFACES OF CONTACT

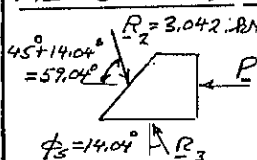
FIND: SMALLEST  $P$  FOR EQUILIBRIUM

FREE BODY: BLOCK A



LAW OF SINES

FREE BODY: WEDGE B

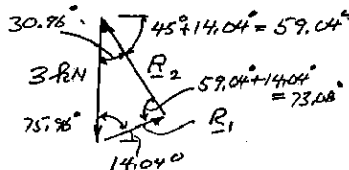


LAW OF SINES

$$\frac{P}{\sin 16.92^\circ} = \frac{3.042 \text{ kN}}{\sin 104.04^\circ}$$

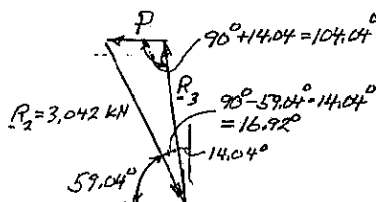
$$P = 0.913 \text{ kN} \quad P = 913 \text{ N} \leftarrow$$

$$\phi_s = \tan^{-1} 0.25 = 14.04^\circ$$

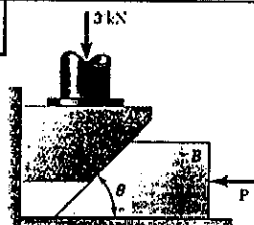


$$\frac{R_2}{\sin 75.96^\circ} = \frac{3 \text{ kN}}{\sin 73.08^\circ}$$

$$R_2 = 3.042 \text{ kN}$$



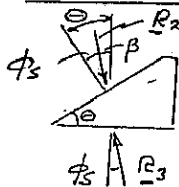
8.70



GIVEN:  $P = 0$ ,  $\mu_s = 0.25$

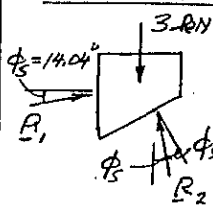
FIND: (a) ANGLE  $\theta$  FOR IMPENDING MOTION  
(b) CORRESPONDING FORCE EXERTED BY WALL.

FREE BODY: WEDGE B



(a) SINCE WEDGE IS A TWO-FORCE BODY,  $R_2$  AND  $R_3$  MUST BE EQUAL AND OPPOSITE, THEREFORE, THEY FORM EQUAL ANGLES WITH VERTICAL ( $\beta = \phi_s$ ) AND  $\theta - \phi_s = \phi_s$   
 $\theta = 2\phi_s = 2(14.04^\circ)$   
 $\theta = 28.1^\circ$

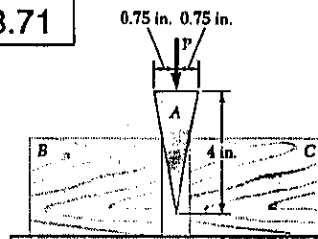
FREE BODY: BLOCK A



$$R_1 = (3 \text{ kN}) \sin 14.04^\circ = 0.7278 \text{ kN}$$

FORCE EXERTED BY WALL:  $R_1 = 728 \text{ N} \angle 14.0^\circ$

8.71



GIVEN: TWO 100-LB BLOCKS  
 $\mu_s = 0.35$  AT ALL SURFACES OF CONTACT

FIND: SMALLEST  $P$  TO START WEDGE MOVING  
(a) IF BOTH B & C CAN MOVE  
(b) IF C CANNOT MOVE

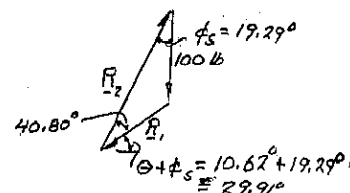
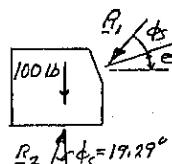


WEDGE ANGLE  $\theta$   
 $\theta = \tan^{-1} \frac{0.75 \text{ in.}}{4 \text{ in.}}$   
 $\theta = 10.62^\circ$

$$\phi_s = \tan^{-1} 0.35 = 19.29^\circ$$

(a)

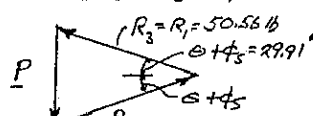
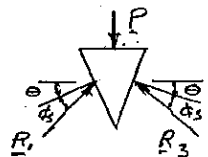
FREE BODY: BLOCK B



$$\frac{P}{\sin 19.29^\circ} = \frac{100 \text{ lb}}{\sin 40.80^\circ}; \quad P = 50.56 \text{ lb}$$

FREE BODY: WEDGE

BY SYMMETRY  $R_3 = R_1$

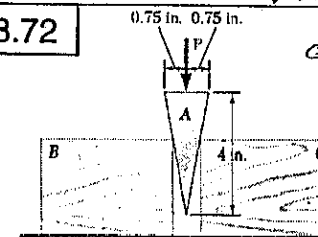


$$P = 2R_1 \sin(\theta + \phi_s) = 2(50.56 \text{ lb}) \sin 29.91^\circ$$

$$P = 50.42 \text{ lb} \quad P = 50.41 \text{ lb} \leftarrow$$

(b) FREE BODIES UNCHANGED. SAME RESULT.  $P = 50.41 \text{ lb} \leftarrow$

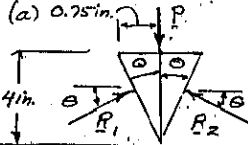
8.72



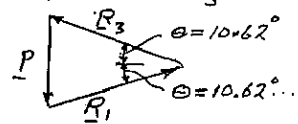
GIVEN: TWO 100-LB BLOCKS  
 $\mu_s = 0.35$  BETWEEN BLOCKS AND FLOOR,  $\mu_s = 0$  AT WEDGE.

FIND: SMALLEST  $P$  TO START WEDGE MOVING  
(a) IF BOTH B AND C CAN MOVE  
(b) IF C CANNOT MOVE

FREE BODY: WEDGE

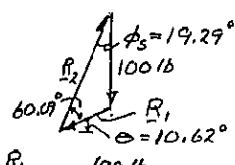
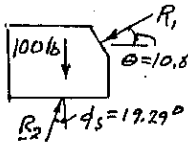


WEDGE ANGLE  $\theta$   
 $\theta = \tan^{-1} \frac{0.75 \text{ in.}}{4 \text{ in.}} = 10.62^\circ$   
 $\phi_s = \tan^{-1} 0.35 = 19.29^\circ$   
BY SYMMETRY  $R_3 = R_1$



$$P = 2R_1 \sin 10.62^\circ \quad (1)$$

FREE BODY: BLOCK B

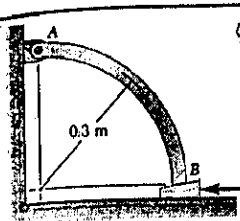


$$\frac{P}{\sin 19.29^\circ} = \frac{100 \text{ lb}}{\sin 60.09^\circ}; \quad R_1 = 38.11 \text{ lb}$$

$$\text{EQ. (1): } P = 2R_1 \sin 10.62^\circ = 2(38.11 \text{ lb}) \sin 10.62^\circ; \quad P = 14.05 \text{ lb}$$

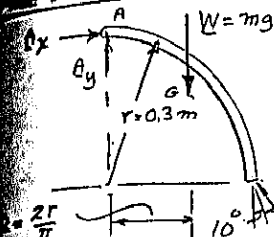
(b) FREE BODIES UNCHANGED. SAME RESULT.  $P = 14.05 \text{ lb} \leftarrow$

8.75



GIVEN: 10° WEDGE  
WEIGHT OF AB = 5 kg  
 $\mu_s = 0.40$  BETWEEN WEDGE AND ROD  
 $\mu_s = 0.20$  BETWEEN WEDGE AND FLOOR  
FIND: SMALLEST P TO MOVE

FREE BODY: ROD AB



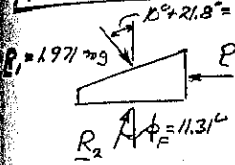
$$\phi_s = \tan^{-1} 0.40 = 21.8^\circ$$

$$\sum M_A = 0: P_1 \cos(10^\circ + \phi_s) r - P_1 \sin(10^\circ + \phi_s) r - mg \left( \frac{2r}{\pi} \right) = 0$$

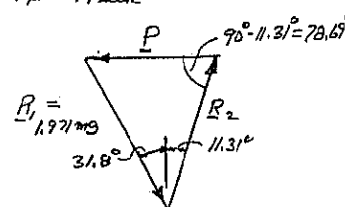
$$P_1 = \frac{2mg}{\pi} \frac{1}{\cos 31.8^\circ - \sin 31.8^\circ}$$

$$P_1 = \frac{2mg}{\pi(0.3229)} = 1.971 mg$$

FREE BODY: WEDGE



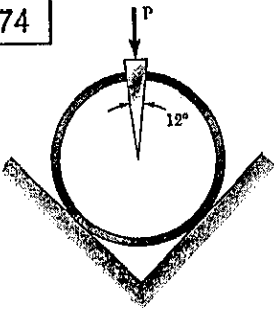
$$\phi_F = \phi_{\text{floor}} = \tan^{-1} 0.20 = 11.31^\circ$$



LAW OF SINES  $\frac{P}{\sin(31.8^\circ + 11.31^\circ)} = \frac{1.971 mg}{\sin 78.69^\circ}$

$$P = 1.374 mg = 1.374(5 \text{ kg})(9.81 \text{ m/s}^2) = 67.4 \text{ N}$$

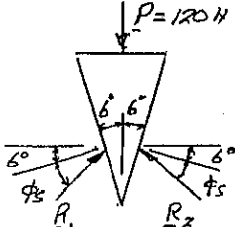
8.74



GIVEN:  $\mu_s = 0.30$ .  
FORCE  $P = 120 \text{ N}$  USED TO INSERT WEDGE

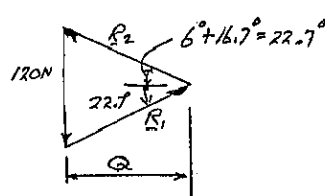
FIND: MAGNITUDE OF FORCES EXERTED ON RING AFTER WEDGE IS INSERTED.

FREE BODY: WEDGE



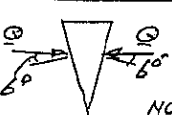
$$\phi_s = \tan^{-1} 0.30 = 16.7^\circ$$

FORCE TRIANGLE



FROM FORCE TRIANGLE: Q = HORIZ. COMPONENT OF R  
 $Q = \frac{1}{2}(120 \text{ N}) / \tan 22.7^\circ = 143.4 \text{ N}$

FREE BODY: AFTER WEDGE HAS BEEN INSERTED

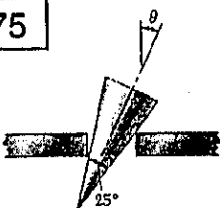


WEDGE IS NOW A TWO-FORCE BODY WITH FORCES SHOWN

$$Q = 143.4 \text{ N}$$

NOTE: SINCE ANGLES BETWEEN FORCES Q AND NORMAL TO WEDGE IS  $6^\circ < \phi_s$ , WEDGE STAYS IN PLACE.

8.75



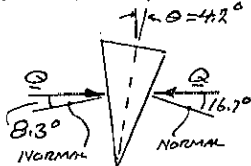
GIVEN: PLATE MOVE TOGETHER

FIND: WHAT HAPPENS TO WEDGE

- (a) IF  $\mu_s = 0.20$ ,  
(b) IF  $\mu_s = 0.30$ .

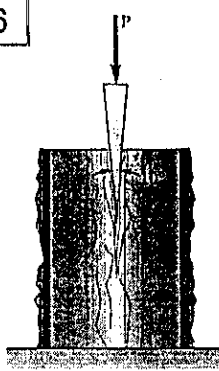
(a) FOR  $\mu_s = 0.20$ ,  $\phi_s = 11.31^\circ$ , REGARDLESS OF HOW WEDGE IS ORIENTED, ON AT LEAST ONE SIDE THE ANGLE BETWEEN THE FACE AND THE HORIZONTAL WILL BE GREATER THAN  $\phi_s$ . THE WEDGE WILL BE FORCED UP AND OUT FROM BETWEEN THE PLATES.

(b) FOR  $\mu_s = 0.30$ ,  $\phi_s = 16.7^\circ$ . AS THE PLATES ARE MOVED TOGETHER,  $\theta$  WILL BECOME SMALLER. AT  $\theta = 4.2^\circ$ , THE POSITION SHOWN IS REACHED.



AT THIS POSITION THE LARGEST ANGLE BETWEEN  $\phi_s$  AND THE NORMAL TO THE WEDGE IS  $16.7^\circ$ . THE WEDGE WILL SELF LOCK.

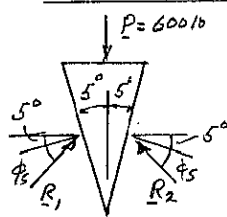
8.76



GIVEN:  $\mu_s = 0.35$   
FORCE  $P = 600 \text{ lb}$  REQUIRED TO INSERT WEDGE

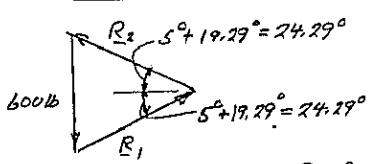
FIND: MAGNITUDE OF FORCES EXERTED ON WOOD BY WEDGE AFTER INSERTION

FREE BODY: WEDGE



$$\phi_s = \tan^{-1} 0.35 = 19.29^\circ$$

FORCE TRIANGLE

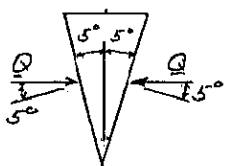


Q = HORIZONTAL COMPONENT OF  $P_1$  &  $R_2$

$$Q = \frac{1}{2}(600 \text{ lb}) / \tan 24.29^\circ$$

$$Q = 664.7 \text{ lb}$$

FREE BODY: AFTER WEDGE HAS BEEN INSERTED

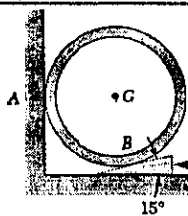


WEDGE IS NOW A TWO-FORCE BODY. FORCE F EXERTED ON WOOD IS EQUAL AND OPPOSITE TO Q.

$$F = 664 \text{ lb}$$

NOTE: SINCE THE  $5^\circ$  ANGLES SHOWN ARE LESS THAN  $\phi_s$ , WEDGE STAYS IN PLACE

8.77



GIVEN: MASS OF PIPE = 50 kg

$$\mu_s = 0.20$$

AT ALL SURFACES

(a) SHOW THAT SLIPPING

OCCURS FIRST AT A

(b) FIND: FORCE P TO CAUSE MOTION

FREE BODY: PIPE

$$+\sum M_B = 0:$$

$$W r \sin \theta + F_A r (1 + \sin \theta) - N_A r \cos \theta = 0$$

ASSUME SLIPPING AT A:  $F_A = \mu_s N_A$ 

$$N_A \cos \theta - \mu_s N_A (1 + \sin \theta) = W \sin \theta$$

$$N_A = \frac{W \sin \theta}{\cos \theta - \mu_s (1 + \sin \theta)}$$

$$N_A = \frac{W \sin 15^\circ}{\cos 15^\circ - (0.20)(1 + \sin 15^\circ)} = 0.3624 W$$

$$+\sum F_x = 0: -F_B - W \sin \theta - F_A \sin \theta + N_A \cos \theta = 0$$

$$F_B = N_A \cos \theta - \mu_s N_A \sin \theta - W \sin \theta$$

$$F_B = 0.3624 W \cos 15^\circ - 0.20(0.3624 W) \sin 15^\circ - W \sin 15^\circ$$

$$F_B = 0.072462 W$$

$$+\sum F_y = 0: N_B - W \cos \theta - F_A \cos \theta - N_A \sin \theta = 0$$

$$N_B = N_A \sin \theta + \mu_s N_A \cos \theta + W \cos \theta$$

$$N_B = (0.3624 W) \sin 15^\circ + 0.20(0.3624 W) \cos 15^\circ + W \cos 15^\circ$$

$$N_B = 1.12974 W$$

$$\text{MAX. AVAILABLE } F_B = \mu_s N_B = 0.22535 W$$

WE NOTE THAT  $F_B < F_{\text{max}}$   $\therefore$  NO SLIP AT B

FREE BODY: WEDGE

$$+\sum F_x = 0: N_2 - N_B \sin \theta + F_B \sin \theta = 0$$

$$N_2 = N_B \cos \theta - F_B \sin \theta$$

$$N_2 = (1.12974 W) \cos 15^\circ - (0.07246 W) \sin 15^\circ$$

$$N_2 = 1.07249 W$$

$$+\sum F_y = 0: F_B \cos \theta + N_B \sin \theta + \mu_s N_2 - P = 0$$

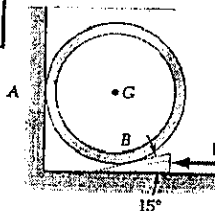
$$P = F_B \cos \theta + N_B \sin \theta + \mu_s N_2$$

$$P = (0.07246 W) \cos 15^\circ + (1.12974 W) \sin 15^\circ + 0.2(1.07249 W)$$

$$P = 0.5769 W$$

$$P = 0.5769(50 \text{ kg})(9.81 \text{ m/s}^2) \quad P = 283 \text{ N}$$

8.78



GIVEN: MASS OF PIPE = 50 kg

$$\mu_s = 0.20$$

FIND: LARGEST  $\mu_s$  AT A

FOR WHICH SLIPPING WILL OCCUR AT A.

FREE BODY: PIPE

$$+\sum M_A = 0$$

$$N_B r \cos \theta - \mu_s N_B r - (\mu_s N_B \sin \theta) r - W r = 0$$

$$N_B = \frac{W}{\cos \theta - \mu_s (1 + \sin \theta)}$$

$$N_B = \frac{W}{\cos 15^\circ - 0.2(1 + \sin 15^\circ)}$$

$$N_B = 1.4002 W$$

$$+\sum F_y = 0: N_A - N_B \sin \theta - \mu_s N_B \cos \theta = 0$$

$$N_A = N_B (\sin \theta + \mu_s \cos \theta)$$

$$= (1.4002 W) (\sin 15^\circ + 0.2 \cos 15^\circ)$$

$$N_A = 0.63292 W$$

(CONTINUED)

8.78 CONTINUED

$$+\sum F_y = 0: -F_A - W + N_B \cos \theta - \mu_s N_B \sin \theta = 0$$

$$F_A = N_B (\cos \theta - \mu_s \sin \theta) - W$$

$$F_A = (1.4002 W) (\cos 15^\circ - 0.2 \sin 15^\circ) - W$$

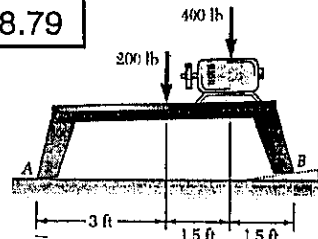
$$F_A = 0.28001 W$$

FOR SLIPPING AT A:  $F_A = \mu_s N_A$ 

$$\mu_s = \frac{F_A}{N_A} = \frac{0.28001 W}{0.63292 W}$$

$$\mu_s = 0.442$$

8.79

GIVEN:  $\mu_s = 0.15$ 

$$\mu_s = 0.15$$

FIND: (a) FORCE P TO

MOVE THE WEDGE.

(b) DOES MACHINE

BASE MOVE?

FREE BODY:

MACHINE BASE

$$+\sum M_B = 0: (200 \text{ lb})(3 \text{ ft}) + (400 \text{ lb})(1.5 \text{ ft}) - A_y(6 \text{ ft}) = 0$$

$$A_y = 200 \text{ lb}$$

$$+\sum F_y = 0: A_y + B_y - 200 \text{ lb} - 400 \text{ lb} = 0$$

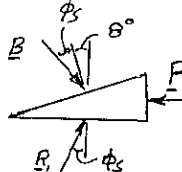
$$200 \text{ lb} + B_y - 200 \text{ lb} - 400 \text{ lb} = 0$$

$$B_y = 400 \text{ lb}$$

FREE BODY: WEDGE

(ASSUME MACHINE BASE WILL NOT MOVE)

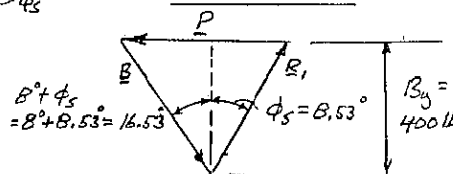
$$\mu_s = 0.15, \phi_s = \tan^{-1} 0.15 = 8.53^\circ$$



WE KNOW THAT

$$B_y = 400 \text{ lb}$$

FORCE TRIANGLE



$$P = (400 \text{ lb}) \tan 16.53^\circ + (400 \text{ lb}) \tan 8.53^\circ; P = 178.7 \text{ lb}$$

TOTAL MAXIMUM FRICTION FORCE AT A AND B

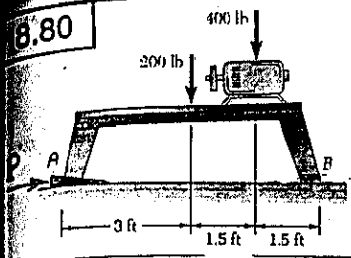
$$F_m = \mu_s W = 0.15(200 \text{ lb} + 400 \text{ lb}) = 90 \text{ lb}$$

 $\therefore$  IF MACHINE MOVES WITH WEDGE  $P = F_m = 90 \text{ lb}$ USING SMALLER  $P$ , WE HAVE:

$$(a) \quad P = 90 \text{ lb}$$

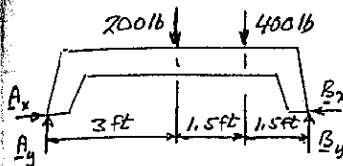
(b) MACHINE BASE MOVES

8.80

GIVEN:  $\theta^\circ$  WEDGE

$$\mu_s = 0.15$$

FIND: (a) FORCE  $P$  TO  
MOVE THE WEDGE  
(b) DOES MACHINE  
BASE MOVE?



FREE BODY: MACHINE BASE

$$+\circlearrowleft \sum M_B = 0: (200 \text{ lb})(3 \text{ ft})$$

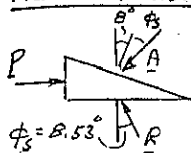
$$+ (400 \text{ lb})(1.5 \text{ ft}) - A_y(6 \text{ ft}) = 0$$

$$A_y = 200 \text{ lb} \uparrow$$

$$+\rightarrow \sum F_x = 0: A_x + B_x - 200 \text{ lb} - 400 \text{ lb} = 0$$

$$B_x = 400 \text{ lb} \rightarrow$$

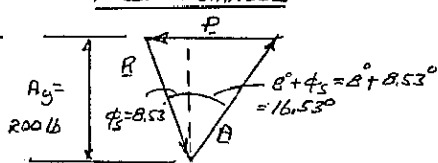
FREE BODY: WEDGE



$$\phi_s = \tan^{-1} 0.15 = 8.53^\circ$$

WE KNOW THAT  $A_y = 200 \text{ lb}$ 

FORCE TRIANGLE



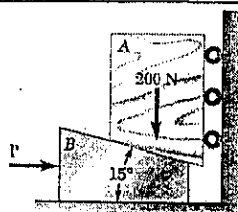
$$P = (200 \text{ lb}) \tan 8.53^\circ + (200 \text{ lb}) \tan 16.53^\circ; \quad P = 89.4 \text{ lb}$$

TOTAL MAX. FRICTION FORCE AT A AND B:

$$F_m = \mu_s (W) = 0.15(200 \text{ lb} + 400 \text{ lb}) = 90 \text{ lb}$$

SINCE  $P < F_m$ , MACHINE BASE WILL NOT MOVE

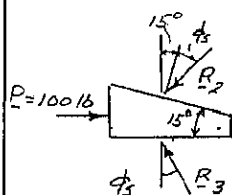
\* 8.81 and 8.82

GIVEN:  $P = 100 \text{ lb}$ 

FIND: VALUE OF  $\mu_s$  FOR  
IMENDING MOTION

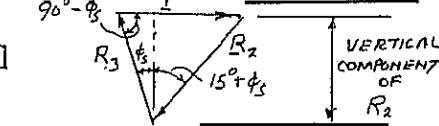
PROB. 8.81: FOR SYSTEM SHOWN

PROB. 8.82: AFTER ROLLERS  
ARE REMOVED



FREE BODY: WEDGE

FORCE TRIANGLE



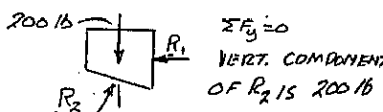
LAW OF SINES

$$\frac{R_2}{\sin(90^\circ - \phi_s)} = \frac{P}{\sin(15^\circ + 2\phi_s)}$$

$$R_2 = P \frac{\sin(90^\circ - \phi_s)}{\sin(15^\circ + 2\phi_s)} \quad (1)$$

PROB. 8.81

FREE BODY: BLOCK

RETURN TO FORCE TRIANGLE OF WEDGE. NOTE  $P = 100 \text{ lb}$ 

$$100 \text{ lb} = (200 \text{ lb}) \tan 15^\circ + (200 \text{ lb}) \tan(15^\circ + \phi_s)$$

$$0.5 = \tan 15^\circ + \tan(15^\circ + \phi_s)$$

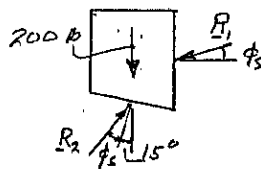
SOLVE BY TRIAL AND ERROR  $\phi_s = 6.301^\circ$ 

$$\mu_s = \tan \phi_s = \tan 6.301^\circ; \quad \mu_s = 0.110$$

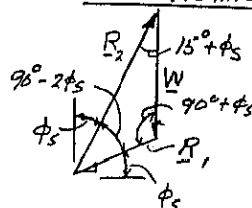
(CONTINUED)

\* 8.81 and 8.82 CONTINUED

PROB. 8.82: FREE BODY: BLOCK (ROLLERS REMOVED)



FORCE TRIANGLE



LAW OF SINES

$$\frac{R_2}{\sin(90^\circ + \phi_s)} = \frac{W}{\sin(90^\circ - 2\phi_s)}$$

$$R_2 = W \frac{\sin(90^\circ + \phi_s)}{\sin(90^\circ - 2\phi_s)} \quad (2)$$

EQUATE  $R_2$  FROM EQ. (1) AND EQ. (2):

$$P \frac{\sin(90^\circ - \phi_s)}{\sin(15^\circ + 2\phi_s)} = W \frac{\sin(90^\circ + \phi_s)}{\sin(90^\circ - 2\phi_s)}$$

$$P = 100 \text{ lb}; \quad W = 200 \text{ lb}; \quad 0.5 = \frac{\sin(90^\circ + \phi_s) \sin(15^\circ + 2\phi_s)}{\sin(90^\circ - 2\phi_s) \sin(90^\circ - \phi_s)}$$

SOLVE BY TRIAL AND ERROR:  $\phi_s = 5.784^\circ$ 

$$\mu_s = \tan \phi_s = \tan 5.784^\circ$$

$$\mu_s = 0.101$$

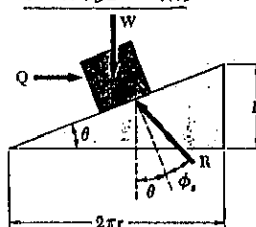
8.83

FOR THE JACK OF SEC. 8.6 (page 41B)

DERIVE FORMULAS FOR FORCE  $P$  FOR  
CASES LISTED BELOW

FROM SEC. 8.6:  $P = \frac{r}{a} Q$ 

(a) TO RAISE LOAD

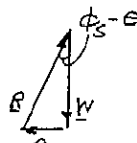
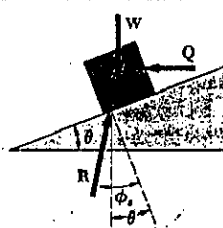


$$Q = W \tan(\theta + \phi_s)$$

$$P = \frac{r}{a} Q$$

$$P = \frac{Wr}{a} \tan(\theta + \phi_s)$$

(b) TO LOWER LOAD

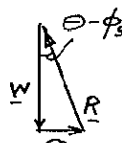
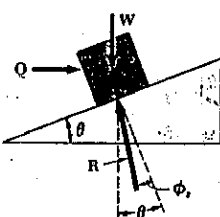


$$Q = W \tan(\phi_s - \theta)$$

$$P = \frac{r}{a} Q$$

$$P = \frac{Wr}{a} \tan(\phi_s - \theta)$$

(c) TO HOLD LOAD (JACK IS NOT SELF LOCKING)

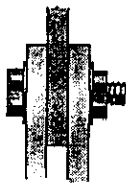


$$Q = W \tan(\theta - \phi_s)$$

$$P = \frac{r}{a} Q$$

$$P = \frac{Wr}{a} \tan(\theta - \phi_s)$$

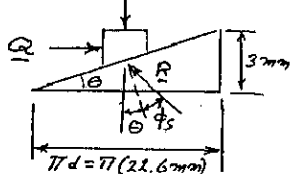
8.84



GIVEN: THREAD DIAMETER  $T_2 = 22.6 \text{ mm}$   
 LEAD  $= 3 \text{ mm}$ ,  $\mu_s = 0.40$ ,  
 TENSION  $= 210 \text{ kN}$   
 FIND: REQUIRED TORQUE

## BLOCK-AND-INCLINE ANALYSIS OF BOLT AND NUT:

$$W = 210 \text{ kN}$$



$$\tan \theta = \frac{3 \text{ mm}}{\pi(22.6 \text{ mm})}$$

$$\theta = 2.42^\circ$$

$$\phi_s = \tan^{-1} 0.40 = 21.8^\circ$$

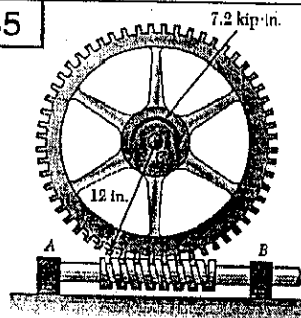
$$Q = (210 \text{ kN}) \tan 24.22^\circ$$

$$Q = 94.47 \text{ kN}$$

$$\text{TORQUE} = QR = (94.47 \text{ kN}) \left( \frac{22.6 \text{ k} \cdot \text{m}}{2} \right)$$

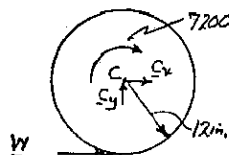
$$\text{TORQUE} = 1068 \text{ N} \cdot \text{m}$$

8.85



GIVEN: MEAN RADIUS  $= 1.5 \text{ in.}$ ,  
 LEAD  $= 0.375 \text{ in.}$ ,  
 $\mu_s = 0.12$ ,  
 FIND: TORQUE APPLIED  
 TO SHAFT REQUIRED  
 TO ROTATE GEAR  
 COUNTERCLOCKWISE.

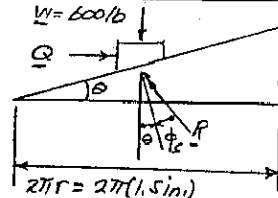
## FREE BODY: LARGE GEAR



$$+\sum M_C = 0: W(12 \text{ in.}) - 7200 \text{ lb} \cdot \text{in.} = 0$$

$$W = 600 \text{ lb}$$

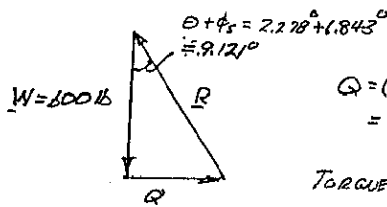
## BLOCK-AND-INCLINE ANALYSIS OF WORM GEAR



$$\tan \theta = \frac{0.375 \text{ in.}}{2\pi(1.5 \text{ in.})}$$

$$\theta = 2.278^\circ$$

$$\phi_s = \tan^{-1} 0.12 = 6.843^\circ$$



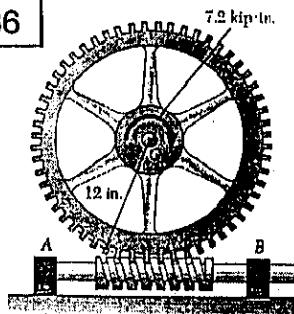
$$Q = (600 \text{ lb}) \tan 9.121^\circ$$

$$= 96.33 \text{ lb}$$

$$\text{TORQUE} = QR = (96.33 \text{ lb})(1.5 \text{ in.})$$

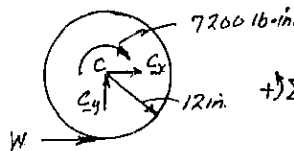
$$\text{TORQUE} = 144.5 \text{ lb} \cdot \text{in.}$$

8.86



GIVEN: MEAN RADIUS  $= 1.5 \text{ in.}$ ,  
 LEAD  $= 0.375 \text{ in.}$ ,  
 $\mu_s = 0.12$ ,  
 FIND: TORQUE APPLIED  
 TO SHAFT REQUIRED  
 TO ROTATE THE GEAR  
 CLOCKWISE.

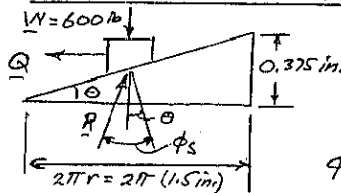
## FREE BODY: LARGE GEAR



$$+\sum M_C = 0: W(12 \text{ in.}) - 7200 \text{ lb} \cdot \text{in.} = 0$$

$$W = 600 \text{ lb}$$

## BLOCK-AND-INCLINE ANALYSIS OF WORM GEAR



$$\tan \theta = \frac{0.375 \text{ in.}}{2\pi(1.5 \text{ in.})}$$

$$\theta = 2.278^\circ$$

$$\phi_s = \tan^{-1} 0.12 = 6.843^\circ$$

$$Q = (600 \text{ lb}) \tan 4.565^\circ$$

$$= 47.91 \text{ lb}$$

$$\text{TORQUE} = QR = (47.91 \text{ lb})(1.5 \text{ in.})$$

$$\text{TORQUE} = 71.9 \text{ lb} \cdot \text{in.}$$

8.87 and 8.88

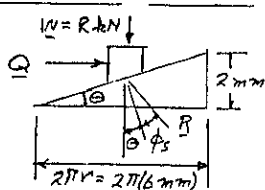
GIVEN: MEAN RADIUS  $= 6 \text{ mm}$ ,  
 PITCH  $= 2 \text{ mm}$ ,  
 $\mu_s = 0.12$ .



## FIND: COUPLE REQUIRED TO ROTATE SLEEVE

PROB 8.87: ROD A, RIGHT-HANDED THREAD, ROD B, LEFT-HANDED  
 PROB 8.88: RIGHT-HANDED THREAD AT BOTH A AND B

## TO DRAW RODS TOGETHER, SCREW AT A



$$\tan \theta = \frac{2 \text{ mm}}{2\pi(6 \text{ mm})} \quad \theta = 3.037^\circ$$

$$\phi_s = \tan^{-1} 0.12 = 6.843^\circ$$

$$Q = (2 \text{ kN}) \tan 9.88^\circ = 348.3 \text{ N}$$

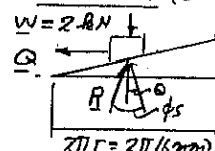
$$\text{TORQUE AT A} = QR$$

$$= (348.3 \text{ N})(6 \text{ mm}) = 2.09 \text{ N} \cdot \text{m}$$

SAME TORQUE REQUIRED AT B

PROB 8.87: TOTAL TORQUE  $= 418 \text{ N} \cdot \text{m}$ 

FOR BOTH THREADS RIGHT HANDED (RODS DO NOT MOVE)

SCREW AT A (SEE ABOVE) TORQUE AT A  $= 2.09 \text{ N} \cdot \text{m}$ 

SCREW AT B (LOOSENING)

SEE ABOVE:  $\theta = 3.037^\circ$ 

$$\phi_s = 6.843^\circ$$

$$\phi_s - \theta = 6.843^\circ - 3.037^\circ = 3.806^\circ \quad Q = (2 \text{ kN}) \tan 3.806^\circ = 133.1 \text{ N}$$

$$\text{TORQUE AT B} = QR$$

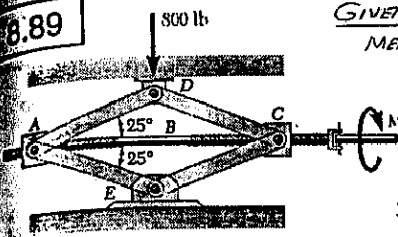
$$= (133.1 \text{ N})(6 \text{ mm}) = 0.798 \text{ N} \cdot \text{m}$$

$$\text{TOTAL TORQUE} = 2.09 \text{ N} \cdot \text{m} + 0.798 \text{ N} \cdot \text{m}$$

$$\text{PROB 8.88: TOTAL TORQUE} = 2.89 \text{ N} \cdot \text{m}$$



8.89



GIVEN: PITCH = 0.1 in.,  
MEAN DIAMETER = 0.375 in.,  
 $\mu_s = 0.15$

FIND: COUPLE  $M$   
REQUIRED TO  
RAISE AUTOMOBILE.

FREE BODY: PARTS A, D, C, E  
TWO-FORCE MEMBERS

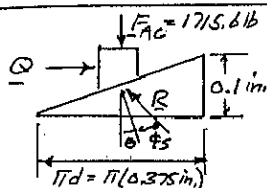
JOINT D:

SYMMETRY:  $F_{AD} = F_{CD}$   
 $\sum F_x = 0: 2F_{ED} \sin 25^\circ - 800 \text{ lb} = 0$   
 $F_{ED} = 946.5 \text{ lb}$

JOINT C:

SYMMETRY:  $F_{CE} = F_{CD}$   
 $\sum F_x = 0: 2F_{ED} \cos 25^\circ - F_{AC} = 0$   
 $F_{AC} = 2(946.5 \text{ lb}) \cos 25^\circ$   
 $F_{AC} = 1715.6 \text{ lb}$

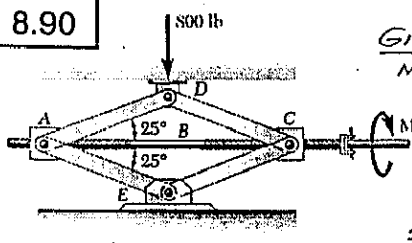
BLOCK-AND-INCLINE ANALYSIS OF ONE SCREW:



$\tan \theta = \frac{0.1 \text{ in.}}{\pi(0.375 \text{ in.})}$ ;  $\theta = 4.852^\circ$   
 $\phi_s = \tan^{-1} 0.15 = 8.531^\circ$

$\theta + \phi_s = 4.852^\circ + 8.531^\circ = 13.383^\circ$   
 $Q = (1715.6 \text{ lb}) \tan 13.383^\circ$   
 $Q = 408.2 \text{ lb}$   
 BUT, WE HAVE TWO SCREWS  
 $\text{TORQUE} = 2QR = 2(408.2 \text{ lb}) \left( \frac{0.375 \text{ in.}}{2} \right)$   
 $\text{TORQUE} = 153.1 \text{ lb} \cdot \text{in.}$

8.90

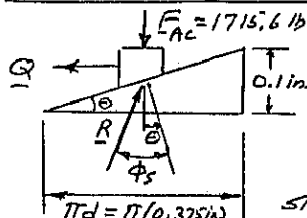


GIVEN: PITCH = 0.1 in.,  
MEAN DIAMETER = 0.375 in.,  
 $\mu_s = 0.15$ .

FIND: COUPLE  $M$   
REQUIRED TO  
LOWER AUTOMOBILE.

SEE SOLUTION OF PROB 8.89 FOR ANALYSIS OF  
LINKAGE ADCE,  $F_{AC} = 1715.6 \text{ lb}$

BLOCK-AND INCLINE ANALYSIS OF ONE SCREW:

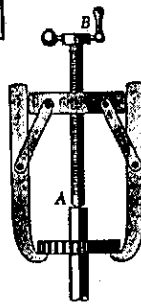


$\tan \theta = \frac{0.1 \text{ in.}}{\pi(0.375 \text{ in.})}$ ;  $\theta = 4.852^\circ$   
 $\phi_s = \tan^{-1} 0.15 = 8.531^\circ$

SINCE  $\phi_s > \theta$ , THE SCREW  
IS SELF LOCKING

$\phi_s - \theta = 8.531^\circ - 4.852^\circ = 3.679^\circ$   
 $Q = (1715.6 \text{ lb}) \tan 3.679^\circ$   
 $Q = 110.3 \text{ lb}$   
 FOR TWO SCREWS:  
 $\text{TORQUE} = 2(110.3 \text{ lb}) \left( \frac{0.375 \text{ in.}}{2} \right)$   
 $\text{TORQUE} = 41.4 \text{ lb} \cdot \text{in.}$

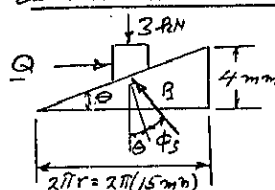
8.91



GIVEN: LEAD = 4 mm,  
MEAN RADIUS = 15 mm,  
 $\mu_s = 0.10$ ,  
FORCE TO BE APPLIED  
TO GEAR = 3 kN.

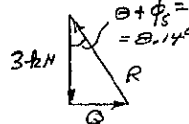
FIND: TORQUE THAT MUST  
BE APPLIED TO SCREW.

BLOCK-AND-INCLINE ANALYSIS OF SCREW



$\tan \theta = \frac{4 \text{ mm}}{2\pi(15 \text{ mm})}$ ;  $\theta = 2.43^\circ$

$\phi_s = \tan^{-1} 0.10 = 5.71^\circ$

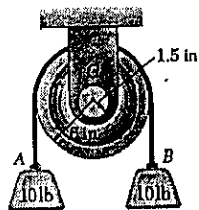


$Q = (3 \text{ kN}) \tan 8.14^\circ = 429 \text{ N}$

$\text{TORQUE} = QR = (429 \text{ N})(0.015 \text{ m})$

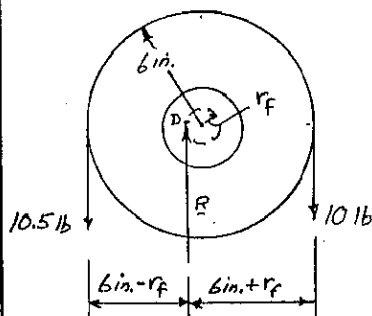
$\text{TORQUE} = 6.44 \text{ N} \cdot \text{m}$

8.92



GIVEN: PULLEY WEIGHS 5 lb.

FIND: COEFFICIENT OF STATIC  
FRICTION IF A 0.5-lb  
WEIGHT ADDED TO BLOCK A  
STARTS ROTATION.



$\sum M_B = 0: (10.5 \text{ lb})(6 \text{ in.} - r_f) - (10 \text{ lb})(6 \text{ in.} + r_f) = 0$   
 $(0.5 \text{ lb})(6 \text{ in.}) = (20.5 \text{ lb})r_f$   
 $r_f = 0.14634 \text{ in.}$

$r_f = r \sin \phi_s$

$\sin \phi_s = \frac{0.14634 \text{ in.}}{1.5 \text{ in.}} = 0.09756$

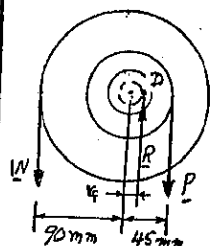
$\phi_s = 5.5987^\circ$

$\mu_s = \tan \phi_s = \tan 5.5987^\circ$   
 $\mu_s = 0.09803$

$\mu_s = 0.098$

# 8.93 through 8.96

FOR EACH PULLEY: RADIUS OF SHAFT,  $r = 10 \text{ mm}$   
 $\mu_s = 0.40$ ,  $\phi_s = \tan^{-1} 0.40 = 21.8^\circ$   
 $r_f = r \sin \phi_s \approx r \mu_s = (10 \text{ mm})(0.40) = 4 \text{ mm}$   
 $W = mg = (20 \text{ kg})(9.81 \text{ m/s}^2) = 196.2 \text{ N}$   
**PROB. 8.93: FIND  $P$  REQUIRED TO START RAISING LOAD**

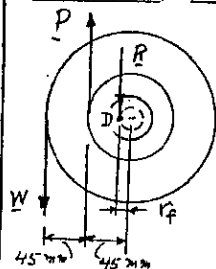


$$+\sum M_D = 0: P(45 - r_f) - W(90 + r_f) = 0$$

$$P = W \frac{90 + r_f}{45 - r_f} = (196.2 \text{ N}) \frac{90 \text{ mm} + 4 \text{ mm}}{45 \text{ mm} - 4 \text{ mm}}$$

$$P = 449.8 \text{ N} \quad \underline{P = 450 \text{ N} \downarrow}$$

**PROB. 8.94 FIND  $P$  REQUIRED TO START RAISING LOAD**

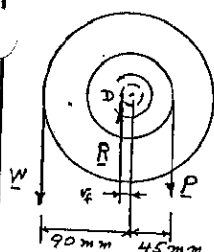


$$+\sum M_D = 0: P(45 - r_f) - W(90 - r_f) = 0$$

$$P = W \frac{90 - r_f}{45 - r_f} = (196.2 \text{ N}) \frac{90 \text{ mm} - 4 \text{ mm}}{45 \text{ mm} - 4 \text{ mm}}$$

$$P = 411.54 \text{ N} \quad \underline{P = 412 \text{ N} \downarrow}$$

**PROB. 8.95 FIND SMALLEST  $P$  TO MAINTAIN EQUILIBRIUM**

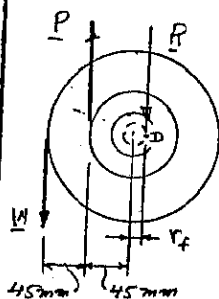


$$+\sum M_D = 0: P(45 + r_f) - W(90 - r_f) = 0$$

$$P = W \frac{90 - r_f}{45 + r_f} = (196.2 \text{ N}) \frac{90 \text{ mm} - 4 \text{ mm}}{45 \text{ mm} + 4 \text{ mm}}$$

$$P = 344.3 \text{ N} \quad \underline{P = 344 \text{ N} \downarrow}$$

**PROB. 8.96 FIND SMALLEST  $P$  TO MAINTAIN EQUILIBRIUM**

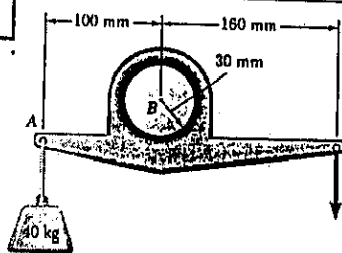


$$+\sum M_D = 0: P(45 + r_f) - W(90 + r_f) = 0$$

$$P = W \frac{90 + r_f}{45 + r_f} = (196.2 \text{ N}) \frac{90 \text{ mm} + 4 \text{ mm}}{45 \text{ mm} + 4 \text{ mm}}$$

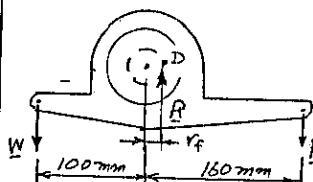
$$P = 376.4 \text{ N} \quad \underline{P = 376 \text{ N} \uparrow}$$

## 8.97



(a) LEVER MOVES  $\rightarrow$  FOR  $P = 275 \text{ N}$ , FIND  $\mu_s$ .  
 (b) FIND SMALLEST  $P$  TO PREVENT ROTATION.

(a) IMPENDING MOTION  $\rightarrow$   
 $r = 30 \text{ mm}$



$$W = (40 \text{ kg})(9.81 \text{ m/s}^2) = 392.4 \text{ N}$$

$$+\sum M_D = 0: P(160 - r_f) - W(100 + r_f) = 0$$

$$r_f = \frac{160P - 100W}{P + W}$$

$$r_f = \frac{(160 \text{ mm})(275 \text{ N}) - (100 \text{ mm})(392.4 \text{ N})}{275 \text{ N} + 392.4 \text{ N}}$$

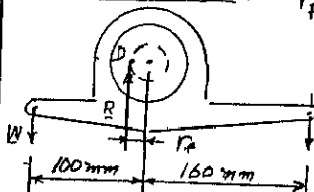
$$r_f = 7.132 \text{ mm}$$

$$r_f = r \sin \phi_s \approx r \mu_s$$

$$\mu_s = \frac{r_f}{r} = \frac{7.132 \text{ mm}}{30 \text{ mm}} = 0.2377$$

$$\mu_s = 0.24$$

(b) IMPENDING MOTION  $\rightarrow$



$$r_f = r \sin \phi_s \approx r \mu_s = (30 \text{ mm})(0.2377)$$

$$r_f = 7.132 \text{ mm}$$

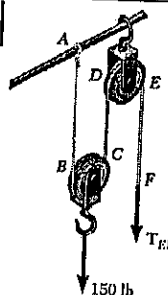
$$+\sum M_D = 0: P(160 + r_f) - W(100 - r_f) = 0$$

$$P = W \frac{100 - r_f}{160 + r_f}$$

$$P = (392.4 \text{ N}) \frac{100 \text{ mm} - 7.132 \text{ mm}}{160 \text{ mm} + 7.132 \text{ mm}}$$

$$P = 218.04 \text{ N} \quad \underline{P = 218 \text{ N} \downarrow}$$

## 8.98

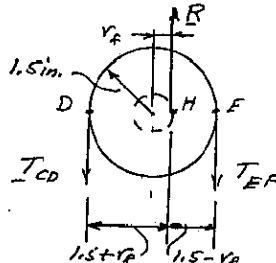
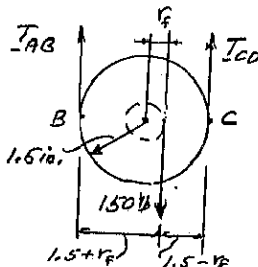


GIVEN: 3-in.-DIAMETER PULLEYS ON 0.5-in.-DIAMETER AXES.  
 $\mu_s = 0.20$

FIND: TENSION IN EACH PORTION OF ROPE AS LOAD IS LOWERED.

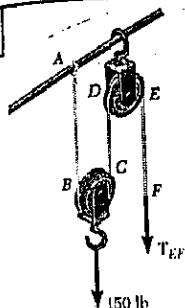
FOR EACH PULLEY: AXLE DIAMETER = 0.5 in.

$$r_f = r \sin \phi_s \approx r \mu_s = 0.20 \left( \frac{0.5 \text{ in.}}{2} \right) = 0.05 \text{ in.}$$



PULLEY BC:  $+\sum M_B = 0: T_{CD}(3 \text{ in.}) - (150 \text{ lb})(1.5 \text{ in.} + r_f) = 0$   
 $T_{CD} = \frac{1}{3}(150 \text{ lb})(1.5 \text{ in.} + 0.05 \text{ in.})$ ;  $T_{CD} = 77.5 \text{ lb}$   
 $+\sum F_y = 0: T_{AB} + 77.5 \text{ lb} - 150 \text{ lb} = 0$ ;  $T_{AB} = 72.5 \text{ lb}$   
 PULLEY DE:  $+\sum M_D = 0: T_{CD}(1.5 + r_f) - T_{EF}(1.5 - r_f) = 0$   
 $T_{EF} = T_{CD} \frac{1.5 + r_f}{1.5 - r_f} = (77.5 \text{ lb}) \frac{1.5 \text{ in.} + 0.05 \text{ in.}}{1.5 \text{ in.} - 0.05 \text{ in.}}$   
 $T_{EF} = 82.8 \text{ lb}$

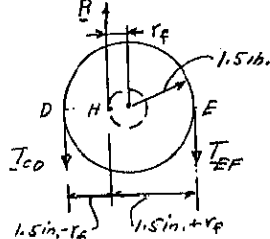
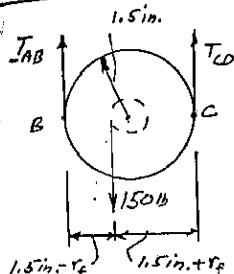
8.99



GIVEN: 3-in. DIAMETER PULLEYS ON 0.5-in. DIAMETER AXES,  $\mu_s = 0.20$ .

FIND: TENSION IN EACH PORTION OF ROPE AS LOAD IS LOWERED.

FOR EACH PULLEY:  $r_f = r \mu_s = \left(\frac{0.5 \text{ in.}}{2}\right) 0.2 = 0.05 \text{ in.}$



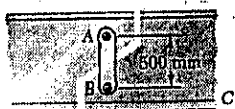
PULLEY BC:  $\sum M_B = 0: T_{CD}(3 \text{ in.}) - (150 \text{ lb})(1.5 \text{ in.} - r_f) = 0$   
 $T_{CD} = \frac{(150 \text{ lb})(1.5 \text{ in.} - 0.05 \text{ in.})}{3 \text{ in.}} = 77.5 \text{ lb}$

$\sum F_y = 0: T_{AB} + 77.5 \text{ lb} - 150 \text{ lb} = 0$   
 $T_{AB} = 72.5 \text{ lb}$

PULLEY DE:  $T_{CD}(1.5 \text{ in.} - r_f) - T_{EF}(1.5 \text{ in.} + r_f) = 0$

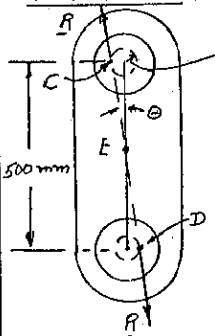
$T_{EF} = T_{CD} \frac{1.5 \text{ in.} - r_f}{1.5 \text{ in.} + r_f} = (77.5 \text{ lb}) \frac{1.5 \text{ in.} - 0.05 \text{ in.}}{1.5 \text{ in.} + 0.05 \text{ in.}} = 67.8 \text{ lb}$

8.100



GIVEN: 60-mm-DIAMETER PINS AT A AND B,  $\mu_s = 0.20$ , LOAD = 200 kN. FIND: (a) HORIZ. FORCE

AT C REQUIRED TO JUST MOVE THE LINK, (b) ANGLE THAT RESULTING FORCE EXERTED ON LINK WILL FORM WITH VERTICAL.

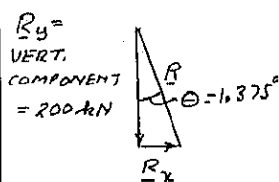
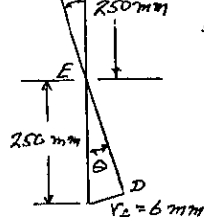


BEARING:  $r = 30 \text{ mm}$

$r_f = \mu_s r = 0.20(30 \text{ mm}) = 6 \text{ mm}$

RESULTANT FORCES  $R$  MUST BE TANGENT TO FRICTION CIRCLES AT POINTS C AND D.

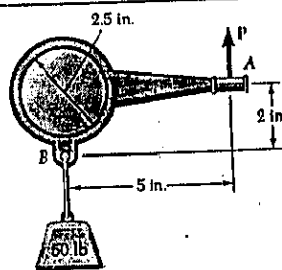
$\sin \theta = \frac{6 \text{ mm}}{250 \text{ mm}}$   
 $\sin \theta = 0.024$   
 $\theta = 1.375^\circ$



$R_x = R_y \tan \theta$   
 $= (200 \text{ kN}) \tan 1.375^\circ$   
 $= 4.80 \text{ kN}$

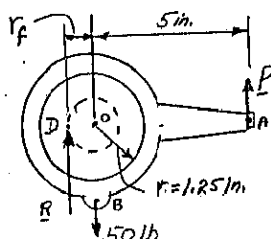
HORIZ. FORCE = 4.80 kN

8.101



GIVEN:  $\mu_s = 0.15$ .

FIND: FORCE  $P$  REQUIRED TO START COUNTERCLOCKWISE ROTATION.



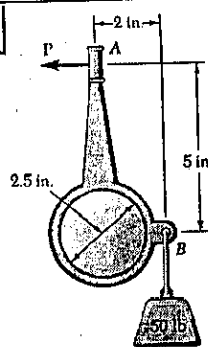
$r_f = \mu_s r = 0.15(1.25 \text{ in.}) = 0.1875 \text{ in.}$

$\sum M_D = 0$   
 $P(5 \text{ in.} + r_f) - (50 \text{ lb})r_f = 0$

$P = \frac{50(0.1875)}{5.1875} = 1.807 \text{ lb}$

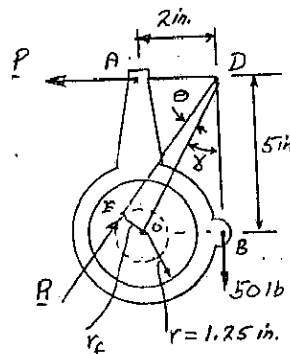
$P = 1.807 \text{ lb} \uparrow$

8.102



GIVEN:  $\mu_s = 0.15$ .

FIND: FORCE  $P$  REQUIRED TO START COUNTERCLOCKWISE ROTATION.



$r_f = \mu_s r = 0.15(1.25 \text{ in.}) = 0.1875 \text{ in.}$

$\tan \gamma = \frac{2 \text{ in.}}{5 \text{ in.}}$   
 $\gamma = 21.80^\circ$

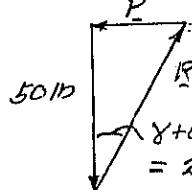
IN  $\triangle EOD$ :

$OD = \sqrt{(2 \text{ in.})^2 + (5 \text{ in.})^2} = 5.385 \text{ in.}$

$\sin \theta = \frac{OE}{OD} = \frac{r_f}{OD} = \frac{0.1875 \text{ in.}}{5.385 \text{ in.}}$

$\theta = 1.995^\circ$

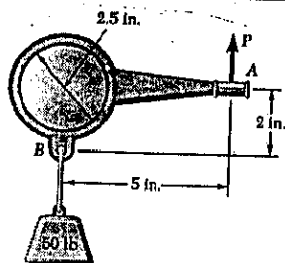
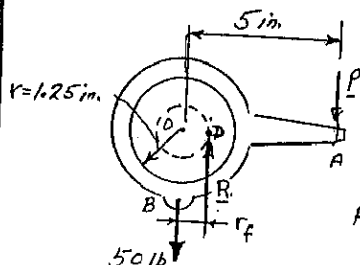
FORCE TRIANGLE



$P = (50 \text{ lb}) \tan(\gamma + \theta)$   
 $= (50 \text{ lb}) \tan 23.796^\circ$   
 $= 22.049 \text{ lb}$

$P = 22.0 \text{ lb} \leftarrow$

8.103

GIVEN:  $\mu_s = 0.15$ .FIND: FORCE P  
REQUIRED TO START  
CLOCKWISE ROTATION.

$$r_f = \mu_s r = 0.15(1.25 \text{ in})$$

$$r_f = 0.1875 \text{ in.}$$

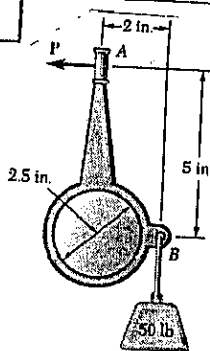
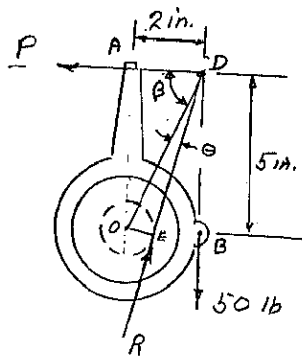
$$+\circlearrowright \Sigma M_D = 0$$

$$P(5 \text{ in.} - r_f) - (50 \text{ lb})r_f = 0$$

$$P = \frac{50(0.1875)}{5 - 0.1875} = 1.942 \text{ lb}$$

$$P = 1.942 \text{ lb} \downarrow$$

8.104

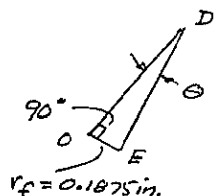
GIVEN:  $\mu_s = 0.15$ .FIND: FORCE P  
REQUIRED TO START  
CLOCKWISE ROTATION

$$r_f = \mu_s r = 0.15(1.25 \text{ in})$$

$$= 0.1875 \text{ in.}$$

$$\tan \beta = \frac{5 \text{ in.}}{2 \text{ in.}}$$

$$\beta = 68.198^\circ$$

IN  $\triangle EOD$ :

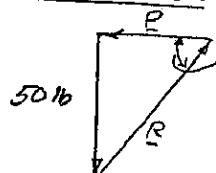
$$OD = \sqrt{(2 \text{ in.})^2 + (5 \text{ in.})^2}$$

$$OD = 5.3852 \text{ in.}$$

$$\sin \theta = \frac{OE}{OD} = \frac{0.1875 \text{ in.}}{5.3852 \text{ in.}}$$

$$\theta = 1.994^\circ$$

FORCE TRIANGLE



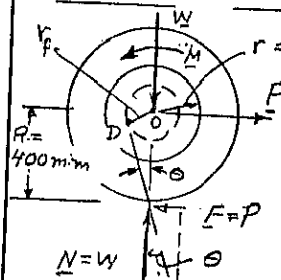
$$\beta + \theta = 68.198^\circ + 1.994^\circ$$

$$= 70.192^\circ$$

$$P = 50 / \tan(\beta + \theta) = \frac{50 \text{ lb}}{\tan 70.192^\circ}$$

$$P = 18.01 \text{ lb} \leftarrow$$

8.105

GIVEN: RAILROAD CAR OF MASS 30 Mg ON  
EIGHT 800-mm-DIAMETER WHEELS WITH  
125-mm-DIAMETER AXLES.  $\mu_s = 0.020$ ,  $\mu_k = 0.015$ .  
FIND: HORIZONTAL FORCE REQUIRED (a) TO START  
CAR MOVING, (b) TO KEEP IT MOVING.

$$\sin \theta = \tan \theta = \frac{r_f}{R} = \frac{r_f}{R}$$

$$P = W \tan \theta = W \frac{r_f}{R}$$

$$P = W \frac{62.5 \text{ mm}}{400 \text{ mm}} = 0.15625 W$$

FOR ONE WHEEL:

$$W = \frac{1}{8}(30 \text{ Mg})(9.81 \text{ m/s}^2) = \frac{1}{8}(294.3 \text{ kN})$$

FOR EIGHT WHEELS OF RAILROAD CAR

$$\Sigma P = 8(0.15625) \left( \frac{1}{8}(294.3 \text{ kN}) \right) = (45.984 \text{ kN})$$

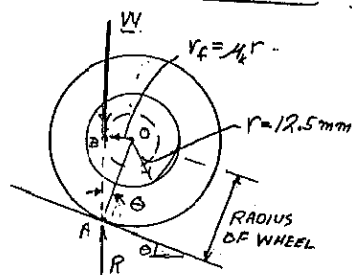
(a) TO START MOTION:  $\mu_s = 0.020$ 

$$\Sigma P = (45.984)(0.020) = 0.9197 \text{ kN}; \quad \Sigma P = 920 \text{ N} \leftarrow$$

(b) TO MAINTAIN MOTION:  $\mu_k = 0.015$ 

$$\Sigma P = (45.984)(0.015) = 0.6897 \text{ kN}; \quad \Sigma P = 690 \text{ N} \leftarrow$$

8.106

GIVEN: SCOOTER IS TO ROLL DOWN  
A 2 PERCENT SLOPE AT CONSTANT SPEED.  
AXLES OF WHEELS ARE 25 mm IN DIAMETER,  $\mu_k = 0.10$ .  
FIND: REQUIRED DIAMETER OF WHEELS.

$$\tan \theta = \frac{2}{100} = 0.02$$

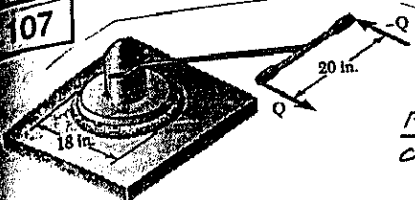
SINCE SCOOTER ROLLS AT CONSTANT SPEED, EACH  
WHEEL IS IN EQUILIBRIUM. THUS W AND R  
MUST HAVE COMMON LINE OF ACTION TANGENT  
TO THE FRICTION CIRCLE.

$$r_f = \mu_k r = (0.10)(12.5 \text{ mm}) = 1.25 \text{ mm}$$

$$OA = \frac{OB}{\tan \theta} = \frac{r_f}{\tan \theta} = \frac{1.25 \text{ mm}}{0.02} = 62.5 \text{ mm}$$

$$\text{DIAMETER OF WHEEL} = 2(OA) = 125 \text{ mm} \leftarrow$$

107



GIVEN:  $\mu_k = 0.25$   
50-lb FLOOR  
POLISHER  
FIND: MAGNITUDE  
OF FORCES  $Q$

SEE FIG. 8.12 (page 343) AND EQ. 8.9 (page 344)  
USING:  $R = 9$  in.,  $P = 50$  lb, AND  $\mu_k = 0.25$

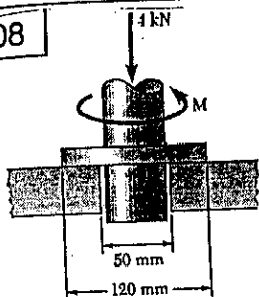
$$M = \frac{2}{3} \mu_k P R = \frac{2}{3} (0.25) (50 \text{ lb}) (9 \text{ in.}) = 75 \text{ lb} \cdot \text{in.}$$

$$\sum M_y = 0 \text{ YIELDS: } M = Q(20 \text{ in.})$$

$$75 \text{ lb} \cdot \text{in.} = Q(20 \text{ in.})$$

$$Q = 3.75 \text{ lb}$$

8.108



GIVEN: COUPLE  
 $M = 30 \text{ N} \cdot \text{m}$   
REQUIRED TO START  
ROTATION

FIND:  $\mu_s$

SEE FIG. 8.12 (page 343) AND EQ. 8.8 (page 344).

$$\text{USING: } R_1 = 25 \text{ mm} = 0.025 \text{ m}$$

$$R_2 = 60 \text{ mm} = 0.060 \text{ m}$$

$$P = 4,000 \text{ N}, \quad M = 30 \text{ N} \cdot \text{m}$$

$$M = \frac{2}{3} \mu_s P \frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}$$

$$30 \text{ N} \cdot \text{m} = \frac{2}{3} \mu_s (4,000 \text{ N}) \frac{(0.060 \text{ m})^3 - (0.025 \text{ m})^3}{(0.060)^2 - (0.025)^2}$$

$$30 \text{ N} \cdot \text{m} = \frac{2}{3} \mu_s (4,000 \text{ N}) (0.00735 \text{ m}); \quad \mu_s = 0.167$$

\* 8.109

FOR SHAFT AND BEARING ASSUME NORMAL  
FORCE PER UNIT AREA IS INVERSELY  
PROPORTIONAL TO  $r$ . SHOW THAT  $M$  IS 75%  
OF VALUE GIVEN BY FORMULA (8.9) ON PAGE 344.

USING FIG. 8.12 (page 343), WE ASSUME

$$\Delta N = \frac{R}{r} \Delta A; \quad \Delta A = r \Delta \theta \Delta r$$

$$\Delta N = \frac{R}{r} r \Delta \theta \Delta r = R \Delta \theta \Delta r$$

WE WRITE,  $P = \sum \Delta N$  OR  $P = \int \Delta N$

$$P = \int_0^{2\pi} \int_{R_1}^{R_2} R \Delta \theta \Delta r = 2\pi R R; \quad R = \frac{P}{2\pi R}$$

$$\Delta N = \frac{P \Delta \theta \Delta r}{2\pi R}$$

$$\Delta M = r \Delta F = \mu_k \Delta N = \mu_k \frac{P \Delta \theta \Delta r}{2\pi R}$$

$$M = \int_0^{2\pi} \int_{R_1}^{R_2} \frac{\mu_k P}{2\pi R} r dr d\theta = \frac{2\pi \mu_k P}{2\pi R} \cdot \frac{R^2}{2} = \frac{1}{2} \mu_k P R$$

FROM EQ. (8.9) FOR A NEW BEARING  $M_{\text{NEW}} = \frac{2}{3} \mu_k P R$

$$\text{THUS } \frac{M}{M_{\text{NEW}}} = \frac{1/2}{2/3} = \frac{3}{4}$$

$$M = 0.75 M_{\text{NEW}}$$

\* 8.110

ASSUMING BEARING WEAR AS GIVEN  
IN PROB. 8.109, SHOW THAT MAGNITUDE  
OF COUPLE TO OVERCOME FRICTION IN A  
WORN-OUT, COLLAR BEARING (SEE FIG. 8.12) IS

$$M = \frac{1}{2} \mu_k P (R_1 + R_2)$$

USING FIG. 8.12 (page 343), WE ASSUME  $\Delta N = \frac{R}{r} \Delta A$

$$\Delta A = r \Delta \theta \Delta r; \quad \Delta N = \frac{R}{r} r \Delta \theta \Delta r = R \Delta \theta \Delta r$$

BUT:  $P = \sum \Delta N$  OR  $P = \int \Delta N$

$$P = \int_0^{2\pi} \int_{R_1}^{R_2} R \Delta \theta \Delta r = 2\pi (R_2 - R_1) R$$

$$\text{THUS, } R = \frac{P}{2\pi (R_2 - R_1)}, \text{ AND } \Delta N = \frac{P \Delta \theta \Delta r}{2\pi (R_2 - R_1)}$$

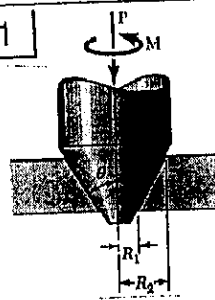
$$\Delta M = r \Delta F = \mu_k \Delta N = \mu_k \frac{P \Delta \theta \Delta r}{2\pi (R_2 - R_1)}$$

$$M = \int_0^{2\pi} \int_{R_1}^{R_2} \frac{\mu_k P}{2\pi (R_2 - R_1)} r dr d\theta = \frac{2\pi \mu_k P}{2\pi (R_2 - R_1)} \cdot \frac{R_2^2 - R_1^2}{2}$$

$$\text{SINCE } R_2^2 - R_1^2 = (R_2 - R_1)(R_2 + R_1)$$

$$M = \frac{1}{2} \mu_k P (R_1 + R_2)$$

\* 8.111



ASSUME: UNIFORM  
PRESSURE BETWEEN  
SURFACES OF CONTACT

SHOW THAT

$$M = \frac{2}{3} \cdot \frac{\mu_k P}{\sin \theta} \cdot \frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}$$

$$\Delta N = R \Delta A$$

$$\Delta A = (r \Delta \phi) \Delta s = (r \Delta \phi) \frac{\Delta r}{\sin \theta}$$

THUS:

$$\Delta N = R \Delta A = \frac{R r}{\sin \theta} \Delta \phi \Delta r$$

VERTICAL COMPONENT OF  $\Delta N$ :

$$(\Delta N)_y = \Delta N \sin \theta = R r \Delta \phi \Delta r$$

$$P = \sum (\Delta N)_y = \sum R r \Delta \phi \Delta r$$

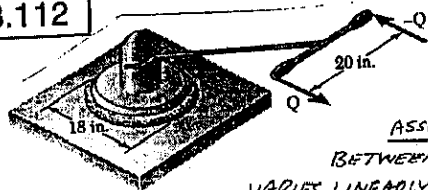
$$\text{OR, USING INTEGRALS } P = \int_0^{2\pi} \int_{R_1}^{R_2} R r dr d\phi = 2\pi R \frac{R^2 - R_1^2}{2}$$

$$\text{THUS, } R = \frac{P}{\pi (R_2^2 - R_1^2)}; \quad \Delta N = \frac{R r}{\sin \theta} \Delta \phi \Delta r = \frac{\mu_k P r^2 \Delta \phi \Delta r}{\pi \sin \theta (R_2^2 - R_1^2)}$$

$$\text{INTEGRATING: } M = \int_0^{2\pi} \int_{R_1}^{R_2} \frac{\mu_k P r^2}{\pi \sin \theta (R_2^2 - R_1^2)} r dr d\phi = \frac{2\pi}{\pi} \cdot \frac{\mu_k P}{\sin \theta} \cdot \frac{R_2^3 - R_1^3}{3(R_2^2 - R_1^2)}$$

$$M = \frac{2}{3} \cdot \frac{\mu_k P}{\sin \theta} \cdot \frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}$$

8.112



GIVEN: POLISHER  
OF WEIGHT = 50 lb  
 $\mu_k = 0.25$

ASSUME: NORMAL FORCE  
BETWEEN FLOOR AND DISK

VARIES LINEARLY FROM A MAXIMUM  
AT CENTER TO ZERO AT EDGE

FIND: MAGNITUDE OF FORCES TO PREVENT MOTION.



$$\frac{\Delta N}{\Delta A} = k \left(1 - \frac{r}{R}\right)$$

$$\Delta A = r \Delta \theta \Delta r$$

$$\Delta N = k \left(1 - \frac{r}{R}\right) r \Delta \theta \Delta r$$

$$P = \sum \Delta N = \int_0^{2\pi} \int_0^R k \left(1 - \frac{r}{R}\right) r dr d\theta = 2\pi k \left[ \frac{r^2}{2} - \frac{r^3}{3R} \right]_0^R$$

$$P = k \frac{\pi R^2}{3}$$

$$\text{THUS: } k = \frac{3P}{\pi R^2} \text{ AND } \Delta N = \frac{3P}{\pi R^2} \left(1 - \frac{r}{R}\right) r \Delta \theta \Delta r$$

MOMENT OF FRICTION FORCE ON  $\Delta A$  IS

$$\Delta M = r \Delta F = \mu_k \Delta N = \frac{3\mu_k P}{\pi R^2} \left(1 - \frac{r}{R}\right) r^2 \Delta \theta \Delta r$$

$$M = \sum \Delta M = \int_0^{2\pi} \int_0^R \frac{3\mu_k P}{\pi R^2} \left(1 - \frac{r}{R}\right) r^2 dr d\theta = \frac{2\pi}{\pi} \frac{3\mu_k P}{R^2} \left[ \frac{r^3}{3} - \frac{r^4}{4R} \right]_0^R$$

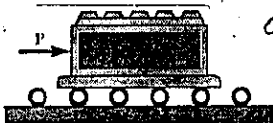
$$M = \frac{1}{2} \mu_k P R$$

$$\mu_k = 0.25, P = 50 \text{ lb}, R = 9 \text{ in}$$

$$M = \frac{1}{2} (0.25) (50 \text{ lb}) (9 \text{ in}) = 56.25 \text{ lb}\cdot\text{in.}$$

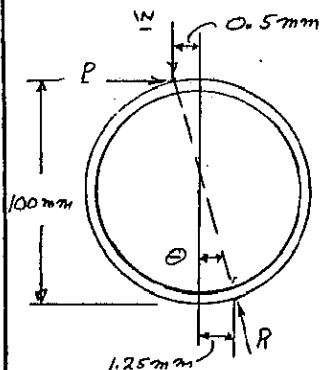
$$\text{BUT: } Q(20 \text{ in.}) = M; Q = \frac{M}{20 \text{ in.}} = \frac{56.25 \text{ lb}\cdot\text{in.}}{20 \text{ in.}}; Q = 2.81 \text{ lb}$$

8.113



GIVEN: 900-lb BASE;  
100-mm DIAMETER  
PIPS, ROLLING  
RESISTANCE IS

0.5 mm BETWEEN PIPS AND BASE + 1.25 mm BETWEEN PIPS  
AND CONCRETE FLOOR. FIND:  $P$  TO MAINTAIN MOTION



$$\tan \theta = \frac{0.5 \text{ mm} + 1.25 \text{ mm}}{100 \text{ mm}}$$

$$\tan \theta = 0.0175$$

$$P = W \tan \theta$$

$$P = 0.0175 W$$

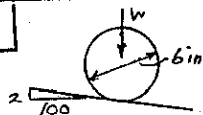
$$W = mg = (900 \text{ kg}) (9.81 \text{ m/s}^2)$$

$$P = (0.0175) (900 \text{ kg}) (9.81 \text{ m/s}^2)$$

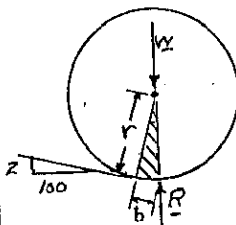
$$P = 154.51 \text{ N}$$

$$P = 154.4 \text{ N}$$

8.114



GIVEN: DISK ROLLS AT  
CONSTANT VELOCITY  
FIND: COEFFICIENT OF  
ROLLING RESISTANCE



DISK IS IN EQUILIBRIUM

SIMILAR TRIANGLES

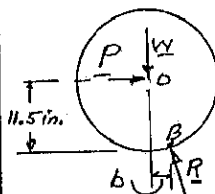
$$\frac{b}{r} = \frac{2}{100}$$

$$b = \frac{2}{100} r = \frac{2}{100} (6 \text{ in.}); b = 0.060 \text{ in.}$$

8.115

GIVEN: 2500-lb AUTOMOBILE WITH  
23-in.-DIAMETER TIRES, COEFFICIENT  
OF ROLLING RESISTANCE = 0.05 in.

FIND: HORIZONTAL FORCE TO MOVE AUTOMOBILE ON  
HORIZONTAL ROAD AT CONSTANT SPEED



$$\sum M_B = 0:$$

$$P(11.5 \text{ in.}) - Wb = 0$$

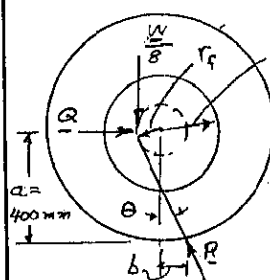
$$P(11.5 \text{ in.}) = (2500 \text{ lb}) (0.05 \text{ in.})$$

$$P = 10.869 \text{ lb}$$

$$P = 10.87 \text{ lb}$$

8.116

GIVEN: 30-Mg RAILROAD CAR ON EIGHT  
800-mm-DIAMETER WHEELS WITH 125-mm AXLES,  
 $\mu_s = 0.020$ ,  $\mu_k = 0.015$ , COEFFICIENT OF ROLLING RESISTANCE 0.5 mm  
FIND: HORIZ. FORCE (a) TO START MOTION, (b) TO MAINTAIN MOTION.



$$r_f = \mu_r r \quad \text{FOR ONE WHEEL}$$

$$\tan \theta \approx \sin \theta = \frac{r_f + b}{a}$$

$$\tan \theta = \frac{r_f + b}{a}$$

$$Q = \frac{W}{8} \tan \theta = \frac{W}{8} \frac{r_f + b}{a}$$

FOR EIGHT WHEELS OF CAR

$$P = W \frac{r_f + b}{a}$$

$$W = mg = (30 \text{ Mg}) (9.81 \text{ m/s}^2) = 294.3 \text{ kN}$$

$$a = 400 \text{ mm}, r = 62.5 \text{ mm}, b = 0.5 \text{ mm}$$

(a) TO START MOTION:  $\mu = \mu_s = 0.020$

$$P = (294.3 \text{ kN}) \frac{(0.020)(62.5 \text{ mm}) + 0.5 \text{ mm}}{400 \text{ mm}}$$

$$P = 1.2876 \text{ kN}$$

$$P = 1.288 \text{ kN}$$

(b) TO MAINTAIN CONSTANT SPEED:  $\mu = \mu_k = 0.015$

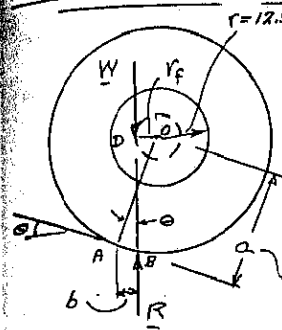
$$P = (294.3 \text{ kN}) \frac{(0.015)(62.5 \text{ mm}) + 0.5 \text{ mm}}{400 \text{ mm}}$$

$$P = 1.0576 \text{ kN}$$

$$P = 1.058 \text{ kN}$$

8.117

GIVEN: SCOOTER IS TO ROLL DOWN A 2 PERCENT SLOPE AT CONSTANT SPEED. RAYLES OF WHEELS ARE 25 mm IN DIAMETER.  $\mu_k = 0.10$ , COEFFICIENT OF ROLLING RESISTANCE = 1.75 mm. FIND: REQUIRED DIAMETER OF WHEELS.



SINCE SCOOTER ROLLS AT CONSTANT SPEED, EACH WHEEL IS IN EQUILIBRIUM. THUS  $W$  AND  $R$  MUST HAVE COMMON LINE OF ACTION TANGENT TO THE FRICTION CIRCLE.

$a$  = RADIUS OF WHEEL

$$\tan \theta = \frac{r_f + b}{a} = 0.02$$

SINCE  $b$  AND  $r_f$  ARE SMALL COMPARED TO  $a$ ,

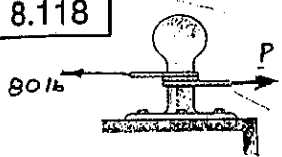
$$\tan \theta \approx \frac{r_f + b}{a} = \frac{\mu_k r + b}{a} = 0.02$$

DATA:  $\mu_k = 0.10$ ,  $b = 1.75 \text{ mm}$ ,  $r = 12.5 \text{ mm}$

$$\frac{(0.10)(12.5 \text{ mm}) + 1.75 \text{ mm}}{a} = 0.02$$

$$a = 150 \text{ mm}; \text{ DIAMETER} = 2a = 300 \text{ mm}.$$

8.118



(a) FOR TWO FULL TURNS OF HAWSER AND

$P = 5000 \text{ lb}$ , FIND  $\mu_s$

(b) FIND NUMBER OF TURNS, IF  $P = 20,000 \text{ lb}$ .

(a)  $\beta = 2 \text{ TURNS} = 2(2\pi) = 4\pi$   
 $T_1 = 80 \text{ lb}$      $T_2 = 5000 \text{ lb}$

$$\ln \frac{T_2}{T_1} = \mu_s \beta \quad \mu_s = \frac{1}{\beta} \ln \frac{T_2}{T_1} = \frac{1}{4\pi} \ln \frac{5000 \text{ lb}}{80 \text{ lb}}$$

$$\mu_s = \frac{1}{4\pi} \ln 62.5 = \frac{4.1351}{4\pi} \quad \mu_s = 0.329$$

(b)  $T_1 = 80 \text{ lb}$ ,  $T_2 = 20,000 \text{ lb}$ ,  $\mu_s = 0.329$

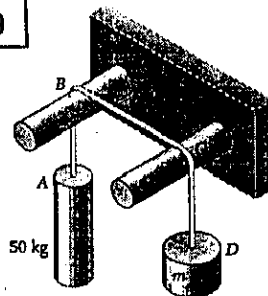
$$\ln \frac{T_2}{T_1} = \mu_s \beta \quad \beta = \frac{1}{\mu_s} \ln \frac{T_2}{T_1} = \frac{1}{0.329} \ln \frac{20,000 \text{ lb}}{80 \text{ lb}}$$

$$\beta = \frac{1}{0.329} \ln(250) = \frac{5.5215}{0.329} = 16.783$$

$$\text{NUMBER OF TURNS} = \frac{16.783}{2\pi}$$

$$\text{NUMBER OF TURNS} = 2.67$$

8.119



GIVEN:  $\mu_s = 0.40$

FIND: RANGE OF MASS  $m$  FOR EQUILIBRIUM

FOR MOTION OF  $A$  IMPENDING DOWNWARD

FOR EACH ROD

$$\beta = \frac{\pi}{2}, \mu_s = 0.4$$

$$W_A = m_A g$$

$$W_D = m g$$

$$\frac{Q}{m_A g} = e^{\mu_s \beta}$$

$$\frac{m g}{Q} = e^{\mu_s \beta}$$

MULTIPLY EQUATIONS MEMBER BY MEMBER

$$\frac{Q}{m_A g} \cdot \frac{m g}{Q} = e^{\mu_s (\beta + \beta)}; \quad \frac{m}{m_A} = e^{0.4(2)\frac{\pi}{2}} = 3.514$$

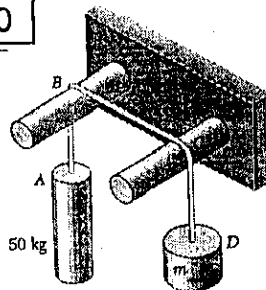
$$m = 3.514 m_A = 3.514(50 \text{ kg}) = 175.7 \text{ kg}$$

FOR MOTION OF  $B$  IMPENDING UPWARD, WE FIND IN A SIMILAR WAY

$$\frac{m_A}{m} = e^{0.4(2)\frac{\pi}{2}} = 3.514; \quad m = \frac{50 \text{ kg}}{3.514} = 14.23 \text{ kg}$$

RANGE FOR EQUILIBRIUM:  $14.23 \text{ kg} \leq m \leq 175.7 \text{ kg}$

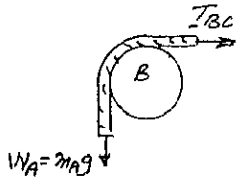
8.120



GIVEN: MOTION OF  $D$  IMPENDS UPWARD WHEN  $m = 20 \text{ kg}$ .

FIND: (a)  $\mu_s$   
 (b) TENSION IN  $BC$

FOR EACH ROD:  $\beta = \frac{\pi}{2}$



$$W_A = m_A g$$

$$\text{EQN: } \frac{m_A g}{T_{BC}} = e^{\mu_s \beta}$$

$$\text{EQ(2): } \frac{T_{BC}}{m g} = e^{\mu_s \beta}$$

MULTIPLY EQUATIONS MEMBER BY MEMBER

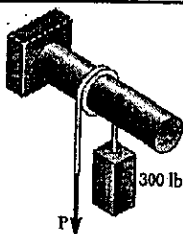
$$\frac{m_A g}{T_{BC}} \cdot \frac{T_{BC}}{m g} = e^{\mu_s (\beta + \beta)}; \quad \frac{m_A}{m} = e^{\mu_s \pi}$$

$$\frac{50 \text{ kg}}{20 \text{ kg}} = e^{\mu_s \pi}; \quad \mu_s \pi = 0.9163; \quad \mu_s = 0.2917$$

$$\text{EQ(2)} \quad \frac{T_{BC}}{(20 \text{ kg})g} = e^{0.2917(\frac{\pi}{2})} = 1.582; \quad T_{BC} = 1.582(20 \text{ kg})(9.81 \text{ m/s}^2)$$

$$T_{BC} = 310 \text{ N}$$

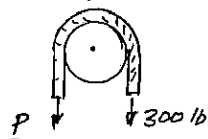
8.121



GIVEN:  $\mu_s = 0.15$   
ROPE WRAPPED  $1\frac{1}{2}$   
TIMES AROUND ROD

FIND: RANGE OF  $P$   
FOR EQUILIBRIUM

$1\frac{1}{2}$  TURNS;  $\beta = 1.5(2\pi) = 3\pi$



FOR MOTION OF 300-LB BLOCK  
IMPENDING UPWARD

$$\frac{P}{300 \text{ lb}} = e^{\mu_s \beta} = e^{0.15(3\pi)}$$

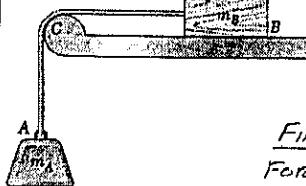
$$\frac{P}{300 \text{ lb}} = 4.111 \quad P = 1233 \text{ lb} \quad \triangleleft$$

FOR MOTION OF BLOCK IMPENDING DOWNWARD

$$\frac{300 \text{ lb}}{P} = e^{\mu_s \beta} = e^{0.15(3\pi)} = 4.111; \quad P = 73.0 \text{ lb} \quad \triangleleft$$

RANGE FOR EQUILIBRIUM:  $73.0 \text{ lb} \leq P \leq 1233 \text{ lb}$   $\triangleleft$

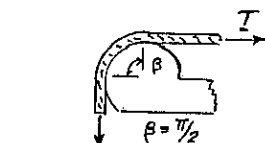
8.122



GIVEN:

$\mu_s = 0.40$   
 $m_A = 12 \text{ kg}$

FIND: SMALLEST  $m_B$   
FOR EQUILIBRIUM.



$$m_A g = m_B g$$

$$\frac{m_A g}{T} = e^{\mu_s \beta}$$

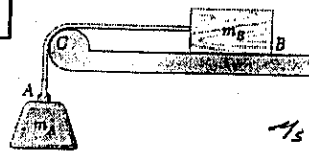
$$T = (m_A g) e^{-\mu_s \beta}$$

$$T = (12 \text{ kg}) g e^{-(0.40) \frac{\pi}{2}} \quad 6.4019 g = (0.40) m_B g$$

$$T = 6.4019 g \quad m_B = \frac{6.4019}{0.40}$$

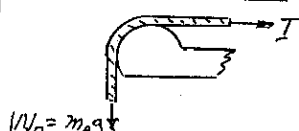
$$m_B = 16.00 \text{ kg} \quad \triangleleft$$

8.123



GIVEN:  $m_A = m_B$

FIND: SMALLEST  
 $\mu_s$  FOR EQUILIBRIUM



$$m_A g = m_B g$$

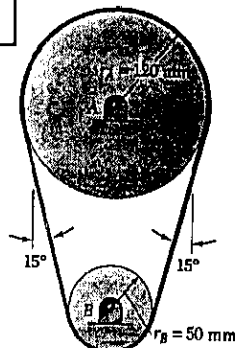
$$\frac{m_A g}{T} = e^{\mu_s \frac{\pi}{2}}$$

$$m_A = m_B = m$$

$$\frac{m}{T} = e^{\mu_s \frac{\pi}{2}}; \quad \mu_s e^{\mu_s \frac{\pi}{2}} = 1$$

SOLVE BY TRIAL AND ERROR:  $\mu_s = 0.475$   $\triangleleft$

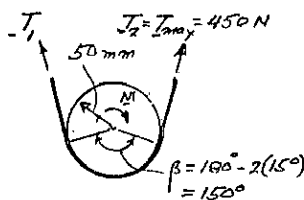
8.124



GIVEN:  $\mu_s = 0.40$   
 $T_{\max} = 450 \text{ N}$

FIND: LARGEST COUPLE  
THAT CAN BE  
EXERTED ON DRUM A

BELT WILL SLIP FIRST AT B, SINCE  $\beta$  AT B IS  
LESS THAN  $\beta$  AT A.



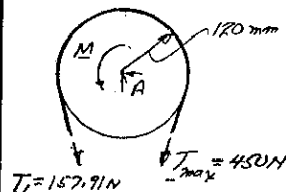
$$\beta = 150^\circ = 150^\circ \frac{\pi}{180^\circ} = \frac{5}{2} \pi \text{ radians}$$

$$\frac{T_2}{T_1} = e^{\mu_s \beta}$$

$$\frac{450 \text{ N}}{T_1} = e^{0.4 \left( \frac{5}{2} \pi \right)} = 2.8497$$

$$T_1 = (450 \text{ N}) / 2.8497 = 157.91 \text{ lb}$$

TORQUE ON DRUM A:



$$+\sum M_A = 0$$

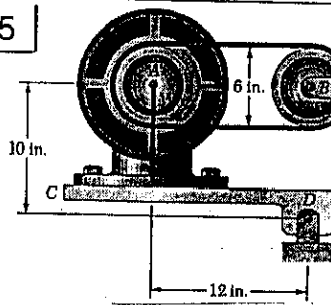
$$M - (T_{\max} - T_1)(0.120 \text{ m})$$

$$M = (450 \text{ N} - 157.91 \text{ N})(0.120 \text{ m})$$

$$M = 35.053 \text{ N}\cdot\text{m}$$

$$M = 35.1 \text{ N}\cdot\text{m} \quad \triangleleft$$

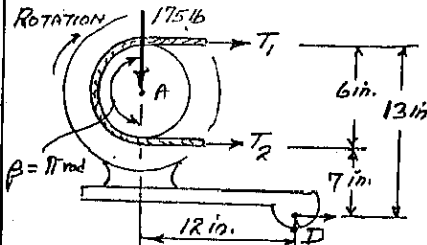
8.125



GIVEN: MOTOR  
MOUNT WEIGHS  
175 lb.

$\mu_s = 0.40$

FIND: LARGEST  
TORQUE TRANS-  
MITTED TO B WHEN  
DRUM A ROTATES  
CLOCKWISE.



$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{0.40 \pi}$$

$$T_2 = 3.5736 T_1$$

$$+\sum M_D = 0: T_1(13 \text{ in.}) + T_2(7 \text{ in.}) - (175 \text{ lb})(12 \text{ in.}) = 0$$

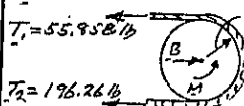
$$T_1(13 \text{ in.}) + 3.5736 T_1(7 \text{ in.}) - 2100 \text{ lb}\cdot\text{in.} = 0$$

$$37.595 T_1 = 2100$$

$$T_1 = 55.858 \text{ lb}$$

$$T_2 = 3.5736(55.858 \text{ lb}) = 196.26 \text{ lb}$$

DRUM B:



$$+\sum M_B = 0$$

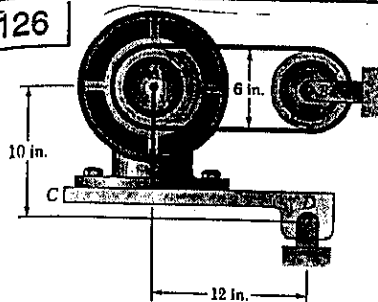
$$M + (55.858 \text{ lb} - 196.26 \text{ lb})(3 \text{ in.}) = 0$$

$$M = 421.2 \text{ lb}\cdot\text{in.}$$

$$M = 421 \text{ lb}\cdot\text{in.} \quad \triangleleft$$

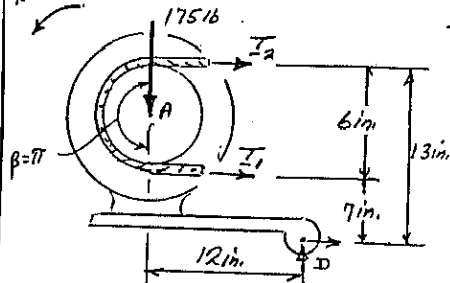


8.126



GIVEN: MOTOR  
MOUNT WEIGHS  
175 lb.  
 $\mu_s = 0.40$   
FIND: LARGEST  
TORQUE TRANS-  
MITTED TO B WHEN  
DRUM A ROTATES  
COUNTERCLOCKWISE

ROTATION



$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{0.4\pi}$$

$$T_2 = 3.5786 T_1$$

$$+\sum M_D = 0: T_1(7 \text{ in.}) + T_2(13 \text{ in.}) - (175 \text{ lb.})(12 \text{ in.}) = 0$$

$$T_1(7 \text{ in.}) + 3.5786 T_1(13 \text{ in.}) - 2100 \text{ lb.} \cdot \text{in.} = 0$$

$$52.677 T_1 = 2100$$

$$T_1 = 39.866 \text{ lb.}$$

$$T_2 = 3.5786(39.866 \text{ lb.}) = 140.072 \text{ lb.}$$

DRUM B:

$$T_2 = 140.072 \text{ lb.}$$

$$T_1 = 39.866 \text{ lb.}$$

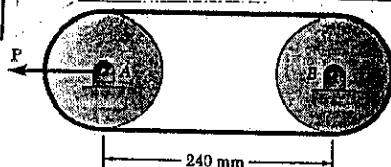
$$+\sum M_B = 0:$$

$$M + (39.866 \text{ lb.} - 140.072 \text{ lb.})(3 \text{ in.}) = 0$$

$$M = 300.6 \text{ lb.} \cdot \text{in.}$$

$$M = 301 \text{ lb.} \cdot \text{in.}$$

8.127



GIVEN:  
60-mm-RADIUS  
PULLEYS,  
 $P = 900 \text{ N}$ ,  
 $\mu_s = 0.35$ .

FIND: (a) LARGEST TORQUE WHICH CAN BE TRANSMITTED.  
(b) MAXIMUM TENSION IN BELT.

DRUM A:

$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{(0.35)\pi}$$

$$T_2 = 3.0028 T_1$$

$$\beta = 180^\circ = \pi \text{ radians}$$

$$+\sum F_x = 0: T_1 + T_2 - 900 \text{ N} = 0$$

$$T_1 + 3.0028 T_1 - 900 \text{ N} = 0$$

$$4.0028 T_1 = 900$$

$$T_1 = 224.841 \text{ N}$$

$$T_2 = 3.0028(224.841 \text{ N}) = 675.15 \text{ N}$$

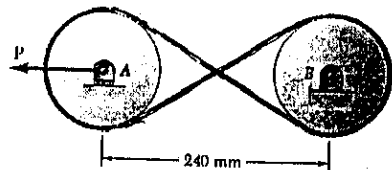
$$T_{\max} = 675 \text{ N}$$

TORQUE  $+\sum M_A = 0:$ 

$$M - (675.15 \text{ N})(0.06 \text{ m}) + (224.841 \text{ N})(0.06 \text{ m}) = 0$$

$$M = 27.0 \text{ N} \cdot \text{m}$$

8.128



GIVEN: 60-mm-RADIUS PULLEYS,  $\mu_s = 0.35$ ,  $P = 900 \text{ N}$   
FIND: (a) LARGEST TORQUE WHICH CAN BE TRANSMITTED  
(b) MAXIMUM TENSION IN BELT.

DRUM A:

$$v = 0.06 \text{ m}$$

$$P = 900 \text{ N}$$

$$\beta = 240^\circ = 240^\circ \frac{\pi}{180^\circ} = \frac{4}{3}\pi$$

$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{0.35(\frac{4}{3}\pi)}$$

$$T_2 = 4.3322 T_1$$

$$+\sum F_x = 0: (T_1 + T_2) \cos 30^\circ - 900 \text{ N} = 0$$

$$(T_1 + 4.3322 T_1) \cos 30^\circ = 900$$

$$T_1 = 194.90 \text{ N}$$

$$T_2 = 4.3322(194.90 \text{ N}) = 844.3 \text{ N}$$

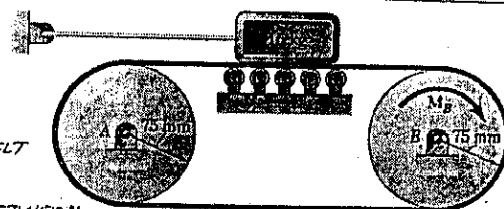
$$T_{\max} = 844 \text{ N}$$

TORQUE:

$$+\sum M_B = 0: M - (844.3 \text{ N})(0.06 \text{ m}) + (194.9 \text{ N})(0.06 \text{ m}) = 0$$

$$M = 39.0 \text{ N} \cdot \text{m}$$

8.129



GIVEN:

$\mu_k = 0.45$   
BETWEEN BELT  
AND BLOCK,  
 $\mu_s = 0.30$  BETWEEN  
BELT AND DRUM. FIND: (a)  $M_B$ , (b)  $T_{\min}$  FOR NO SLIPPING.

BLOCK

$$W = mg = (15 \text{ kg})(9.81 \text{ m/s}^2) = 147.15 \text{ N}$$

$$N = W$$

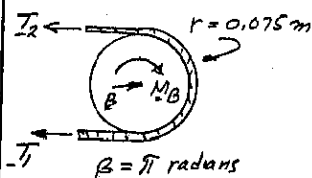
$$F = \mu_k W = 0.45(147.15 \text{ N}) = 66.217 \text{ N}$$

$$T_1 = N = W$$

$$T_2 = N + F$$

PORTION OF BELT  
LOCATED UNDER BLOCK

$$+\sum F_x = 0: T_2 - T_1 - 66.217 \text{ N} = 0 \quad (1)$$



DRUM B:

$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{0.3\pi} = 2.5663$$

$$T_2 = 2.5663 T_1 \quad (2)$$

$$EQ(1): 2.5663 T_1 - T_1 - 66.217 \text{ N} = 0$$

$$1.5663 T_1 = 66.217 \text{ N}$$

$$T_1 = 42.276 \text{ N}; T_{\min} = 42.3 \text{ N}$$

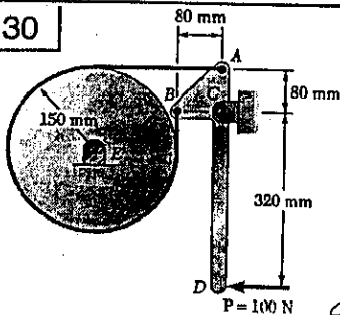
EQ(2):

$$T_2 = 2.5663(42.276 \text{ N}) = 108.493 \text{ N}$$

$$+\sum M_B = 0: M_B - (108.493 \text{ N})(0.075 \text{ m}) + (42.276 \text{ N})(0.075 \text{ m}) = 0$$

$$M_B = 4.97 \text{ N} \cdot \text{m}$$

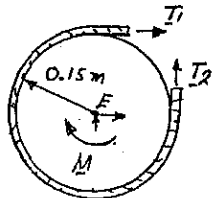
8.130

GIVEN:  $\mu_k = 0.25$ 

FIND: MAGNITUDE OF COUPLE APPLIED TO FLYWHEEL FOR CLOCKWISE ROTATION  
SHOW THAT RESULT IS SAME FOR COUNTERCLOCKWISE ROTATION

FREE BODY: FLYWHEEL

FOR CLOCKWISE ROTATION OF FLYWHEEL  $T_2$  AND  $T_1$  ARE LOCATED AS SHOWN.

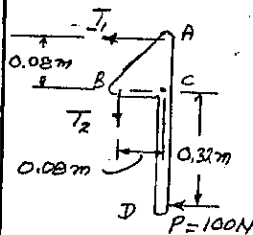


$$\beta = \frac{3}{4}(360^\circ) = \frac{3}{4}(2\pi) = \frac{3}{2}\pi \text{ radians}$$

$$\frac{T_2}{T_1} = e^{\mu_k \beta} = e^{0.25(\frac{3}{2}\pi)} = 3.2482$$

$$T_2 = 3.2482 T_1 \quad (1)$$

FREE BODY: HANDLE



$$+\uparrow \Sigma M_C = 0 \quad (2)$$

$$(T_1 + T_2)(0.08 \text{ m}) - (100 \text{ N})(0.32 \text{ m}) = 0$$

$$(T_1 + 3.2482 T_1) = 400 \text{ N}$$

$$T_1 = (400 \text{ N}) / 4.2482 = 94.157 \text{ N}$$

$$T_2 = 3.2482(94.157 \text{ N}) = 305.842 \text{ N}$$

RETURN TO FREE BODY OF FLYWHEEL

$$+\uparrow \Sigma M_E = 0: M + (T_1 - T_2)(0.15 \text{ m}) = 0$$

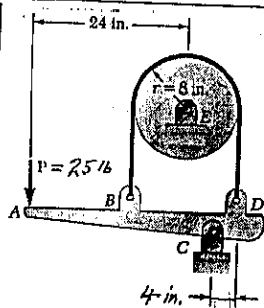
$$M + (94.157 \text{ N} - 305.842 \text{ N})(0.15 \text{ m}) = 0$$

$$M = 31.752 \text{ N}\cdot\text{m}$$

$$M = 31.8 \text{ N}\cdot\text{m}$$

IF ROTATION IS REVERSED (TO BE  $\beta$ )  $T_2$  AND  $T_1$  ARE INTERCHANGED; EQS. (1) AND (2) ARE NOT CHANGED, THUS VALUES OF  $T_1$ ,  $T_2$ , AND  $M$  ARE THE SAME.

8.131

GIVEN:  $\mu_k = 0.25$ 

FIND: MAGNITUDE OF COUPLE APPLIED TO DRUM FOR ROTATION  
(a) COUNTERCLOCKWISE  
(b) CLOCKWISE

(a) COUNTERCLOCKWISE ROTATION FREE BODY DRUM

$$r = 8 \text{ in.}, \beta = 180^\circ = \pi \text{ radians}$$

$$\frac{T_2}{T_1} = e^{\mu_k \beta} = e^{0.25\pi} = 2.1933$$

$$T_2 = 2.1933 T_1$$

FREE BODY: CONTROL BAR

$$+\uparrow \Sigma M_C = 0$$

$$T_1(12 \text{ in.}) - T_2(4 \text{ in.}) - (25 \text{ lb})(28 \text{ in.}) = 0$$

$$T_1(12) - 2.1933 T_1(4) - 700 = 0$$

$$T_1 = 216.93 \text{ lb}$$

$$T_2 = 2.1933(216.93 \text{ lb}) = 475.80 \text{ lb}$$

(CONTINUED)

8.131 CONTINUED

RETURN TO FREE BODY OF DRUM

$$+\uparrow \Sigma M_E = 0: M + T_1(8 \text{ in.}) - T_2(8 \text{ in.}) = 0$$

$$M + (216.93 \text{ lb})(8 \text{ in.}) - (475.80 \text{ lb})(8 \text{ in.}) = 0$$

$$M = 2070.9 \text{ lb}\cdot\text{in.}$$

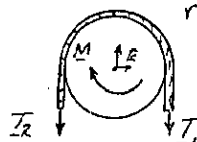
$$M = 2070 \text{ lb}\cdot\text{in.}$$

(b) CLOCKWISE ROTATION

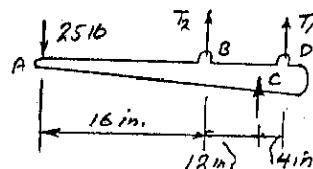
$$r = 8 \text{ in.}, \beta = \pi \text{ rad}$$

$$\frac{T_2}{T_1} = e^{\mu_k \beta} = e^{0.25\pi} = 2.1933$$

$$T_2 = 2.1933 T_1$$



FREE BODY: CONTROL ROD



$$+\uparrow \Sigma M_C = 0$$

$$T_2(12 \text{ in.}) - T_1(4 \text{ in.}) - (25 \text{ lb})(28 \text{ in.}) = 0$$

$$2.1933 T_1(12) - T_1(4) - 700 = 0$$

$$T_1 = 31.363 \text{ lb}$$

$$T_2 = 2.1933(31.363 \text{ lb})$$

$$T_2 = 68.788 \text{ lb}$$

RETURN TO FREE BODY OF DRUM

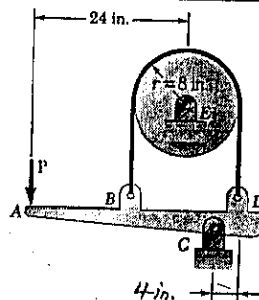
$$+\uparrow \Sigma M_E = 0: M + T_1(8 \text{ in.}) - T_2(8 \text{ in.}) = 0$$

$$M + (31.363 \text{ lb})(8 \text{ in.}) - (68.788 \text{ lb})(8 \text{ in.}) = 0$$

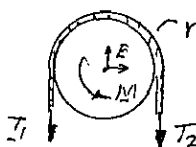
$$M = 299.4 \text{ lb}\cdot\text{in.}$$

$$M = 299 \text{ lb}\cdot\text{in.}$$

8.132



FIND: MAXIMUM  $\mu_s$  FOR BRAKE TO BE SELF LOCKING FOR COUNTERCLOCKWISE ROTATION OF DRUM

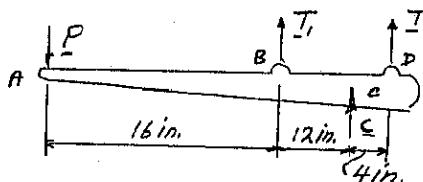


$$r = 8 \text{ in.}, \beta = 180^\circ = \pi \text{ radians}$$

$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{\mu_s \pi}$$

$$T_2 = e^{\mu_s \pi} T_1$$

FREE BODY: CONTROL ROD



$$+\uparrow \Sigma M_C = 0: P(28 \text{ in.}) - T_1(12 \text{ in.}) + T_2(4 \text{ in.}) = 0$$

$$28P - 12T_1 + e^{\mu_s \pi} T_1(4) = 0$$

FOR SELF-LOCKING BRAKE  $P = 0$ 

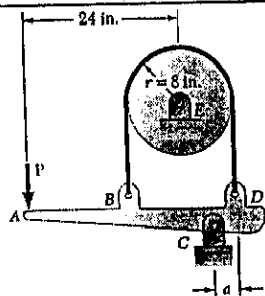
$$12T_1 = 4T_1 e^{\mu_s \pi}$$

$$e^{\mu_s \pi} = 3 \quad \mu_s \pi = \ln 3 = 1.0986$$

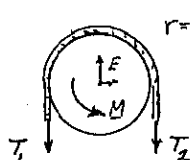
$$\mu_s = \frac{1.0986}{\pi} = 0.3497$$

$$\mu_s = 0.350$$

8.133



GIVEN:  $\mu_s = 0.30$   
ROTATION.  
FIND: MINIMUM  
VALUE OF  $a$  FOR  
WHICH BRAKE  
IS NOT SELF-  
LOCKING.

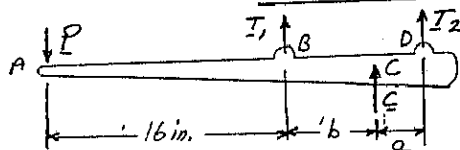


$r = 8 \text{ in.}$ ,  $\beta = \pi \text{ radians}$

$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{0.30\pi} = 2.5663$$

$$T_2 = 2.5663 T_1$$

FREE BODY: CONTROL ROD



$$b = 16 \text{ in.} - a$$

$$+\sum M_C = 0: P(16 \text{ in.} + b) - T_1 b + T_2 a = 0$$

FOR BRAKE TO BE SELF LOCKING,  $P = 0$

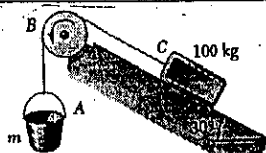
$$T_2 a = T_1 b; 2.5663 T_1 a = T_1 (16 - a)$$

$$2.5663 a = 16 - a$$

$$3.5663 a = 16$$

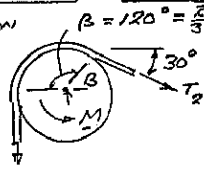
$$a = 4.49 \text{ in.}$$

8.134



GIVEN:  $\mu_s = 0.35$   
 $\mu_k = 0.25$   
FIND: SMALLEST  $m$   
FOR WHICH BLOCK C  
(A) REMAINS AT REST, (B) STARTS  
MOVING UP, (C) CONTINUES MOVING UP.

ROTATION



$$T_1 = mg$$

FREE BODY: DRUM

$$\frac{T_2}{T_1} = e^{\mu_s \beta}$$

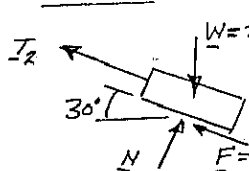
$$T_2 = mg e^{2.47773} \quad (1)$$

(a) SMALLEST  $m$  FOR BLOCK C TO REMAIN AT REST

CABLE SLIPS ON DRUM

$$\text{EQ(1) WITH } \mu_k = 0.25; T_2 = mg e^{2(0.25)\pi/3} = 1.6881 mg$$

BLOCK C: AT REST, MOTION IMPENDING



$$+\sum F = 0: N - m_c g \cos 30^\circ$$

$$N = m_c g \cos 30^\circ$$

$$F = \mu_s N = 0.35 m_c g \cos 30^\circ$$

$$m_c = 100 \text{ kg}$$

$$+\sum F = 0: T_2 + F - m_c g \sin 30^\circ = 0$$

$$1.6881 mg + 0.35 m_c g \cos 30^\circ - m_c g \sin 30^\circ = 0$$

$$1.6881 m = 0.19689 m_c$$

$$m = 0.11663 m_c = 0.11663(100 \text{ kg}); m = 11.66 \text{ kg}$$

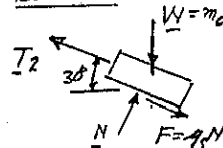
(CONTINUED)

8.134 CONTINUED

(b) SMALLEST  $m$  TO START  
BLOCK MOVING UP

NO SLIPPING AT BOTH DRUM AND BLOCK  $\mu_s = 0.35$   
EQ(1):  $T_2 = mg e^{2(0.35)\pi/3} = 2.0814 mg$

BLOCK C:



MOTION IMPENDING  $m_c = 100 \text{ kg}$

$$+\sum F = 0: N - m_c g \cos 30^\circ$$

$$N = m_c g \cos 30^\circ$$

$$F = \mu_s N = 0.35 m_c g \cos 30^\circ$$

$$+\sum F = 0: T_2 - F - m_c g \sin 30^\circ = 0$$

$$2.0814 mg - 0.35 m_c g \cos 30^\circ - m_c g \sin 30^\circ = 0$$

$$2.0814 m = 0.80311 m_c$$

$$m = 0.38585 m_c = 0.38585(100 \text{ kg})$$

$$m = 38.6 \text{ kg}$$

(c) SMALLEST  $m$  TO KEEP BLOCK MOVING UP

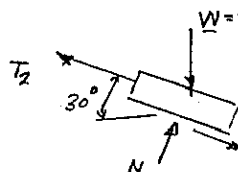
DRUM: NO SLIPPING  $\mu_s = 0.35$

EQ(1) WITH  $\mu_s = 0.35$

$$T_2 = mg e^{2.47773} = mg e^{2(0.35)\pi/3}$$

$$T_2 = 2.0814 mg$$

BLOCK C: MOVING UP PLANE, THUS  $\mu_k = 0.25$



MOTION UP

$$+\sum F = 0$$

$$N - m_c g \cos 30^\circ = 0$$

$$N = m_c g \cos 30^\circ$$

$$F = \mu_k N = 0.25 m_c g \cos 30^\circ$$

$$+\sum F = 0: T_2 - F - m_c g \sin 30^\circ = 0$$

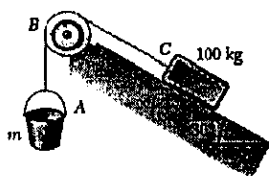
$$2.0814 mg - 0.25 m_c g \cos 30^\circ - m_c g \sin 30^\circ = 0$$

$$2.0814 m = 0.71651 m_c$$

$$m = 0.34424 m_c = 0.34424(100 \text{ kg})$$

$$m = 34.4 \text{ kg}$$

8.135



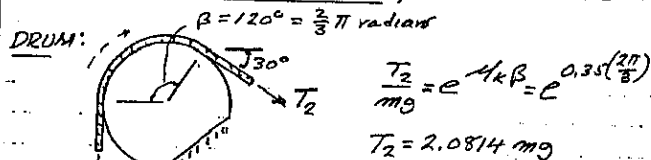
GIVEN: DRUM B IS FIXED.

$$\mu_s = 0.35$$

$$\mu_k = 0.25$$

FIND: SMALLEST  $m$  FOR WHICH BLOCK C  
(a) REMAINS AT REST, (b) STARTS MOVING UP,  
(c) CONTINUES MOVING UP.

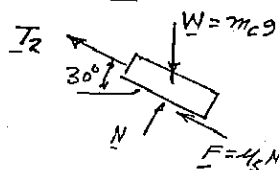
(a) BLOCK C REMAINS AT REST, MOTION IMPENDS



$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{0.35 \left(\frac{2\pi}{3}\right)}$$

$$T_2 = 2.0814 m g$$

BLOCK C



MOTION IMPENDS

$$\uparrow \Sigma F = 0: N - m_c g \cos 30^\circ = 0$$

$$N = m_c g \cos 30^\circ$$

$$F = \mu_s N = 0.35 m_c g \cos 30^\circ$$

$$+\uparrow \Sigma F = 0: T_2 + F - m_c g \sin 30^\circ = 0$$

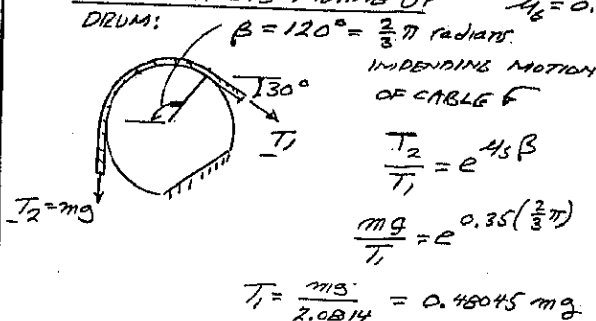
$$2.0814 m g + 0.35 m_c g \cos 30^\circ - m_c g \sin 30^\circ = 0$$

$$2.0814 m g = 0.19689 m_c$$

$$m = 0.09459 m_c = 0.09459 (100 \text{ kg})$$

$$m = 9.46 \text{ kg}$$

(b) BLOCK C STARTS MOVING UP



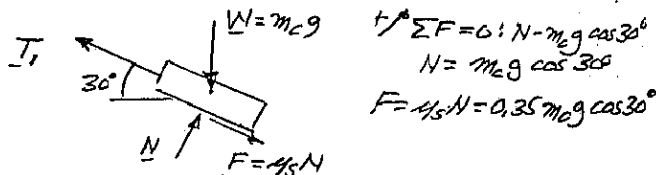
IMENDING MOTION  
OF CABLE C

$$\frac{T_2}{T_1} = e^{\mu_k \beta}$$

$$\frac{m g}{T_1} = e^{0.25 \left(\frac{2\pi}{3}\right)}$$

$$T_1 = \frac{m g}{2.0814} = 0.48045 m g$$

BLOCK C MOTION IMPENDS



$$+\uparrow \Sigma F = 0: T_1 - F - m_c g \sin 30^\circ = 0$$

$$0.48045 m g - 0.35 m_c g \cos 30^\circ - 0.5 m_c g = 0$$

$$0.48045 m = 0.80311 m_c$$

$$m = 1.67158 m_c = 1.67158 (100 \text{ kg})$$

$$m = 167.2 \text{ kg}$$

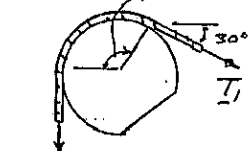
(CONTINUED)

8.135 CONTINUED

(c) SMALLEST  $m$  TO  
KEEP BLOCK MOVING

DRUM: MOTION OF CABLE C  $\mu_k = 0.25$

$$\beta = 120^\circ = \frac{2}{3} \pi \text{ radians}$$

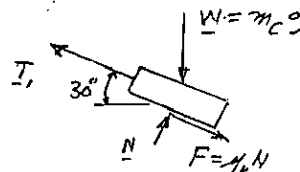


$$\frac{T_2}{T_1} = e^{\mu_k \beta} = e^{0.25 \left(\frac{2\pi}{3}\right)}$$

$$\frac{m g}{T_1} = 1.6881$$

$$T = \frac{m g}{1.6881} = 0.59238 m g$$

BLOCK C: BLOCK MOVES



$$+\uparrow \Sigma F = 0: N - m_c g \cos 30^\circ = 0$$

$$N = m_c g \cos 30^\circ$$

$$F = \mu_k N = 0.25 m_c g \cos 30^\circ$$

$$+\uparrow \Sigma F = 0: T_1 - F - m_c g \sin 30^\circ = 0$$

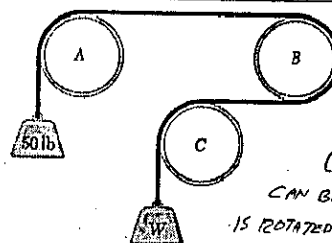
$$0.59238 m g - 0.25 m_c g \cos 30^\circ - 0.5 m_c g = 0$$

$$0.59238 m = 0.71651 m_c$$

$$m = 1.20954 m_c = 1.20954 (100 \text{ kg})$$

$$m = 121.0 \text{ kg}$$

8.136



GIVEN:  $\mu_s = 0.25$   
 $\mu_k = 0.20$

FIND: (a) SMALLEST  
W FOR EQUILIBRIUM

(b) LARGEST W THAT  
CAN BE RAISED IF PIPE B  
IS ROTATED WITH A+C FIXED.

$$\frac{T_2}{T_1} = e^{\mu \beta}$$

(a)  $\mu = \mu_s = 0.25$  AT ALL PIPES

$$\beta = \frac{\pi}{2}$$

$$\frac{T_2}{T_1} = e^{\mu \beta}$$

$$\frac{50 \text{ lb}}{T_{AB}} = e^{0.25 \frac{\pi}{2}}$$

$$\frac{T_{AB}}{T_{BC}} = e^{0.25 \pi}$$

$$\frac{T_{BC}}{W} = e^{0.25 \frac{\pi}{2}}$$

$$\frac{50 \text{ lb}}{T_{AB}} \cdot \frac{T_{AB}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\frac{\pi}{2}} \cdot e^{\pi} \cdot e^{\frac{\pi}{2}} = e^{\frac{\pi}{2} + \pi + \frac{\pi}{2}} = e^{2\pi} = 4.8105$$

$$\frac{50 \text{ lb}}{W} = 4.8105; W = 10.3946 \text{ lb}$$

(b) PIPE B ROTATED

$$\beta = \frac{\pi}{2}; \mu = \mu_k$$

$$\beta = \pi; \mu = \mu_s$$

$$\beta = \frac{\pi}{2}; \mu = \mu_k$$

$$\frac{T_2}{T_1} = e^{\mu \beta}$$

$$\frac{50 \text{ lb}}{T_{AB}} = e^{0.25 \frac{\pi}{2}}$$

$$\frac{T_{AB}}{T_{BC}} = e^{0.25 \pi}$$

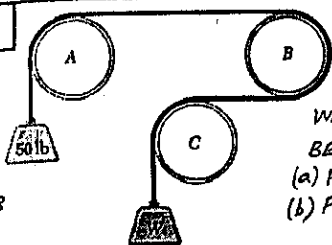
$$\frac{T_{BC}}{W} = e^{0.25 \frac{\pi}{2}}$$

$$\frac{50 \text{ lb}}{T_{AB}} \cdot \frac{T_{AB}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\frac{\pi}{2}} \cdot e^{\pi} \cdot e^{\frac{\pi}{2}} = e^{\frac{\pi}{2} + \pi + \frac{\pi}{2}} = e^{2\pi} = 4.8105$$

$$\frac{50 \text{ lb}}{W} = 0.85464; W = \frac{50 \text{ lb}}{0.85464} = 58.5041 \text{ lb}$$

$$W = 58.51 \text{ lb}$$

8.137

GIVEN:  $\mu_s = 0.25$  $\mu_k = 0.20$ 

FIND: LARGEST

WEIGHT  $W$  THAT CAN

(a) PIPE A IS ROTATED

(b) PIPE C IS ROTATED

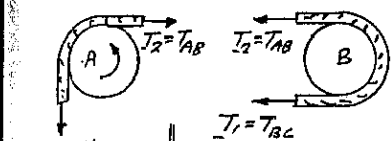
$$\frac{T_2}{T_1} = e^{\mu \theta}$$

(a) PIPE A ROTATES

$$\beta = \frac{\pi}{2}; \mu = \mu_s$$

$$\beta = \pi; \mu = \mu_k$$

$$\beta = \frac{\pi}{2}; \mu = \mu_k$$



$$\frac{T_2}{T_1} = e^{\mu \theta} \Rightarrow \frac{T_{AB}}{50 \text{ lb}} = e^{0.25 \frac{\pi}{2}}$$

$$\frac{T_{AB}}{T_{BC}} = e^{0.2\pi}$$

$$\frac{T_{BC}}{W} = e^{0.2 \frac{\pi}{2}}$$

$$\frac{T_{AB}}{50 \text{ lb}} \cdot \frac{T_{BC}}{T_{AB}} \cdot \frac{W}{T_{BC}} = e^{\frac{\pi}{2}} \cdot e^{-\frac{\pi}{5}} \cdot e^{-\frac{\pi}{10}} = e^{\pi(\frac{1}{2} - \frac{1}{5} - \frac{1}{10})} = e^{\frac{\pi}{10}} = 0.57708$$

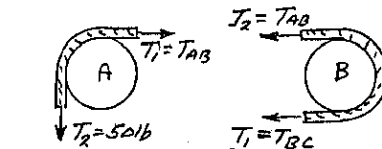
$$\frac{W}{50 \text{ lb}} = 0.57708; W = 28.854 \text{ lb}; W = 28.9 \text{ lb}$$

(b) PIPE C ROTATES

$$\beta = \frac{\pi}{2}; \mu = \mu_k$$

$$\beta = \pi; \mu = \mu_k$$

$$\beta = \frac{\pi}{2}; \mu = \mu_s$$



$$\frac{50 \text{ lb}}{T_{AB}} = e^{0.2 \frac{\pi}{2}}$$

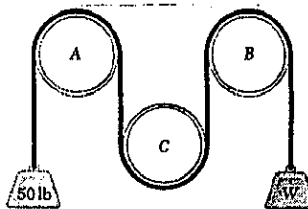
$$\frac{T_{AB}}{T_{BC}} = e^{0.2\pi}$$

$$\frac{W}{T_{BC}} = e^{0.25 \frac{\pi}{2}}$$

$$\frac{50 \text{ lb}}{T_{AB}} \cdot \frac{T_{AB}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\frac{\pi}{10}} \cdot e^{\frac{\pi}{5}} \cdot e^{-\frac{\pi}{5}} = e^{\frac{2\pi}{10}} = 0.57708$$

$$\frac{50 \text{ lb}}{W} = 0.57708; W = 28.854 \text{ lb}; W = 28.9 \text{ lb}$$

8.138

GIVEN:  $\mu_s = 0.25$  $\mu_k = 0.20$ 

FIND: (a) SMALLEST

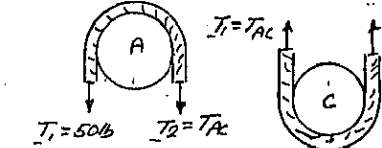
WEIGHT  $W$  FOR

EQUILIBRIUM,

(b) LARGEST  $W$ 

WHICH CAN BE

(RAISED IF PIPE B IS ROTATED) WHILE A AND C ARE FIXED.

(a) SMALLEST  $W$  FOR EQUILIBRIUM;  $\beta = \pi, \mu = \mu_s$ 

$$\frac{T_{AC}}{50 \text{ lb}} = e^{0.25\pi}$$

$$\frac{T_{BC}}{T_{AC}} = e^{0.25\pi}$$

$$\frac{W}{T_{BC}} = e^{0.25\pi}$$

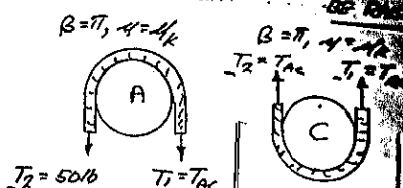
$$\frac{T_{AC}}{50 \text{ lb}} \cdot \frac{T_{BC}}{T_{AC}} \cdot \frac{W}{T_{BC}} = e^{\frac{\pi}{4}} \cdot e^{\frac{\pi}{4}} \cdot e^{\frac{\pi}{4}} = e^{\frac{3\pi}{4}} = 10.551$$

$$\frac{W}{50 \text{ lb}} = 10.551; W = 47.739 \text{ lb}; W = 47.7 \text{ lb}$$

(CONTINUED)

8.138 CONTINUED

(b)



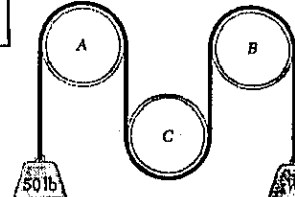
$$\frac{50 \text{ lb}}{T_{AC}} = e^{0.2\pi}$$

$$\frac{T_{AC}}{T_{BC}} = e^{0.2\pi}$$

$$\frac{50 \text{ lb}}{T_{AC}} \cdot \frac{T_{AC}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\frac{\pi}{5}} \cdot e^{\frac{\pi}{5}} \cdot e^{-\frac{\pi}{5}} = e^{\frac{\pi}{5}} = 1.602$$

$$\frac{50 \text{ lb}}{W} = 1.602; W = \frac{50 \text{ lb}}{1.602} = 31.21 \text{ lb}; W = 31.2 \text{ lb}$$

8.139

GIVEN:  $\mu_s = 0.25$  $\mu_k = 0.20$ 

FIND: LARGEST

WEIGHT  $W$  THAT CAN BE

RAISED IF ONLY

(a) PIPE A IS ROTATED

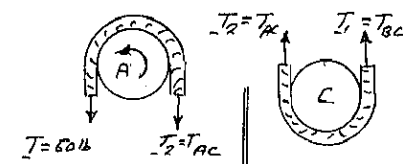
(b) PIPE C IS ROTATED

(a) PIPE A ROTATES

$$\beta = \pi, \mu = \mu_s$$

$$\beta = \pi, \mu = \mu_k$$

$$\beta = \pi, \mu = \mu_k$$



$$\frac{T_{AC}}{50 \text{ lb}} = e^{0.25\pi}$$

$$\frac{T_{AC}}{T_{BC}} = e^{0.2\pi}$$

$$\frac{T_{BC}}{W} = e^{0.2\pi}$$

$$\frac{T_{AC}}{50 \text{ lb}} \cdot \frac{T_{BC}}{T_{AC}} \cdot \frac{W}{T_{BC}} = e^{\frac{\pi}{4}} \cdot e^{-\frac{\pi}{5}} \cdot e^{-\frac{\pi}{5}} = e^{\pi(\frac{1}{4} - \frac{1}{5} - \frac{1}{5})} = e^{-\frac{3\pi}{20}} = 0.62423$$

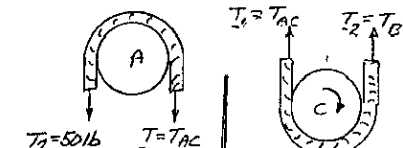
$$\frac{W}{50 \text{ lb}} = 0.62423; W = 31.21 \text{ lb}; W = 31.2 \text{ lb}$$

(b) PIPE C ROTATES

$$\beta = \pi, \mu = \mu_k$$

$$\beta = \pi, \mu = \mu_s$$

$$\beta = \pi, \mu = \mu_k$$



$$\frac{50 \text{ lb}}{T_{AC}} = e^{0.2\pi}$$

$$\frac{T_{BC}}{T_{AC}} = e^{0.25\pi}$$

$$\frac{T_{BC}}{W} = e^{0.2\pi}$$

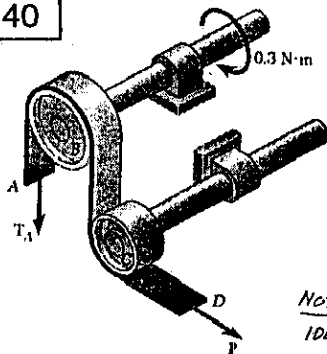
$$\frac{50 \text{ lb}}{T_{AC}} \cdot \frac{T_{AC}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\frac{\pi}{5}} \cdot e^{-\frac{\pi}{4}} \cdot e^{\frac{\pi}{5}} = e^{\pi(\frac{1}{5} - \frac{1}{4} + \frac{1}{5})} = e^{\frac{3\pi}{20}} = 1.602$$

$$\frac{50 \text{ lb}}{W} = 1.602$$

$$W = \frac{50 \text{ lb}}{1.602} = 31.21 \text{ lb}; W = 31.2 \text{ lb}$$

$$W = 31.2 \text{ lb}$$

8.140

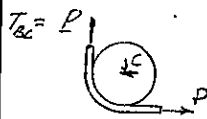


GIVEN:  $\mu_s = 0.40$ ,  $\mu_k = 0.30$   
 DRUM B,  $r = 20 \text{ mm}$

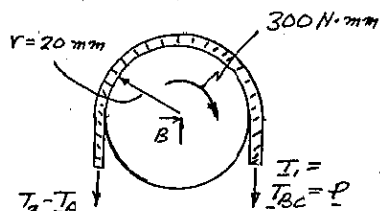
FIND: SMALLEST  $P$   
 IF SLIPPING IS NOT  
 TO OCCUR ON DRUM B.

NOTE: DRUM C IS AN  
 IDLER WITH NO FRICTION

DRUM C: IDLER



DRUM B



FOR SLIPPING IMPENDING:

$$\mu = \mu_s = 0.40$$

$$\beta = \pi \text{ radians}$$

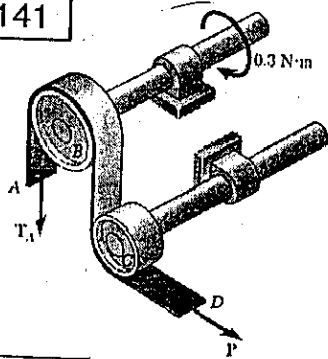
$$\frac{T_2}{T_1} = e^{\mu\beta}: \frac{T_A}{P} = e^{0.4\pi} = 3.5136$$

$$T_A = 3.5136 P$$

$$\begin{aligned} +\sum M_B = 0: T_A(20 \text{ mm}) - P(20 \text{ mm}) - 300 \text{ N}\cdot\text{mm} &= 0 \\ (3.5136 P - P)(20 \text{ mm}) &= 300 \text{ N}\cdot\text{mm} \\ 2.5136 P &= 15 \text{ N} \\ P &= 5.967 \text{ N} \end{aligned}$$

$$P = 5.97 \text{ N}$$

8.141

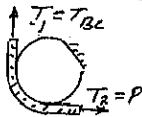


GIVEN:  $\mu_s = 0.40$ ,  $\mu_k = 0.30$   
 DRUM B,  $r = 20 \text{ mm}$   
 DRUM C IS FROZEN  
 AND CANNOT ROTATE

FIND: SMALLEST  $P$   
 IF SLIPPING IS NOT  
 TO OCCUR ON DRUM B.

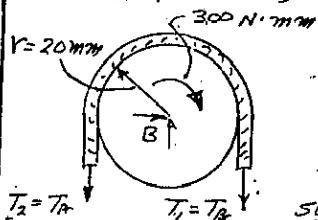
$$\frac{T_2}{T_1} = e^{\mu\beta}$$

DRUM C:  $\beta = \frac{\pi}{2}$   
 SLIPPING OCCURS  
 $\mu = \mu_k = 0.30$



$$\begin{aligned} \frac{P}{T_{BC}} &= e^{0.3(\frac{\pi}{2})} \\ T_{BC} &= 1.602 P \\ P &= 1.602 T_{BC} \quad (1) \end{aligned}$$

DRUM B:  $\beta = \pi$ ,  $\mu = \mu_s = 0.40$



$$\begin{aligned} \frac{T_2}{T_1} &= e^{\mu\beta}: \frac{T_A}{T_{BC}} = e^{0.4\pi} = 3.5136 \\ T_A &= 3.5136 T_{BC} \quad (2) \end{aligned}$$

$$+\sum M_B = 0:$$

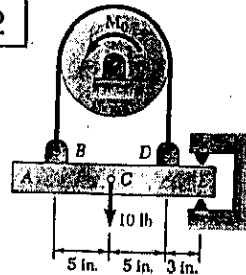
$$\begin{aligned} T_A(20 \text{ mm}) - T_{BC}(20 \text{ mm}) - 300 \text{ N}\cdot\text{mm} &= 0 \\ \text{SUBSTITUTE FOR } T_A \text{ FROM EQ. (2):} \\ (3.5136 T_{BC} - T_{BC})(20 \text{ mm}) &= 300 \text{ N}\cdot\text{mm} \\ T_{BC} &= 5.967 \text{ N} \end{aligned}$$

$$\text{EQ. (1): } P = 1.602 T_{BC} = 1.602(5.967 \text{ N})$$

$$P = 9.559 \text{ N}$$

$$P = 9.56 \text{ N}$$

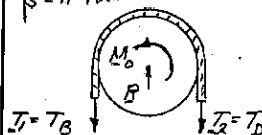
8.142



GIVEN:  $\mu_s = 0.30$   
 $M_0$  ACTS

FIND: (a)  $M_0$  FOR  
 WHICH SLIPPING  
 IMPENDS.  
 (b) FORCE  $E$  EXERTED  
 ON BAR AE

$$\beta = \pi \text{ rad.}$$

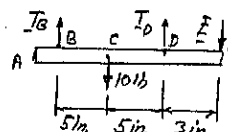


DRUM: SLIPPING IMPENDS

$$\mu_s = 0.30$$

$$\frac{T_2}{T_1} = e^{\mu\beta}: \frac{T_D}{T_B} = e^{0.30\pi} = 2.5663$$

$$T_D = 2.5663 T_B$$

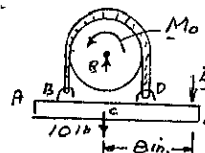


BAR AE:

$$\begin{aligned} +\sum F_y = 0: T_B + T_D - E - 10 \text{ lb} &= 0 \\ T_B + 2.5663 T_B - E - 10 \text{ lb} &= 0 \\ 3.5663 T_B - E - 10 \text{ lb} &= 0 \\ E &= 3.5663 T_B - 10 \text{ lb} \quad (1) \end{aligned}$$

$$\begin{aligned} +\sum M_D = 0: E(3 \text{ in.}) - (10 \text{ lb})(5 \text{ in.}) + T_B(10 \text{ in.}) &= 0 \\ (3.5663 T_B - 10 \text{ lb})(3 \text{ in.}) - 50 \text{ lb}\cdot\text{in.} + T_B(10 \text{ in.}) &= 0 \\ 20.699 T_B &= 80 \\ T_B &= 3.8649 \text{ lb} \end{aligned}$$

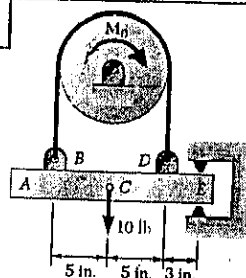
$$\text{EQ. (1): } E = 3.5663(3.8649 \text{ lb}) - 10 \text{ lb}; E = 3.78 \text{ lb}$$



FREE BODY: DRUM AND BAR

$$\begin{aligned} +\sum M_C = 0: M_0 - E(8 \text{ in.}) &= 0 \\ M_0 &= (3.78 \text{ lb})(8 \text{ in.}) = 30.27 \text{ lb}\cdot\text{in.} \\ M_0 &= 30.3 \text{ lb}\cdot\text{in.} \end{aligned}$$

8.143

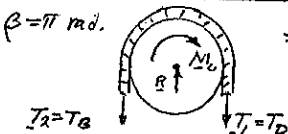


GIVEN:  $\mu_s = 0.30$   
 $M_0$  ACTS

FIND: (a)  $M_0$  FOR  
 WHICH SLIPPING  
 IMPENDS.

(b) FORCE  $E$   
 EXERTED ON BAR AE

$$\beta = \pi \text{ rad.}$$

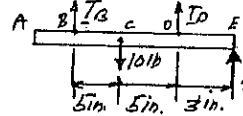


DRUM: SLIPPING IMPENDS

$$\mu_s = 0.30$$

$$\frac{T_2}{T_1} = e^{\mu\beta}: \frac{T_D}{T_B} = e^{0.30\pi} = 2.5663$$

$$T_D = 2.5663 T_B$$

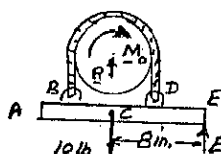


BAR AE

$$\begin{aligned} +\sum F_y = 0: T_B + T_D + E - 10 \text{ lb} &= 0 \\ 2.5663 T_B + T_B + E - 10 \text{ lb} &= 0 \\ E &= -3.5663 T_B + 10 \text{ lb} \quad (1) \end{aligned}$$

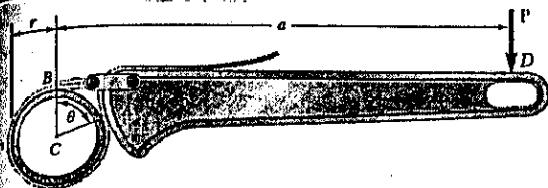
$$\begin{aligned} +\sum M_D = 0: T_D(10 \text{ in.}) - (10 \text{ lb})(5 \text{ in.}) + E(13 \text{ in.}) &= 0 \\ T_D(10 \text{ in.}) - 50 \text{ lb}\cdot\text{in.} + (-3.5663 T_D + 10 \text{ lb})(13 \text{ in.}) &= 0 \\ -36.362 T_D + 80 \text{ lb}\cdot\text{in.} &= 0; T_D = 2.200 \text{ lb} \end{aligned}$$

$$\text{EQ. (1): } E = -3.5663(2.200 \text{ lb}) + 10 \text{ lb}; E = 2.15 \text{ lb}$$



FREE BODY: DRUM AND BAR

$$\begin{aligned} +\sum M_C = 0: M_0 - E(8 \text{ in.}) &= 0 \\ M_0 &= (2.15 \text{ lb})(8 \text{ in.}) = 17.23 \text{ lb}\cdot\text{in.} \\ M_0 &= 17.23 \text{ lb}\cdot\text{in.} \end{aligned}$$



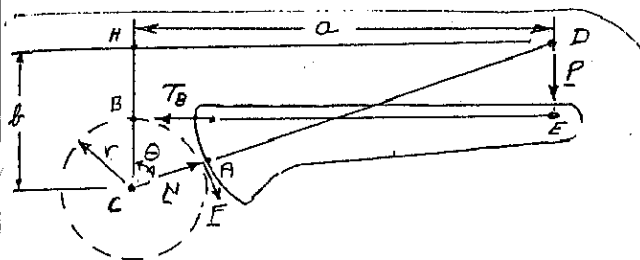
GIVEN:  $a = 200 \text{ mm}$ ,  $r = 30 \text{ mm}$ .

ASSUME VALUE OF  $\mu_s$  IS THE SAME AT ALL SURFACES OF CONTACT

FIND: SMALLEST VALUE OF  $\mu_s$  FOR WHICH THE WRENCH IS SELF-LOCKING IF IN  
 PROB. 8.144  $\theta = 65^\circ$   
 PROB. 8.145  $\theta = 75^\circ$ .

FOR WRENCH TO BE SELF-LOCKING ( $P=0$ ), THE VALUE OF  $\mu_s$  MUST PREVENT SLIPPING OF STRAP WHICH IS IN CONTACT WITH THE PIPE FROM POINT A TO POINT B AND MUST BE LARGE ENOUGH SO THAT AT POINT A THE STRAP TENSION CAN INCREASE FROM ZERO TO THE MINIMUM TENSION REQUIRED TO DEVELOP "BELT FRICTION" BETWEEN STRAP AND PIPE.

FREE BODY: WRENCH HANDLE



GEOMETRY IN  $\triangle CDH$ :  $CH = a / \tan \theta$ ,  $CD = a / \sin \theta$

$$DE = BH = CH - BC$$

$$DE = \frac{a}{\tan \theta} - r$$

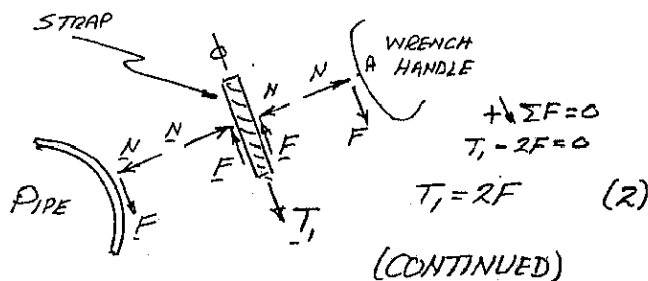
$$AD = CD - CA = \frac{a}{\sin \theta} - r$$

ON WRENCH HANDLE

$$+\circlearrowleft \sum M_D = 0: T_B(DE) - F(AD) = 0$$

$$\frac{T_B}{F} = \frac{AD}{DE} = \frac{\frac{a}{\sin \theta} - r}{\frac{a}{\tan \theta} - r} \quad (1)$$

FREE BODY: STRAP AT POINT A

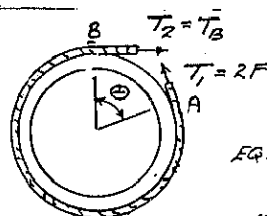


$$+\circlearrowleft \sum F = 0$$

$$T_1 - 2F = 0$$

$$T_1 = 2F \quad (2)$$

(CONTINUED)



PIPE AND STRAP

$$\beta = (2\pi - \theta) \text{ radians}$$

$$\text{EQ. (B.13): } \mu_s \beta = \ln \frac{T_2}{T_1}$$

$$\mu_s = \frac{1}{\beta} \ln \frac{T_2}{2F} \quad (3)$$

RETURN TO FREE BODY OF WRENCH HANDLE

$$+\circlearrowleft \sum F_y = 0: N \sin \theta + F \cos \theta - T_B = 0$$

$$\frac{N}{F} \sin \theta = \frac{T_B}{F} - \cos \theta$$

SINCE  $F = \mu_s N$ , WE HAVE

$$\frac{1}{\mu_s} \sin \theta = \frac{T_B}{F} - \cos \theta$$

$$\mu_s = \frac{\sin \theta}{\frac{T_B}{F} - \cos \theta} \quad (4)$$

NOTE: FOR A GIVEN SET OF DATA, WE SEEK THE LARGER OF THE VALUES OF  $\mu_s$  FROM EQS. (3) AND (4)

PROB. 8.144:  $a = 200 \text{ mm}$ ,  $r = 30 \text{ mm}$ ,  $\theta = 65^\circ$

$$\text{EQ. (1): } \frac{T_B}{F} = \frac{\frac{200 \text{ mm}}{\sin 65^\circ} - 30 \text{ mm}}{\frac{200 \text{ mm}}{\tan 65^\circ} - 30 \text{ mm}} = \frac{190.676 \text{ mm}}{63.268 \text{ mm}} = 3.0141$$

$$\beta = 2\pi - \theta = 2\pi - 65^\circ \frac{\pi}{180^\circ} = 5.1487 \text{ radians}$$

$$\text{EQ. (3): } \mu_s = \frac{1}{5.1487 \text{ rad}} \ln \frac{3.0141}{2} = \frac{0.41015}{5.1487} = 0.0797 \quad \triangle$$

$$\text{EQ. (4): } \mu_s = \frac{\sin 65^\circ}{3.0141 - \cos 65^\circ} = \frac{0.90631}{2.1575} = 0.3497 \quad \triangle$$

WE CHOOSE THE LARGER VALUE:  $\mu_s = 0.350$   $\blacktriangle$

PROB. 8.145:  $a = 200 \text{ mm}$ ,  $r = 30 \text{ mm}$ ,  $\theta = 75^\circ$

$$\text{EQ. (1): } \frac{T_B}{F} = \frac{\frac{200 \text{ mm}}{\sin 75^\circ} - 30 \text{ mm}}{\frac{200 \text{ mm}}{\tan 75^\circ} - 30 \text{ mm}} = \frac{177.055 \text{ mm}}{23.570 \text{ mm}} = 7.5056$$

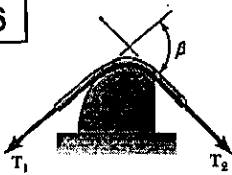
$$\beta = 2\pi - \theta = 2\pi - 75^\circ \frac{\pi}{180^\circ} = 4.9742$$

$$\text{EQ. (3): } \mu_s = \frac{1}{4.9742 \text{ rad}} \ln \frac{7.5056}{2} = \frac{1.3225}{4.9742} = 0.2659 \quad \triangle$$

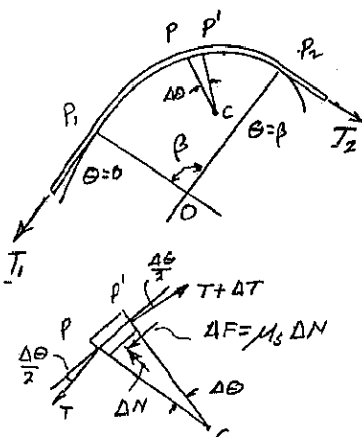
$$\text{EQ. (4): } \mu_s = \frac{\sin 75^\circ}{7.5056 - \cos 75^\circ} = \frac{0.96593}{7.2462} = 0.1333 \quad \triangle$$

WE CHOOSE THE LARGER VALUE:  $\mu_s = 0.266$   $\blacktriangle$

8.146



PROVE THAT  
EQS. (8.13) AND (8.14)  
ARE VALID FOR  
ANY SHAPE SURFACE



NOTE  $\beta$  IS THE  
ANGLE BETWEEN  
BOTH TANGENTS  
AT  $P_1$  AND  
NORMALS AT  $P_1$  AND  $P_2$ .

NEXT, NOTE THAT THE  
DERIVATION OF

$$\frac{dT}{T} = \mu_s d\theta \quad (1)$$

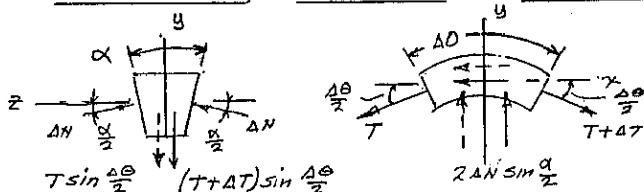
ON PAGES 436 AND 437  
DID NOT DEPEND ON  
THE RADIUS OF CURVATURE  
BEING CONSTANT. THEREFORE  
THIS EQUATION MAY BE OBTAINED  
FROM THE FREE-BODY DIAGRAM  
SHOWN HERE.

INTEGRATING EQ.(1) IN  $\theta$  FROM 0 TO  $\beta$  AND IN  
 $T$  FROM  $T_1$  TO  $T_2$ , WE OBTAIN AGAIN

$$\ln \frac{T_2}{T_1} = \mu_s \beta \quad \text{AND} \quad \frac{T_2}{T_1} = e^{\mu_s \beta}$$

8.147

COMPLETE DERIVATION OF EQ. 8.15



$$\pm \sum F_x = 0: (T + \Delta T) \cos \frac{\Delta \theta}{2} - T \cos \frac{\Delta \theta}{2} - 2\mu_s \Delta N = 0 \quad (1)$$

$$+\uparrow \sum F_y = 0: 2\Delta N \sin \frac{\alpha}{2} - (T + \Delta T) \sin \frac{\Delta \theta}{2} - T \sin \frac{\Delta \theta}{2} = 0 \quad (2)$$

SOLVE (1) FOR  $\Delta N$  AND SUBSTITUTE IN (2):

$$\Delta T \cos \frac{\Delta \theta}{2} \sin \frac{\alpha}{2} - \mu_s (2T + \Delta T) \sin \frac{\Delta \theta}{2} = 0$$

DIVIDE ALL TERMS BY  $\Delta \theta$ :

$$\frac{\Delta T}{\Delta \theta} \cos \frac{\Delta \theta}{2} \sin \frac{\alpha}{2} - \mu_s (T + \frac{\Delta T}{2}) \frac{\sin \frac{\Delta \theta}{2}}{\frac{\Delta \theta}{2}} = 0$$

LET  $\Delta \theta$  APPROACH ZERO

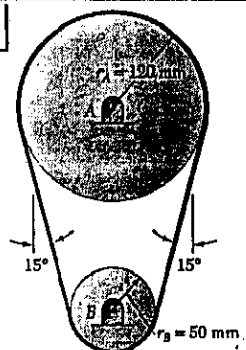
$$\frac{dT}{d\theta} \sin \frac{\alpha}{2} - \mu_s T = 0$$

$$\frac{dT}{T} = \frac{\mu_s}{\sin \frac{\alpha}{2}} d\theta$$

INTEGRATE IN  $\theta$  FROM 0 TO  $\beta$  AND IN  $T$  FROM  
 $T_1$  TO  $T_2$ :

$$\ln \frac{T_2}{T_1} = \frac{\mu_s \beta}{\sin \frac{\alpha}{2}} \quad \text{OR} \quad \frac{T_2}{T_1} = e^{\frac{\mu_s \beta}{\sin \frac{\alpha}{2}}}$$

8.148



GIVEN:  $\mu_s = 0.40$   
 $T_{\max} = 450 \text{ N}$   
V-BELT WITH  $\alpha = 36^\circ$

FIND: LARGEST COUPLE  
THAT CAN BE  
EXERTED ON PULLEY A

SINCE  $\beta$  IS SMALLER FOR PULLEY B, THE BELT  
WILL SLIP FIRST AT B.

$$I_2 = T_{\max} = 450 \text{ N}$$

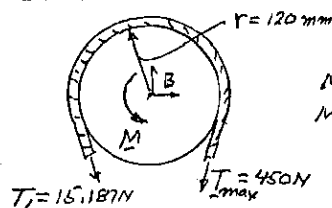
$$\beta = 150^\circ \left( \frac{\pi \text{ rad}}{180^\circ} \right) = \frac{5}{8} \pi \text{ rad}$$

$$\frac{T_2}{T_1} = e^{\mu_s \beta / \sin \frac{\alpha}{2}}$$

$$\frac{450 \text{ N}}{T_1} = e^{(0.4) \left( \frac{5}{8} \pi \right) / \sin 18^\circ} = e^{3.397}$$

$$\frac{450 \text{ N}}{T_1} = 29.63; \quad T_1 = 15.187 \text{ N}$$

TORQUE ON PULLEY A



$$+\circlearrowleft \sum M_B = 0$$

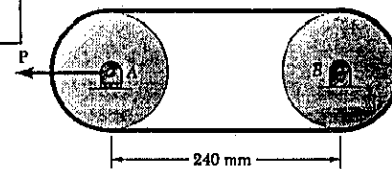
$$M - (T_{\max} - T_1)(0.12 \text{ m}) = 0$$

$$M - (450 \text{ N} - 15.187 \text{ N})(0.12 \text{ m}) = 0$$

$$M = 52.18 \text{ N} \cdot \text{m}$$

$$M = 52.2 \text{ N} \cdot \text{m}$$

8.149



GIVEN: 60-mm-RADIUS V-BELT PULLEYS WITH  $\alpha = 36^\circ$   
 $P = 900 \text{ N}$ ,  $\mu_s = 0.35$

FIND: LARGEST TORQUE WHICH CAN BE TRANSMITTED,  
MAXIMUM TENSION IN V-BELT

PULLEY A:  $\beta = \pi \text{ rad}$

$$\frac{T_2}{T_1} = e^{\mu_s \beta / \sin \frac{\alpha}{2}}$$

$$\frac{T_2}{T_1} = e^{0.35 \pi / \sin 18^\circ}$$

$$\frac{T_2}{T_1} = e^{3.558} = 35.1$$

$$T_2 = 35.1 T_1$$

$$\pm \sum F_x = 0: T_1 + T_2 + 900 \text{ N} = 0$$

$$T_1 + 35.1 T_1 + 900 \text{ N} = 0$$

$$T_1 = 24.93 \text{ N}; \quad T_2 = 35.1(24.93 \text{ N}) = 875.03 \text{ N}$$

$$+\circlearrowleft \sum M_A = 0: M - T_2(0.06 \text{ m}) + T_1(0.06 \text{ m}) = 0$$

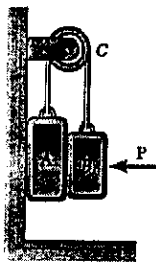
$$M - (875.03 \text{ N})(0.06 \text{ m}) + (24.93 \text{ N})(0.06 \text{ m}) = 0$$

$$M = 51.0 \text{ N} \cdot \text{m}$$

$$T_{\max} = T_2 \quad T_{\max} = 875 \text{ N}$$



50



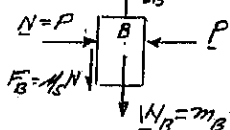
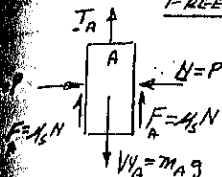
GIVEN:  $m_A = 12 \text{ kg}$ ,  $m_B = 6 \text{ kg}$   
 $\mu_s = 0.12$

FIND: SMALLEST VALUE OF  $P$  FOR EQUILIBRIUM

NOTE: PULLEY CAN FREELY ROTATE

IMPENDING MOTION: BLOCK A  $\downarrow$  BLOCK B  $\uparrow$

FREE-BODY DIAGRAMS



$$\begin{aligned} \uparrow \Sigma F_y = 0: T_A + 2F_A - W_A &= 0 \\ T_A + 2\mu_s N - m_A g &= 0 \\ T_A &= m_A g - 2\mu_s N \end{aligned}$$

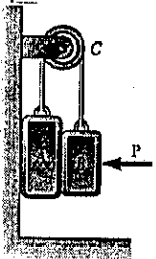
$$\begin{aligned} \uparrow \Sigma F_y = 0: T_B - F_B - W_B &= 0 \\ T_B - \mu_s N - m_B g &= 0 \\ T_B &= m_B g + \mu_s N \end{aligned}$$

NOT  $T_A = T_B$ :  $m_A g - 2\mu_s N = m_B g + \mu_s N$   
 $(m_A - m_B)g = 3\mu_s N$   
 $(12 \text{ kg} - 6 \text{ kg})g = 3(0.12)N$

$$N = \frac{6g}{0.36} = 16.667g = 16.667(9.81 \text{ m/s}^2) = 163.5 \text{ N}$$

SINCE  $P = N$ , WE HAVE  $P = 163.5 \text{ N}$

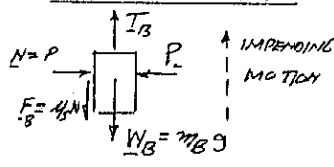
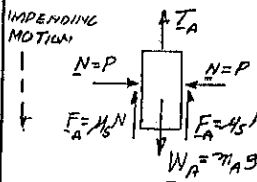
8.151



GIVEN:  $m_A = 12 \text{ kg}$ ,  $m_B = 6 \text{ kg}$   
 ROTATION OF PULLEY IS PREVENTED.  
 $\mu_s = 0.12$  AT ALL SURFACES AND BETWEEN CABLE AND PULLEY

FIND: SMALLEST VALUE OF  $P$  FOR EQUILIBRIUM

FREE-BODY DIAGRAM

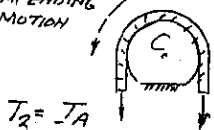


$$\begin{aligned} \uparrow \Sigma F_y = 0: T_A + 2F_A - W_A &= 0 \\ T_A + 2\mu_s N - m_A g &= 0 \\ T_A &= m_A g - 2\mu_s N \end{aligned}$$

$$\begin{aligned} \uparrow \Sigma F_y = 0: T_B - F_B - W_B &= 0 \\ T_B - \mu_s N - m_B g &= 0 \\ T_B &= m_B g + \mu_s N \end{aligned}$$

FIXED PULLEY:  $\beta = \pi$

$$\frac{T_2}{T_1} = e^{\mu_s \beta}; \frac{T_A}{T_B} = e^{0.12\pi} = 1.4579$$



$$T_2 = T_A$$

$$T_A = 1.4579 T_B$$

SUBSTITUTE FOR  $T_A$  AND  $T_B$

$$(m_A g - 2\mu_s N) = 1.4579(m_B g + \mu_s N)$$

$$(m_A - 1.4579 m_B)g = 3.4579 \mu_s N$$

$$[12 \text{ kg} - 1.4579(6 \text{ kg})]9.81 \text{ m/s}^2 = 3.4579(0.12)N$$

$$N = 76.998 \text{ N}$$

SINCE  $P = N$ , WE HAVE

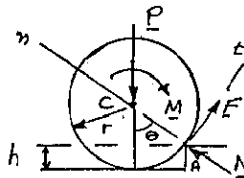
$$P = 76.9 \text{ N}$$

8.152



GIVEN:  $\mu_s = 0.90$ ,  
 12-in.-RADIUS WHEELS,  
 60% OF WEIGHT  
 IS ON FRONT WHEELS.

FIND: LARGEST  $h$  FOR AUTO TO CLIMB CURB  
 (a) FRONT-WHEEL DRIVE, (b) REAR-WHEEL DRIVE



(a) FRONT-WHEEL DRIVE

ONE FRONT WHEEL:  $r = 12 \text{ in.}$

$$\uparrow \Sigma F_x = 0: F - P \sin \theta = 0$$

$$\uparrow \Sigma F_y = 0: N - P \cos \theta = 0$$

SLIDING IMPENDS:

$$\mu_s = \frac{F}{N} = \frac{P \sin \theta}{P \cos \theta} = \tan \theta$$

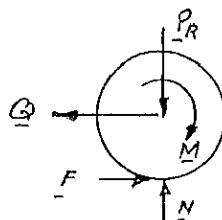
$$\tan \theta = \mu_s = 0.90; \theta = 41.987^\circ$$

$$h = r - r \cos \theta = r(1 - \cos \theta) = (12 \text{ in.})(1 - \cos 41.987^\circ)$$

$$h = 3.0805 \text{ in.} \quad h = 3.08 \text{ in.}$$

(b) REAR WHEEL DRIVE

EACH REAR WHEEL CARRIES 0.2W AND EACH FRONT WHEEL CARRIES 0.3W. LET  $Q$  BE FORCE EXERTED BY CHASSIS ON EACH WHEEL



FREE BODY: REAR WHEEL

$$P_R = 0.2W$$

$$\uparrow \Sigma F_y = 0: N - 0.2W = 0$$

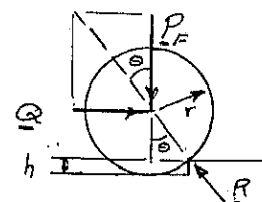
$$N = 0.2W$$

$$F = F_m = \mu_s N = 0.90(0.2W)$$

$$F = 0.18W$$

$$\uparrow \Sigma F_x = 0: F - Q = 0$$

$$Q = F = 0.18W$$



FREE BODY: FRONT WHEEL

$$P_F = 0.3W$$

$$r = 12 \text{ in.}$$

FRONT WHEEL IS A TWO-FORCE BODY

$$\tan \theta = \frac{Q}{P_F} = \frac{0.18W}{0.3W} = 0.6$$

$$\theta = 30.96^\circ$$

$$\begin{aligned} h &= r - r \cos \theta = r(1 - \cos \theta) \\ &= (12 \text{ in.})(1 - \cos 30.96^\circ) \\ &= 1.7101 \text{ in.} \end{aligned}$$

$$h = 1.710 \text{ in.}$$

NOTE: COMPARING PROBS 8.152 AND 8.153, WE NOTE THAT -

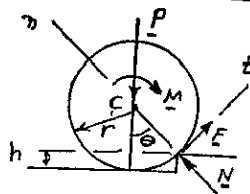
FOR FRONT WHEEL DRIVE THE RESULT IS INDEPENDENT OF WEIGHT DISTRIBUTION FOR REAR-WHEEL DRIVE THE HEAVIER THE LOAD ON THE REAR WHEELS, THE LARGER THE CURB HEIGHT  $h$  WILL BE

8.153



GIVEN:  $\mu_s = 0.90$ ,  
12-in. RADIUS WHEELS  
EQUAL WEIGHT ON  
EACH WHEEL.

FIND: LARGEST  $h$  FOR AUTO CLIMB CURB  
(a) FRONT-WHEEL DRIVE, (b) REAR-WHEEL DRIVE



(a) FRONT-WHEEL DRIVE

ONE FRONT WHEEL  $r = 12$  in.

$$+\uparrow \Sigma F_x = 0: F - P \sin \theta = 0$$

$$+\uparrow \Sigma F_y = 0: N - P \cos \theta = 0$$

SLIPPING IMPEXDS:

$$\mu_s = \frac{F}{N} = \frac{P \sin \theta}{P \cos \theta} = \tan \theta$$

$$\tan \theta = \mu_s = 0.901 \quad \theta = 41.987^\circ$$

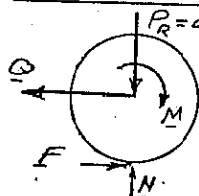
$$h = r - r \cos \theta = r(1 - \cos \theta) = (12 \text{ in.})(1 - \cos 41.987^\circ)$$

$$h = 3.0805 \text{ in.}$$

$$h = 3.08 \text{ in.}$$

(b) REAR-WHEEL DRIVE

FREE BODY: REAR WHEEL



LET  $Q$  BE FORCE EXERTED  
BY CHASSIS ON EACH WHEEL

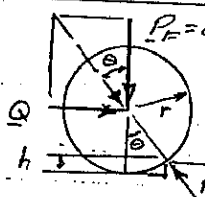
$$+\uparrow \Sigma F_y = 0: N - 0.25W = 0$$

$$N = 0.25W$$

$$F = \mu_s N = 0.90(0.25W) = 0.225W$$

$$\Sigma F_x = 0: Q = 0.225W$$

FREE BODY: FRONT WHEEL



$r = 12$  in.

TWO-FORCE BODY

$$\tan \theta = \frac{Q}{P_F} = \frac{0.225W}{0.25W} = 0.9$$

$$\theta = 41.987^\circ$$

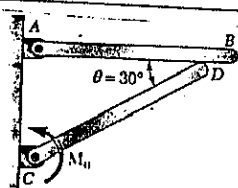
$$h = r - r \cos \theta = r(1 - \cos \theta)$$

$$h = (12 \text{ in.})(1 - \cos 41.987^\circ) = 3.0805 \text{ in.}$$

$$h = 3.08 \text{ in.}$$

[SEE NOTE AT END OF SOLUTION OF PROB 8.152]

8.154



GIVEN: EACH ROD  
IS OF LENGTH  $L$   
AND WEIGHT  $W$ ,  
 $\mu_s = 0.40$

FIND: RANGE OF VALUES  
OF  $M_0$  FOR EQUILIBRIUM

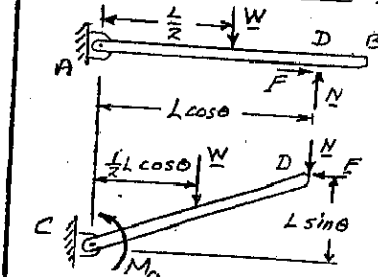
FOR IMPENDING  
CLOCKWISE MOTION

$$+\uparrow \Sigma M_A = 0$$

$$N(L \cos \theta) - W\left(\frac{L}{2}\right) = 0$$

$$N = \frac{W}{2 \cos \theta}$$

$$F = \mu_s N = \frac{\mu_s W}{2 \cos \theta}$$



$$+\uparrow \Sigma M_C = 0: M_0 - W\left(\frac{1}{2}L \cos \theta\right) - \frac{W}{2 \cos \theta}(L \cos \theta) + \frac{\mu_s W}{2 \cos \theta}(L \sin \theta) = 0$$

$$M_0 = \frac{1}{2}WL(\cos \theta + 1 - \mu_s \tan \theta) \quad (1)$$

$$M_0 = \frac{1}{2}WL(\cos 30^\circ + 1 - 0.40 \tan 30^\circ)$$

$$M_0 = 0.81754WL$$

$$M_0 = 0.818WL$$

(CONTINUED)

8.154 CONTINUED

FOR IMPENDING  
COUNTERCLOCKWISE

MOTION OF THE RODS, WE CHANGE THE  
SIGN OF  $\mu_s$  IN EQ(1).

$$M_0 = \frac{1}{2}WL(\cos \theta + 1 + \mu_s \tan \theta)$$

$$= \frac{1}{2}WL(\cos 30^\circ + 1 + 0.40 \tan 30^\circ)$$

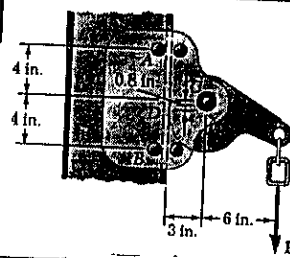
$$M_0 = 1.0484WL$$

$$M_0 = 1.048WL$$

RANGE OF  $M_0$  FOR EQUILIBRIUM:

$$0.818WL \leq M_0 \leq 1.048WL$$

8.155



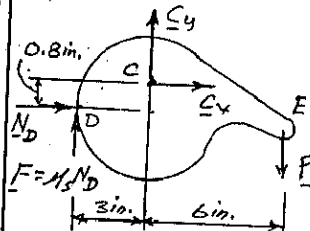
FIND: SMALLEST  
BETWEEN RAIL  
AND CAM AND  
BETWEEN RAIL  
AND PINS FOR  
EQUILIBRIUM

FREE BODY: CAM

$$+\uparrow \Sigma M_C = 0:$$

$$N_D(0.8 \text{ in.}) - \mu_s N_D(3 \text{ in.}) - P(6 \text{ in.}) = 0$$

$$N_D = \frac{6P}{0.8 - 3\mu_s} \quad (1)$$



FREE BODY: SLEEVE AND CAM

$$+\uparrow \Sigma F_x = 0: N_D - N_A - N_B = 0$$

$$N_A + N_B = N_D \quad (2)$$

$$+\uparrow \Sigma F_y = 0: F_A + F_B + F_D - P = 0$$

$$\text{OR } \mu_s(N_A + N_B + N_D) = P \quad (3)$$

SUBSTITUTE FROM (2) INTO (3)

$$\mu_s(2N_D) = P \quad N_D = \frac{P}{2\mu_s} \quad (4)$$

EQUATE EXPRESSIONS FOR  $N_D$  FROM (1) AND (4)

$$\frac{P}{2\mu_s} = \frac{6P}{0.8 - 3\mu_s}; \quad 0.8 - 3\mu_s = 12\mu_s$$

$$\mu_s = \frac{0.8}{15}$$

$$\mu_s = 0.0533$$

NOTE: TO VERIFY THAT CONTACT AT PINS A AND B  
TAKES PLACE AS ASSUMED WE SHALL  
CHECK THAT  $N_A > 0$  AND  $N_B = 0$ .

$$\text{FROM (4): } N_D = \frac{P}{2\mu_s} = \frac{P}{2(0.0533)} = 9.375P$$

FROM FREE BODY OF CAM AND SLEEVE

$$+\uparrow \Sigma M_B = 0: N_A(8 \text{ in.}) - N_D(4 \text{ in.}) - P(9 \text{ in.}) = 0$$

$$8N_A = (9.375P)(4) + 9P$$

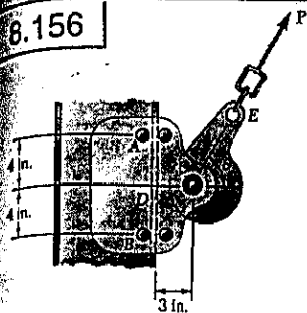
$$N_A = 5.8125P > 0 \quad \text{OK}$$

FROM (2):  $N_A + N_B = N_D$

$$5.8125P + N_B = 9.375P$$

$$N_B = 3.5625P > 0 \quad \text{OK}$$

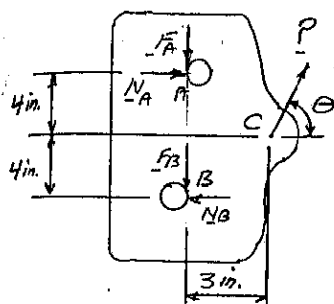
8.156



FIND: LARGEST  $\mu_s$  BETWEEN RAIL AND PINS A AND B IF SLEEVE IS TO MOVE UP WHEN

- (a)  $\theta = 60^\circ$ ;  
(b)  $\theta = 50^\circ$ ;  
(c)  $\theta = 40^\circ$ .

NOTE THE CAM IS A TWO-FORCE MEMBER



FREE BODY: SLEEVE

WE ASSUME CONTACT BETWEEN RAIL AND PINS AS SHOWN.

$$+\sum M_C = 0$$

$$F_A(3\text{ in.}) + F_B(3\text{ in.}) - N_A(4\text{ in.}) - N_B(4\text{ in.}) = 0$$

$$\text{BUT: } F_A = \mu_s N_A \quad F_B = \mu_s N_B$$

WE FIND

$$3\mu_s(N_A + N_B) - 4(N_A + N_B) = 0$$

$$\mu_s = \frac{4}{3} = 1.333$$

WE NOW VERIFY THAT OUR ASSUMPTION WAS CORRECT.

$$+\sum F_x = 0: N_A - N_B + P \cos \theta = 0$$

$$N_B - N_A = P \cos \theta$$

$$+\sum F_y = 0: -F_A - F_B + P \sin \theta = 0$$

$$\mu_s N_A + \mu_s N_B = P \sin \theta$$

$$N_A + N_B = \frac{P \sin \theta}{\mu_s}$$

$$\text{ADD (1) AND (2): } 2N_B = P \left( \cos \theta - \frac{\sin \theta}{\mu_s} \right) > 0 \quad \text{OK}$$

$$\text{SUBTRACT (1) FROM (2): } 2N_A = P \left( \frac{\sin \theta}{\mu_s} - \cos \theta \right)$$

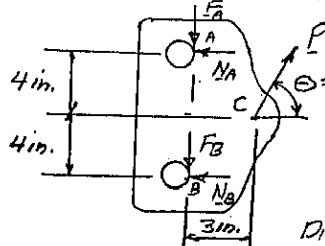
$$N_A > 0 \text{ ONLY IF } \frac{\sin \theta}{\mu_s} - \cos \theta > 0$$

$$\tan \theta > \mu_s = 1.333; \quad \theta = 53.13^\circ$$

THUS FOR (a) AND (b) CONDITION IS SATISFIED, CONTACT TAKES PLACE AS SHOWN. ANSWER IS CORRECT

$$(a) \text{ AND } (b) \quad \mu_s = 1.333$$

BUT FOR (c)  $\theta = 50^\circ < 53.13^\circ$  AND OUR ASSUMPTION IS WRONG,  $N_A$  IS DIRECTED TO LEFT



$$+\sum F_x = 0:$$

$$-N_A - N_B + P \cos 50^\circ = 0$$

$$N_A + N_B = P \cos 50^\circ \quad (3)$$

$$+\sum F_y = 0:$$

$$-F_A - F_B + P \sin 50^\circ = 0$$

$$\mu_s(N_A + N_B) = P \sin 50^\circ \quad (4)$$

DIVIDE (4) BY (3):

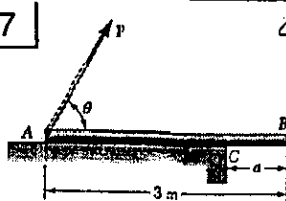
$$\mu_s = \tan 50^\circ = 1.192$$

$$(c) \mu_s = 1.192$$

NOTE: FOR  $\theta > 53.13^\circ$ ,  $\mu_s$  IS INDEPENDENT OF  $\theta$ .

FOR  $\theta < 53.13^\circ$ ,  $\mu_s$  DEPENDS ON  $\theta$  AND IS  $\mu_s = \tan \theta$

8.157



GIVEN: 20-kg TUBE AB,  $\mu_s = 0.30$ .

FIND: LARGEST  $\theta$  FOR TUBE TO SLIDE HORIZONTALLY WHEN (a)  $a = 0$ , (b)  $a = 0.75\text{ m}$ .

FOR MAX  $\theta$ , SLIDING AND ROTATION ABOUT C BOTH IMPEPEND

(a)

THREE-FORCE BODY

FORCE P MUST PASS THROUGH POINT D WHERE  $W$  AND  $C$  INTERSECT. SINCE SLIPPING IMPEPENDS  $\angle$  FORM ANGLE  $\phi_s$  WITH TUBE

$$\phi_s = \tan^{-1} \mu_s = \tan^{-1} 0.30$$

$$\phi_s = 16.70^\circ$$

$$\theta = 73.3^\circ$$

ISOSCELES TRIANGLE

$$\theta = 90^\circ - \phi_s = 90^\circ - 16.7^\circ$$

$$+\sum M_C = 0: (P \cos \phi_s)L + W \frac{L}{2} = 0$$

$$P = \frac{W}{2 \cos \phi_s} = \frac{(20\text{ kg})(9.81\text{ m/s}^2)}{2 \cos 16.7^\circ} \quad P = 102.4\text{ N}$$

(b)

THREE-FORCE BODY (SEE ABOVE IN A CDG:)

$$DG = \frac{0.75\text{ m}}{\cos 16.7^\circ} = 2.50\text{ m}$$

IN AADG:

$$\tan \theta = \frac{DG}{AG} = \frac{2.5\text{ m}}{1.5\text{ m}}$$

$$\tan \theta = 1.667, \quad \theta = 59.04^\circ$$

$$\theta = 59.0^\circ$$

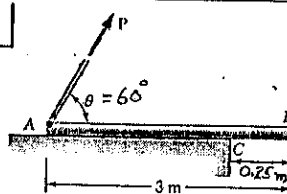
$$+\sum M_C = 0: (P \sin \theta)(2.25\text{ m}) - W(0.75\text{ m}) = 0$$

$$P = \frac{0.333}{\sin \theta} W = \frac{0.333}{\sin 59.04^\circ} (20\text{ kg})(9.81\text{ m/s}^2)$$

$$P = 76.27\text{ N}$$

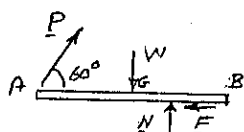
$$P = 76.3\text{ N}$$

8.158



GIVEN:  $\mu_s = 0.30$  20-kg TUBE AB

FIND: (a) SMALLEST P TO MOVE TUBE, (b) WHETHER TUBE SLIDES OR ROTATES.



ASSUME SLIDING

$$\sum F_y = 0: N = W - P \sin 60^\circ$$

$$F = \mu_s N = \mu_s (W - P \sin 60^\circ)$$

$$\sum F_x = 0: P \cos 60^\circ = F = \mu_s (W - P \sin 60^\circ)$$

$$P = \frac{\mu_s W}{\cos 60^\circ + \mu_s \sin 60^\circ} = \frac{0.3 W}{\cos 60^\circ + 0.3 \sin 60^\circ} = 0.3948 W$$

ASSUME ROTATION ABOUT C

$$+\sum M_C = 0$$

$$(P \sin 60^\circ)(2.75\text{ m}) - W(1.75\text{ m}) = 0$$

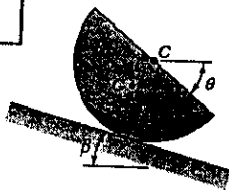
$$P = 0.5249 W$$

TUBE SLIDES

$$\text{FOR SLIDING: } P = 0.3948 W = 0.3948 (20\text{ kg})(9.81\text{ m/s}^2)$$

$$P = 77.5\text{ N}$$

8.159



GIVEN: HOMOGENEOUS  
HEMISPHERE

$$\mu_s = 0.30$$

FIND: (a) VALUE OF  $\theta$  FOR  
WHICH SLIDING IMPENDS.  
(b) CORRESPONDING  
VALUE OF  $\theta$ .

$r = \text{RADIUS}$

WE HAVE A TWO-FORCE BODY  
FOR SLIDING TO IMPEND  
 $R$  FORMS ANGLE  $\phi_s$  WITH  
INCLINE.

$$\phi_s = \tan^{-1} 0.30 = 16.70^\circ$$

$$\beta = 16.70^\circ$$

GEOMETRY:

$$GC = \frac{3}{8}r \text{ (See Fig. 5.21)}$$

$$AC = r$$

TRIANGLE ACG:  $\angle ACG = \theta - \phi$

$$\angle AGC = 180^\circ - \theta$$

LAW OF SINES

$$\frac{\sin(180^\circ - \theta)}{AC} = \frac{\sin \phi_s}{GC}$$

$$\sin(180^\circ - \theta) = \frac{AC}{GC} \sin \phi_s = \frac{r}{3/8r} \sin 16.70^\circ$$

$$\sin(180^\circ - \theta) = 0.76629$$

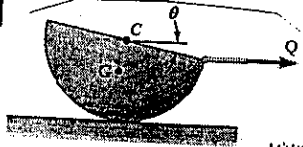
$$180^\circ - \theta = 50.0^\circ \text{ AND } 130.0^\circ$$

$$\theta = 130.0^\circ \text{ AND } 50.0^\circ$$

$\theta = 130.0^\circ$  IMPOSSIBLE

$$\theta = 50.0^\circ$$

8.160

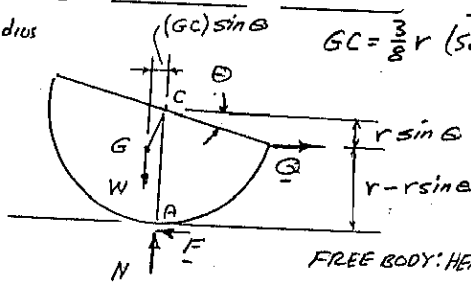


GIVEN: HOMOGENEOUS  
HEMISPHERE

$$\mu_s = 0.30$$

FIND: VALUE OF  $\theta$  FOR  
WHICH SLIDING IMPENDS.

$r = \text{radius}$



$$GC = \frac{3}{8}r \text{ (See Fig. 5.21)}$$

FREE BODY: HEMISPHERE

$$\uparrow \Sigma F_y = 0: N - W = 0; N = W$$

$$\text{SLIDING IMPENDS: } F = \mu_s N = \mu_s W$$

$$\rightarrow \Sigma F_x = 0: Q - F = 0; Q = \mu_s W$$

$$\uparrow \Sigma M_A = 0: W(GC) \sin \theta - Q(r - r \sin \theta) = 0$$

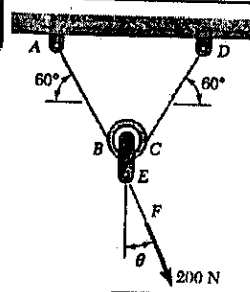
$$\sin \theta = \frac{Q}{\frac{3}{8}W + Q} = \frac{\mu_s W}{\frac{3}{8}W + \mu_s W}$$

$$\sin \theta = \frac{\mu_s}{\frac{3}{8} + \mu_s} = \frac{0.30}{0.375 + 0.30} = \frac{4}{9}$$

$$\theta = 26.39^\circ$$

$$\theta = 26.4^\circ$$

8.161

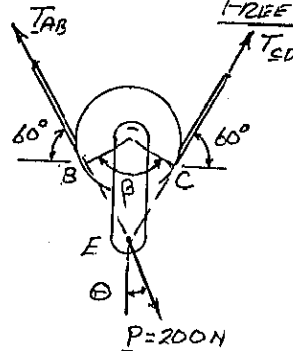


GIVEN: AXLE OF  
PULLEY IS FROZEN AND  
CANNOT ROTATE WITH  
RESPECT TO BLOCK

$$\mu_s = 0.30$$

FIND: (a) MAXIMUM  
VALUE OF  $\theta$  FOR  
EQUILIBRIUM.  
(b) REACTIONS AT  
SUPPORTS A AND D

FREE BODY: BLOCK AND PULLEY



SINCE 200-N FORCE  
TENDS TO ROTATE PULLEY,  
CABLE TENDS TO SLIP  
RELATIVE TO PULLEY.  $\therefore$

$$T_1 = T_{CD} \quad T_2 = T_{AB}$$

$$\beta = 120^\circ = \frac{2\pi}{3} \text{ rad}$$

$$\mu_s = 0.30$$

$$\frac{T_1}{T_2} = e^{\mu_s \beta}$$

$$\frac{T_{AB}}{T_{CD}} = e^{0.30 \left( \frac{2\pi}{3} \right)} = e^{0.2\pi} = 1.8745$$

$$T_{AB} = 1.8745 T_{CD} \quad (1)$$

FORCE TRIANGLE

LAW OF COSINES

$$P^2 = T_{AB}^2 + T_{CD}^2 - 2T_{AB}T_{CD} \cos 120^\circ$$

$$= (1.8745 T_{CD})^2 + T_{CD}^2$$

$$- 2(1.8745 T_{CD}) T_{CD} (-0.5)$$

$$= [(1.8745)^2 + 1 + 1.8745] T_{CD}^2$$

$$P^2 = 6.3880 T_{CD}^2$$

$$T_{CD} = 0.39565 P$$

(2)

(a) MAXIMUM ALLOWABLE VALUE OF  $\theta$ :

$$\text{LAW OF SINES: } \frac{\sin \gamma}{T_{CD}} = \frac{\sin 120^\circ}{P}; \sin \gamma = \frac{T_{CD}}{P} \sin 120^\circ$$

RECALLING EQ(2):

$$\sin \gamma = \frac{0.39565 P}{P} \sin 120^\circ = 0.34264; \gamma = 20.04^\circ$$

$$\theta = 90^\circ - (60^\circ + 20.04^\circ)$$

$$\theta = 9.96^\circ$$

(b) REACTIONS AT A AND D.  $P = 200 \text{ N}$

$$\text{EQ(2): } T_{CD} = 0.39565(200 \text{ N}) = 79.13 \text{ N}$$

$$\text{EQ(1): } T_{AB} = 1.8745 T_{CD} = 1.8745(79.13 \text{ N}) = 148.33 \text{ N}$$

THUS

$$A = 148.3 \text{ N} \angle 60^\circ$$

$$D = 79.1 \text{ N} \angle 60^\circ$$